Results in Physics 8 (2018) 592–597

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Nonextensive GES instability with nonlinear pressure effects

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ARTICLE INFO

Article history:
Received 5 November 2017
Received in revised form 21 December 2017
Accepted 22 December 2017
Available online 29 December 2017

ABSTRACT

We herein analyze the instability dynamics associated with the nonextensive nonthermal gravito-electrostatic sheath (GES) model for the perturbed solar plasma portraiture. The usual neutral gas approximation is herewith judiciously relaxed and the laboratory plasma-wall interaction physics is procedurally incorporated amid barotropic nonlinearity. The main motivation here stems from the true nature of the solar plasma system as a set of concentric nonlocal nonthermal sub-layers as evidenced from different multi-space satellite probes and missions. The formalism couples the solar interior plasma (SIP, bounded) and solar wind plasma (SWP, unbounded) via the diffused solar surface boundary (SSB) formed due to an exact long-range gravito-electrostatic force-equilibration. A linear normal mode ansatz reveals both dispersive and non-dispersive features of the modified GES collective wave excitations. It is seen that the thermostatistical GES stability depends solely on the electron-to-ion temperature ratio. The damping behavior on both the scales is more pronounced in the acoustic domain, $K \rightarrow \infty$, than the gravitational domain, $K \rightarrow 0$; where, K is the Jeans-normalized angular wave number. It offers a unique quasi-linear coupling of the gravitational and acoustic fluctuations amid the GES force action. The results may be useful to see the excitation dynamics of natural normal modes in bounded nonextensive astero-environs from a new viewpoint of the plasma-wall coupling mechanism.

Introduction

The standard solar model (SSM) has widely been successful in explaining many fundamental issues related to both the equilibrium as well as fluctuation dynamics of the solar plasma system for the last few decades [1,2]. The plasma wall-interaction processes, however, have most probably never been addressed to that extent on the astrophysical scales of space and time, except in the experimental laboratory domains, in this context of the gravitationally bounded Sun. In order to apply the laboratory sheath physics on the astrophysical scales, a few remarkable points are to be noted. Normally, the plasmas on laboratory scales are confined in a vacuum chamber of finite size. Thus, there appears a thin non-neutral space charge layer at the vicinity of the wall, whenever plasmas interact with absorbing boundary walls [3]. This non-neutral space charge layer (dark) formed in the vicinity of confining wall is termed as the *plasma sheath*. The underlying condition required to be fulfilled in the non-neutral lab-sheath formation processes is called the Bohm ionic-flow criterion [3]. This is a local criterion stating that the ion flow speed at the sheath entrance zone must be at least comparable with the ion sound phase speed. Applying the basic physics of laboratory plasma sheath formation [4], the GES model has recently been reported, emphasizing on both the equilibrium [5–8] and fluctuation [9] eigen-structures of the Sun and its circumvent atmosphere.

An analogous Bohm flow condition needed for the quasineutral isothermal GES formation in terms of the ion escape velocity (M_∞) across the gravitational potential barrier as an indicative strength of the self-gravitational potential wall is given as $M_\infty \geqslant \sqrt{2}$ [6]. The strength of the gravitational potential barrier is such that the heavier ions (colder) cannot overcome it at the solar surface boundary (SSB); but, the lighter electrons (hotter) can. Consequently, the thermal leakage of the electrons against the ions at the SSB develops an electrostatic polarization-induced electric charge evolving as the GES. A more detailed description on the GES structure formation could be found in Refs. [5,6]. In contrast, if the ionic temperature varies in the presence of ion-neutral collisions, this equivalent Bohm condition might have two extreme flow limits, with the lower limit affected by the ionic temperature [10]. Besides, the perturbed GES model formulation relative to a nonlocal equilibrium has successfully proposed the excitation of the *GES-oscillator mode*, *GES-wave mode* and *electrostatic acoustic mode* having time periods in agreement with the earlier SSM-based predictions [9] These modes, in turn, might enlighten the internal structure of the Sun and its morphodynamic evolution [1,2,11,12].

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https://doi.org/10.1016/j.rinp.2017.12.063
2211-3797/

It is well known on a broader horizon that most of the astrophysical bounded structures and ambient atmospheres are usually the consequences of naturalistic gravito-electrostatic or gravito-acoustic interactions [1,2]. The precise gravito-electrostatic coupling in the presence of large-scale nonlocal effects caused by the long-range interparticle functional processes is ensured by the nonextensivity nonthermal parameter, also known as entropic index [13–15], given in the *Tsallis thermostatistical framework* by a quasi-hydrostatic equilibrium condition (i.e., nonequilibrium stationary state) as $q = 1 + (\nabla T/\nabla U_G)$; where, T is the temperature (in energy units) and U_G the gravitational potential energy (in the same units). Thus, $q = 1$ only when $\nabla T = 0$, which corresponds to a thermalized state (constant thermodynamic potential). This nonextensivity parameter not only modifies the basic fluctuation characteristics, but also the polarities of the evolving diversified eigen-mode structures in diversified astrophysical environs in the form of bipolar, quadrupolar, multipolar patterns, and so forth [16]. In addition to the q-exponential nonextensive distribution, it may be pertinent to add here that there exists another nonthermal type of thermostatistical distribution law, termed as the kappa-distribution (κ), bearing a fine equivalent interrelationship, $q = (1 + 1/\kappa)$ [17]. At this backdrop, the true nature of the solar plasma system as a nonthermal typecast has been confirmed and validated by various multi-space satellites, such as *Ulysses, Voyager*, *Helios*, *Cluster*, and so on [17,18]. So, the nonthermal distributions are quite significant in the realistic thermostatistical study of diversified astro-cosmic environs, such as stellar polytropes, velocity distribution of galaxy clusters, instability of self-gravitating systems, and so forth [13–15].

In recent past, the plasma-wall interaction-based GES model structure has been revisited by considering the q-nonextensive nonthermal electron density distribution law [19]. It has been semi-analytically shown that the equilibrium GES structure gets considerably modified due to the inclusion of the nonextensive nonequilibrium thermostatistical distribution of the electrons. As an instant illustrative outcome, the nonextensive GES field strength for the bounded solar structure solution to exist is found to decrease by 21.67%, and the electrostatic potential by 30%. The GES fluctuations in the framework of the nonextensive electrons have, however, been never analyzed in the past, although it is likely to be important in the astroplasma community, particularly, from a theoretical helioseismic viewpoint [11]. Evidently, there has been a long-sought necessity for a theoretical model formulation to investigate the true nature of diversified collective waves and oscillations in self-gravitating nonthermal plasmas. In addition to molecular clouds [20–22], it includes the gravitationally bounded Sun and its unbounded atmosphere in the presence of realistic thermostatistically modified nonthermal plasma-wall interaction processes.

In this report, the nonthermal GES model fluctuation dynamics is studied in the framework of the q-nonextensive nonthermal electrons under the auspice of the basic plasma-wall interaction physics insofar applied in the laboratory confined plasmas solely [3–9]. It considers the nonlinear logatropic pressure effects arising due to erratic plasma dynamics in the mathematical setup. The standard technique of a Fourier-based normal mode analysis [1–3,9] is employed over the derived linearized GES structure equations to arrive at linear dispersion relations interconnected via the GES force field action [9]. The propagatory, dispersive and growth-related characteristics are analyzed in a constructive numerical illustrative standpoint. It is reported that, in both the solar interior plasma (SIP) and solar wind plasma (SWP) coupled via the diffuse SSB amid the GES force fields, the fluctuations show sharp damping behavior of unique type with the wave number variation; but with stable oscillatory characters with no damping at all in the coordination space. It specifically reveals a unique form of quasi-linear dynamic coupling mechanism existing between the gravitational (Jeansean) and acoustic (electrostatic) modes surviving in the proposed GES-based nonextensive solar plasma system on the astrophysical fluid scales of space and time. Lastly, a synopsis on the non-trivial implications and applications of the study is briefly outlined.

Model and formalism

A simplified bi-fluidic quasineutral solar plasma model is considered in the spherically symmetric GES model (radial, 1-D, reduced degrees of freedom) framework [5–9]. It dynamically couples the bounded SIP and unbounded SWP scales via the interfacial SSB formed under the action of the long-range non-local gravito-electrostatic force balancing. The precise location of the SSB on the radial space is determined by the maximization of the SIP self-gravity [5–9]. The considered model consists of the q-nonextensive electrons (nonthermal) and constitutive inertial ions (fluid) in the presence of nonlinear logatropic fluid pressure effects. The presence of neutrals and heavier species is ignored for idealization. The origin of such logatropic pressure effects may be due to the energy cascading processes via fluid turbulence. The turbulence effects may be observable only at the higher orders of perturbation. The existence of multi-layered magnetically coupled structures [1,2] is excluded for analytic simplicity. The presence of any other constituent massive neutral and charged species is ignored to construct a pure form of the GES structure fluctuations. The global electrical quasineutrality is justifiable on the ground that the asymptotic value of the Debye-to-Jeans length ratio is almost zero ($\lambda_{De}/\lambda_J \sim 10^{-20}$) [5–9]. It may be noted furthermore that, even after the Sun (SIP) is formed via the Jeansean dynamic molecular cloud collapse [1,2], it continues dynamically to remain self-gravitating in nature. It allows us to choose the Jeansean scales as a standard normalization for the investigation. The outward randomizing thermal and organizing electrostatic forces (non-gravitational) jointly prevent the Sun to undergo any further self-gravitational collapse by defeating the inward self-gravity towards a balanced condition. Thus, the gravitating SIP (subsonic) is closed by the gravitational Poisson equation describing the gravitational potential distribution (non-Newtonian) arising due to the collective material density fields. In contrast, for the SWP, the bounded SIP acts as a source of transformed external gravity to the outflowing (supersonic) SWP. Thus, the solar self-gravity is naturalistically switched off and the Poisson equation gets redundant for the SWP description (Newtonian) without violation of any generality [6–9].

SIP calculation scheme

The electron dynamics in the spherically symmetric SIP with all the usual notations is governed by the nonextensive nonthermal population distribution law [17,18] with entropic index q_e in a standard astrophysical normalized form [19] as

$$N_e = [1 + (1 - q_e)\theta]^{1/(1-q_e)}. \tag{1}$$

The ion continuity equation for flux density conservation and momentum equation describing the force density conservation for the spherically symmetric gravitating plasma continuum along with the closing gravito-electrostatic Poisson equations for potential distributions sourced in density fields are derived in the same normalized form, respectively, given as

$$\frac{\partial N_i}{\partial \tau} + M\frac{\partial N_i}{\partial \xi} + N_i\frac{\partial M}{\partial \xi} + \frac{2}{\xi}N_i M = 0, \tag{2}$$

$$\frac{\partial M}{\partial \tau}+M\frac{\partial M}{\partial \xi}=-\in_T\left[\frac{1}{N_i}\frac{\partial N_i}{\partial \xi}+\frac{1}{N_i^2}\frac{\partial N_i}{\partial \xi}\right]-\frac{\partial \theta}{\partial \xi}-g_s, \quad (3)$$

$$\frac{\partial g_s}{\partial \xi}+\frac{2}{\xi}g_s=N_i, \quad (4)$$

$$\left(\frac{\lambda_{De}}{\lambda_J}\right)^2\left[\frac{\partial^2\theta}{\partial \xi^2}+\frac{2}{\xi}\frac{\partial \theta}{\partial \xi}\right]=N_e-N_i. \quad (5)$$

The adopted standard astrophysical normalization scheme with all the customary *Jeansean* notations [5–9] are reproducably described and recast as

$$\left.\begin{array}{l}\xi=\frac{r}{\lambda_J}, \quad \tau=t\omega_J, \quad N_e=\frac{n_e}{n_{e0}}, \quad N_i=\frac{n_i}{n_{i0}}, \quad M=\frac{v_i}{c_S}, \\ \theta=\frac{e\varphi}{T_e}, \quad \eta=\frac{\psi}{c_S^2}, \quad g_s=\frac{d\eta}{d\xi},\end{array}\right\} \quad (6)$$

where $\omega_J=c_S/\lambda_J=\sqrt{4\pi n_0 m_i G}$ is the Jeans frequency and $c_S=\omega_J\lambda_J$ is the plasma sound phase speed. Here, ξ is the normalized position coordinate relative to the centre of the entire SIP mass distribution, τ the normalized time, N_e (N_i) the normalized electron (ion) number density, M the normalized ion Mach number, θ the normalized electrostatic potential and $g_s(=d\eta/d\xi)$ the normalized nonlocal self-gravity. In addition, the other relevant parameters λ_J, ρ_Θ, and T_e (T_i) represent the Jeans scale length, mean solar material density, and electron (ion) temperature (in eV); respectively. Then, $G=6.67\times10^{-11}$ m^3 kg^{-1} s^{-2} denotes the universal gravitational coupling constant signifying the strength of gravitational interaction of matter [1–3]. Moreover, $\in_T=(T_i/T_e)$ stands for the ion-to-electron temperature (in eV) ratio signifying parametrically the electronic and ionic dynamics in the GES-based solar plasma system.

It may be noted here that the appearance of the $2/\xi$-terms (in Eqs. (2), (4), (5)) is due to the consideration of a spherically symmetric geometry, which would clearly, otherwise, remain absent in the case of a planar geometry approximation ($\xi\to\infty$). The spherically symmetric Poisson equations (Eqs. (4), (5)) are in time-stationary form due to the fact that the gravito-electrostatic interplay in the presented classical non-relativistic fabric is time-independent. The appearance of the term, $1/N_i^2(\partial N_i/\partial\xi)$, in Eq. (3) is due to the presence of the nonlinear barotropic (pressure-density correlation) effects given in the logatropic pressure form as $p_{Turb}=p_0\,\log(\rho/\rho_c)$; where, p_0 is the mean pressure, ρ is the material density and ρ_c is the heliospheric core (reference) material density [20,21]. The application of such pressure effects are realizable in a wide class of astrophysical inhomogeneous fluid environs [19–22].

In the solar plasma environments, as already mentioned, the Jeans spatial scale length is very much larger than the plasma Debye length ($\lambda_{De}/\lambda_J\sim10^{-20}$). As a result, the entire solar plasma system can be treated as a quasineutral fluid [5–9]. Thus, Eq. (1) now interestingly yields the electron-ion quasineutrality condition on the astrophysical scales of space and time as

$$N_i\approx N_e=[1+(1-q_e)\theta]^{1/(1-q_e)}. \quad (7)$$

Applying the quasineutrality condition (Eq. (7)), the SIP equations (Eqs. (2)–(4)) in the presence of nonlocality-driven nonextensivity and barometric effects transform respectively as

$$\frac{\partial}{\partial\tau}[1+(1-q_e)\theta]^{1/(1-q_e)}+M\frac{\partial}{\partial\xi}[1+(1-q_e)\theta]^{1/(1-q_e)} + [1+(1-q_e)\theta]^{1/(1-q_e)}\frac{\partial M}{\partial\xi}+\frac{2}{\xi}M[1+(1-q_e)\theta]^{1/(1-q_e)}=0, \quad (8)$$

$$\frac{\partial M}{\partial\tau}+M\frac{\partial M}{\partial\xi}=-\frac{\partial\theta}{\partial\xi}-g_s-\frac{\in_T}{[1+(1-q_e)\theta]^{1/(1-q_e)}}\times\left[\frac{\partial}{\partial\xi}[1+(1-q_e)\theta]^{1/(1-q_e)} +\frac{1}{[1+(1-q_e)\theta]^{1/(1-q_e)}}\frac{\partial}{\partial\xi}[1+(1-q_e)\theta]^{1/(1-q_e)}\right], \quad (9)$$

$$\frac{\partial g_s}{\partial\xi}+\frac{2}{\xi}g_s=[1+(1-q_e)\theta]^{1/(1-q_e)}. \quad (10)$$

We now consider linear (first-order) perturbations in the relevant solar plasma parameters around the assumed homogeneous hydrostatic equilibrium as, $N=1+N_1$, $M=0+M_1$, $\theta=0+\theta_1$ and $g_s=0+g_{s_1}$. The linearized set of the perturbed equations now obtained from Eqs. (8)–(10) are respectively given as

$$\frac{\partial\theta_1}{\partial\tau}+\frac{\partial M_1}{\partial\xi}+\frac{2}{\xi}M_1=0, \quad (11)$$

$$\frac{\partial M_1}{\partial\tau}=-\frac{\partial\theta_1}{\partial\xi}(1+2\in_T)-g_{s1}, \quad (12)$$

$$\frac{\partial g_{s1}}{\partial\xi}+\frac{2}{\xi}g_{s1}-\theta_1=0. \quad (13)$$

The linear perturbations along the radial direction relative to the SIP center are now assumed for simplicity to evolve crudely in the form of plane waves as $\sim\exp[-i(\Omega\tau-K\xi)]$, where Ω is the Jeans-normalized angular frequency and K the Jeans-normalized angular wave number of the considered perturbations. Approximating the radial sinusoidal fluctuations in the form of plane waves, with the exclusion of polar and azimuthal counterparts, is based on a quasi-classic local approximation, mathematically given as $K\xi\gg1$ indicating short-wavelength fluctuations relative to the plasma system size [1,2]. It validates the proposed perturbation dynamics in the adopted simplified framework of normal mode (local) analysis. As an outcome, Eqs. (9)–(11) get respectively transformed from the coordination space (ξ,τ) into the wave space (K,Ω) as

$$i\Omega\theta_1+iKM_1+\frac{2}{\xi}M_1=0, \quad (14)$$

$$-i\Omega M_1=-iK\theta_1(1+2\in_T)-g_{s1}, \quad (15)$$

$$iKg_{s1}+\frac{2}{\xi}g_{s1}-\theta_1=0. \quad (16)$$

By the method of decomposition and elimination over Eqs. (14)–(16), the linear quadratic dispersion relation is procedurally derived and presented as

$$\Omega^2-K^2-2K^2\in_T+\frac{2}{\xi}iK(1+2\in_T)+1=0. \quad (17)$$

Thus, it is seen that the considered fluctuations are governed by a generalized dispersion relation of quadratic form modified by curvature effects. It may be noted that the instability here is triggered by the free energy source lying in the non-zero plasma currents themselves. It is against the conventional picture of sinusoidal eigenmodes of a linear dynamical system with a fixed frequency unaffected by such $(\xi,\in_T)$-parametric agencies [1,2].

Now, substituting $\Omega=\Omega_r+i\Omega_i$ in Eq. (17) for stability analysis, the respective real and imaginary parts are given as

$$\Omega_r^2-\Omega_i^2-K^2-2K^2\in_T+1=0, \quad (18)$$

$$2\Omega_r\Omega_i + \frac{2K}{\xi}(1 + 2\in_T) = 0. \quad (19)$$

The roots (from Eqs. (18) and (19)) for the real and imaginary frequency parts are respectively given as

$$\Omega_r(\xi, K, \in_T) = 0.707 \times (K^2 + 2K^2\in_T - 1)^{1/2} + \left[\sqrt{(K^2 + 2K^2\in_T - 1)^2 + 4(1 + 4\in_T + 4\in_T^2)\left(\frac{K}{\xi}\right)^{1/2}}\right]^{1/2}, \quad (20)$$

$$\Omega_i(\xi, K, \in_T) = -1.414 \times (1 + 2\in_T) \times (K^2 + 2K^2\in_T - 1)^{-1/2} + \left[\sqrt{(K^2 + 2K^2\in_T - 1)^2 + 4(1 + 4\in_T + 4\in_T^2)\left(\frac{K}{\xi}\right)^2}\right]^{-1/2} \times \left(\frac{K}{\xi}\right). \quad (21)$$

It is clearly evident from Eqs. (20) and (21) that the thermostatistically developed GES stability model depends purely on the electron-to-ion temperature ratio.

SWP calculation scheme

The normalized set of the SWP governing equations comprises of the nonextensivity-modified continuity and reduced momentum equations respectively presented as

$$\frac{\partial N_i}{\partial \tau} + M\frac{\partial N_i}{\partial \xi} + N_i\frac{\partial M}{\partial \xi} + \frac{2}{\xi}N_iM = 0, \quad (22)$$

$$\frac{\partial M}{\partial \tau} + M\frac{\partial M}{\partial \xi} = -\in_T\left[\frac{1}{N_i}\frac{\partial N_i}{\partial \xi} + \frac{1}{N_i^2}\frac{\partial N_i}{\partial \xi}\right] - \frac{\partial \theta}{\partial \xi} - \frac{a_0}{\xi^2}, \quad (23)$$

where $a_0 = GM_\Theta/c_S^2\lambda_J \sim 95$ is a constant giving an indirect measure of the solar wind temperature [5–9], and M_Θ is the mean (equilibrium) solar mass. The quasineutrality condition (Eq. (7)) applied in Eqs. (22) and (23) yields the respective perturbed forms as

$$\frac{\partial \theta_1}{\partial \tau} + \frac{\partial M_1}{\partial \xi} + \frac{2}{\xi}M_1 = 0, \quad (24)$$

$$\frac{\partial M_1}{\partial \tau} = -\frac{\partial \theta_1}{\partial \xi}(1 + 2\in_T). \quad (25)$$

Applying the same technique as on the SIP scale, Eqs. (24) and (25) get transformed from the coordination space (ξ, τ) into the wave space (K, Ω) respectively given as

$$-i\Omega\theta_1 + iKM_1 + \frac{2}{\xi}M_1 = 0, \quad (26)$$

$$-i\Omega M_1 = -iK\theta_1(1 + 2\in_T). \quad (27)$$

Following the same procedure of decoupling over Eqs. (26) and (27), the linear generalized dispersion relation is derived and written as

$$\Omega^2 - K^2 - 2K^2\in_T + \frac{2}{\xi}iK(1 + 2\in_T) = 0, \quad (28)$$

where the real and imaginary parts of Eq. (28) with $\Omega = \Omega_r + i\Omega_i$ are respectively cast as

$$\Omega_r^2 - \Omega_i^2 - K^2 - 2K^2\in_T = 0, \quad (29)$$

$$2\Omega_r\Omega_i + \frac{2K}{\xi}(1 + 2\in_T) = 0. \quad (30)$$

The finally obtained roots for the frequency (from Eqs. (29) and (30)) are respectively presented as

$$\Omega_r(\xi, K, \in_T) = 0.707 \times \{K^2(1 + 2\in_T)\}^{1/2} + \left[\sqrt{\{K^2(1 + 2\in_T)\}^2 + 4(1 + 4\in_T + 4\in_T^2)\left(\frac{K}{\xi}\right)^2}\right]^{1/2}, \quad (31)$$

$$\Omega_i(\xi, K, \in_T) = -1.414 \times (1 + 2\in_T) \times \{K^2(1 + 2\in_T)\}^{-1/2} + \left[\sqrt{\{K^2(1 + 2\in_T)\}^2 + 4(1 + 4\in_T + 4\in_T^2)\left(\frac{K}{\xi}\right)^2}\right]^{-1/2} \times \left(\frac{K}{\xi}\right). \quad (32)$$

It is manifest from Eqs. (31) and (32) that the thermostatistically developed GES stability of the SWP depends solely on the electron-to-ion temperature ratio as a plausible parameter. The frequency parts are functions of the position coordinate due to geometric curvature effects. This is again against the conventional picture of sinusoidal eigenmodes of a linear dynamical system with a fixed frequency free from such $(\xi, \in_T)$-parametric agencies [1,2].

Results and discussions

A pair of dispersion relations is already obtained for both the SIP (Eq. (17)) and SWP (Eq. (28)) scales by employing the normal mode analysis with some radial restrictions. As already mentioned, the bi-scale relationships are analytically and structurally auto-coupled via the long-range non-local gravito-electrostatic force fields [5–9]. In a broader sense, the auto-coupling is established principally with the help of the solar internal self-gravity (non-Newtonian, non-uniform, SIP) converted into an external gravity (Newtonian, uniform, SWP) via the interfacial SSB. The pair of the dispersion relations is now numerically analyzed in the framework of judicious set of input initial values relevant in the real solar plasma system [5–9].

In Fig. 1, we show the profile of the normalized (a) real frequency part (Ω_r) and (b) imaginary frequency part (Ω_i) associated with the SIP fluctuation dynamics on the bounded SIP scale with variation in the normalized position (ξ) and normalized wave number (K). The fixed input physical parameter is $\in_T = (T_i/T_e) = 0.4$, a value chosen for exact hydrostatic force balancing for the SSB to form [5–9]. It is seen that, near the vicinity of wave number $K = 0$, the waves show dispersive nature; while, as K increases, the waves turn purely acoustic-like (non-dispersive), as confirmed by the linear $\Omega_r = f(K)$ dependencies (Fig. 1(a)). The microscopic appearance of dispersive characteristics in the gravitational waves is due to the joint action of nonlinear pressure field and strong self-gravitational pressure effects. On the other hand, the real frequency sharply decreases and remains almost constant with ξ, signifying stable oscillatory propagation (Fig. 1(a)). Moreover, the waves show a sharp damping behavior up to the vicinity of $K = 50$, beyond which there is a slight damping. It is furthermore speculated that the imaginary frequency is found to remain almost constant along ξ, thereby typifying quasi-stable harmonic oscillations of the solar plasma in a linear order (Fig. 1(b)).

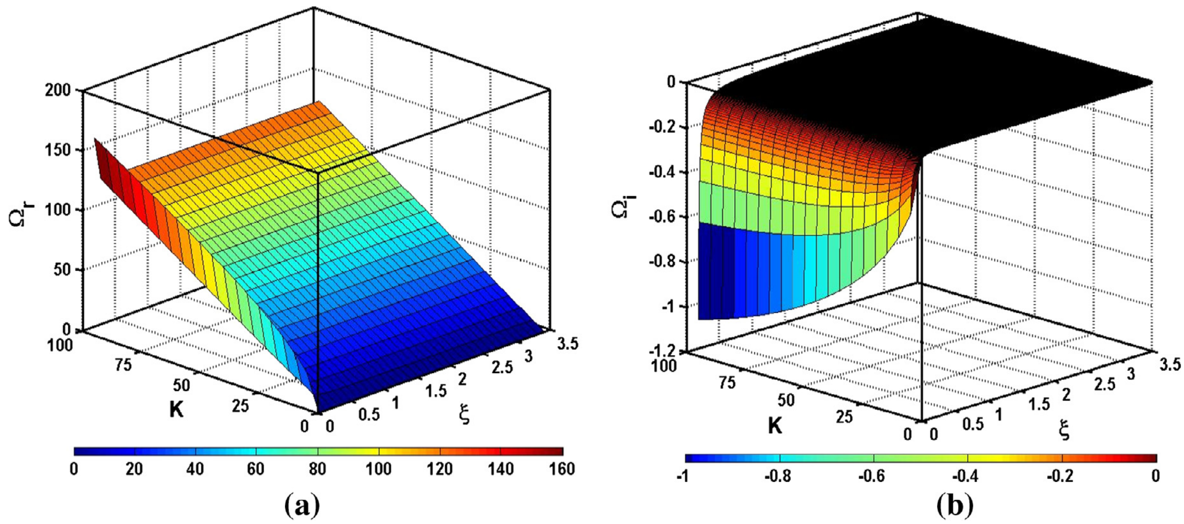

Fig. 1. Profile of the normalized (a) real frequency part (Ω_r) and (b) imaginary frequency part (Ω_i) associated with the SIP fluctuation dynamics with variation in normalized position (ξ) and normalized wave number (K) on the bounded scale. The fine details are given in the text.

Next, in Fig. 2, we depict the profile of the normalized (a) real frequency part (Ω_r) and (b) imaginary frequency part (Ω_i) associated with the SWP fluctuation dynamics on the unbounded SWP scale with variation in the normalized position (ξ) and normalized wave number (K). Here, the input parameters kept fixed are $\in_T = (T_i/T_e) = 0.1$, and $a_0 = 95$. It is observed that, on the unbounded scale, the perturbation waves are purely acoustic-like in nature, without any strong dispersive property exhibited. It implicates that in the SWP regime, the effect of fluid pressure is too weak to introduce any observable effect in the wave propagation dynamics. It is also seen that there is no significant variation of the real frequency in the ξ-space (Fig. 2(a)). Thus, the SWP fluctuations propagate as stable oscillatory modes. Furthermore, the GES waves show sharp damping behavior while going from the large-wavelength regime to the short-wavelength one (Fig. 2(b)). The imaginary frequency variation with respect to ξ (Fig. 2(b)) is similar to that observed in Fig. 1(b), signifying the existence of stable oscillations on both the spatial scales, both bounded and unbounded. It may therefore be summarily inferred that the oscillatory behaviors of the solar plasma perturbations in a linear order are scale-invariant under the balanced GES force fields.

The results numerically presented above (Figs. 1–2) are purely based on a standard scale-free mathematical analysis in the framework of Jeansean calculation scheme to study the nonextensive nonthermal GES perturbations on both the interior and exterior. It is able to reveal a unique form of quasi-linear coupling of the long-wavelength gravitational and short-wavelength acoustic mode fluctuations naturalistically existing in the GES-based solar plasma system (Fig. 1(a)–Fig. 2(a)). We now, for a quantitative glimpse, use the standard estimated value of $\omega_J = \sqrt{4\pi n_0 m_i G} \sim 1.08 \times 10^{-3}$ rad s^{-1} [1,2]. The unnormalized value of the q-modified GES mode periods can now be roughly estimated as $\tau \sim 1$ min. The modes with such time periods approximately match with the average helioseismic modes and oscillations (~3–5 min in reality as per standard solar model predictions [1,2,11]). Thus, it can be seen that, although in an infancy stage yet to be well concretized, the reliability of the investigated results can be approximately validated in reproducing the average

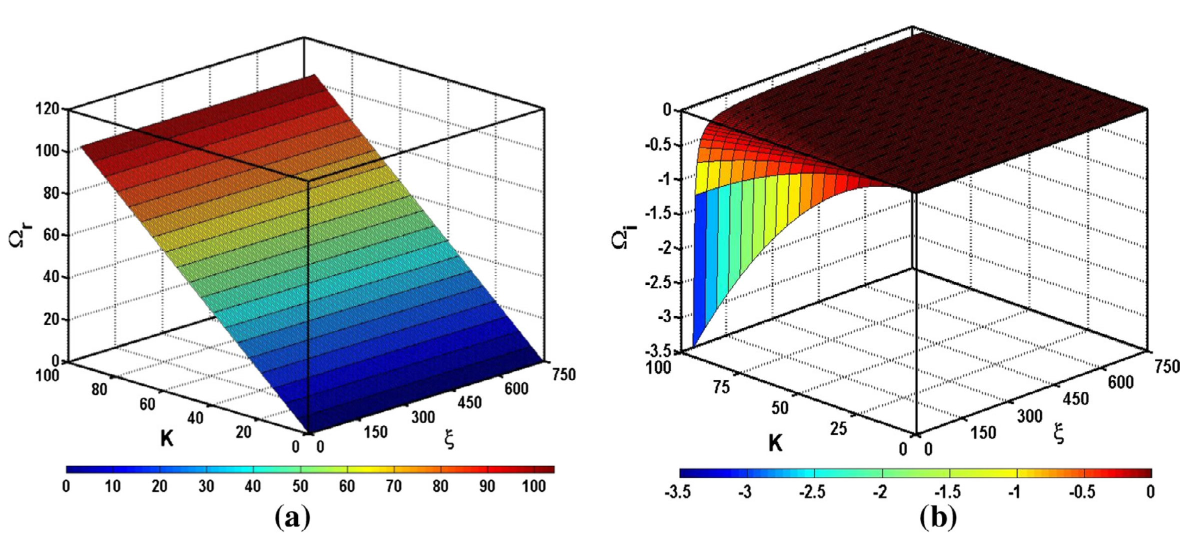

Fig. 2. Profile of the normalized (a) real frequency part (Ω_r) and (b) imaginary frequency part (Ω_i) associated with the SWP fluctuation dynamics with variation in normalized position (ξ) and normalized wave number (K) on the unbounded scale. The fine details are presented in the text.

picture of the astro-plasmic fluctuations and wave activites reported previously elsewhere [9,11].

Conclusions

This paper reports a semi-analytic classical study on the GES stability behaviors conjointly modified by the nonthermal electron dynamics and nonlinear logatropic fluid pressure effects on the solar hydrodynamic scales of space and time. A quasi-linear coupling of the long-wavelength gravitational waves and short-wavelength electrostatic waves in the presence of all the possible key realistic agencies relevant for the solar plasma sector is analyzed. In particular, it incorporates the basic physics of laboratory-confined plasma sheath formation and fluctuations via the nonlinear plasma wall-interaction processes. A normal mode analysis in the form of plane radial waves with some indicated restrictions reduces the solar plasma model into a unique pair of generalized quadratic dispersion relations with unique K-dependent coefficients. After a constructive canonical illustrative analysis, it is shown that the thermostatistically developed lowest-order GES stability depends solely on the electron-ion temperature ratio and the radial position coordinate. The damping behavior of the fluctuations on both the SIP and SWP scales is more pronounced in the acoustic-like domain than the gravitational-like one. The inter-coupling mechanism between the two separate scales in terms of dispersion features is attributable to the existence of the long-range non-local gravito-electrostatic force field action dynamically evolving via the interfacial SSB. A unique conformation of quasi-linear interaction of the gravitational (SIP) and acoustic (SWP) modes is reported to exist. This is in good agreement with the previous theoretical results, but as distinctive special cases of the thermalized electronic species instead of the nonthermal nonextensive ones [9]. The effects of fluid turbulence probably contributing to the nonlinear logatropic barotropic law do not appear in our linear analysis thereby indicating a fully nonlinear spectral power-law treatment as a futuristic refinement.

It may be further noted that the mathematical strategy constructed here is a simplified one owing to the consideration of a spherically symmetric geometrical configuration with the polar and azimuthal perspectives fully ignored. It means that a plane-wave analysis in the radial direction is employed, which is based on a quasi-classic local (short wavelength) approximation, given as, $K\xi \gg 1$ [1,2]. In other words, it depicts the fluctuation spectra of shorter wavelengths (relative to the plasma characteristic length) in the radial direction alone. Thus, the signatures of spherical waves are to be investigated in the framework of non-planar harmonics and special functions [1,2]. In order to see the actual picture, we must note that the application of the WKB method [2] is the only remedial measure. The basic model setups to handle the nonuniform (inhomogeneous) solar dynamics are still necessary without invoking any kind of conventional approximations in the light of non-local theory, proper energy transport equation, solar differential rotation, radiation hydrodynamics, etc [1,2]. The proposed study, despite facts and faults, may be useful as a first elemental step to see the collective fluctuation dynamics in the bounded nonthermal GES-based Sun and its unbounded atmosphere from a new viewpoint of nonextensive nonthermal plasma-wall interaction processes remaining previously employed only on the laboratory scales of space and time.

Acknowledgements

It is our great pleasure to acknowledge the learned anonymous referees for constructive comments, valuable remarks and insightful suggesstions. Active cooperation from Tezpur University is duly acknowledged. The financial support, DST SERB Fast Track Project (Grant: 021/2011), is thankfully recognized.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.rinp.2017.12.063.

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