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# Mathematical methods via the nonlinear two-dimensional water waves of Olver dynamical equation and its exact solitary wave solutions

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ABSTRACT

The problem formulations of the nonlinear for the small-long amplitude two-dimensional water waves propagation with free surface are studied. The water wave problem leads to the nonlinear Olver dynamical equation. By applying the extended mapping method, We derive the solitary wave solutions of the nonlinear Olver dynamical equation. These solutions for the nonlinear Olver dynamical equation are obtained efficiency and precisely of the method can be demonstrated. The movement role of the waves by making the graphs of the exact solutions and the stability of these solutions are analyzed and discussed. All solutions are stable and exact.


## Problem formulation of derivation dynamical equation

The $(2+1)$-dimensions interaction of a shear flow with a elevation surface in a stratified fluid was discussed. The nonlinear evolution equations were derived govern $(2+1)$-dimensions waves in a fluid of arbitrary depth. For unidirectional waves, the Korteweg-de Vries dynamical equation for shallow water [1–4]; and the intermediate Long Wave dynamical equation for the Benjamin-Ono dynamical equation for deep water are well known [5–8]. The $(2+1)$-dimensions water waves have been deduced only for the case of the Boussinesq equations although equations for weakly $(2+1)$-dimensions (but uni-directional) waves, in the Kadomtsev–Petviashvili dynamical class, which have been proposed for deep water [9–11].

The mathematical physics governing by nonlinear partial deferential dynamical equations have applications in physical science [12]. The analytical solutions for these dynamical equations play an important role in many phenomenons in optics; fluid mechanics; plasma physics and hydrodynamics. By using the improved modified extended tanh-function method, soliton solutions were derived [13]. One-dimensional soliton, N-soliton solutions and periodic solutions have been deduced [14–16]. The explicit solutions for the KdV equation was obtained by Hirota using inverse scattering [17–20]. By implementing the direct algebraic method, complex solutions of some nonlinear partial differential equations have been derived [21–25]. Exact solutions of the generalized $(2+1)$-dimensions nonlinear evolution equations via the modified simple equation method were obtained [26]. Classification of travelling wave solutions for Schrodinger-KdV, combined KdV-modified KdV and coupled Burgers dynamical equations have been found [27–30].

We consider the irrotational motion of the two-dimensional water waves is incompressible and inviscid fluid flow under the influence of a gravitational force and the elevation free surface $\eta(x,t)$. The model equations for long and small amplitude $(2+1)$-dimensions waves over a shallow horizontal bottom are presented. The basic dynamical system governing the velocity potential $\phi(x,y,t)$ are the Laplace; Euler and continuity equations as following [31,32]

$$\delta\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0, \qquad 0<y<1+\epsilon\eta, \tag{1}$$

$$\frac{\partial\phi}{\partial y}=0, \qquad y=0; \qquad |\nabla\phi|\longrightarrow 0, \qquad |x|\longrightarrow 0 \tag{2}$$

$$\frac{\partial\phi}{\partial t}+\frac{\epsilon}{2}\left(\frac{\partial\phi}{\partial x}\right)^2+\frac{\epsilon}{2\delta}\left(\frac{\partial\phi}{\partial y}\right)^2+\eta-\tau\delta\frac{\partial^2\eta}{\partial x^2}\left(1+\epsilon^2\left(\frac{\partial\eta}{\partial x}\right)^2\right)^{-3/2}=0; \tag{3}$$

$$\frac{\partial\eta}{\partial t}+\epsilon\frac{\partial\eta}{\partial x}\frac{\partial\phi}{\partial x}-\frac{1}{\delta}\frac{\partial\phi}{\partial y}=0, \qquad y=1+\alpha\eta \tag{4}$$

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where $\phi(x,y,t)$ is the velocity potential; $x, y$ are the horizontal and vertical coordinates; $\eta$ is the free surface elevation; $\epsilon$ is the ratio of wave amplitude to undisturbed fluid depth; $\delta$ represent the square of the ratio of fluid depth to wave length; $\tau$ is a dimensionless surface tension. By the standard perturbation expansion method (Boussinesq's scheme), the formal series solution to the elliptic boundary value problem (1-2) with the velocity potential at height $0 \leqslant h \leqslant 1$ and $\phi(x,h,t) = \psi(x,t)$, the solution can be constructed as

$$\phi = \psi + \frac{\delta}{2}\left(h^2 - y^2\right)\psi_{xx} + \frac{\delta^2}{24}\left(5h^4 - 6h^2y^2 + y^4\right)\psi_{xxxx} + \frac{\delta^3}{120}\left(61h^6 - 75h^4y^2 + 15h^2y^4 - y^6\right)\psi_{xxxxxx} + \ldots\ldots, \tag{5}$$

By substituting from (5) into (3-4), expanding in power $\epsilon$ and $\delta$, by truncating second order and differentiating the first equation yields the dynamical system

$$u_t + \eta_x + \epsilon u u_x + \frac{\delta}{2}(h^2 - 1)u_{xxt} - \delta\tau\eta_{xxx} + \frac{\epsilon\delta}{2}(h^2 - 1)u u_{xxx} + \frac{\epsilon\delta}{2}(h^2 + 1)u_x u_{xx} - \epsilon\delta(\eta u_{xt})_x + \frac{\delta^2}{24}(5h^4 - 6h^2 + 1)u_{xxxxt} = 0, \tag{6}$$

$$\eta_t + u_x + \epsilon(\eta u)_x + \frac{\delta}{6}(3h^2 - 1)u_{xxx} + \frac{\epsilon\delta}{2}(h^2 - 1)(\eta u_{xx})_x + \frac{\delta^2}{120}(25h^4 - 10h^2 + 1)u_{xxxxx} = 0 \tag{7}$$

where $u(x,t) = \phi_x(x,h,t) = \psi_x(x,t)$ is the horizontal velocity at depth $h$. The Boussinesq dynamical system is valid for waves moving in both directions. Taking the elevation surface $\eta = u$ as the principle variable, Olver [31] derived an unidirectional model to describe the long and small amplitude two-dimensional waves over a shallow water. The second order perturbation expansion for unidirectional waves leads to the equation

$$\eta_t + \eta_x + \frac{3}{2}\epsilon\eta\eta_x + \left(\frac{1}{6} - \frac{1}{2}\tau\right)\epsilon\eta_{xxx} - \frac{3}{8}\epsilon^2\eta^2\eta_x + \left(\frac{5}{12} - \frac{1}{2}\tau\right)\epsilon\delta\eta\eta_{xxx} + \left(\frac{23}{24} + \frac{5}{8}\tau\right)\epsilon\delta\eta_x\eta_{xx} + \left(\frac{19}{360} - \frac{1}{12}\tau - \frac{1}{8}\tau^2\right)\delta^2\eta_{xxxxx} = 0, \tag{8}$$

To simplify the Olver equation, let the new dependent variable $\eta(x,t) = v(x,t) + \frac{2}{\epsilon}$, the Olver Eq. (8) leads to

$$v_t + \frac{5}{2}v_x + (1-\tau)\delta v_{xxx} - \frac{3}{8}\epsilon^2 v^2 v_x + \left(\frac{5}{12} - \frac{1}{2}\tau\right)\epsilon\delta v v_{xxx} + \left(\frac{23}{24} + \frac{5}{8}\tau\right)\epsilon\delta v_x v_{xx} + \left(\frac{19}{360} - \frac{1}{12}\tau - \frac{1}{8}\tau^2\right)\delta^2 v_{xxxxx} = 0, \tag{9}$$

Consider the traveling wave solution as

$$v(x,t) = v(\xi), \quad and \quad \xi = x - \omega t, \tag{10}$$

where $\omega$ is the wave velocity. Then Eq. (9) becomes

$$\alpha_1 v_\xi + \alpha_2 v_{\xi\xi\xi} + \alpha_3 v^2 v_\xi + \alpha_4 v v_{\xi\xi\xi} + \alpha_5 v_\xi v_{\xi\xi} + \alpha_6 v_{\xi\xi\xi\xi\xi} = 0, \tag{11}$$

where the coefficients $\alpha_1 = \left(\frac{5}{2} - \omega\right)$; $\alpha_2 = (1-\tau)\delta$; $\alpha_3 = -\frac{3}{8}\epsilon^2$; $\alpha_4 = \left(\frac{5}{12} - \frac{1}{2}\tau\right)\epsilon\delta$; $\alpha_5 = \left(\frac{23}{24} + \frac{5}{8}\tau\right)\epsilon\delta$; $\alpha_6 = \left(\frac{19}{360} - \frac{1}{12}\tau - \frac{1}{8}\tau^2\right)\delta^2$. By integration Eq. (11), we obtain

$$\alpha_1 v + \alpha_2 v_{\xi\xi} + \frac{\alpha_3}{3}v^3 + \alpha_4 v v_{\xi\xi} + \left(\frac{\alpha_5 - \alpha_4}{2}\right)v_\xi^2 + \alpha_6 v_{\xi\xi\xi\xi} + \sigma = 0, \tag{12}$$

where $\sigma$ is the constant of integration. We have two cases: $\alpha_3\alpha_6 > 0$ depth of water less than about 3—5 mm and surface tension dominates gravity; $\alpha_3\alpha_6 < 0$ depth of water more than about 3—5 mm and gravity dominates surface tension. Assuming $v(\xi) = \sqrt{\pm\frac{6\alpha_6}{\alpha_3}}U(\xi)$ and substitute into Eq. (12), we get

$$U_{\xi\xi\xi\xi} \pm U^3 + \frac{2}{3}s_1 U U_{\xi\xi} + s_2 U_\xi^2 + \frac{1}{3}s_3 U_{\xi\xi} + s_4 U + s_5 = 0, \tag{13}$$

where $s_1 = 3\frac{\sqrt{3}\alpha_4}{\sqrt{\pm 2\alpha_3\alpha_6}}$; $s_2 = \frac{\sqrt{3}(\alpha_5 - \alpha_4)}{\sqrt{\pm 2\alpha_3\alpha_6}}$; $s_3 = \frac{3\alpha_2}{\alpha_6} - \frac{3\alpha_1\alpha_4}{2\alpha_3\alpha_6}$; $s_4 = \frac{1-\omega}{\alpha_6} - \frac{\alpha_1^2}{4\alpha_3\alpha_6}$; $s_5 = \frac{\sigma}{\alpha_6}\sqrt{\pm\frac{\alpha_3}{6\alpha_6}}$ Eq. (13) describes capillary waves in very thin layers of water (effects of gravity is negligible) and gravitational waves (effects of surface tension is negligible) respectively.

## Exact solitary wave solutions

We found the families of exact solitary wave solutions for the nonlinear two-dimensional shallow water waves dynamical equation by applying the extended direct algebraic mapping method. By

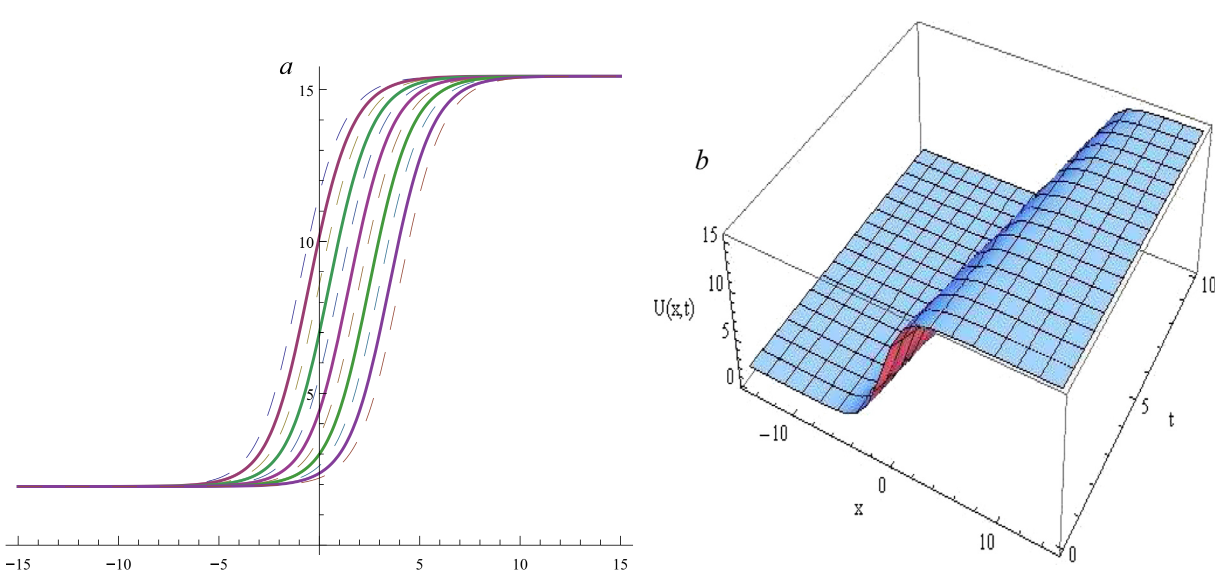


**Fig. 1.** (a, b) Velocity potential (18) with various different shapes are plotted as half bright solitary wave in the intervals $[-10, 10]$ and $[0, 10]$.

Applying different analytic solutions of dynamical Eq. (13), which gives the different values for the velocity potential $U$ as follows:

*Families I*

By applying the extended mapping method, the nonlinear two-dimensional Olver dynamical equation has solitary wave solutions as:

$$U(\xi)=\sum_{i=0}^{n}a_iF^i(\xi)+\sum_{i=-1}^{-n}b_{-i}F^i(\xi)+\sum_{i=2}^{n}c_iF^{i-2}(\xi)F'(\xi)+\sum_{i=-1}^{-n}d_{-i}F^i(\xi)F'(\xi), \quad (14)$$

where $a_0,a_1,\ldots\ldots\ldots a_n,b_1,\ldots\ldots\ldots b_n,c_2,\ldots\ldots\ldots c_n,d_1,\ldots\ldots\ldots d_n$ are arbitrary constants, the value of $F(\theta)$ and $F'(\theta)$ satisfy the following

$$F'(\xi)=\sqrt{pF^2(\xi)+qF^3(\xi)+rF^4(\xi)};\quad F''(\xi)=pF(\xi)+\frac{3}{2}qF^2(\xi)+2rF^3(\xi); \quad (15)$$

By balancing the highest order linear partial derivative term and the highest order nonlinear term in dynamical Eq. (13) gives the value of $m=2$. The solitary wave solutions of dynamical Eq. (13) has the form

$$U(\xi)=a_0+a_1F(\xi)+a_2F^2(\xi)+\frac{b_1}{F(\xi)}+\frac{b_2}{F^2(\xi)}+c_2F'(\xi)+d_1\frac{F'(\xi)}{F(\xi)}+d_2\frac{F'(\xi)}{F^2(\xi)}. \quad (16)$$

By inserting from Eq. (16) to dynamical Eq. (13) and combine coefficients of $F'^j(\xi)F^i(\xi)(j=0,1;i=0,1,2,3,\ldots n)$, then put all coefficients equal to zero to obtain a system of algebraic equations. We get the parameters $a_0,a_1,a_2,b_1,b_2,c_2,d_1,d_2$ by solving the obtained system as

$$a_0=\frac{1}{12}\left(-p(s_1+s_2)\pm p\sqrt{(s_1+s_2)^2-60}-\frac{3s_3(s_1+3s_2)\pm 9s_3\sqrt{(s_1+s_2)^2-60}}{s_1(2s_1+3s_2)-135}\right),$$
$$a_1=\frac{q}{4}\left(-(s_1+s_2)\pm\sqrt{(s_1+s_2)^2-60}\right),$$
$$a_2=\frac{r}{2}\left(-(s_1+s_2)\pm\sqrt{(s_1+s_2)^2-60}\right),$$
$$c_2=\pm\sqrt{\frac{r\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)-30\right)}{2}},$$
$$b_1=b_2=d_1=d_2=0, \quad (17)$$

Substituting from Eqs. (17) into (16), the velocity potential of Eq. (13) can be obtained as a solitary wave solutions as:

$$U_1(x,t)=\frac{1}{12}\left(-p(s_1+s_2)\pm p\sqrt{(s_1+s_2)^2-60}-\frac{3s_3(s_1+3s_2)\pm 9s_3\sqrt{(s_1+s_2)^2-60}}{s_1(2s_1+3s_2)-135}\right.$$
$$+3p\left(s_1+s_2\pm\sqrt{(s_1+s_2)^2-60}\right)\times\left(1+\tanh\left[\frac{\sqrt{p}}{2}(x-\omega\tau)+\xi_0\right]\right)$$
$$-\frac{6rp^2}{q^2}\left(s_1+s_2\pm\sqrt{(s_1+s_2)^2-60}\right)\times\left(1+\tanh\left[\frac{\sqrt{p}}{2}(x-\omega\tau)+\xi_0\right]\right)^2$$
$$\pm\frac{3p\sqrt{2p}}{q}\left(\sqrt{\frac{r\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)-30\right)}{2}}\right.$$
$$\left.\times sech^2\left[\frac{\sqrt{p}}{2}(x-\omega\tau)+\xi_0\right]\right) \quad (18)$$

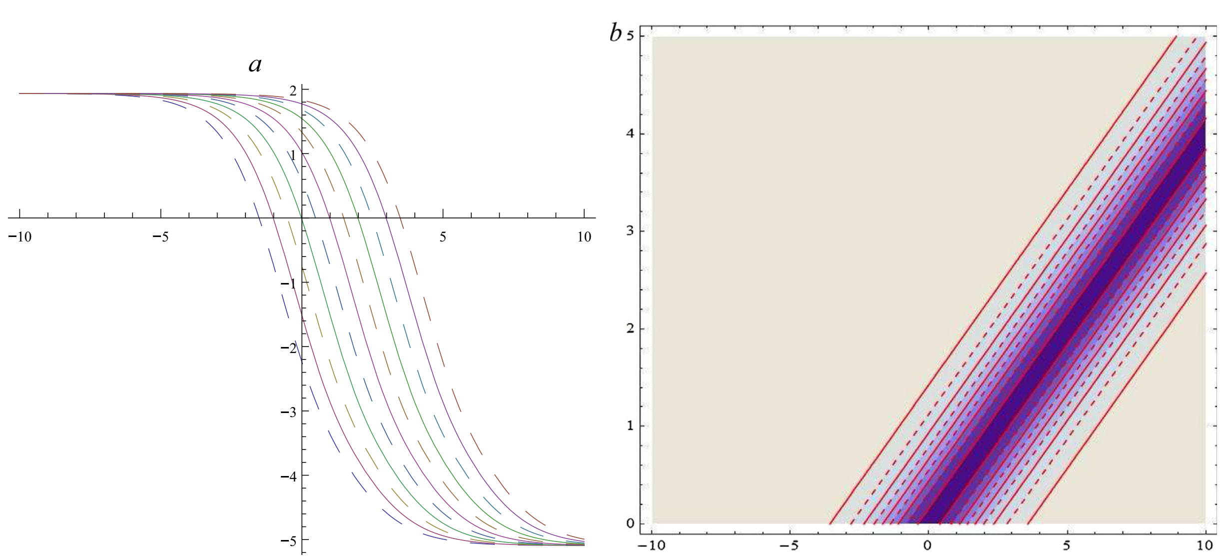


**Fig. 2.** (a, b) present half bright solitary wave of the velocity potential (19) with various different shapes are plotted in the intervals [−10, 10] and [0, 5].

$$
\begin{aligned}
U_2(x,t) = \frac{1}{24}\Bigg( & -2p(s_1+s_2) \pm 2p\sqrt{(s_1+s_2)^2-60} \\
& - \frac{6s_3(s_1+3s_2) \pm 18s_3\sqrt{(s_1+s_2)^2-60}}{s_1(2s_1+3s_2)-135} \\
& + 3q\sqrt{\frac{p}{r}}\left(s_1+s_2 \pm \sqrt{(s_1+s_2)^2-60}\right) \\
& \times \left(1+\frac{\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}{\rho+\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}\right) \\
& - 3p\left(s_1+s_2 \pm \sqrt{(s_1+s_2)^2-60}\right) \\
& \times \left(1+\frac{\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}{\rho+\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}\right)^2 \\
& \pm 6p\left(\sqrt{(s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)-30}\right) \\
& \times \frac{1+\rho\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}{\left(\rho+\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\right)^2}\Bigg)
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
U_3(x,t) = \frac{1}{12}\Bigg( & -2p(s_1+s_2) \pm 2p\sqrt{(s_1+s_2)^2-60} \\
& - \frac{6s_3(s_1+3s_2) \pm 18s_3\sqrt{(s_1+s_2)^2-60}}{s_1(2s_1+3s_2)-135} \\
& + 3p\left(s_1+s_2 \pm \sqrt{(s_1+s_2)^2-60}\right) \\
& \times \left(1+\frac{\rho\sqrt{1+k^2}+\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}{k+\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}\right) \\
& - \frac{6rp^2}{q^2}\left(s_1+s_2 \pm \sqrt{(s_1+s_2)^2-60}\right) \\
& \times \left(1+\frac{\rho\sqrt{1+k^2}+\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}{k+\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}\right)^2 \\
& \pm 6p\left(\sqrt{2pr\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)-30\right)}\right) \\
& \times \frac{k\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]-\rho\sqrt{1+k^2}\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]-1}{q\left(k+\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\right)^2}\Bigg)
\end{aligned} \tag{20}
$$

The sufficient conditions for solitary wave solutions (18)–(20) are stable:

$$
\begin{aligned}
& p>0, \qquad (s_1+s_2)^2>60, \\
& r\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)\right)>30 \\
& s_1(2s_1+3s_2)-135\neq 0, \qquad q\neq 0, \\
& \rho+\cosh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\neq 0 \\
& k+\sinh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\neq 0
\end{aligned} \tag{21}
$$

*Families II*

By implementing the direct algebraic mapping method, the nonlinear two-dimensional Olver water wave dynamical equation has solitary wave solution as in the following form: (see Figs. 1–3)

$$
\begin{aligned}
U(\xi) = & \sum_{i=0}^{n} a_i F^i(\xi) + \sum_{i=-1}^{-n} b_{-i} F^i(\xi) + \sum_{i=2}^{n} c_i F^{i-2}(\xi) F'(\xi) \\
& + \sum_{i=-1}^{-n} d_{-i} F^i(\xi) F'(\xi),
\end{aligned} \tag{22}
$$

where $a_0, a_1, \ldots\ldots\ldots a_n, b_1, \ldots\ldots\ldots b_n, c_2, \ldots\ldots\ldots c_n, d_1, \ldots\ldots\ldots d_n$ are arbitrary constants, the value of $F(\xi)$ and $F'(\xi)$ satisfy the following

$$
\begin{aligned}
& F'(\xi) = \sqrt{pF^2(\xi)+qF^4(\xi)+rF^6(\xi)}; \\
& F''(\xi) = pF(\xi)+2qF^3(\xi)+3rF^5(\xi);
\end{aligned} \tag{23}
$$

We deduced the value of $m = 4$, by balancing the highest order linear partial derivative term and the highest order nonlinear term in Eq. (13). The solitary wave solutions of Eq. (13) has the form

$$
\begin{aligned}
U(\xi) = & a_0 + a_1F(\xi) + a_2F^2(\xi) + a_3F^3(\xi) + a_4F^4(\xi) \\
& + \frac{b_1}{F(\xi)} + \frac{b_2}{F^2(\xi)} + \frac{b_3}{F^3(\xi)} + \frac{b_4}{F^4(\xi)} + c_2F'(\xi) + c_3F(\xi)F'(\xi) \\
& + c_4F^2(\xi)F'(\xi) + d_1\frac{F'(\xi)}{F(\xi)} + d_2\frac{F'(\xi)}{F^2(\xi)} + d_3\frac{F'(\xi)}{F^3(\xi)} + d_4\frac{F'(\xi)}{F^4(\xi)}.
\end{aligned} \tag{24}
$$

By inserting from Eq. (24) into Eq. (13) and collect coefficients of $F'^j(\xi)F^i(\xi)(j=0,1; i=0,1,2,3,\ldots n)$ and solving algebraic system, we obtained the constants $a_0, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, c_2, c_3, c_4, d_1, d_2, d_3, d_4$. By substituting from values of these parameters into Eq. (24), the velocity potential of Eq. (13) can be derived as a exact solitary wave solutions:

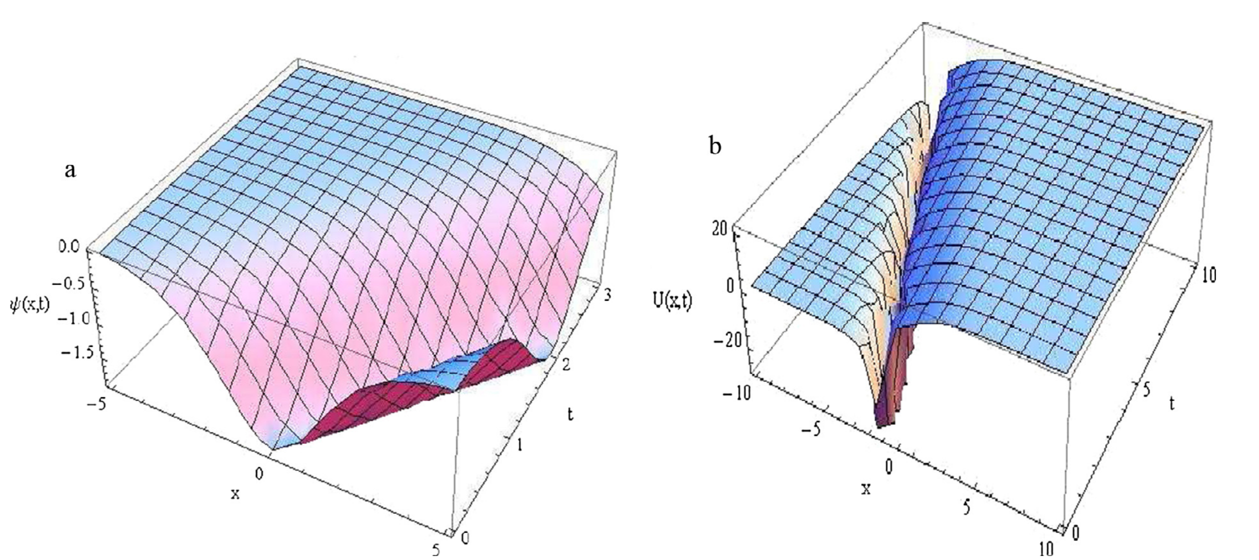


**Fig. 3.** The velocity potential (20) of the Olver equation shows as dark solitary wave solutions in Fig. (3a, b).

$$
\begin{aligned}
U_4(x,t)=\frac{1}{12}\Bigg(&\frac{96rp^2\left(-(s_1+s_2)\pm\sqrt{(s_1+s_2)^2-60}\right)}{\left(q-\sqrt{q^2-4pr}\cosh\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]\right)^2}\\
&-4p\left(s_1+s_2\pm\sqrt{(s_1+s_2)^2-60}\right)\\
&-\frac{3s_3(s_1+3s_2)\pm 9s_3\sqrt{(s_1+s_2)^2-60}}{s_1(2s_1+3s_2)-135}\\
&-\frac{24pq\left(-(s_1+s_2)\pm\sqrt{(s_1+s_2)^2-60}\right)}{q-\sqrt{q^2-4pr}\cosh\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]}\\
&\pm 96p\left(\sqrt{2pr\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)-30\right)}\right)\\
&\cdot\frac{sech^2\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\tanh\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]}{\sqrt{q^2-4pr}\left(-2+\left(1+\frac{q}{\sqrt{q^2-4pr}}\right)sech^2\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\right)^2}\Bigg)
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
U_5(x,t)=\frac{1}{12}\Bigg(&\frac{96rp^2\left(-(s_1+s_2)\pm\sqrt{(s_1+s_2)^2-60}\right)}{\left(q-\sqrt{q^2-4pr}\sin\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]\right)^2}\\
&-4p\left(s_1+s_2\pm\sqrt{(s_1+s_2)^2-60}\right)\\
&-\frac{3s_3(s_1+3s_2)\pm 9s_3\sqrt{(s_1+s_2)^2-60}}{s_1(2s_1+3s_2)-135}\\
&+\frac{24pq\left(-(s_1+s_2)\pm\sqrt{(s_1+s_2)^2-60}\right)}{\sqrt{q^2-4pr}\sin\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]-q}\\
&\pm 48p\left(\sqrt{2pr\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)-30\right)}\right)\\
&\cdot\frac{\sqrt{q^2-4pr}\cos\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]}{\left(q-\sqrt{q^2-4pr}\sin\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]\right)^2}\Bigg)
\end{aligned}
\tag{26}
$$

The exact solitary wave solutions (25) and (26) have the sufficient conditions for stability as:

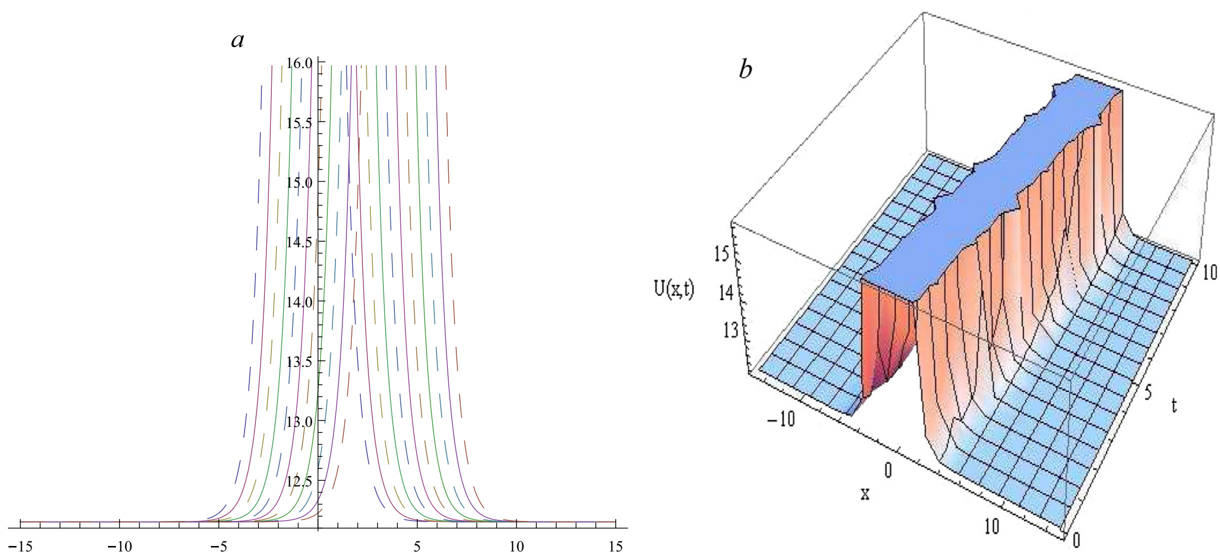


**Fig. 4.** (a, b), The velocity potential (25) with various different shapes are plotted as bright solitary wave in the intervals $[-10, 10]$ and $[0, 10]$.

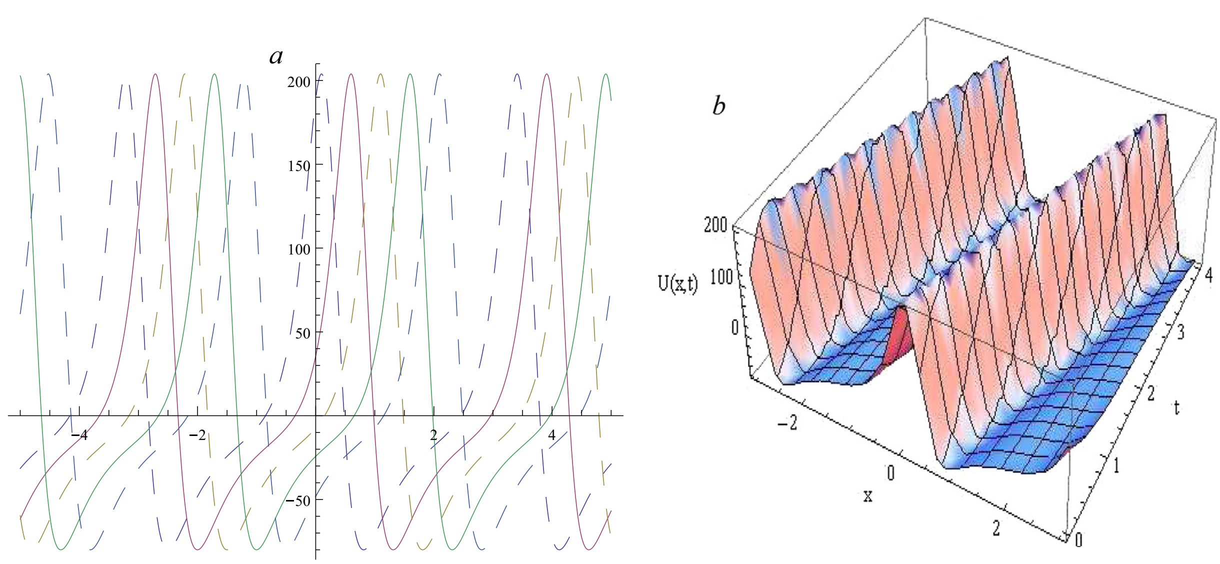


**Fig. 5.** Periodic solitary wave solution (26) of the Olver equation represented in Fig. (5a, b) in the intervals $[-3, 3]$ and $[0, 4]$.

$$
\begin{aligned}
&q^2-4pr>0,\quad (s_1+s_2)^2>60,\\
&q-\sqrt{q^2-4pr}\cosh\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]\neq 0,\\
&r\left((s_1+s_2)\left(s_1+s_2-\sqrt{(s_1+s_2)^2-60}\right)\right)>30,\\
&s_1(2s_1+3s_2)-135\neq 0,\quad q\neq 0,\\
&q-\sqrt{q^2-4pr}\sin\left[2\sqrt{p}(x-\omega\tau)+\xi_0\right]\neq 0,\\
&2+\left(1+\frac{q}{\sqrt{q^2-4pr}}\right)sech^2\left[\sqrt{p}(x-\omega\tau)+\xi_0\right]\neq 0
\end{aligned}
\tag{27}
$$

The generalized higher order Olver Eq. (8) has the steady state solution

$$\eta(x,t)=Ae^{\psi},\qquad \psi=kx-\omega t, \tag{28}$$

where $k,\omega$ are the normalized wave number and frequency of perturbation. The dispersion relation $\omega=\omega(k)$ of a constant coefficient linear evolution equation determines how time oscillations $e^{kx}$ are linked to spatial oscillations $e^{\omega t}$ of wave number $k$, substituting from Eq. (27) in equation (8), we obtain the following dispersion relation as

$$\omega=k+\frac{1}{6}\epsilon(1-3\tau)k^3+\frac{1}{360}\delta^2(19-30\tau-45\tau^2)k^5, \tag{29}$$

The dispersion relation (28) shows the steady state stability depends on the group velocity dispersion, self-phase modulation and stimulated Raman scattering. In the case the wave number $\omega$ is real for all $k$ and the steady state is stable against small perturbations. (See Figs. 4 and 5).

## Conclusion

In this research, we succeed to reductive perturbation method to study the problem formulations of the nonlinear for long and small amplitude two-dimensional water waves propagation with free surface. The nonlinear Olver dynamical equation deduced from the water wave problem. We derived the solitary wave solutions of the nonlinear Olver dynamical equation by applying by the extended mapping method. We obtained many new form of solutions for the model in fluid mechanic. The obtained solutions for the nonlinear Olver dynamical equation were efficiency and precisely of this method. The movement role of the waves by making the graphs of the exact solutions and the stability of these solutions were discussed and analyzed. This show how these methods is very effective, efficacious, influential, very simple and direct methods to apply them for many nonlinear evolution equations.

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