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Impact of two relaxation times on thermal, P and SV waves at interface with magnetic field and temperature dependent elastic moduli

Ambreen Asfar Khan, Akbar Zaman *, Sundas Yaseen

Department of Mathematical and Statistics, International Islamic University, Islamabad 44000, Pakistan

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ABSTRACT

In this article, two models of the generalized thermo-elastic theory are used to see the influence on the refraction and reflection of the plane waves at the interface under a constant magnetic field. The elasticity modulus depends on the reference temperature. The elasticity modulus is considered as a linear function of reference temperature. The resulting problem is solved by using the boundary conditions at the interface. The matrix equations have been solved numerically.

Introduction

The infinite velocity of the thermal wave is used in the classical theory of thermo-elasticity. This assumption may be useful for many engineering problems, but practically it is unacceptable approximation. In some experiments finite speed of the thermal waves are observed, so to remove this difference generalized thermo-elastic theories LS and GL was proposed by Lord and Shulman [1]. Green and Lindsay [2] developed generalized thermo-elastic theory involving one thermal relaxation time. Lindsay and Green [2] derived a temperature dependent thermo-elasticity involving two relaxation times without violating the classical Fourier law of heat conduction. Because, propagation of wave in thermo-elastic media plays a vital role in several fields such as solid dynamics, earth quake engineering, nuclear reactors and aeronautic etc. Various authors considered the propagation of wave in thermo-elastic an isotropic medium. Parfitt and Eringen [3] considered the plane waves reflection from the flat wall of a micro-polar elastic half space. Ariman [4] studied the propagation of wave in a micro-polar elastic half space. For some relevant work of interest, we refer the readers to study the work of Kumar and Singh [5], Singh [6] and Deswal and Kumar [7]. But some papers described the influence of reference temperature elastic modulus. In this reference, Othman and Song [8] viewed the influence of temperature dependent elastic moduli on the reflection magneto thermo-elastic waves with two relaxation times.

Moreover, Abd-Alla et al. [9] considered the refraction and reflection of SV waves at the solid liquid interface by considering primary stress and three thermo-elastic theories. Kumar and Saini [10] illustrated the effect of refraction and reflection of waves at the interface between two different porous solids. Wei et al. [11] investigated the refraction and reflection of P waves at thermo-elastic and porous thermo-elastic medium.

The magneto thermo-elastic theory includes the impact of magnetic field on the thermo-elastic waves. This theory has achieved more importance in various industrial appliances, especially in nuclear devices. The connection of magnetic field with strain and thermal field has been discussed by many researchers; these include Sinha and Elsibai [12], Deresiewicz [13], Tuncay and Corapcioglu [14], Achenbach [15] and Z.D. Zhou et al. [16]. In this paper, we have considered with influence of two relaxation times on the refraction and reflection of thermo-elastic plane waves at the solid liquid interface. The refraction and reflection coefficient ratios of different refracted and reflected waves with the incident angle θ have been observed by Green Lindsay (GL) theory and dynamical coupling (CD) theory.

The current article is organized in the following order: Sectio n "Formulation of the problem" described the formulation of the problem. Method of solution is explained in Section "Methods of Solutions". Detail descriptions of the boundary conditions for the current scenario are given in Section "Boundary Conditions". Sect ion "Expressions for the refraction and reflection coefficients" is devoted to obtain the expressions for the refraction and reflection

* Corresponding author.
E-mail addresses: ambreen.afsar@iiu.edu.pk (A.A. Khan), akbarzaman75@yahoo.com (A. Zaman), sundasyaseen92@gmail.com (S. Yaseen).

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coefficients. Numerical results and discussion is given in Section " Numerical Results and Discussions".

Formulation of the problem

Let us assume an isotropic, linear, homogeneous, perfectly conducting and thermally elastic medium with temperature dependent mechanical characteristics covering at the interface of the two half-spaces.

We kept constant temperature T_0 throughout the body with uniform magnetic field $H_0 = (0, H, 0)$, which is applied in the positive direction of y-axis.

Basic equations

The electromagnetic field is controlled by the following Maxwell equations.

$$curl \circ = \mathbf{J} + \frac{\partial \mathcal{B}}{\partial \boldsymbol{t}}, \qquad \mathcal{B} = \epsilon_0 \mathbf{E}, \tag{1}$$

$$curl\ \mathbf{E} = -\mu_0 \frac{\partial \hbar}{\partial t}, \tag{2}$$

$$\boldsymbol{E} = -\mu_0 \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0 \right), \tag{3}$$

$$div \circ = 0. \tag{4}$$

Here E is an induced electric field, $\boldsymbol{H}_0$ is initial uniform magnetic intensity vector, ϵ_0 is electric permeability and J is the current density vector.

The generalized thermo-elastic differential equations under *GL* theory, in the absence of heat source and body force, has the form

1. Equation of motion

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i \tag{5}$$

here f_i is the Lorentz force is given as under

$$f_i = \mu_0 (\mathbf{J} \times \mathbf{H}_0)_i \tag{6}$$

$$f_1 = -\mu_0 H \frac{\partial \hbar}{\partial x} - \epsilon_0 \mu_0^2 H^2 \ddot{u}, \quad f_2 = 0, \quad f_3 = -\mu_0 H \frac{\partial \hbar}{\partial z} - \epsilon_0 \mu_0^2 H^2 \ddot{w} \tag{7}$$

2. The constitutive law for the generalized thermo-elasticity theory under the GL theory has the form

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij} \left[\gimel \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \eta \left(T - T_0 + v_0 \frac{\partial T}{\partial t} \right) \right]. \tag{8}$$

3. Under GL theory, the heat conduction equation is

$$\mathrm{K} \nabla^2 T = \rho \tau_E \frac{\partial T}{\partial t} \left(1 + v_1 \frac{\partial}{\partial t} \right) + \eta T_0 \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right). \tag{9}$$

4. Strain-displacement relation

$$e_{ii} = u_{i,i}, \qquad e_{jj} = u_{j,j}, \qquad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{10}$$

here $\gimel$, μ are lame's constants, K is thermal conductivity, ρ is density, τ_E is specific heat at constant strain, σ_{ij} is components of stress tensor, u_i is components of displacement vector, T is absolute temperature, t is time and v_0, v_1 are two relaxation times.

Where the derivative with respect to time is represented by a superposed dot and a comma after suffix shows material derivatives $i, j = x, z$.

The displacement components in two dimensional forms can be written as

$$u_x = u(x, z, t), \quad u_y = 0, \quad u_z = w(x, z, t). \tag{11}$$

Where, Helmolz's representations of the displacement components u_x and u_z in terms of scalar potential functions Φ and Ψ,

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \quad w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}. \tag{12}$$

We define temperature dependent parameters as follow:

$$\mathrm{E} = E_0 f(T), \ \gimel = \gimel_0 E_0 f(T), \quad \mu = \mu_0 E_0 f(T), \quad \eta = \eta_0 E_0 f(T) \tag{13}$$

The non-dimensional function of temperature is $f(T)$. When the modulus of elasticity is temperature independent then $f(T) = 1$ and $E = E_0$.

Putting, Eqs. (7), (8), (10) and (13) into Eq. (5) yield

$$\rho \frac{\partial^2 u_x}{\partial t^2} = E_0 f(T) \left[\gimel_0 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + 2\mu_0 \frac{\partial e_{xx}}{\partial x} - \eta_0 \frac{\partial}{\partial x}(T + v_0 \dot{T}) \right] + 2E_0 f(T) \mu_0 \frac{\partial e_{xz}}{\partial z} - \mu_0 H \frac{\partial \hbar}{\partial x} - \mu_0^2 H^2 \epsilon_0 \frac{\partial^2 u}{\partial t^2}, \tag{14}$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = E_0 f(T) \left[\gimel_0 \left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 w}{\partial z^2} \right) + 2\mu_0 \frac{\partial e_{zz}}{\partial z} - \eta_0 \frac{\partial}{\partial z}(T + v_0 \dot{T}) \right] + 2E_0 f(T) \mu_0 \frac{\partial e_{zx}}{\partial x} - \mu_0 H \frac{\partial \hbar}{\partial z} - \mu_0^2 H^2 \epsilon_0 \frac{\partial^2 w}{\partial t^2}. \tag{15}$$

Putting, Eq. (12) in Eqs. (1)–(4), we can obtain

$$\hbar = -H \nabla^2 \Phi. \tag{16}$$

We introduce the different non dimensional variables are follow:

$$x_i^* = \frac{x_i}{\omega_1 C_t}, \ u_i^* = \frac{u_i}{\omega_1 C_t}, \ t^* = \frac{t}{\omega_1 C_t}, \ v_0^* = \frac{v_0}{\omega_1}, \ v_1^* = \frac{v_1}{\omega_1}, \ \hbar^* = \frac{\hbar}{H},$$
$$\sigma_{ij}^* = \frac{\sigma_{ij}}{\rho C_t^2}, T^* = \frac{\eta_0 E_0 (T - T_0)}{\rho C_t^2}, \ \beta = 1 + \frac{c_a^2}{c^2}, \ \beta_1 = \frac{1}{1 - \beta^* T_0} = \frac{1}{f(T_0)}. \tag{17}$$

After non-dimensionalize, the Eqs. (8), (9), (14), (15) and (16) taken the following forms

$$\beta \beta_1 \frac{\partial^2 \Phi}{\partial t^2} = (1 + \beta_1 r_H) \nabla^2 \Phi - \left(\mathrm{T} + v_0 \frac{\partial T}{\partial t} \right), \tag{18}$$

$$\beta \beta_1 \frac{\partial^2 \Psi}{\partial t^2} = (1 - \alpha) \nabla^2 \Psi, \tag{19}$$

$$\nabla^2 \mathrm{T} = \left(\frac{\partial T}{\partial t} + v_1 \frac{\partial^2 T}{\partial t^2} \right) + \varepsilon \nabla^2 \dot{\Phi}, \tag{20}$$

$$\hbar = -\nabla^2 \Phi. \tag{21}$$

where ∇^2 is the Laplace's operator.

The constitutive equations reduce to

$$\beta_1 \sigma_{ij} = (1 - \alpha)(u_{i,j} + u_{j,i}) + \delta_{ij} \left((2\alpha - 1) \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \left(\mathrm{T} + v_0 \frac{\partial T}{\partial t} \right) \right). \tag{22}$$

Where, $\alpha = E_0(\gimel_0 + \mu_0)/\rho C_t^2$, $r_H = \frac{c_a^2}{C_t^2}$, $\varepsilon^* = \frac{\eta_0 T_0}{\rho^2 \tau_E C_t^2}$, $c_a^2 = \frac{\mu_0 H^2}{\rho}$, $C_t^2 = E_0(\gimel_0 + 2\mu_0)/\rho$, $c^2 = \frac{1}{\mu_0 \epsilon_0}$, $\omega_1 = K/\rho C_t \tau_E^2$.

Here r_H is the amount of magnetic pressure. E_0 is constant modulus of elasticity at $\beta^*=0$, C_a is the supposed Alfven speed, ε is the standard thermo-elastic coupling parameters.

Now for the medium M', we will use prime to describe all the quantities of basic Eqs. (1)–(4). Taking,

$$u' = \frac{\partial \Phi'}{\partial x} - \frac{\partial \Psi'}{\partial z}, \qquad w = \frac{\partial \Phi'}{\partial z} + \frac{\partial \Psi'}{\partial x}. \tag{23}$$

After non-dimensionalize, we obtain

$$\beta' \beta_1' \frac{\partial^2 \Phi'}{\partial t^2} = (1 + \beta_1' r_H')\nabla^2 \Phi' - \left(\mathrm{T}' + v_0' \frac{\partial T'}{\partial t} \right), \tag{24}$$

$$\beta' \beta_1' \frac{\partial^2 \Psi'}{\partial t^2} = (1 - \alpha')\nabla^2 \Psi', \tag{25}$$

$$\nabla^2 \mathrm{T}' = \left(\frac{\partial T'}{\partial t} + v_1' \frac{\partial^2 T'}{\partial t^2} \right) + \varepsilon' \nabla^2 \dot{\Phi}', \tag{26}$$

$$\hbar' = -\nabla^2 \Phi', \tag{27}$$

$$\beta_1' e_{ij}' = (1 - \alpha')(u_{i,j}' + u_{j,i}') + \delta_{ij}' \left((2\alpha' - 1)\left(\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} \right) - \left(\mathrm{T}' + v_0' \frac{\partial T'}{\partial t} \right) \right). \tag{28}$$

Methods of solutions

Plane propagation in the x direction makes an angle θ with the z-axis, we introduce

$$\{\Phi, \mathrm{T}, \hbar\}(x, z, t) = [\Phi_1, \mathrm{T}_1, \hbar_1] exp\{i\xi(x \sin\theta + z\cos\theta) - \varpi t\}, \tag{29}$$

$$\Psi(x, z, t) = \Psi_1 exp\{i\ell(x \sin\theta + z\cos\theta) - \varpi t\}, \tag{30}$$

where ξ and ℓ are the wave numbers and ϖ is the complex frequency.

Putting Eqs. (28) and (29) into Eqs. (18)–(21), we get a system of three homogeneous equations.

$$(\xi^2\alpha_1 + \beta\beta_1\varpi^2)\Phi_1 + \mathrm{p}T_1 = 0, \tag{31}$$

$$\varpi\varepsilon\xi^2\Phi_1 + (\xi^2 - \varpi \mathrm{q})T_1 = 0, \tag{32}$$

$$-\xi^2\Phi_1 + \hbar_1 = 0, \tag{33}$$

in which $\alpha_1 = 1 + \beta_1 r_H$, $\mathrm{p} = 1 - \varpi v_0$, $\mathrm{q} = 1 - \varpi v_1$.

The system of Eqs. (31)–(33) has non-trivial solutions if and only if the determinant of factor matric vanishes. So

$$\begin{vmatrix} (\xi^2\alpha_1 + \beta\beta_1\varpi^2) & p & 0 \\ \varpi\varepsilon\xi^2 & (\xi^2 - \varpi q) & 0 \\ -\xi^2 & 0 & 1 \end{vmatrix} = 0, \tag{34}$$

This yield,

$$\mathrm{v}^4 - \frac{\beta\beta_1\varpi - \alpha_1 q - p\varepsilon}{\beta\beta_1 q}\mathrm{v}^2 - \frac{\varpi\alpha_1}{\beta\beta_1 q} = 0, \tag{35}$$

in which $\mathrm{v} = \frac{\varpi}{\xi}$ is the velocity of reflected P-waves.

From Eqs. (19), (28) and (29), we get

$$W^2 + \frac{(1-\alpha)}{\beta\beta_1} = 0, \;\; w = \sqrt{\frac{\alpha - 1}{\beta\beta_1}}, \tag{36}$$

in which $W = \frac{\varpi}{\ell}$ is the velocity of reflected SV-waves.

Similarly for medium M',

$$\mathrm{v}'^4 - \frac{\beta'\beta_1'\varpi - \alpha_1' n - m\varepsilon'}{\beta'\beta_1' n}\mathrm{v}'^2 - \frac{\varpi\alpha_1'}{\beta'\beta_1' n} = 0, \tag{37}$$

where $\mathrm{m} = 1 - \varpi v_0'$, $\mathrm{n} = 1 - \varpi v_1'$.

in which $\mathrm{v}' = \frac{\varpi}{\xi'}$ is the velocity of refracted P-waves.

$$W'^2 + \frac{(1-\alpha')}{\beta'\beta_1'} = 0, \quad W' = \sqrt{\frac{\alpha' - 1}{\beta'\beta_1'}}, \tag{38}$$

in which $W' = \frac{\varpi}{\ell'}$ is the velocity of refracted SV-waves.

For incident SV-wave

Consider a plane SV wave propagating through a medium M and is incident at z = 0, three waves (SV, P and thermal) are reflected in the same medium M by making an angles θ, θ_1 and θ_2 with z-axis and P-wave, Thermal waves are transmitted into medium M' by making an angles θ_1' and θ_2'. The displacement potentials Φ, Ψ for medium M and Φ′, Ψ′ for medium M' will take the forms:

$$\Phi = A_1 exp\{i\xi_1(x\sin\theta_1 - z\cos\theta_1) - \varpi t\} + A_2 exp\{i\xi_2(x\sin\theta_2 - z\cos\theta_2) - \varpi t\}. \tag{39}$$

$$\Psi = B_1 exp\{i\ell(x\sin\theta + z\cos\theta) - \varpi t\} + B_2 exp\{i\ell(x\sin\theta - z\cos\theta) - \varpi t\}, \tag{40}$$

$$\Phi' = A_1' exp\{i\xi_1'(x\sin\theta_1' + z\cos\theta_1') - \varpi t\} + A_2' exp\{i\xi_2'(x\sin\theta_2' + z\cos\theta_2') - \varpi t\}, \tag{41}$$

$$\Psi' = 0. \tag{42}$$

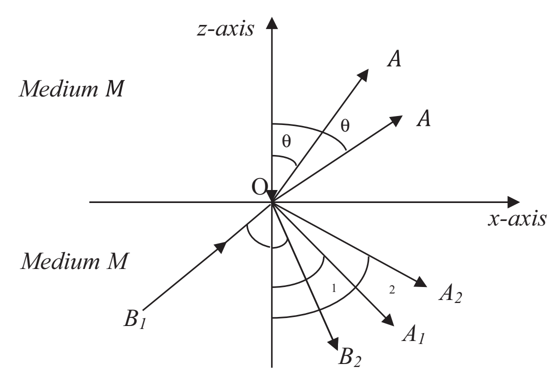

Fig. 1. Relation between incident angle of SV-wave, reflect and the refract angles.

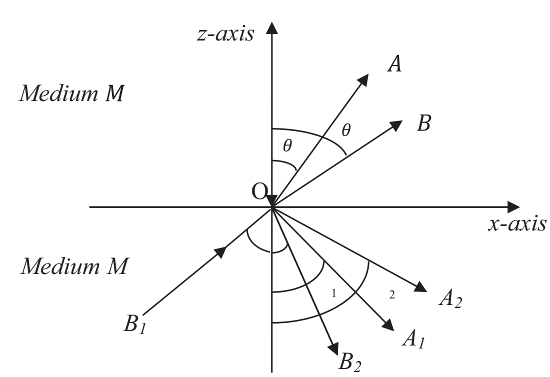

Fig. 2. Relation between incident angle of P-wave, reflect and the refract angles.

where the angles θ, θ_1, θ_2, θ'_1, θ'_2 and the corresponding numbers ℓ, ξ_1, ξ_2, ξ'_1, ξ'_2 are joined with the following relation:

$$\xi_1 \sin\theta_1 = \xi_2 \sin\theta_2 = \ell \sin\theta = \xi'_1 \sin\theta'_1 = \xi'_2 \sin\theta'_2. \tag{43}$$

On the interface z = 0,

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta}{c'} = \frac{\sin\theta'_1}{v'_1} = \frac{\sin\theta'_2}{v'_2}. \tag{44}$$

in which

$$v_1 = \frac{\varpi}{\xi_1}, v_2 = \frac{\varpi}{\xi_2}, c' = \frac{\varpi}{\ell}, v'_1 = \frac{\varpi}{\xi'_1}, v'_2 = \frac{\varpi}{\xi'_2}$$

v_1, v_2 are the roots of Eq. (35) and v'_1, v'_2 are the roots of Eq. (37).

For incident P-wave

A plane P-wave propagating through a medium M and is incident at z = 0, three waves (P, SV and thermal) are reflected in the same medium M by making an angles θ, θ_1 and θ_2 with z-axis and P-wave, Thermal waves are transmitted into medium M' by making an angles θ' and θ'_1. The displacement potentials Φ, Ψ for medium M and Φ', Ψ' for medium M' will take the forms:

$$\begin{aligned}\Phi = {} & B_1 exp\{i\xi_1(x\sin\theta + z\cos\theta) - \varpi t\} \\ & + B_2 exp\{i\xi_1(x\sin\theta - z\cos\theta) - \varpi t\} \\ & + A_1 \exp\{i\xi_2(x\sin\theta_1 - z\cos\theta_1) - \varpi t\},\end{aligned} \tag{45}$$

$$\Psi = A_2 exp\{i\ell(x\sin\theta_2 - z\cos\theta_2) - \varpi t\}, \tag{46}$$

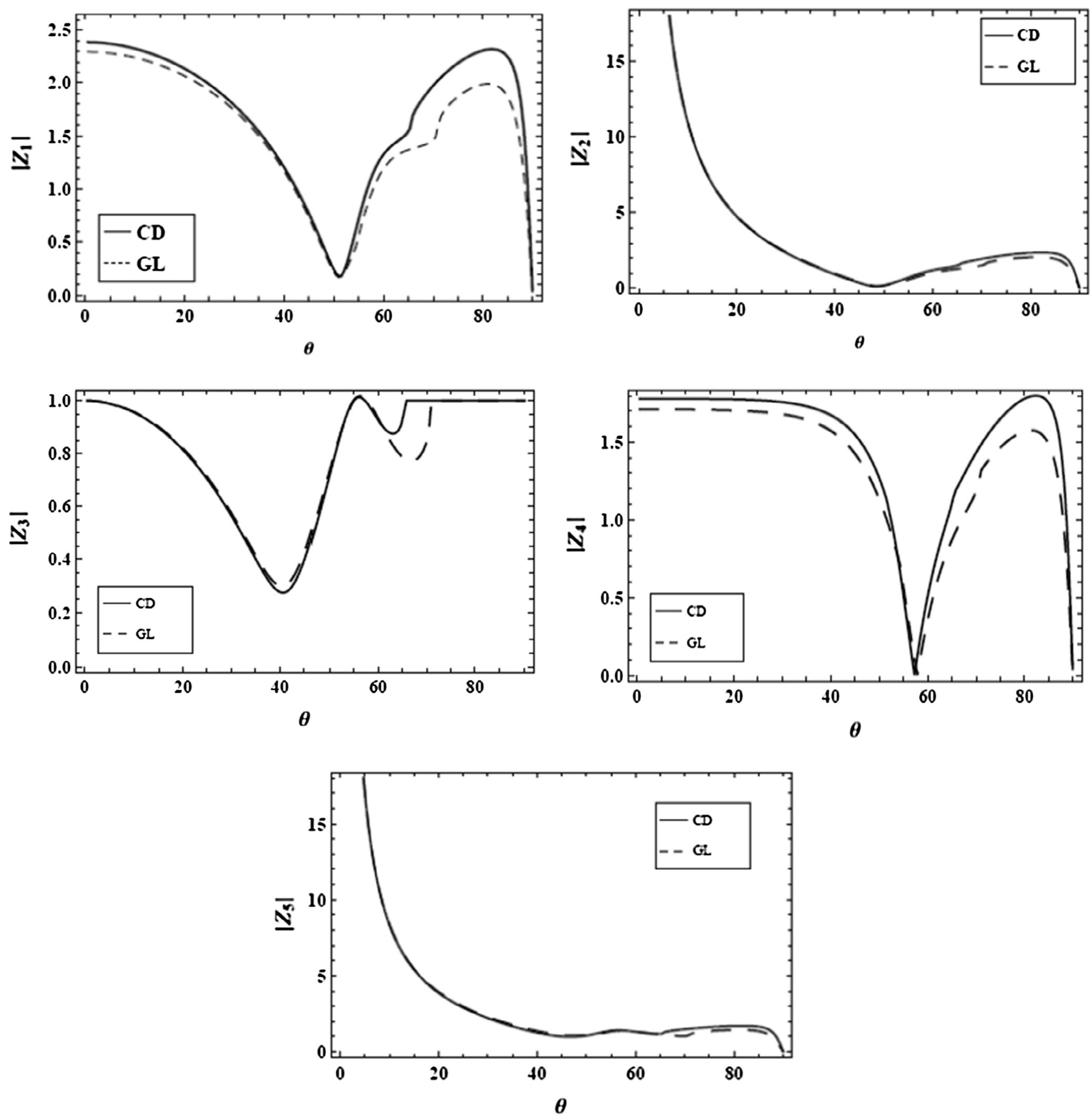

Fig. 3. Difference of the amplitudes $|Z_i|(i = 1, 2, \ldots, 5)$ making an incident angle of SV waves under different theory, H = 0.4, $\varepsilon = 0.08$, $\beta^* = 0.001$.

$$\Phi' = B_1' exp\{i\xi_1'(x\sin\theta' + z\cos\theta') - \varpi t\} + A_1' exp\{i\xi_2'(x\sin\theta_1' + z\cos\theta_1') - \varpi t\}, \tag{47}$$

$$\Psi' = 0, \tag{48}$$

where the angles θ, θ_1, θ_2, θ', θ_1' and the corresponding numbers ℓ, ξ_1, ξ_2, ξ_1', ξ_2' are joined with the following relation:

$$\xi_1 \sin\theta = \xi_2 \sin\theta_1 = \ell \sin\theta_2 = \xi_1' \sin\theta' = \xi_2' \sin\theta_1', \tag{49}$$

On the interface z = 0,

$$\frac{\sin\theta}{v_1} = \frac{\sin\theta_1}{v_2} = \frac{\sin\theta_2}{c'} = \frac{\sin\theta'}{v_1'} = \frac{\sin\theta_1'}{v_2'}, \tag{50}$$

in which

$$v_1 = \frac{\varpi}{\xi_1}, \quad v_2 = \frac{\varpi}{\xi_2}, \quad c' = \frac{\varpi}{\ell}, \quad v_1' = \frac{\varpi}{\xi_1'}, \quad v_2' = \frac{\varpi}{\xi_2'}.$$

v_1, v_2 are the roots of Eq. (35) and v_1', v_2' are the roots of Eq. (36).

Boundary conditions

1) At the interface, the continuity of normal displacement i.e. $w = w'$.

$$\frac{\partial\Phi}{\partial z} + \frac{\partial\Psi}{\partial x} = \frac{\partial\Phi'}{\partial z} + \frac{\partial\Psi'}{\partial x}, \text{ at } z = 0. \tag{51}$$

2) At the interface, the tangential displacement must disappear i.e. *u = 0.*

$$\frac{\partial\Phi}{\partial x} - \frac{\partial\Psi}{\partial z} = 0, \text{ at } z = 0. \tag{52}$$

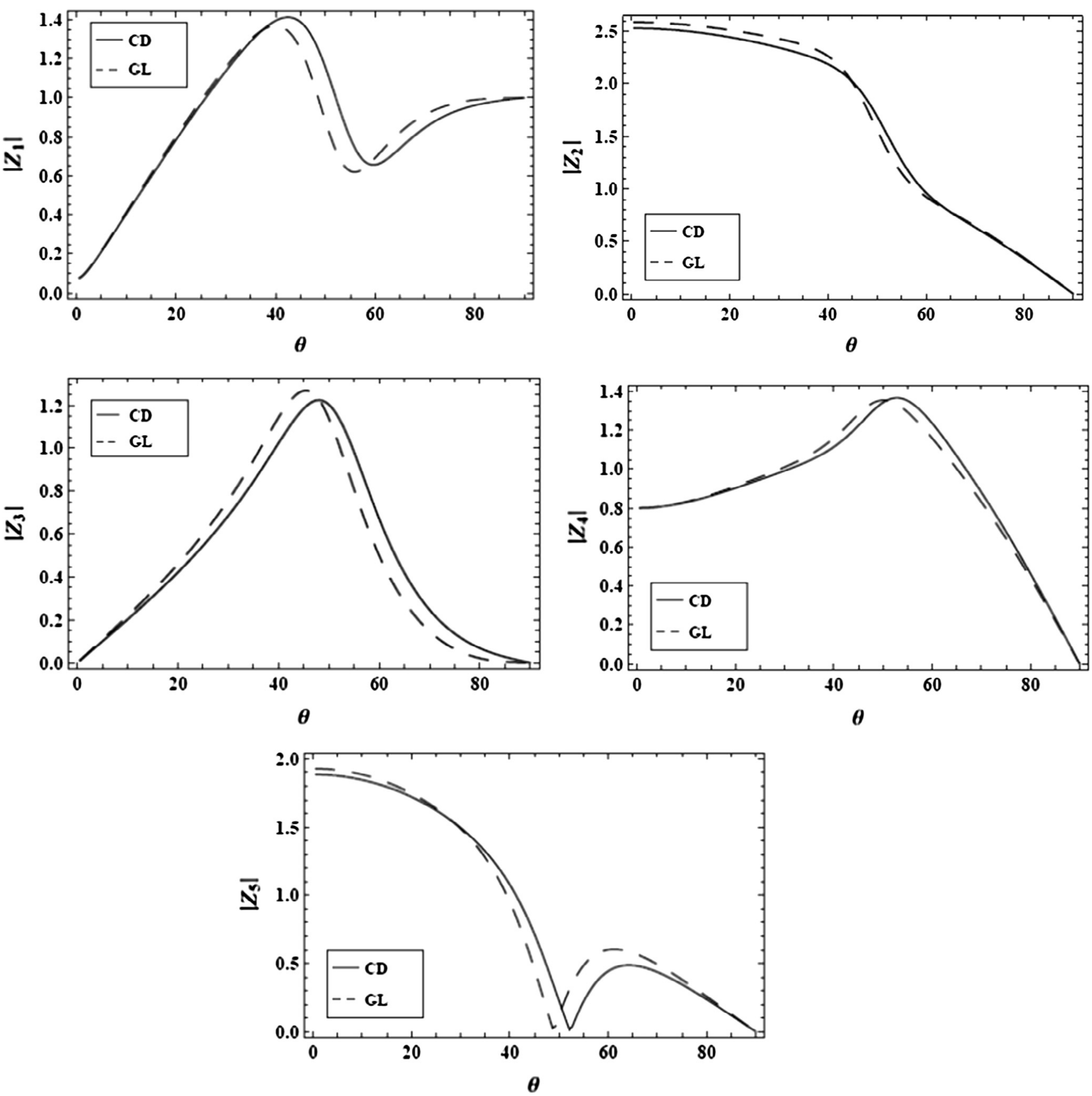

Fig. 4. Difference of the amplitudes $|Z_i| (i = 1, 2, \ldots, 5)$ making an incident angle of P-waves for under different theory, H = 0.4, $\varepsilon = 0.08$, $\beta^* = 0.001$.

3) At the interface, normal force per unit primary area is continuous i.e. $\sigma_{33} = \sigma'_{33}$.

$$\frac{1}{\beta_1}\left[(2\alpha-2-\beta_1 r_H)\nabla^2\Phi+2(1-\alpha)\left(\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Psi}{\partial x\partial z}\right)+\beta\beta_1\frac{\partial^2\Phi}{\partial t^2}\right] = \frac{1}{\beta_1'}\left[(2\alpha'-2-\beta_1' r_H')\nabla^2\Phi'+2(1-\alpha')\left(\frac{\partial^2\Phi'}{\partial z^2}+\frac{\partial^2\Psi'}{\partial x\partial z}\right)+\beta'\beta_1'\frac{\partial^2\Phi'}{\partial t^2}\right], \text{ at } z=0. \tag{53}$$

4) At the interface, tangential force per unit primary area must disappear i.e. $\sigma_{13} = 0$.

$$2\frac{\partial^2\Phi}{\partial x\partial z}+\frac{\partial^2\Psi}{\partial x^2}-\frac{\partial^2\Psi}{\partial z^2}=0, \text{ at } z=0. \tag{54}$$

5) At the interface, continuity of temperature i.e. $T = T'$.

$$m\left[\alpha_1\nabla^2\Phi-\beta\beta_1\frac{\partial^2\Phi}{\partial t^2}\right]=p\left[\alpha_1'\nabla^2\Phi'-\beta'\beta_1'\frac{\partial^2\Phi'}{\partial t^2}\right], \text{ at } z=0. \tag{55}$$

Expressions for the refraction and reflection coefficients

For incident SV-wave

Eq. (39)–(42) reduces after applying the boundary conditions (51)–(55).

$$\frac{A_1}{B_1}\left(\frac{c'}{v_1}\cos\theta_1\right)+\frac{A_2}{B_1}\left(\frac{c'}{v_2}\cos\theta_2\right)-\frac{B_2}{B_1}(\sin\theta)+\frac{A_1'}{B_1}\left(\frac{c'}{v_1'}\cos\theta_1'\right)+\frac{A_2'}{B_1}\left(\frac{c'}{v_2'}\cos\theta_2'\right)=\sin\theta \tag{56}$$

$$\frac{A_1}{B_1}\left(\frac{c'}{v_1}\sin\theta_1\right)+\frac{A_2}{B_1}\left(\frac{c'}{v_2}\sin\theta_2\right)+\frac{B_2}{B_1}(\cos\theta)=\cos\theta \tag{57}$$

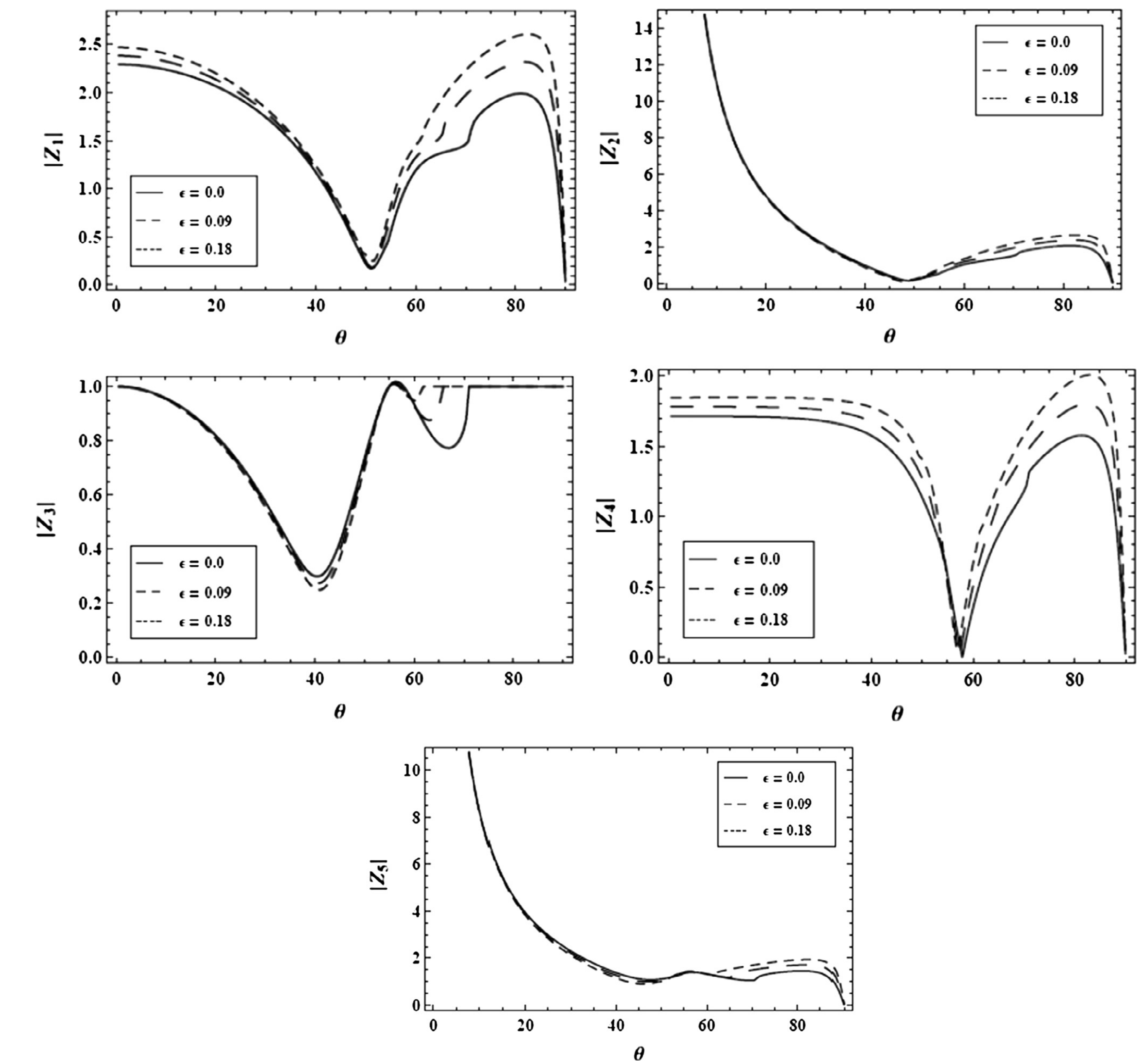

Fig. 5. Difference of the amplitudes $|Z_i|(i = 1, 2, \ldots, 5)$ making an incident angle of SV waves for effect of coupling parameter, H = 0.5, $\beta^* = 0.001$.

$$\frac{A_1}{B_1}\left(\frac{c'^2}{v_1^2}\frac{m_1}{\beta_1}\right)+\frac{A_2}{B_1}\left(\frac{c'^2}{v_2^2}\frac{m_2}{\beta_1}\right)+\frac{B_2}{B_1}\left(\frac{(1-\alpha)}{\beta_1}\sin 2\theta\right)-\frac{A_1'}{B_1}\left(\frac{c'^2}{v_1'^2}\frac{m_3}{\beta_1'}\right)-\frac{A_2'}{B_1}\left(\frac{c'^2}{v_2'^2}\frac{m_4}{\beta_1'}\right)=\frac{(1-\alpha)}{\beta_1}\sin 2\theta \tag{58}$$

$$\frac{A_1}{B_1}\left(\frac{c'^2}{v_1^2}\sin 2\theta_1\right)+\frac{A_2}{B_1}\left(\frac{c'^2}{v_2^2}\sin 2\theta_2\right)+\frac{B_2}{B_1}(\cos 2\theta)=-\cos 2\theta \tag{59}$$

$$\frac{A_1}{B_1}\left(\frac{n_1}{v_1^2}\right)+\frac{A_2}{B_1}\left(\frac{n_2}{v_2^2}\right)-\frac{A_1'}{B_1}\left(\frac{n_3}{v_1'^2}\right)-\frac{A_2'}{B_1}\left(\frac{n_4}{v_2'^2}\right)=0 \tag{60}$$

where,

$$m_1=\left[(2+\beta_1 r_H-2\alpha)+(2\alpha-2)\cos^2\theta_1+\frac{\beta\beta_1}{\xi_1^2}\varpi^2\right],$$

$$m_2=\left[(2+\beta_1 r_H-2\alpha)+(2\alpha-2)\cos^2\theta_2+\frac{\beta\beta_1}{\xi_2^2}\varpi^2\right],$$

$$m_3=\left[(2+\beta_1' r_H'-2\alpha')+(2\alpha'-2)\cos^2\theta_1'+\frac{\beta'\beta_1'}{\xi_1'^2}\varpi^2\right],$$

$$m_4=\left[(2+\beta_1' r_H'-2\alpha')+(2\alpha'-2)\cos^2\theta_2'+\frac{\beta'\beta_1'}{\xi_2'^2}\varpi^2\right].$$

$$n_1=m\varpi^2(\alpha_1+\beta\beta_1 v_1^2),\ n_2=m\varpi^2(\alpha_1+\beta\beta_1 v_2^2),\ n_3=p\varpi^2(\alpha_1'+\beta'\beta_1' v_1'^2),$$

$$n_4=p\varpi^2(\alpha_1'+\beta'\beta_1' v_2'^2),\ Z_1=\frac{A_1}{B_1},Z_2=\frac{A_2}{B_1},Z_3=\frac{B_2}{B_1},Z_4=\frac{A_1'}{B_1}\text{ and }Z_5=\frac{A_2'}{B_1}.$$

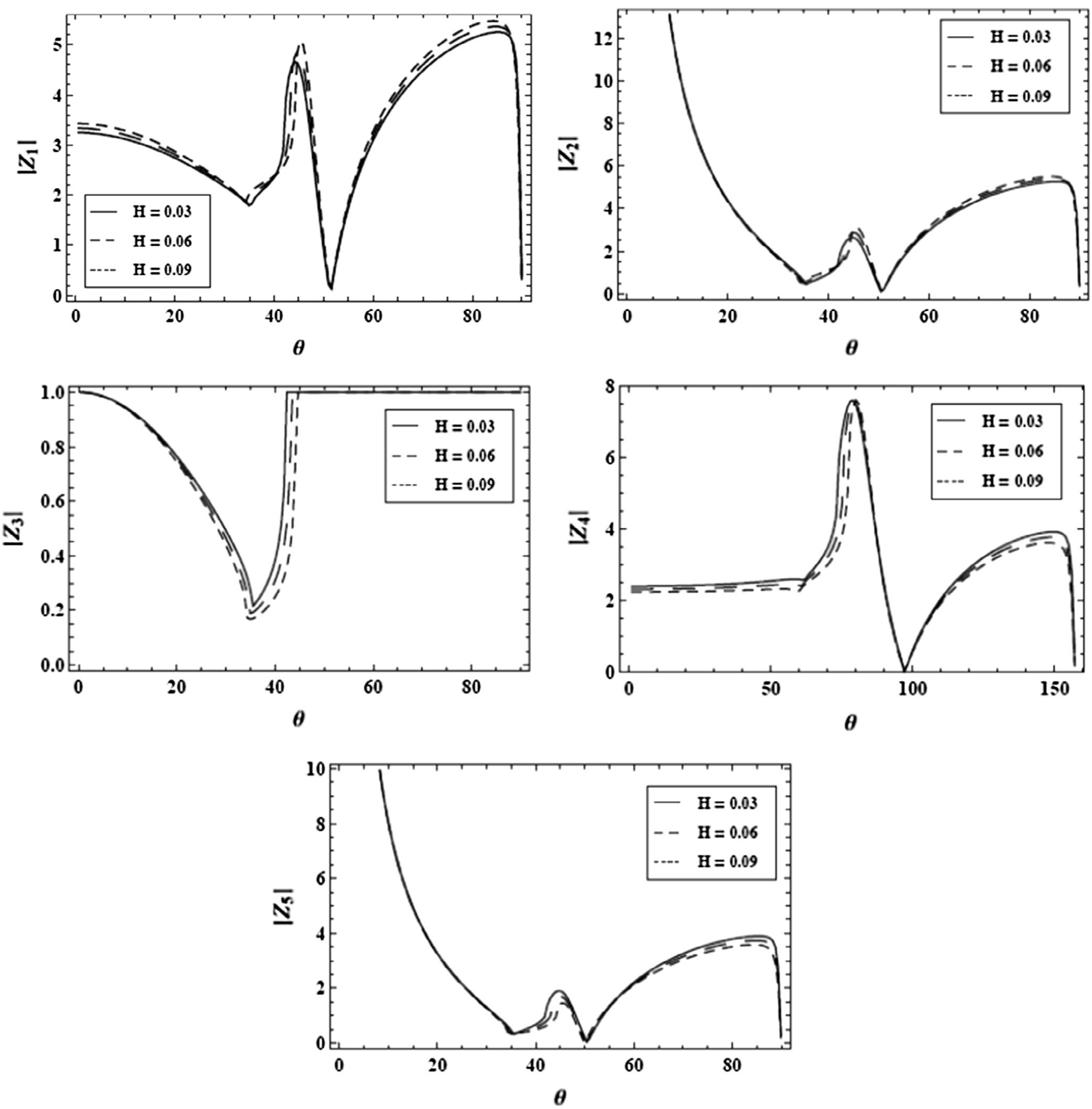

Fig. 6. Difference of the amplitudes $|Z_i|(i=1,2,\ldots,5)$ making an incident angle of SV waves for effect of magnetic field, $\varepsilon=0.7$, $\beta^*=0.001$.

where $(Z_i, i = 1, 2, \ldots, 5)$ represents the amplitude ratios of reflected P, T, SV waves and refracted P, T waves.

For incident P-wave

Eqs. (45)–(48) reduces after applying the boundary conditions (51)–(55).

$$\frac{B_2}{B_1}\left(\frac{\cos\theta}{v_1}\right)+\frac{A_1}{B_1}\left(\frac{\cos\theta_1}{v_2}\right)-\frac{A_2}{B_1}\left(\frac{\sin\theta_2}{c'}\right)+\frac{B_1'}{B_1}\left(\frac{\cos\theta'}{v_1'}\right)+\frac{A_1'}{B_1}\left(\frac{\cos\theta_1'}{v_2'}\right)=\frac{\cos\theta}{v_1}, \tag{61}$$

$$\frac{B_2}{B_1}\left(\frac{\sin\theta}{v_1}\right)+\frac{A_1}{B_1}\left(\frac{\sin\theta_1}{v_2}\right)+\frac{A_2}{B_1}\left(\frac{\cos\theta_2}{c'}\right)=-\frac{\sin\theta}{v_1}, \tag{62}$$

$$\frac{B_2}{B_1}\left(\frac{m_2'}{\beta_1 v_1^2}\right)+\frac{A_1}{B_1}\left(\frac{m_3'}{\beta_1 v_2^2}\right)+\frac{A_2}{B_1}\left(\frac{(\alpha-1)}{\beta_1 c'^2}\sin 2\theta_2\right)-\frac{B_1'}{B_1}\left(\frac{m_4'}{\beta_1' v_1'^2}\right)-\frac{A_1'}{B_1}\left(\frac{m_5'}{\beta_1' v_2'^2}\right)=-\frac{m_1'}{\beta_1 v_1^2}, \tag{63}$$

$$\frac{B_2}{B_1}\left(\frac{\sin 2\theta}{v_1^2}\right)+\frac{A_1}{B_1}\left(\frac{\sin 2\theta_1}{v_2^2}\right)+\frac{A_2}{B_1}\left(\frac{\cos 2\theta_2}{c'^2}\right)=\frac{\sin 2\theta}{v_1^2}, \tag{64}$$

$$\frac{B_2}{B_1}\left(\frac{n_1}{v_1^2}\right)+\frac{A_1}{B_1}\left(\frac{n_2}{v_2^2}\right)-\frac{B_1'}{B_1}\left(\frac{n_3}{v_1'^2}\right)-\frac{A_1'}{B_1}\left(\frac{n_4}{v_2'^2}\right)=-\frac{m\varpi^2}{v_1^2}\left(\alpha_1+\beta\beta_1 v_1^2\right). \tag{65}$$

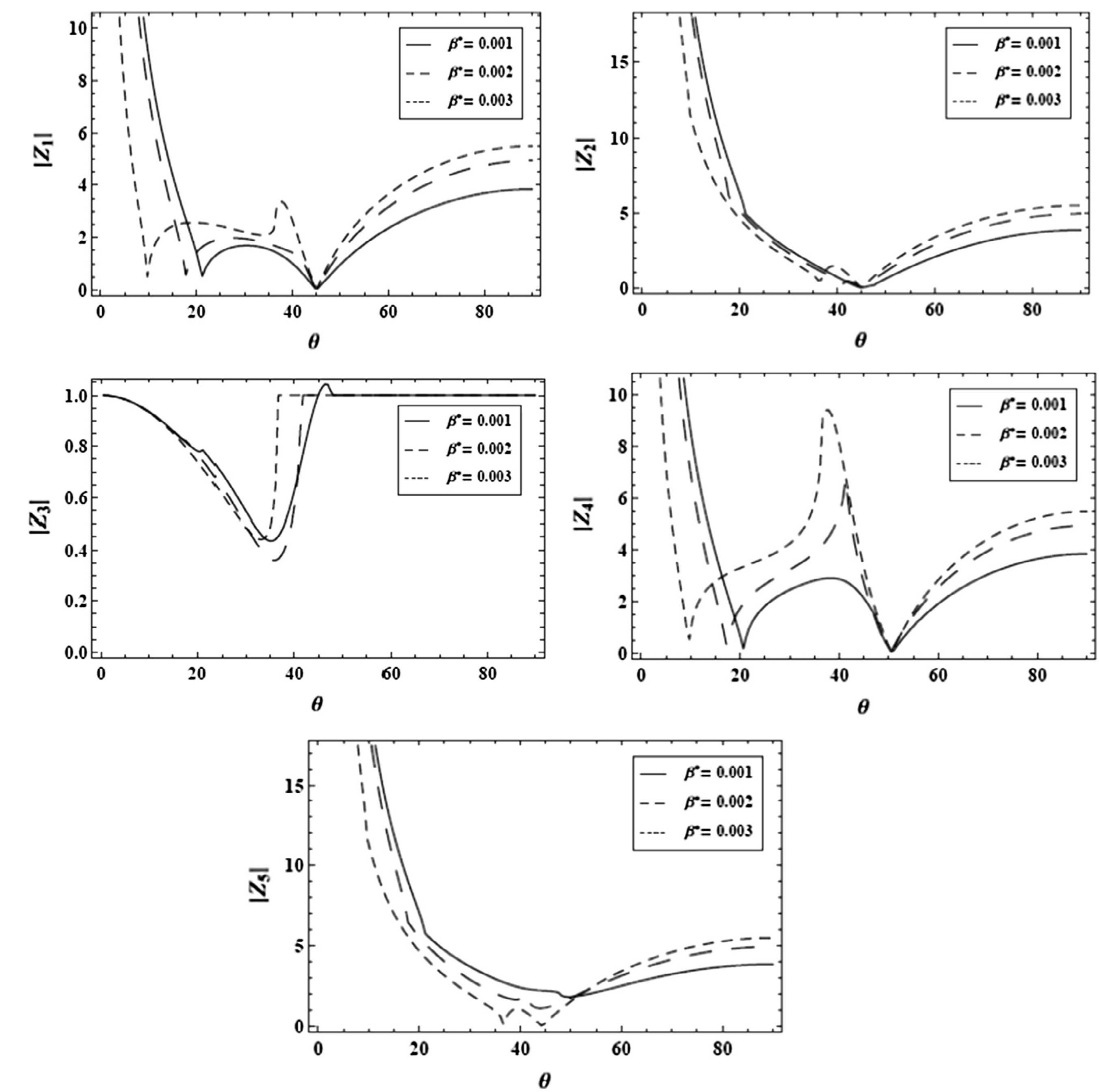

Fig. 7. Difference of the amplitudes $|Z_i| (i = 1, 2, \ldots, 5)$ making an incident angle of SV waves for temperature dependent modulus effect, H = 0.0, $\varepsilon = 0.7$.

Where,

$$m'_1 = m'_2 = \left[(2+\beta_1 r_H - 2\alpha) + (2\alpha - 2)\cos^2\theta + \frac{\beta\beta_1}{\xi_1^2}\varpi^2\right],$$

$$m'_3 = \left[(2+\beta_1 r_H - 2\alpha) + (2\alpha - 2)\cos^2\theta_1 + \frac{\beta\beta_1}{\xi_2^2}\varpi^2\right],$$

$$m'_4 = \left[(2+\beta'_1 r'_H - 2\alpha') + (2\alpha' - 2)\cos^2\theta' + \frac{\beta'\beta'_1}{\xi_1'^2}\varpi^2\right],$$

$$m'_5 = \left[(2+\beta'_1 r'_H - 2\alpha') + (2\alpha' - 2)\cos^2\theta'_1 + \frac{\beta'\beta'_1}{\xi_2'^2}\varpi^2\right].$$

$$Z_1 = \frac{B_2}{B_1}, Z_2 = \frac{A_1}{B_1}, Z_3 = \frac{A_2}{B_1}, Z_4 = \frac{B'_1}{B_1} \text{ and } Z_5 = \frac{A'_1}{B_1}.$$

where $(Z_i, i = 1, 2, \ldots, 5)$ represents the amplitude ratios of reflected P, T, SV waves and refracted P, T waves.

Numerical results and discussions

We have considered the data for solid medium as crust and liquid medium as water following Singh and Chakraborty [17], for the numerical analysis of previous section.

For solid medium (*M* crust "Granite"): which in geology is the uppermost solid shell of a rocky planet or natural satellite, and is chemically different from the underlying mantle.

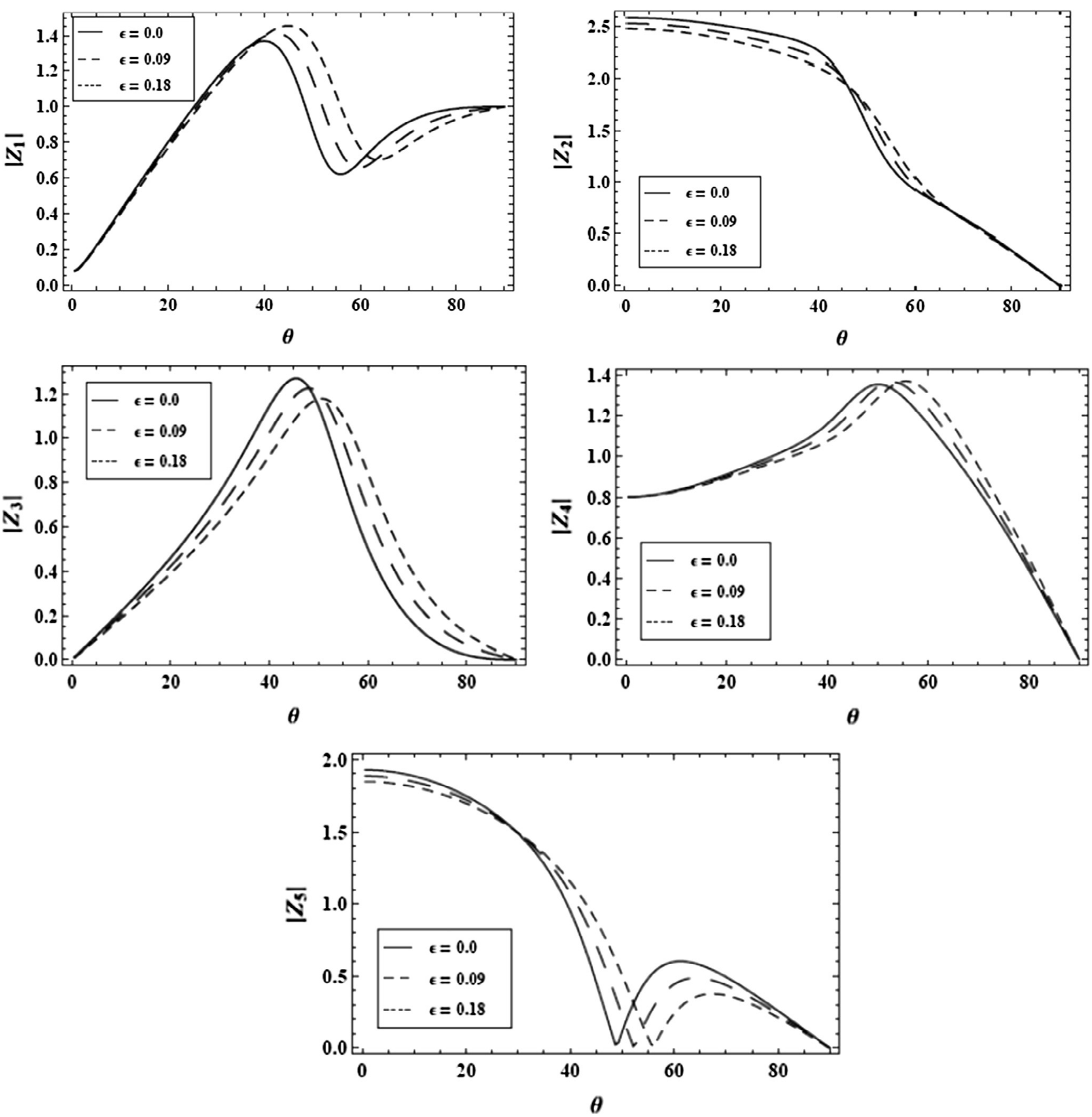

Fig. 8. Difference of the amplitudes $|Z_i|(i = 1, 2, \ldots, 5)$ making an incident angle of P waves for coupling parameter, H = 0.5, $\beta^* = 0.001$.

$\gimel = \mu = 3 \times 10^{10} \mathrm{N} \cdot \mathrm{m}^{-2}, \quad T_0 = 300\,\mathrm{K}, \quad \varpi = 7.5 \times 10^{13}\ \mathrm{S}^{-1},$
$\tau_E = 1100\,\mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}, \quad \rho = 2900\,\mathrm{kg} \cdot \mathrm{m}^{-3},$
$\mathrm{K} = 3\,\mathrm{W} \cdot \mathrm{m}^{-1} \cdot \mathrm{K}^{-1}, \quad E_0 = 2.6 \times 10^5.$

For fluid medium (M' "Water"):

$$\gimel' = \mu' = 20.4 \times 10^9\ \mathrm{N} \cdot \mathrm{m}^{-2}, \quad \tau'_E = 4187\,\mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}, \quad \rho' = 1000\,\mathrm{kg} \cdot \mathrm{m}^{-3}, \quad K' = 0.6\,\mathrm{W} \cdot \mathrm{m}^{-1} \cdot \mathrm{K}^{-1}, \quad E'_0 = 2.2 \times 10^9.$$

Considering $v_0 = v'_0 = 0.8$, $v_1 = v'_1 = 0.9$ and $\epsilon_0 = \epsilon'_0 = 0.2$. (See Figs. 1,2).

Figs. 3 and 4 gives the effects of amplitude ratio with the incident angle for the SV and the P waves under two theories. In the situation of SV wave, $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ commenced from the maximum values and goes to zero at $\theta = 90^\circ$ but for $|Z_3|$, it begins from unity and ends on as well a unity at $\theta = 90^\circ$. It also indicates that GL theory in $|Z_1|$, $|Z_2|$ and $|Z_4|$ have the smaller values than CD theory, whereas GL theory in $|Z_3|$ and $|Z_5|$ have smaller values than CD theory after $\theta = 65^\circ$.

In the case of P wave, $|Z_1|$ begins from zero and reaches to unity at $\theta = 90^\circ$ $|Z_2|$ and $|Z_5|$ begins from its extreme values and reaches to zero at $\theta = 90^\circ$. Whereas for $|Z_2|$ =0 at $\theta = 90^\circ$ $|Z_3|$ =0 when

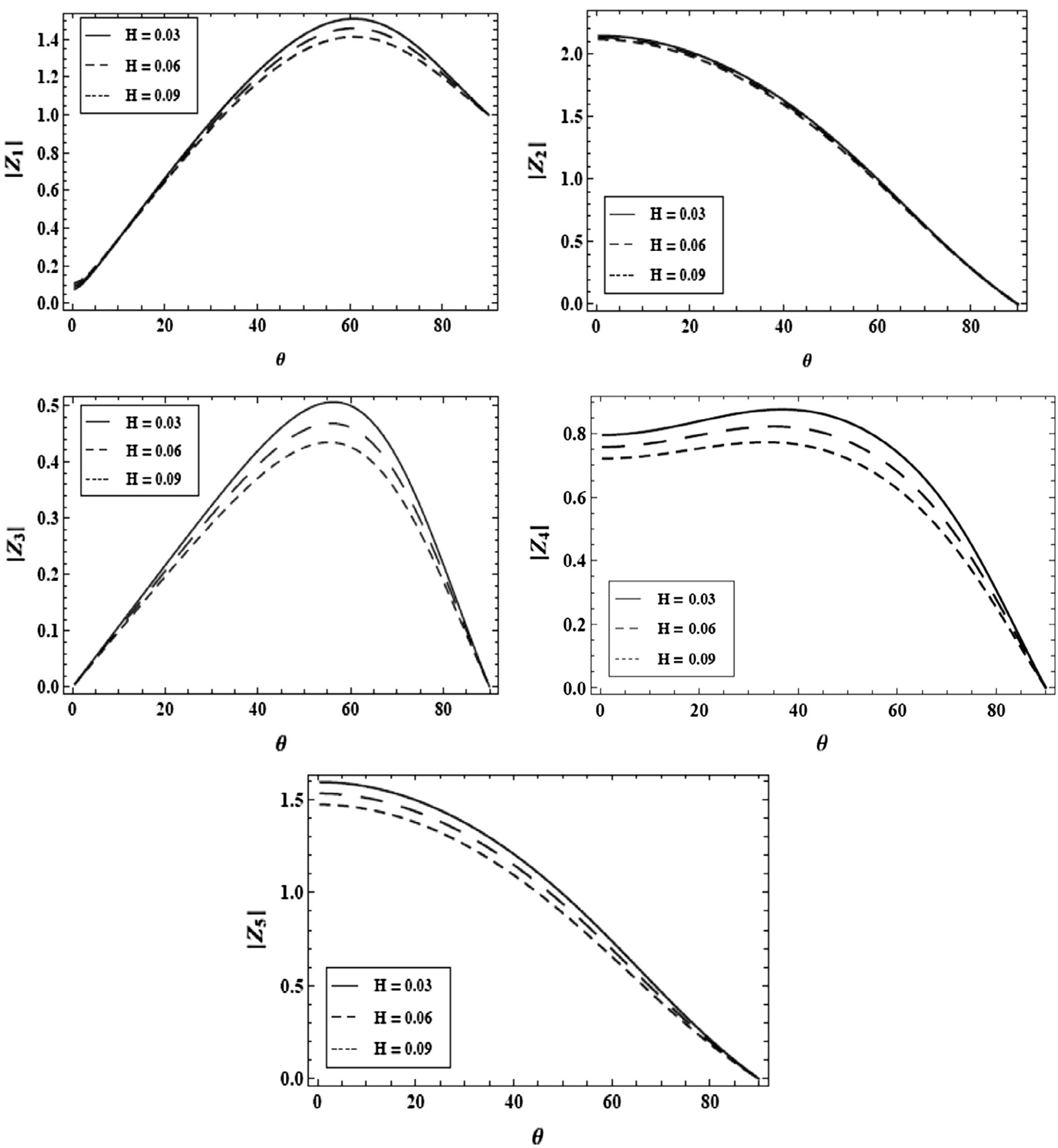

Fig. 9. Difference of the amplitudes $|Z_i| (i = 1, 2, \ldots, 5)$ making an incident angle of P waves for effect of magnetic field, $\varepsilon = 0.7$, $\beta^* = 0.001$.

$\theta = 0^{\circ}$ and $\theta = 90^{\circ}$. $|Z_4|$ gets its highest value after $\theta = 50^{\circ}$ and gradually it goes to zero at $\theta = 90^{\circ}$.

Figs. 5–7 depicts the effect of amplitudes with incident angle of SV wave under the variation of two relaxation times to GL theory. Fig. 5 exhibits the variation of incident angle of SV wave with the amplitude ratio under various values of ε. The amplitude ratio $|Z_1|$, $|Z_2|$ and $|Z_4|$ increases with increase of ε whereas amplitude ratio $|Z_3|$ and $|Z_5|$ initially decreases by increasing ε and after $\theta = 60^{\circ}$, the amplitude ratio increases by increasing ε.

Fig. 6 gives the difference of magnetic field on the amplitude ratio of SV wave. It is seem that $|Z_1|$ and $|Z_2|$ increases with an increase of H, but $|Z_3|$, $|Z_4|$ and $|Z_5|$ decreases by increasing H. $|Z_4|$ has maximum value at $\theta = 45^{\circ}$.

Fig. 7 shows the influence of reference temperature modulus on amplitude ratio. We can see that the amplitude ratio $|Z_1|$ and $|Z_4|$ rises with rising β^* after $\theta = 20^{\circ}$, $|Z_3|$ starts from unity and end on unity as well with increasing β^* and all the curve mix with each other after $\theta = 45^{\circ}$. While $|Z_2|$ and $|Z_5|$ decreases with increasing β^* before $\theta = 45^{\circ}$ and $\theta = 45^{\circ}$ it has opposite effect.

Figs. 8 and 9 gives the difference of amplitude with the incident angle of P wave under the influence of two relaxation times to GL-theory. Fig. 8 shows the effect of ε on the amplitude ratio. $|Z_1|$, $|Z_2|$ and $|Z_5|$ decreases before $\theta = 30^{\circ}$ and after $\theta = 45^{\circ}$ increases by increasing ε, while $|Z_3|$ and $|Z_4|$ decreases before $\theta = 50^{\circ}$ and after $\theta = 50^{\circ}$ it start increasing and moves toward zero at $\theta = 90^{\circ}$.

We observed from Fig. 9 that $|Z_1|$ to $|Z_5|$ decreases as H increases. For $|Z_1|$ it moves towards unity at $\theta = 90^{\circ}$ where as in $|Z_2|$, $|Z_3|$, $|Z_4|$ and $|Z_5|$ it moves toward zero at $\theta = 90^{\circ}$. Fig. 10 exhibits the difference of reference temperature modulus on the

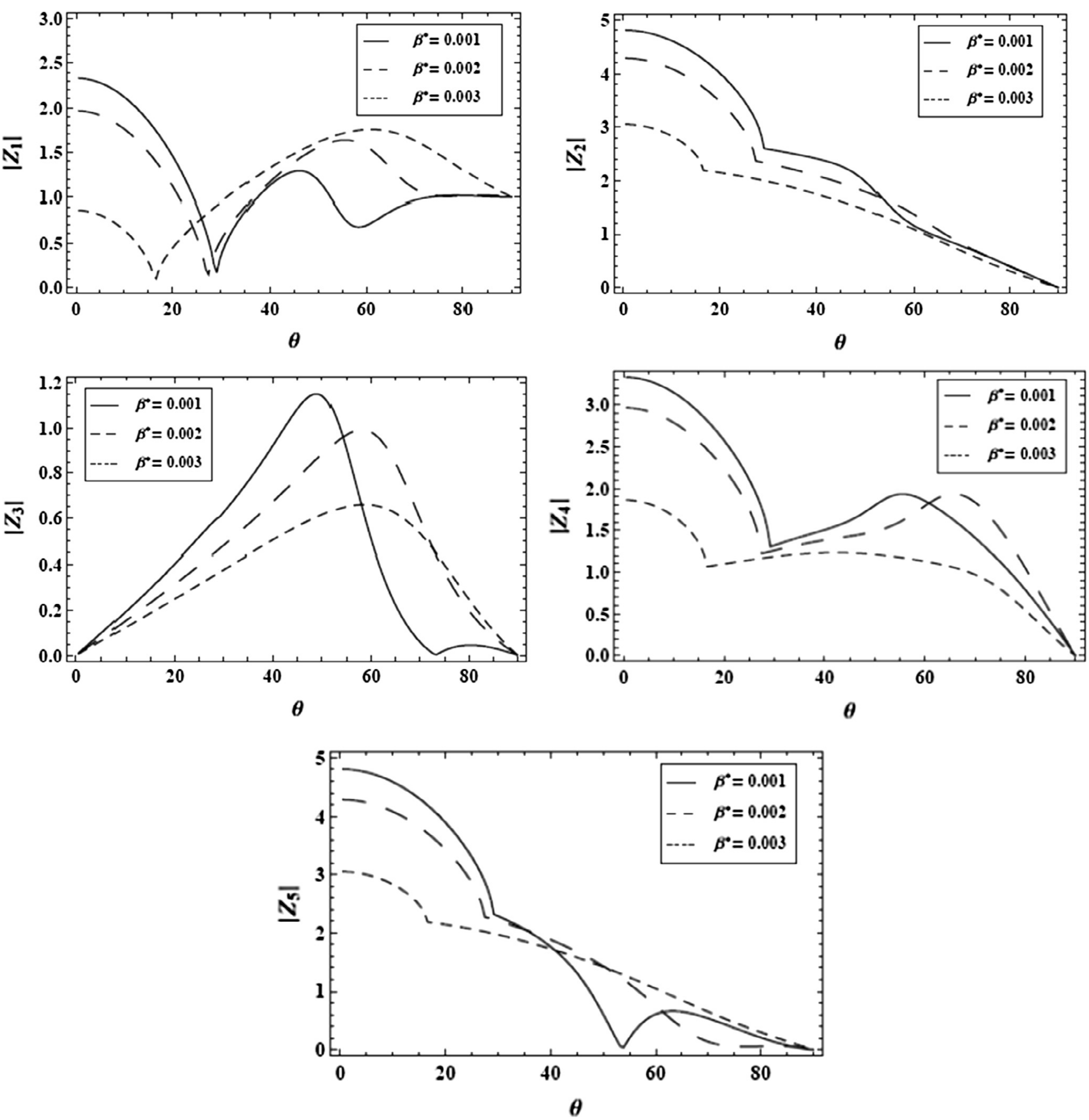

Fig. 10. Difference of the amplitudes $|Z_i|(i = 1, 2, \ldots, 5)$ making an incident angle of P waves for temperature dependent modulus effect, H = 0.0, $\varepsilon = 0.7$.

amplitude ratio. We see that $|Z_1|$ to $|Z_5|$ have increasing and decreasing behavior for all values of β^*.

Conclusion

In this paper, we discussed the effect of temperature dependent elastic moduli, coupling parameter and magnetic field on the refraction and reflection at the interface. For SV and P waves incident at the solid liquid interface, the effect of variation of temperature dependent modulus is more prominent than that of coupling parameter and magnetic field on the amplitude ratios of refracted and reflected P and thermal waves.

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