Results in Physics 8 (2018) 642–653

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Laser pulse, initial stress and modified Ohm's law in micropolar thermoelasticity with microtemperatures

Mohamed I.A. Othman [a,b], Ramadan S. Tantawi [a], Mohamed I.M. Hilal [a,c,*]

[a] Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt
[b] Department of Mathematics, Faculty of Science, Taif University 888, Taif, Saudi Arabia
[c] Department of Basic Sciences, Faculty of Engineering Sciences, Sinai University, Al-Arish, Egypt

ARTICLE INFO

Article history:
Received 26 October 2017
Received in revised form 8 December 2017
Accepted 21 December 2017
Available online 29 December 2017

Keywords:
Initial stress
Laser pulse
Magneto-thermoelasticity
Modified Ohm's law
Micropolar
Microtemperatures

ABSTRACT

The present manuscript studies the effect of the initial stress in micropolar magneto-thermoelasticity with microtemperatures heated by a laser pulse. The modified Ohm's law illustrates the temperature gradient and the charge density effects in the governing equations of the studied problem. The used analytical method was the normal modes. The physical quantities are established numerically and represented graphically.

Introduction

The response of the material to the external stimuli depends heavily on the motions of its inner structures. The classical elasticity does not contain this effect, where only translational degrees of freedom of the material point of the body are considered. The micropolar continuum is defined as the collection of the interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. The granular materials and multimolecular bodies, whose microstructure act as an evident part in their macroscopic responses are typical examples of the micropolar materials such as the composites with rigid chopped fibers, elastic solids with rigid granular inclusion and other industrial materials such as liquid crystal. Micropolar theory of elasticity introduced by Eringen [1,2] insures the local deformation and rotation of the material points of a body. This theory provides a model that can support the body and surfaces couples and display a high frequency optical branch of the wave spectrum. Some problems of micropolar thermoelasticity are discussed in [3–5].

The microtemperatures theory is considered as the theory which deals with the temperature, wave propagation in a rigid heat conductor and allows for variation of thermal properties at a microstructure level. Cryogenic liquids are heavily involved in space research and such liquids must be stored in stainless steel vessels known as run-tanks. The nanostructures in solids are also important, and the large thermal stresses placed in the solid vessels may be associated with thermal microstructure effects and hence there is certainly a need for a well-structured theory for a rigid solid which allows for microtemperatures effects. Grot [6] established the thermodynamic theory of elastic materials with inner structures contains the microdeformations while the microelements possess microtemperatures. Riha [7] presented a study of the heat conduction in materials with inner structures. Iesan and Quintanilla [8] constructed the linear theory of thermo-elasticity of materials with inner structure. Iesan [9] presented the mathematical model of the theory of micromorphic elastic solids with microtemperatures since the micro-elements possess microtemperatures and can stretch and contract independently of their translations. Casas and Quintanilla [10] studied the exponential stability in thermo-elasticity with microtemperatures. Scalia and Svandze [11] discussed the solutions of the theory of thermoelasticity with microtemperatures. Iesan [12] studied the thermoelastic bodies with microstructure and microtemperatures.

In geophysics, the initial stress is very important mechanical effect in solids. Due to the high stresses; the Earth's surface due to the gravity has a strong influence on the propagation speed of the elastic waves. While in the soft biological tissues initial (or

* Corresponding author at: Department of Basic Sciences, Faculty of Engineering Sciences, Sinai University, Al-Arish, Egypt.

E-mail address: mimhilal@yahoo.com (M.I.M. Hilal).

https://doi.org/10.1016/j.rinp.2017.12.059
2211-3797/© 2017 Published by Elsevier B.V.

residual); stresses in artery walls ensure that the circumferential stress distribution through the thickness of the artery wall is close to uniform at typical physiological blood pressures. Initial stresses may arise, for example, from applying loads, as in the case of gravity, processes of growth and development in living tissue or, in the case of engineering components, from the manufacturing process. Ames and Straughan [13] checked out the continuous dependence results for initially pre-stressed thermoelastic bodies. Montanaro [14] investigated the linear thermoelasticity with hydrostatic initial stress. The laser beam at a high intensity when interacts with the solid surface, the absorption takes place. This in turn causes an internal energy gain of the substrate material and heat release from the irradiated region. For the ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam have introduced situations where very large thermal gradients or an ultra-high heating speed may exist on the boundaries as in Sun et al. [15]. Othman et al. [16] discussed the effect of initial stress on thermoelastic rotating medium with laser pulse heating.

The interaction between the magnetic field and stress and strain in the thermoelastic medium is very remarkable due to its application in geophysics, plasma physics, related topics essentially in the nuclear fields, since the extremely high temperatures and the temperature gradients, in addition to the magnetic fields originating inside the nuclear reactors, also for the emissions of electromagnetic radiations from nuclear devices and for understanding the effect of the Earth's magnetic field on seismic waves. Othman et al. [17] investigated the effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation. It was assumed that the interactions between the magnetic field and the electric field take place by means of the Lorentz forces appearing in the equations of the motion and by means of a term entering Ohm's law and describing the electric field produced by the velocity of a material particle, moving in a magnetic field. Ohm's law was modified by the inclusion of the temperature gradient, this modification for the temperature gradient stated that the strength of the current at each point is proportional to the gradient of electric potential. The accuracy of the assumptions that flow proportional to the gradient is more readily tested, using modern measurement methods, for the electrical case than in the heat case. Othman et al. [18,19] discussed two problems involve the effect of the magnetic field in two different cases on micropolar thermoelastic solids with microtemperatures.

This article studies the initially stressed linear, isotropic, homogeneous magneto-micropolar thermoelasticity with microtemperatures heated by a laser pulse. The normal mode method was used to get the solution of the physical quantities of the very field. These quantities are calculated analytically and numerically then represented graphically.

Basic equations

According to the linear theory of thermodynamics for isotropic elastic materials with inner structure, and due to Eringen [1], Iesan [12] and Montanaro [14], the field equations and the constitutive relations for a stressed, linear, isotropic, homogeneous, micropolar, magneto-thermoelastic material with microtemperatures, heat sources and first heat source moment, without body forces, body couples, can be considered as:

$$\sigma_{ij,i} + F_i = \rho u_{i,tt}, \tag{1}$$

$$m_{ij,i} + \varepsilon_{ijr}\sigma_{ir} - \mu_1(\nabla\times w)_i = j\rho\phi_{i,tt}, \tag{2}$$

$$k_6 w_{i,jj} + (k_4+k_5)w_{j,ij} + \mu_1(\nabla\times\dot{\phi})_i - k_2 w_i - b w_{i,t} - k_3 T_{,i} = 0, \tag{3}$$

$$kT_{,ii} - \rho C_e\dot{T} - \gamma_1 T_0\dot{u}_{i,i} + k_1 w_{i,i} = -\rho Q, \tag{4}$$

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j}+u_{j,i}) + k^*(u_{j,i} - \varepsilon_{ijr}\phi_r) - \gamma_1 T\delta_{ij} - p(\delta_{ij}+\omega_{ij}), \tag{5}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{6}$$

$$q_i = kT_{,i} + k_1 w_i, \tag{7}$$

$$q_{ij} = -k_4 w_{k,k}\delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i}, \tag{8}$$

$$Q_i = (k_1 - k_2)w_i + (k - k_3)T_{,i}, \tag{9}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{10}$$

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right), \tag{11}$$

$$F_i = \mu_0(\mathbf{J}\times\mathbf{H})_i, \tag{12}$$

where λ, μ are the Lamé constants, α, β, γ and k^* are the micropolar constants, $\gamma_1 = (3\lambda + 2\mu + k^*)\alpha_t$, α_t is the linear thermal expansion coefficient, ρ is the density, C_e is the specific heat, k is the thermal conductivity, u_i is the displacement vector, T is the absolute temperature, T_0 is the reference temperature chosen so that $|(T-T_0)/T_0| \ll 1$, ϕ_i is the microrotation vector, σ_{ij} are the components of the stresses, e_{ij} are the components of strains, ω_{ij} are the rotation of vector, δ_{ij} is the Kronecker delta, ε_{ijr} is the permutation symbol, p is the pressure, e is the dilation, m_{ij} is the couple stresses, j is the micro-inertia, w_i is the microtemperatures vector, μ_1, b, k_i $(i = 1, 2, \ldots, 6)$ are the constitutive coefficients, F_i is the Lorentz force, q_i is the heat flux moment, q_{ij} the first heat flux moment, Q_i is the mean heat flux vector, and Q is the heat source. In the previous equations a comma denotes the coordinate system derivatives.

Formulation and solution of the problem

Consider an isotropic, linear, homogeneous, micropolar thermoelastic solid with microtemperatures and a half-space $(y \geqslant 0)$, the rectangular Cartesian coordinate system (x, y, z) has originated on the surface when $z = 0$. For the 2-D problem then, we would need to assume the dynamic displacement vector as $u = (u, v, 0)$. The microrotation vector ϕ will be $\phi = (0, 0, \phi_3)$, consequently the microtemperatures vector w will be $w = (w_1, w_2, 0)$. A magnetic field with a constant intensity $\mathrm{H}_i = (0, 0, \mathrm{H}_0 + \mathbf{h}(x, y, t)$ acts on the z axis. All quantities considered will be a function of the time variable t and of the coordinates x and y. Application of the initial magnetic field H_0, produces an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$. The simplified linear equations of electrodynamics of slowly moving medium for a homogenous, thermally and electrically conducting elastic solid can be written on the form

$$\nabla\times\mathbf{h} = \mathbf{J} + \dot{\mathbf{D}}, \tag{13}$$

$$\nabla\times\mathbf{E} = -\dot{\mathbf{B}}, \tag{14}$$

$$\nabla.\mathbf{B} = 0, \quad \nabla.\mathbf{D} = \rho_e, \tag{15}$$

$$\mathbf{D} = \varepsilon_0\mathbf{E}, \quad \mathbf{B} = \mu_0(\mathbf{H}_0 + \mathbf{h}), \tag{16}$$

with the modified Ohm's law for the media has a finite conductivity

$$\mathbf{J} = \sigma_0(\mathbf{E} + \mu_0\dot{\mathbf{u}}\times\mathbf{H}) - k_0\nabla T, \tag{17}$$

where **B** is the magnetic induction field vector, **J** is the current density vector, **D** is the electric displacement vector, μ_0 is the magnetic permeability, ρ_e is the charge density, ε_0 is the electric permittivity, σ_0 is the electric conductivity and k_0 is the coefficient connecting the temperature gradient and the electric current density.

From Eqs. (11)–(16), one can obtain the Lorentz force components in the form

$$F_x = \sigma_0\mu_0 H_0\left(E_y - \mu_0 H_0\frac{\partial u}{\partial t}\right) - k_0\frac{\partial T}{\partial y}, \tag{18}$$

$$F_y = -\sigma_0\mu_0 H_0\left(E_x + \mu_0 H_0\frac{\partial v}{\partial t}\right) + k_0\frac{\partial T}{\partial x}, \tag{19}$$

$$F_z = 0. \tag{20}$$

Also, we can deduce that

$$\frac{\partial h}{\partial y} = \sigma_0\left(E_x + \mu_0 H_0\frac{\partial v}{\partial t}\right) + \varepsilon_0\frac{\partial E_x}{\partial t} - k_0\frac{\partial T}{\partial x}, \tag{21}$$

$$\frac{\partial h}{\partial x} = -\sigma_0\left(E_y - \mu_0 H_0\frac{\partial u}{\partial t}\right) - \varepsilon_0\frac{\partial E_y}{\partial t} + k_0\frac{\partial T}{\partial y}, \tag{22}$$

$$\mu_0\frac{\partial h}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}. \tag{23}$$

The plate surface illuminated by the heat impulse from the laser beam can be formulated in the form as in Ref. [16]

$$Q = \frac{I_0\gamma^* t}{2\pi r^2 t_0^2}\exp\left(-\frac{x^2}{r^2} - \frac{t}{t_0}\right)\exp(-\gamma^* y), \tag{24}$$

where I_0 is the absorbed energy, r is the radius of the laser beam, t_0 is the characteristic time of the laser pulse or the time duration of the laser pulse, and γ^* is a constant.

Eqs. (1)–(4) will be

$$\left(\mu + k^* - \frac{p}{2}\right)\nabla^2 u + \left(\lambda + \mu + \frac{p}{2}\right)\frac{\partial e}{\partial x} + k^*\frac{\partial \phi_3}{\partial y} - \gamma_1\frac{\partial T}{\partial x} + \sigma_0\mu_0 H_0\left[E_y - \mu_0 H_0\frac{\partial u}{\partial t}\right] - k_0\frac{\partial T}{\partial y} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{25}$$

$$\left(\mu + k^* - \frac{p}{2}\right)\nabla^2 v + \left(\lambda + \mu + \frac{p}{2}\right)\frac{\partial e}{\partial y} - k^*\frac{\partial \phi_3}{\partial x} - \gamma_1\frac{\partial T}{\partial y} - \sigma_0\mu_0 H_0\left[E_x + \mu_0 H_0\frac{\partial v}{\partial t}\right] + k_0\frac{\partial T}{\partial x} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{26}$$

$$\gamma\nabla^2\phi_3 - 2k^*\phi_3 + (k^* - p)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \mu_1\left(\frac{\partial w_2}{\partial x} - \frac{\partial w_1}{\partial y}\right) = j\rho\frac{\partial^2\phi_3}{\partial t^2}, \tag{27}$$

$$k_6\nabla^2 w_1 + (k_4 + k_5)\frac{\partial}{\partial x}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) + \mu_1\frac{\partial}{\partial t}\frac{\partial \phi_3}{\partial y} - k_2 w_1 - b\frac{\partial w_1}{\partial t} - k_3\frac{\partial T}{\partial x} = 0, \tag{28}$$

$$k_6\nabla^2 w_2 + (k_4 + k_5)\frac{\partial}{\partial y}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) - \mu_1\frac{\partial}{\partial t}\frac{\partial \phi_3}{\partial x} - k_2 w_2 - b\frac{\partial w_2}{\partial t} - k_3\frac{\partial T}{\partial y} = 0, \tag{29}$$

$$k\nabla^2 T - \rho C_e\frac{\partial T}{\partial t} - \gamma_1 T_0\frac{\partial e}{\partial t} + k_1\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) = -\rho Q. \tag{30}$$

Use the following non-dimensional variables

$$x_i' = \frac{\omega_1^*}{c_0}x_i,\quad u_i' = \frac{\rho c_0\omega_1^*}{\gamma_1 T_0}u_i,\quad \phi_3' = \frac{\rho c_0^2}{\gamma_1 T_0}\phi_3,\quad w_i' = \frac{c_0}{\omega_1^*}w_i,$$
$$m_{ij}' = \frac{\omega_1^*}{\gamma_1 c_0 T_0}m_{ij},\quad q_{ij}' = \frac{\mu c_0^2}{\omega_1^*}q_{ij},\quad T' = \frac{1}{T_0}T,$$
$$(\sigma_{ij}', p_1') = \frac{1}{\gamma_1 T_0}(\sigma_{ij}, p_1),\quad t' = \omega_1^* t,\quad k_0' = \frac{\mu_0 H_0}{\gamma_1}k_0,$$
$$Q' = \frac{Q}{T_0 C_e\omega_1^*},\quad h' = \frac{\omega_1^*}{\mu_0 H_0\sigma_0}h,\quad E_i' = \frac{\omega_1^* c_0}{\sigma_0 H_0\mu_0^2}E_i,$$
$$\omega_1^* = \frac{\rho C_e c_0^2}{k},\quad c_0^2 = \frac{\lambda + 2\mu + k^*}{\rho},\quad c^2 = \frac{1}{\mu_0\varepsilon_0}. \tag{31}$$

where c is the speed of light.

Substituting the non-dimensional relations (31) (dropping the prime for the simplicity) into Eqs. (25)–(30), we get

$$\nabla^2 u + c_1\frac{\partial e}{\partial x} + c_2\frac{\partial\phi_3}{\partial y} - c_3\frac{\partial T}{\partial x} + c_4 E_y - c_5\frac{\partial u}{\partial t} - c_6\frac{\partial T}{\partial y} = c_3\frac{\partial^2 u}{\partial t^2}, \tag{32}$$

$$\nabla^2 v + c_1\frac{\partial e}{\partial y} - c_2\frac{\partial\phi_3}{\partial x} - c_3\frac{\partial T}{\partial y} - c_4 E_x - c_5\frac{\partial v}{\partial t} + c_6\frac{\partial T}{\partial x} = c_3\frac{\partial^2 v}{\partial t^2}, \tag{33}$$

$$\nabla^2\phi_3 - c_7\phi_3 + c_8\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - c_9\left(\frac{\partial w_2}{\partial x} - \frac{\partial w_1}{\partial y}\right) = c_{10}\frac{\partial^2\phi_3}{\partial t^2}, \tag{34}$$

$$\nabla^2 w_1 + c_{11}\frac{\partial}{\partial x}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) + c_{12}\frac{\partial}{\partial t}\frac{\partial\phi_3}{\partial y} - c_{13}w_1 - c_{14}\frac{\partial w_1}{\partial t} - c_{15}\frac{\partial T}{\partial x} = 0, \tag{35}$$

$$\nabla^2 w_2 + c_{11}\frac{\partial}{\partial y}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) - c_{12}\frac{\partial}{\partial t}\frac{\partial\phi_3}{\partial x} - c_{13}w_2 - c_{14}\frac{\partial w_2}{\partial t} - c_{15}\frac{\partial T}{\partial y} = 0, \tag{36}$$

$$c_{16}\nabla^2 T - \frac{\partial T}{\partial t} - c_{17}\frac{\partial e}{\partial t} + c_{18}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) = -Q, \tag{37}$$

The same procedure for the Eqs. (21)–(23) gives

$$\nabla^2 h = -\left(c_{29} + \varepsilon_1\frac{\partial}{\partial t}\right)\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) + c_{30}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \tag{38}$$

$$\left(c_{29} + \varepsilon_1\frac{\partial}{\partial t}\right)\left(\frac{\partial E_1}{\partial x} + \frac{\partial E_2}{\partial y}\right) = c_{31}\nabla^2 T - c_{30}\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right), \tag{39}$$

$$\nabla^2 h - \frac{c_{29}}{c_{32}}\frac{\partial h}{\partial t} - \varepsilon_1\frac{\partial^2 h}{\partial t^2} - \frac{c_{30}}{c_{32}}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0. \tag{40}$$

Suppose that the potential functions $\psi_1(x,y,t)$, $\psi_2(x,y,t)$, $q_1(x,y,t)$, and $\boldsymbol{q}_2(x,y,t)$, of the dimensionless form

$$u = \frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y},\quad v = \frac{\partial\psi_1}{\partial y} - \frac{\partial\psi_2}{\partial x},\quad w_1 = \frac{\partial q_1}{\partial x} + \frac{\partial q_2}{\partial y},\quad \text{and}\quad w_2 = \frac{\partial q_1}{\partial y} - \frac{\partial q_2}{\partial x}. \tag{41}$$

To obtain the physical quantities of the studied problem, it is appropriate to assume the solution in the form

$$[\psi_1, \psi_2, h, \phi_3, q_1, q_2, T](x,y,t) = [\psi_1^*, \psi_2^*, h^*, \phi_3^*, q_1^*, q_2^*, T^*](y)e^{i(ax - \xi t)}. \tag{42}$$

Where $[\psi_1^*, \psi_2^*, h^*, \phi_3^*, q_1^*, q_2^*, T^*](y)$ are the amplitude of $[\psi_1, \psi_2, h, \phi_3, q_1, q_2, T]$, ξ is the angular frequency, and a is the wave number.

Using Eqs. (40) and (41) into Eqs. (32)–(36) produces the following system

$$[\mathrm{D}^2 - N_3]\psi_1^* - N_4 h^* - N_5 T^* = 0, \tag{43}$$

$$[\mathrm{D}^2 - N_6]\psi_2^* + c_2\phi_3^* + N_7 T^* = 0, \tag{44}$$

$$N_8[\mathrm{D}^2 - a^2]\psi_1^* + [\mathrm{D}^2 - N_9]h^* = 0, \tag{45}$$

$$-c_8[\mathrm{D}^2 - a^2]\psi_2^* + [\mathrm{D}^2 - N_{10}]\phi_3^* + c_{10}[\mathrm{D}^2 - a^2]q_2^* = 0, \tag{46}$$

$$[\mathrm{D}^2 - N_{11}]q_1^* - N_{12}T^* = 0, \tag{47}$$

$$N_{13}\phi_3^* + [\mathrm{D}^2 - N_{14}]q_2^* = 0, \tag{48}$$

$$N_{15}[\mathrm{D}^2 - a^2]\psi_1^* + c_{16}[\mathrm{D}^2 - a^2]q_1^* + [\mathrm{D}^2 - N_{17}]T^* = -N_{18}f(x,t)\exp(-\gamma^* y). \tag{49}$$

All the constants $N_3 - N_{18}$, $c_7 - c_{17}$ are given in the Appendix A, where $\mathrm{D} = d/dy$.

Eliminate the functions ψ_1^*, ψ_2^*, h^*, ϕ_3^*, q_1^*, q_2^* and T^* among Eqs. (43)–(49), yields to the following differential equation

$$[\mathrm{D}^{14} - \lambda_1\mathrm{D}^{12} + \lambda_2\mathrm{D}^{10} - \lambda_3\mathrm{D}^8 + \lambda_4\mathrm{D}^6 - \lambda_5\mathrm{D}^4 + \lambda_6\mathrm{D}^2 - \lambda_7] \times \{\psi_1^*, \psi_2^*, h^*, \phi_3^*, q_1^*, q_2^*, T^*\}(y) = -N_{18}L_1, L_2, L_3, L_4, L_5, L_6, L_7 f(x,t)\exp(-\gamma^* y) \tag{50}$$

where $\lambda_{\mathrm{n}}(\mathrm{n} = 1, 2, \ldots, 7)$ and $L_{\mathrm{n}}(\mathrm{n} = 1, 2, \ldots, 7)$ are constants.

Eq. (50) can be factored as

$$[(\mathrm{D}^2 - S_1^2)(\mathrm{D}^2 - S_2^2)(\mathrm{D}^2 - S_3^2)(\mathrm{D}^2 - S_4^2)(\mathrm{D}^2 - S_5^2)(\mathrm{D}^2 - S_6^2)(\mathrm{D}^2 - S_7^2)]\psi_1^*(y) = -N_{18}L_1 f(x,t)\exp(-\gamma^* y), \tag{51}$$

where $S_{\mathrm{n}}^2(\mathrm{n} = 1, 2, \ldots, 7)$ are the roots of the characteristic equation of Eq. (51).

The general solution of the Eq. (51), bounded as $y \to \infty$, gives the solution of the physical quantities in the forms:

$$u(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{7\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) - A_1 Q_1\left(\frac{2xL_1}{r^2} + \gamma^* L_2\right)\exp(-\gamma^* y), \tag{52}$$

$$v(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{8\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1\left(\frac{2xL_2}{r^2} - \gamma^* L_1\right)\exp(-\gamma^* y), \tag{53}$$

$$w_1(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{9\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) - A_1 Q_1\left(\frac{2xL_5}{r^2} + \gamma^* L_6\right)\exp(-\gamma^* y), \tag{54}$$

$$w_2(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{10\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1\left(\frac{2xL_6}{r^2} - \gamma^* L_5\right)\exp(-\gamma^* y), \tag{55}$$

$$h(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{2\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 L_3 \exp(-\gamma^* y), \tag{56}$$

$$\phi_3(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{3\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 L_4 \exp(-\gamma^* y), \tag{57}$$

$$T(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{6\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 L_7 \exp(-\gamma^* y), \tag{58}$$

$$\sigma_{xx}(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{11\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 A_2 \exp(-\gamma^* y), \tag{59}$$

$$\sigma_{yy}(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{12\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 A_3 \exp(-\gamma^* y), \tag{60}$$

$$\sigma_{xy}(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{13\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 A_4 \exp(-\gamma^* y), \tag{61}$$

$$q_{yy}(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{17\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 A_8 \exp(-\gamma^* y), \tag{62}$$

$$q_{xy}(x,y,t) = \sum_{\mathrm{n}=1}^{7} R_{\mathrm{n}} B_{18\mathrm{n}} \exp(-S_{\mathrm{n}} y + \mathrm{i}(ax - \xi t)) + A_1 Q_1 A_9 \exp(-\gamma^* y), \tag{63}$$

where $R_{\mathrm{n}}(\mathrm{n} = 1, 2, \ldots, 7)$ are constants. The other field quantities are given in Appendix B.

Consider the magnetic and the electric field intensities in a free space, which are denoted by h_0, E_{10}, and E_{20}, respectively. These variables satisfy the field equations:

$$\frac{\partial h_0}{\partial y} = \varepsilon_1 \frac{\partial E_{x0}}{\partial t}, \tag{64}$$

$$\frac{\partial h_0}{\partial x} = -\varepsilon_1 \frac{\partial E_{y0}}{\partial t}, \tag{65}$$

$$\frac{\partial h_0}{\partial t} = \frac{\partial E_{x0}}{\partial y} - \frac{\partial E_{y0}}{\partial x}. \tag{66}$$

Similarly, these variables can be decomposed in terms of normal modes in the form

$$[h_0, E_{x0}, E_{y0}](x,y,t) = [h_0^*, E_{x0}^*, E_{y0}^*](y)\exp(\mathrm{i}(ax - \xi t)). \tag{67}$$

Using Eq. (67) into Eqs. (64)–(76) gives

$$[\mathrm{D}^2 - a^2 + \xi^2\varepsilon_1]h_0^* = 0. \tag{68}$$

The general solutions of the quantities h_0, E_{x0}, and E_{y0} which are bounded as $y \to \infty$ are

$$h_0(x,y,t) = A_{12}\exp(-S_8 y + \mathrm{i}(ax - \xi t)), \tag{69}$$

$$E_{x0}(x,y,t) = \frac{S_8}{\mathrm{i}\xi\varepsilon_1} A_{12}\exp(-S_8 y + \mathrm{i}(ax - \xi t)), \tag{70}$$

$$E_{y0}(x,y,t) = \frac{a}{\xi\varepsilon_1} A_{12}\exp(-S_8 y + \mathrm{i}(ax - \xi t)), \tag{71}$$

where A_{12} is a constant and $S_8 = \sqrt{a^2 - \xi^2\varepsilon_1}$.

Boundary conditions

Consider the following non-dimensional boundary conditions neglecting the positive exponentials to avoid the unbounded solutions at infinity, the surface at $y = 0$, satisfies:

(1) The mechanical boundary conditions:
 (a) The normal stress is mechanically stressed by a constant force p_1 i.e.

$$\sigma_{yy}(x, 0, t) = -p_1 \exp(i(ax - \xi t)) - p, \tag{72}$$

 (b) The shearing stress is traction-free, so that

$$\sigma_{xy}(x, 0, t) = 0, \tag{73}$$

 (c) The condition of the couple stress (which is constant in y-direction). Then

$$m_{yz}(x, 0, t) = 0, \tag{74}$$

(2) The thermal condition (the half-space is free). This leads to

$$T(x, 0, t) = 0, \tag{75}$$

(3) The transverse components of the electric field intensity are continuous across the surface of the half-space. This means

$$E_y(x, 0, t) = E_{y0}(x, 0, t), \tag{76}$$

(4) The transverse components of the magnetic field intensity are continuous across the surface of the half-space. This implies that

$$h(x, 0, t) = h_0(x, 0, t), \tag{77}$$

(5) The normal and the tangential heat flux moments are free.

$$q_{xx}(x, 0, t) = q_{xy}(x, 0, t) = 0. \tag{78}$$

Substitute the expressions of the considered physical quantities into the previous boundary conditions to give the equations satisfied by the parameters with the method of the matrix inverse to the raised system of equations to obtain the values of constants.

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ A_{12} \end{pmatrix} = \begin{pmatrix} B_{121} & B_{122} & B_{123} & B_{124} & B_{125} & B_{126} & B_{127} & 0 \\ B_{131} & B_{132} & B_{133} & B_{134} & B_{135} & B_{136} & B_{137} & 0 \\ B_{141} & B_{142} & B_{143} & B_{144} & B_{145} & B_{146} & B_{147} & 0 \\ B_{61} & B_{62} & B_{63} & B_{64} & B_{65} & B_{66} & B_{67} & 0 \\ B_{191} & B_{192} & B_{193} & B_{194} & B_{195} & B_{196} & B_{197} & L \\ B_{21} & B_{22} & B_{23} & B_{24} & B_{25} & B_{26} & B_{27} & -1 \\ B_{171} & B_{172} & B_{173} & B_{174} & B_{175} & B_{176} & B_{177} & 0 \\ B_{181} & B_{182} & B_{183} & B_{184} & B_{185} & B_{186} & B_{187} & 0 \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{79}$$

In the present work, we consider the following particular cases:

(i) Absence of the initial stress: by taking $p = 0$ in the constitutive Eq. (5).
(ii) For the non-connecting temperature and the electric current density: by taking $k_0 = 0$ in Eq. (17).
(iii) Absence of the micropolar: by taking $\alpha, \beta, \gamma, k^*, j = 0$ into Eqs. (1)–(5).

Numerical results and discussion

According to Eringen [20], the magnesium crystal-like material was chosen for purposes of numerical calculations. The used parameters are given in SI units; $\lambda = 9.4 \times 10^{10}\ \mathrm{N/m^2}$, $\mu = 4 \times 10^{10}\ \mathrm{N/m^2}$, $k = 1.7 \times 10^2\ \mathrm{N/s \cdot K}$, $\rho = 1.74 \times 10^3\ \mathrm{kg/m^3}$, $\alpha_t = 7.4033 \times 10^{-7}/\mathrm{K}$, $C_e = 1.04 \times 10^3\ \mathrm{J/kg \cdot K}$, $k^* = 1 \times 10^{10}\ \mathrm{N/m^2}$, $\gamma = 7.779 \times 10^{-8}\ \mathrm{N}$, $j = 2 \times 10^{-20}\ \mathrm{m^2}$, $T_0 = 298\ \mathrm{K}$, $k_1 = 0.0035\ \mathrm{N/s}$, $k_2 = 0.0045\ \mathrm{N/s}$, $k_3 = 0.0055\ \mathrm{N/s\ K}$, $k_4 = 0.065\ \mathrm{N/s\ m^2}$, $k_5 = 0.076\ \mathrm{N/s\ m^2}$, $k_6 = 0.096\ \mathrm{N/s\ m^2}$, $\mu_1 = 0.0085\ \mathrm{N}$, $b = 0.15 \times 10^{-9}\ \mathrm{N}$, $p_1 = 0.5\ \mathrm{K}$, $a = 0.01\ \mathrm{m}$, $\varepsilon_0 = 10^{-9}/(36\pi)\ \mathrm{F/m}$, $t = 0.009\ \mathrm{s}$, $I_0 = 10^{-6}\ \mathrm{J/m^2}$, $\xi = 4\ \mathrm{rad/s}$, $x = 0.5\ \mathrm{m}$, $0 \leqslant y \leqslant 5\ \mathrm{m}$.

The variation of each of the real parts of the displacement u, the microtemperatures vector w_1, the temperature T, the stress σ_{yy}, the microrotation ϕ_3, the first heat flux moment q_{yy} and the induced magnetic field h are represented by the distance y for different comparisons.

Figs. 1–6 represent the behavior of the physical quantities against the distance y, when $k_0 = 10^6\ \mathrm{m/K}$, $t = 0.009\ \mathrm{s}$, $I_0 = 10^{-6}\ \mathrm{J/m^2}$, in the case of $p = 0,\ 0.7$.

Fig. 1 shows that the variation of the displacement component u decreases with the increase of initial stress for $0 \leqslant y \leqslant 5$. Fig. 2 clarifies that the microtemperatures vector w_1 decreases in the range $0 \leqslant y \leqslant 0.8$, then increases in the range $0.8 \leqslant y \leqslant 5$, with the increase of initial stress. That is clear that the initial stress an important effect in displacement and stresses as its importance in the variation of the temperature from Fig. 3 that the temperature T decreases with the increase of initial stress in the range $0 \leqslant y \leqslant 2$, however it increases in the range $2 \leqslant y \leqslant 5$. Fig. 4 depicts that the normal stress σ_{yy} decreases with the increase of initial stress in the range $0 \leqslant y \leqslant 2$, then it increases in the range $2 \leqslant y \leqslant 5$. Fig. 5 explains that the microrotation vector ϕ_3 increases with the increase of initial stress in the range $0 \leqslant y \leqslant 1.5$, then it decreases in the range $1.5 \leqslant y \leqslant 5$. Fig. 6 shows that the first heat flux moment q_{yy} increases with the increase of initial stress in the range $0 \leqslant y \leqslant 1$, then it decreases in the range $1 \leqslant y \leqslant 5$. The initial stress is an effective mechanical operator in the problem. Figs. 7–11 represent the behavior of the physical quantities against the distance y, when $k_0 = 10^6\ \mathrm{m/K}$, $p = 0.7\ \mathrm{N/m}$ $I_0 = 10^{-6}\ \mathrm{J/m^2}$, in the case of $t = 0.009,\ 0$.

Fig. 7 shows that the variation of the displacement u increases in the range $0 \leqslant y \leqslant 0.4$, then it decreased in the range

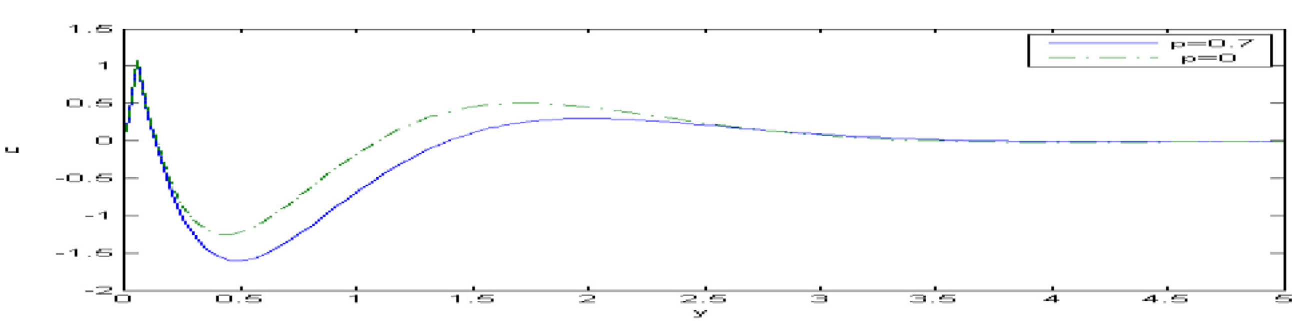

Fig. 1. Variation of the displacement component u against y.

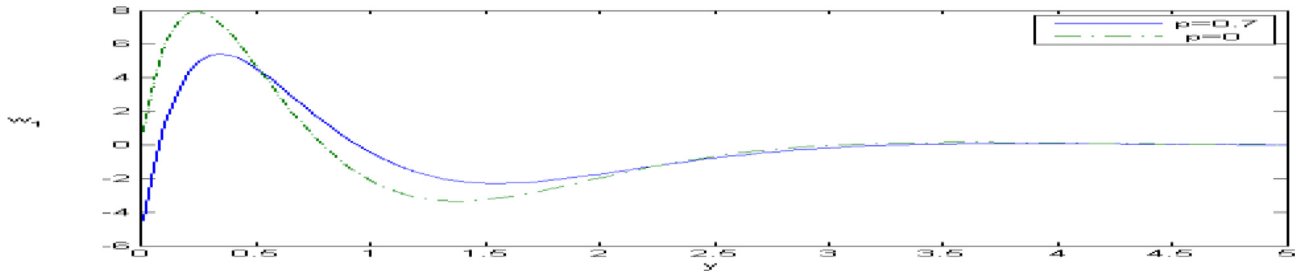

Fig. 2. Variation of microtemperatures vector w_1 against y.

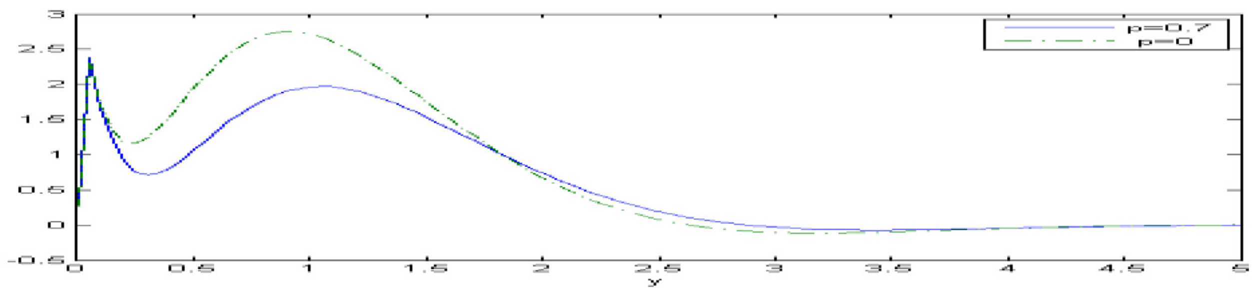

Fig. 3. Variation of the temperature T against y.

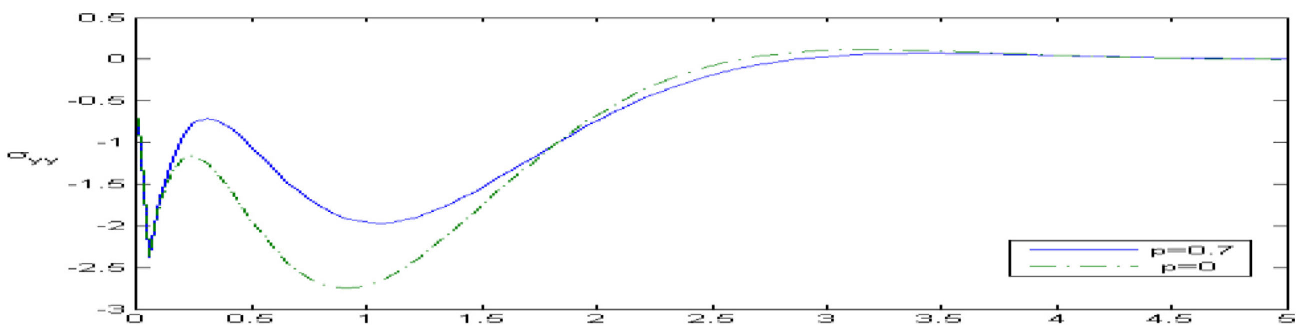

Fig. 4. Variation of the normal stress σ_{yy} against y.

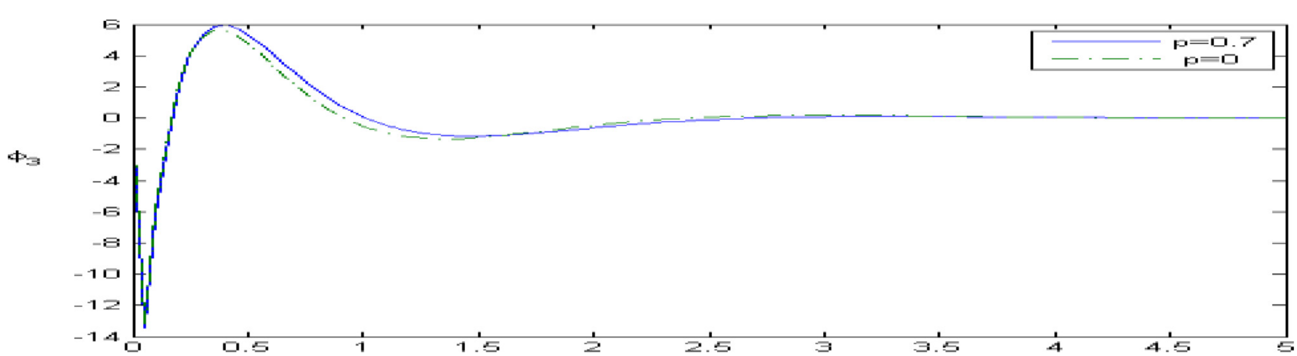

Fig. 5. Variation of the microrotation vector ϕ_3 against y.

$0.4 \leqslant y \leqslant 5$, with the increase of the time. Fig. 8 clarifies that the microtemperatures vector w_1 decreases in the range $0 \leqslant y \leqslant 0.8$, then increases in the range $0.8 \leqslant y \leqslant 5$, with the increase of the time. It is clear from Fig. 9 that the temperature T decreases with the increase of the time in the range $0 \leqslant y \leqslant 1.5$, however it increases in the range $1.5 \leqslant y \leqslant 5$. It can have deduced that the time is important factor in the problem. Fig. 10 depicts that the normal stress σ_{yy} increases with the increase of the time in the range $0 \leqslant y \leqslant 1.4$, then it increases in the range $1.4 \leqslant y \leqslant 5$. Fig. 11 explains that the microrotation vector ϕ_3 decreases with

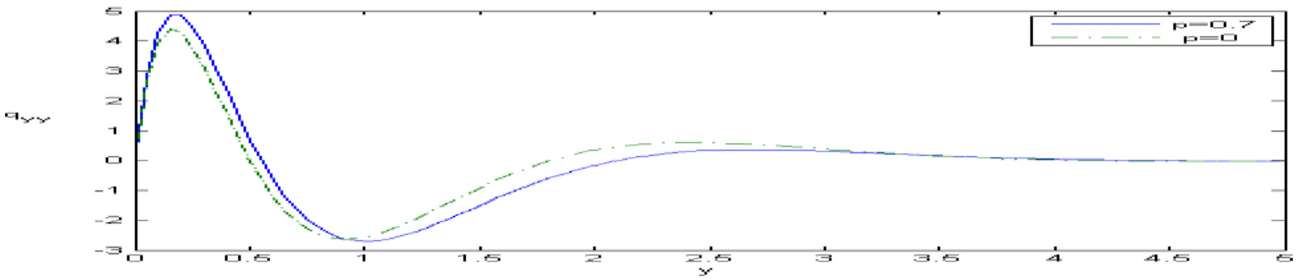

Fig. 6. Variation of the first heat flux moment component q_{yy} against y.

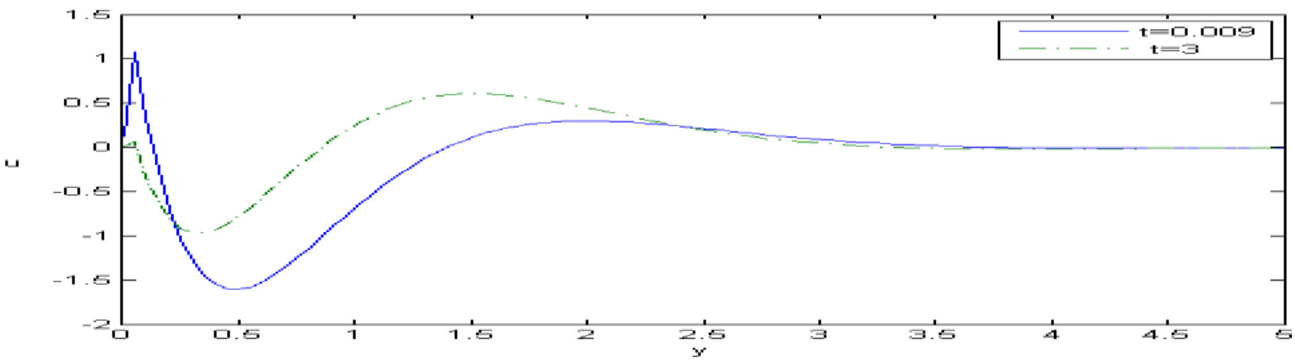

Fig. 7. Variation of the displacement component u against y.

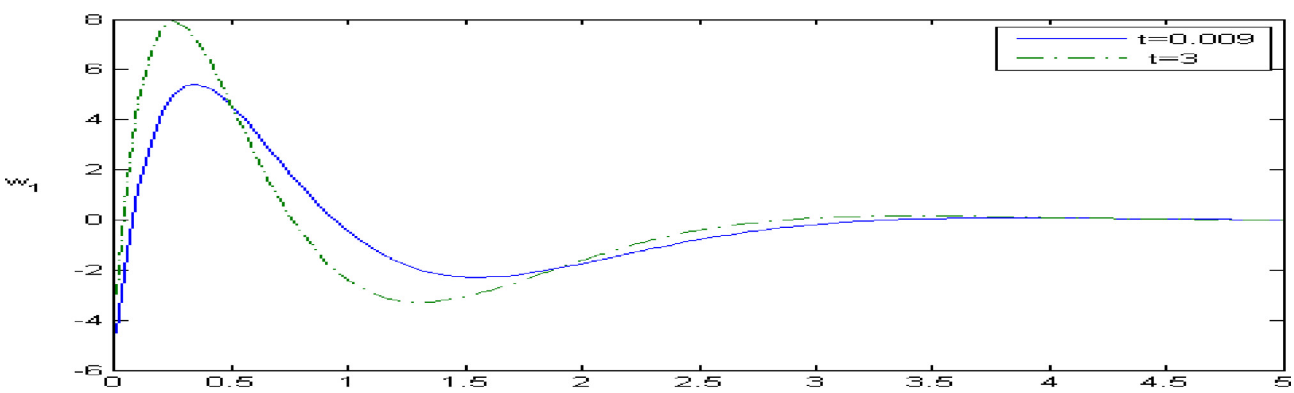

Fig. 8. Variation of microtemperatures vector w_1 against y.

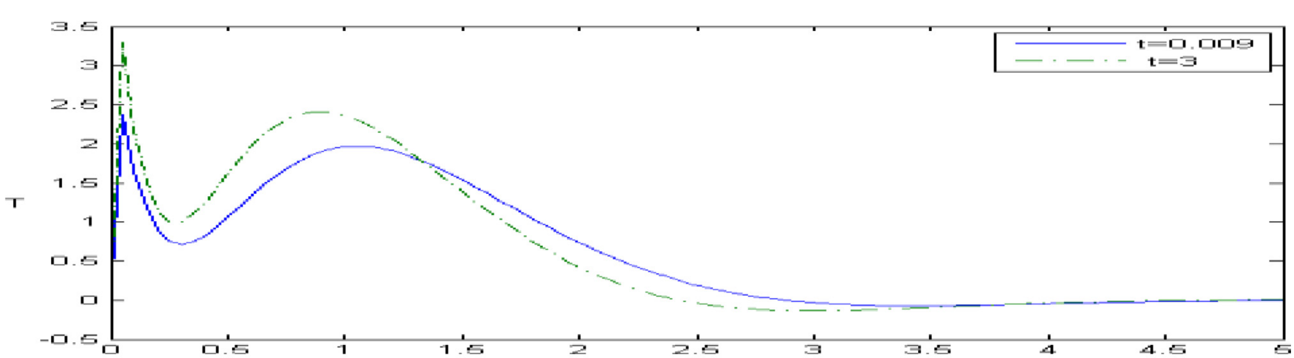

Fig. 9. Variation of the temperature T against y.

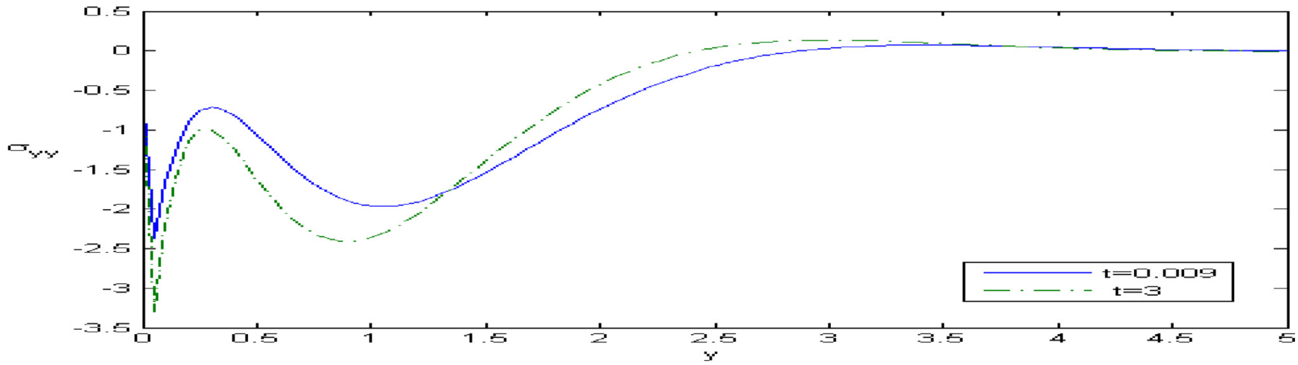

Fig. 10. Variation of the normal stress σ_{yy} against y.

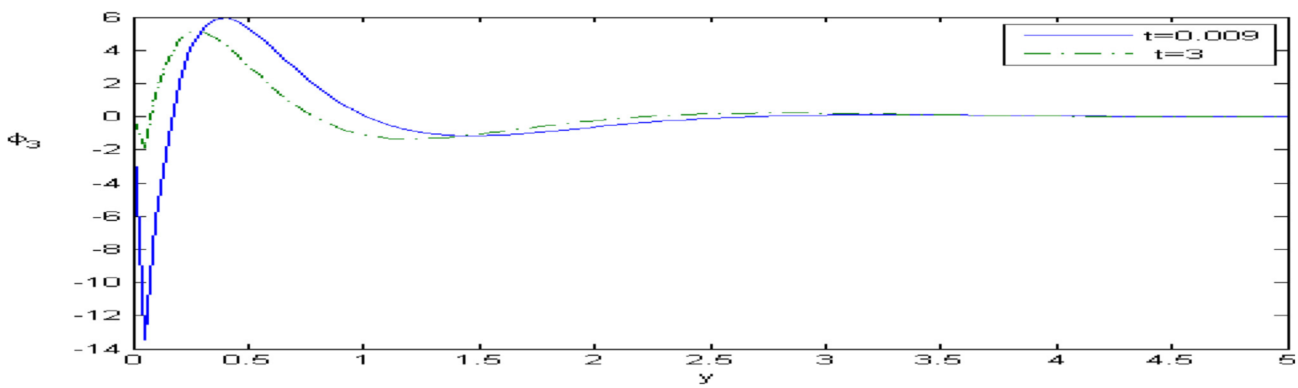

Fig. 11. Variation of the microrotation vector ϕ_3 against y.

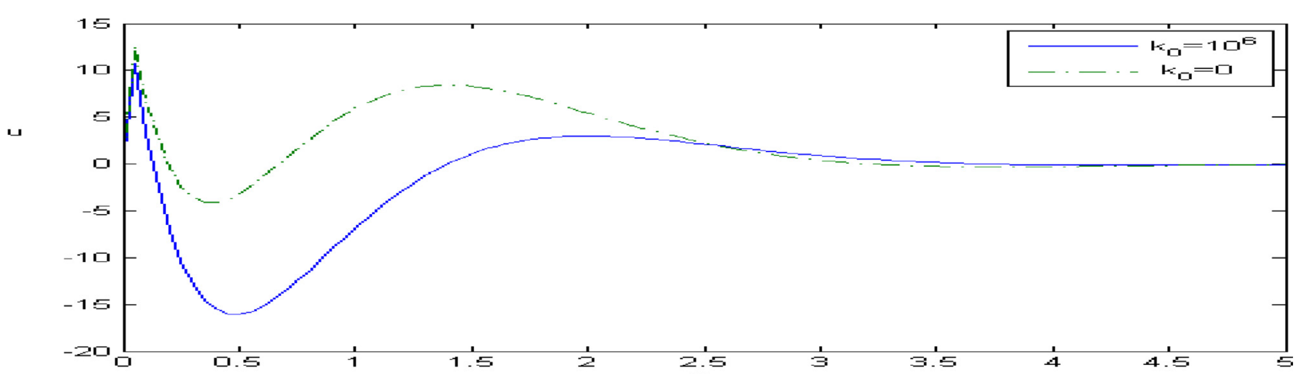

Fig. 12. Variation of the displacement component u against y.

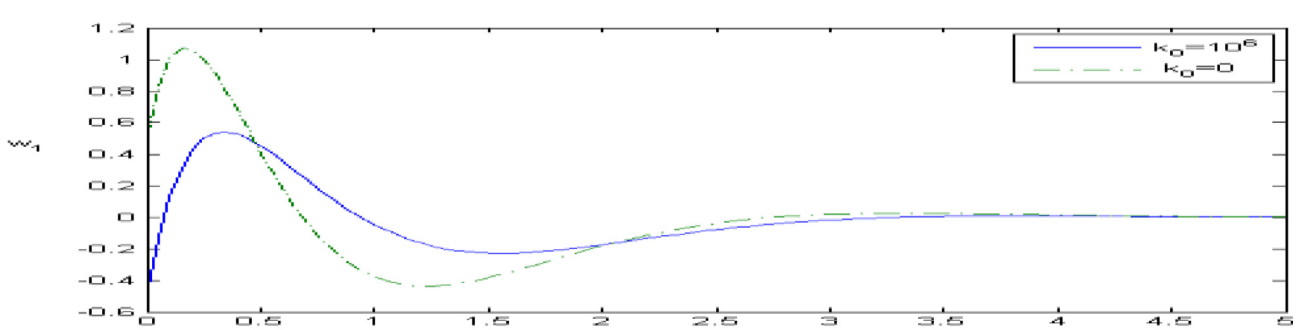

Fig. 13. Variation of the microtemperatures vector w_1 against y.

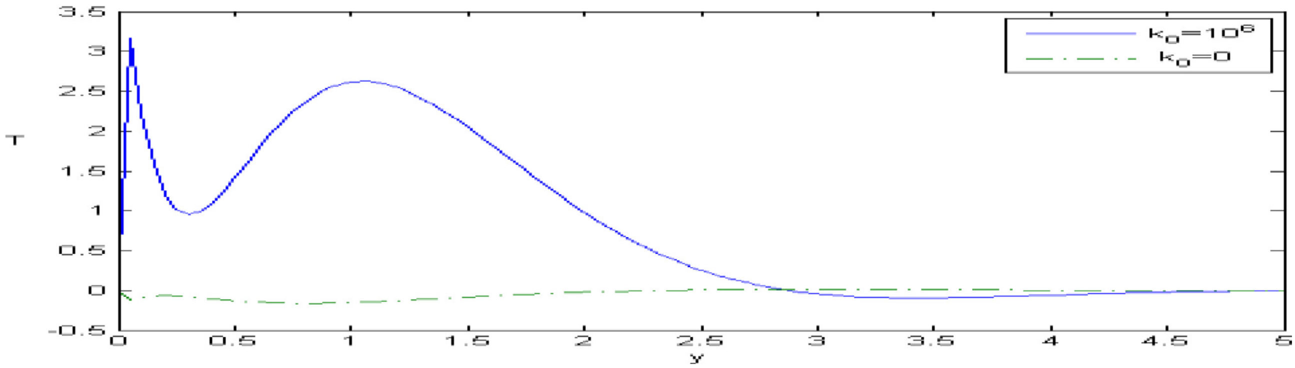

Fig. 14. Variation of the temperature T against y.

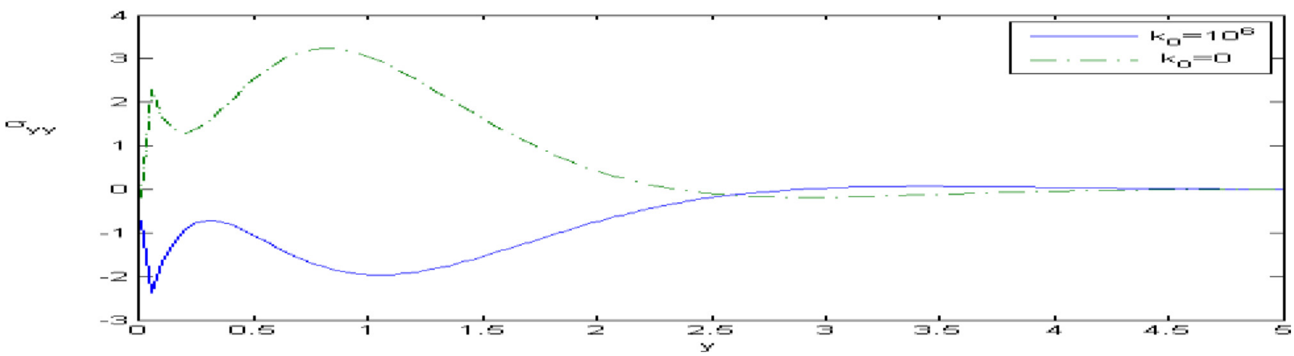

Fig. 15. Variation of the normal stress component σ_{yy} against y.

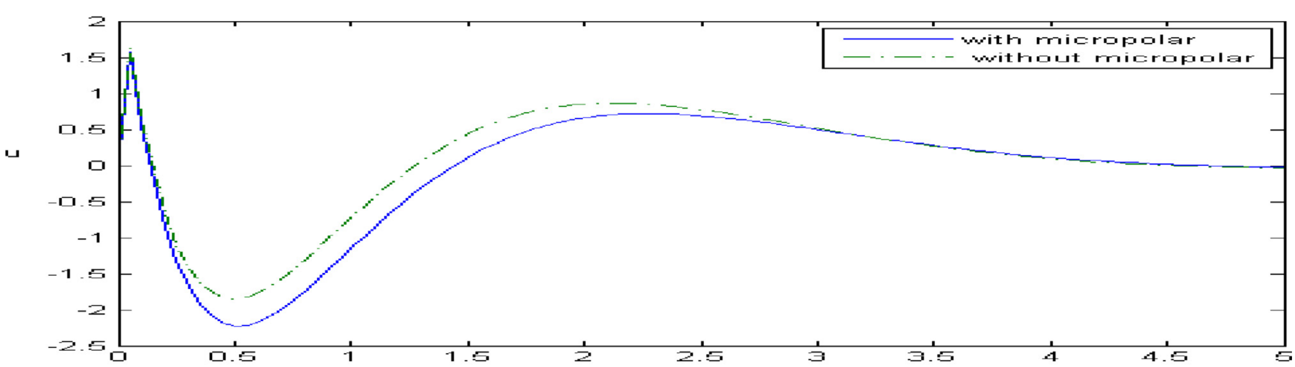

Fig. 16. Variation of the displacement component u against y with and without micropolar.

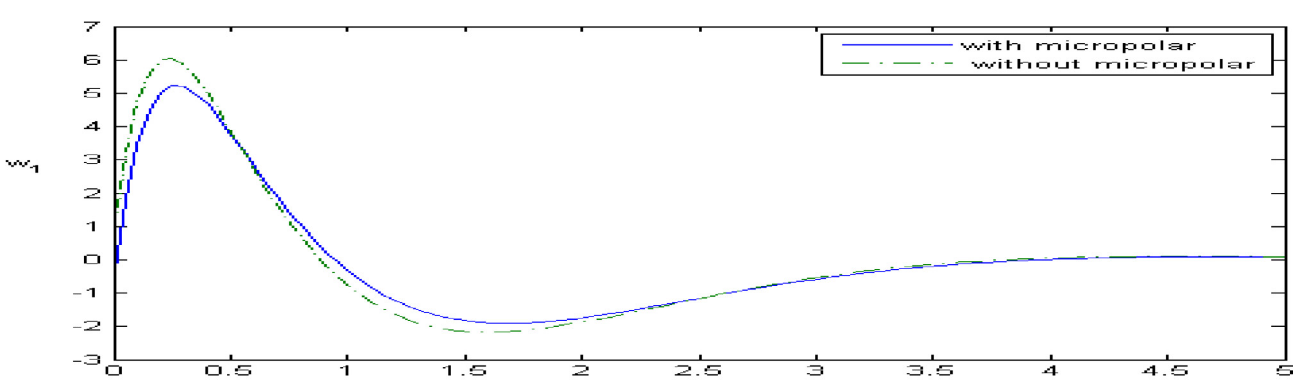

Fig. 17. Variation of microtemperatures vector w_1 against y with and without micropolar.

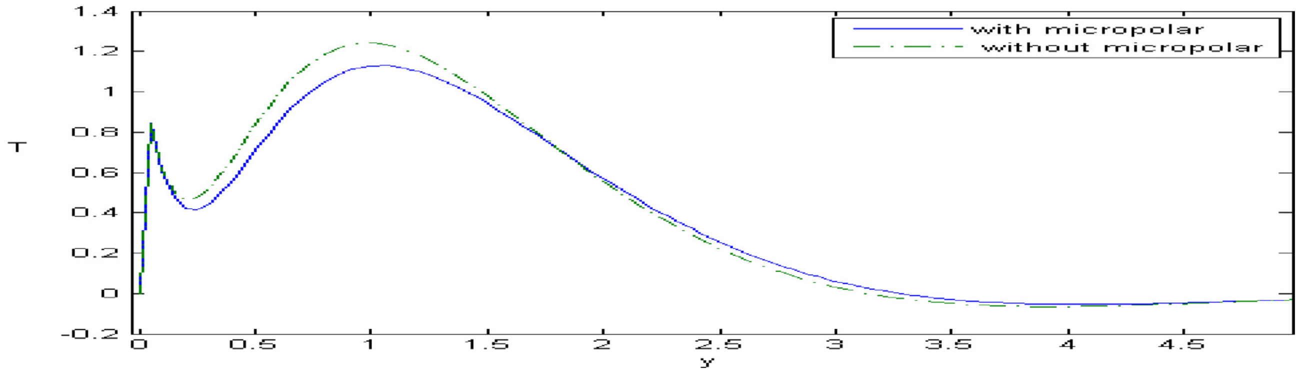

Fig. 18. Variation of the temperature T against y with and without micropolar.

the increase of the time in the range $0 \leqslant y \leqslant 0.4$, then it increases in the range $0.4 \leqslant y \leqslant 5$. The time plays an important role in the variation of previous physical quantities. Figs. 12–16 represent the behavior of the physical quantities against the distance y when $p = 0.7\ \mathrm{N/m}$, $t = 0.009\ \mathrm{s}$, $I_0 = 10^{-6}\ \mathrm{J/m^2}$, in the case of $k_0 = 10^6,\ 0\ \mathrm{m/K}$.

Fig. 12 shows that the variation of the displacement component u decreases in the range $0 \leqslant y \leqslant 2.5$, then it increased in the range $2.5 \leqslant y \leqslant 5$, with the increase of k_0. Fig. 13 clarifies that the microtemperatures vector w_1 decreases in the ranges $0 \leqslant y \leqslant 0.7$, and $2 \leqslant y \leqslant 5$, however, it increases in the range $0.7 \leqslant y \leqslant 2$, with the increase of k_0. It is clear from Fig. 14 that the temperature T increases with the increase of k_0 in the range $0 \leqslant y \leqslant 2.7$, however it decreases in the range $2.7 \leqslant y \leqslant 5$. Fig. 15 depicts that the normal stress component decreases in the range $0 \leqslant y \leqslant 2.5$, while it increases in the range $2.5 \leqslant y \leqslant 5$ with the increase of k_0. The temperature gradient coefficient k_0 plays an important role in the variation of previous physical quantities. Figs. 16–18 represent the behavior of the physical quantities against the distance y, in the presence and absence of micropolar. Fig. 16 shows that the variation of the displacement component u decreases for $y > 0$, with the increase of micropolar. Fig. 17 clarifies that the microtemperatures vector w_1 decreases in the range $0 \leqslant y \leqslant 0.7$, and increases in the range $0.7 \leqslant y \leqslant 5$, with the increase of micropolar. It is clear from Fig. 18 that the temperature T decreases with the increase of micropolar in the range $0 \leqslant y \leqslant 1.8$, however it increases in the range $1.8 \leqslant y \leqslant 5$. It deduced that all functions are continuous and all the curves converge to zero.

Concluding remarks

1. The effect of the coefficient of modified Ohm's law is observed from the behavior of the physical quantities also within the used mechanical and thermal loadings.
2. The micropolarity is an important property in the variation of the functions and useful in factoring material such as polymers and foundations materials in civil engineering.
3. The microtemperatures configuration a great importance for the continuous medium mechanics, earthquake engineering and seismologist for mining tremors and drilling into the earth's crust.
4. All functions are continuous and all the curves converge to zero with the increase of the value of the physical operators and also with the increase of the distance y, that mean; all physical quantities propagate as wave function in the plane with the time.

Appendix A

$$N_1 = c_1 + 1, \quad N_2 = c_{11} + 1,$$

$$N_3 = a^2 + \frac{\mathrm{i}\xi}{N_1}\left(\frac{c_4 c_{30}}{c_{29} - \mathrm{i}\xi\varepsilon_1} - c_5\right) - \frac{c_3\xi^2}{N_1}, N_4 = \frac{c_4}{N_1(c_{29} - \mathrm{i}\xi\varepsilon_1)},$$

$$N_5 = \frac{c_3}{N_1}, \quad N_6 = a^2 + \mathrm{i}\xi\left(\frac{c_4 c_{30}}{c_{29} - \mathrm{i}\xi\varepsilon_1} - c_5\right) - c_3\xi^2,$$

$$N_7 = \frac{c_4 c_{31}}{c_{29} - \mathrm{i}\xi\varepsilon_1} - c_6, \quad N_8 = \frac{\mathrm{i}\xi c_{30}}{c_{32}}, \quad N_9 = a^2 - \frac{\mathrm{i}\xi c_{29}}{c_{32}} - \varepsilon_1\xi^2,$$

$$N_{10} = a^2 + c_7 - c_{10}\xi^2, N_{11} = a^2 + \frac{c_{13} - \mathrm{i}\xi c_{14}}{N_2}, \quad N_{12} = \frac{c_{15}}{N_2},$$

$$N_{13} = -\mathrm{i}\xi c_{12}, \quad N_{14} = a^2 + c_{13} - \mathrm{i}\xi c_{14}, \quad N_{15} = \frac{\mathrm{i}\xi c_{17}}{c_{16}},$$

$$N_{16} = \frac{c_{18}}{c_{16}}, \quad N_{17} = a^2 - \frac{\mathrm{i}\xi}{c_{16}}, \quad N_{18} = \frac{Q_0}{c_{16}},$$

$$Q_0 = \frac{I_0\gamma^*}{2\pi r^2 t_0^2}, \quad f(x,t) = t\exp\left(-\frac{x^2}{r^2} - \frac{t}{t_0} - iax - \xi t\right),$$

$$L = \frac{-S_8}{\mathrm{i}\xi\varepsilon_1}, B_{1\mathrm{n}} = \frac{-B_{6\mathrm{n}}[N_7 S_\mathrm{n}^4 - \delta_1 S_\mathrm{n}^2 + \delta_2]}{[S_\mathrm{n}^6 - \delta_3 S_\mathrm{n}^4 - \delta_4 S_\mathrm{n}^2 - \delta_5]}, \quad B_{2\mathrm{n}} = \frac{-N_6(S_\mathrm{n}^2 - a^2)}{(S_\mathrm{n}^2 - N_9)},$$

$$B_{3\mathrm{n}} = \frac{-B_{1\mathrm{n}}(S_\mathrm{n}^2 - N_6) - N_7 B_{6\mathrm{n}}}{c_2}, \quad B_{4\mathrm{n}} = \frac{N_{12}B_{6\mathrm{n}}}{(S_\mathrm{n}^2 - N_{11})},$$

$$B_{5\mathrm{n}} = \frac{-N_{13}B_{3\mathrm{n}}}{(S_\mathrm{n}^2 - N_{14})}, \quad B_{6\mathrm{n}} = \frac{S_\mathrm{n}^4 - (N_3 + N_9 - N_4 N_8)S_\mathrm{n}^2 + (N_3 N_9 - N_4 N_8 a^2)}{N_5(S_\mathrm{n}^2 - N_9)},$$

$$\delta_1 = N_7(N_{10} + N_{14} + c_{10}N_{13}), \quad \delta_2 = N_7(N_{10}N_{14} + c_{10}N_{13}a^2),$$

$$\delta_3 = (N_{10} + N_{14} + c_{10}N_{13} + c_2 c_8 - N_6),$$

$$\delta_4 = c_2 c_8(N_{14} + a^2) + N_6(N_{10} + N_{14} + c_{10}N_{13}) + N_{10}N_{14} + c_{10}N_{13}a^2,$$

$$\delta_5 = c_2 c_8 N_{14} + N_6 N_{10} N_{14} + c_6 N_6 N_{13} a^2,$$

$$Q_1 = -N_{18} f_1(x,t), f_1(x,t) = t\ \exp\left(-\frac{x^2}{r^2} - \frac{t}{t_0}\right),$$

$$A_1 = \frac{1}{\gamma^{*14} - \lambda_1\gamma^{*12} + \lambda_2\gamma^{*10} - \lambda_3\gamma^{*8} + \lambda_4\gamma^{*6} - \lambda_5\gamma^{*4} + \lambda_6\gamma^{*2} - \lambda_7},$$

$$L_1 = N_5 N_{18}(\gamma^{*2} - N_{11})(\gamma^{*2} - N_9),$$

$$L_2 = \frac{-N_7 L_7[\gamma^{*4} - (N_{10} + N_{14} + c_{10}N_{13})\gamma^{*2} + (N_{10}N_{14} + c_{10}N_{13}a^2)]}{\gamma^{*6} - \beta_1\gamma^{*4} + \beta_2\gamma^{*2} - \beta_3},$$

$$L_3 = \frac{N_8 L_1(\gamma^{*2} - a^2)}{(\gamma^{*2} - N_9)}, \quad L_4 = \frac{-L_2(\gamma^{*2} - N_6) - N_7 L_7}{c_2},$$

$$L_5 = \frac{N_{12}L_7}{(\gamma^{*2} - N_{11})}, \quad L_6 = \frac{-N_{13}L_4}{(\gamma^{*2} - N_{14})},$$

$$L_7 = N_{18}(\gamma^{*2} - N_{11})[\gamma^{*4} - (N_3 + N_9 - N_4 N_8)\gamma^{*2} + (N_3 N_9 - N_4 N_8 a^2)],$$

$$\beta_1 = 2N_6 + N_{14} + c_{10}N_{13} - c_2c_8,$$
$$\beta_2 = N_{14}(N_{10} + N_6 - c_2c_8) + c_{10}(N_{13}a^2 + N_6N_{13}) + N_6^2 - c_2c_8a^2,$$

$$\beta_3 = N_6N_{10}N_{14} + c_{10}N_6N_{13}a^2 - c_2c_8N_{14}a^2, \quad B_{7n} = \mathrm{i}a - S_nB_{1n},$$
$$B_{8n} = -S_n - \mathrm{i}aB_{1n}, \quad B_{9n} = \mathrm{i}aB_{4n} - S_nB_{5n},$$

$$B_{10n} = -S_nB_{4n} - \mathrm{i}aB_{5n},$$
$$B_{11n} = c_{19}(\mathrm{i}aB_{7n} - S_nB_{8n}) + \mathrm{i}ac_{20}B_{7n} - B_{6n} - p,$$
$$B_{12n} = c_{19}(\mathrm{i}aB_{7n} - S_nB_{8n}) - S_nc_{20}B_{8n} - B_{6n} - p,$$
$$B_{13n} = -c_{21}S_nB_{7n} + \mathrm{i}ac_{22}B_{8n} - c_{23}B_{8n}, \quad B_{14n} = -S_nB_{3n},$$
$$B_{15n} = -\mathrm{i}aB_{3n}, \quad B_{16n} = c_{25}(\mathrm{i}aB_{9n} - S_nB_{10n}) + \mathrm{i}ac_{26}B_{9n},$$
$$B_{17n} = c_{25}(\mathrm{i}aB_{9n} - S_nB_{10n}) - S_nc_{26}B_{10n},$$
$$B_{18n} = -c_{27}S_nB_{9n} + \mathrm{i}ac_{28}B_{10n},$$
$$B_{19n} = \frac{1}{(c_{29} - \mathrm{i}\xi\varepsilon_1)}[-S_nB_{2n} + \mathrm{i}\xi c_{30}B_{8n} + \mathrm{i}ac_{31}B_{6n}],$$
$$B_{20n} = \frac{-1}{(c_{29} - \mathrm{i}\xi\varepsilon_1)}[\mathrm{i}aB_{2n} + \mathrm{i}\xi c_{30}B_{7n} + S_nc_{31}B_{6n}],$$

$$A_2 = \frac{2c_{19}}{r^2}\left\{\left[L_1(1 - \frac{2x^2}{r^2}) - \gamma^*xL_2\right] - \gamma^*\left(-\gamma^*L_1 + \frac{2xL_2}{r^2}\right)\right\} + \frac{2c_{20}}{r^2}\left[L_1\left(1 - \frac{2x^2}{r^2}\right) - \gamma^*xL_2\right] - L_7,$$

$$A_3 = \frac{2c_{19}}{r^2}\left\{\left[L_1(1 - \frac{2x^2}{r^2}) - \gamma^*xL_2\right] - \gamma^*\left(-\gamma^*L_1 + \frac{2xL_2}{r^2}\right)\right\} - c_{20}\gamma^*\left(-\gamma^*L_1 + \frac{2xL_2}{r^2}\right) - L_7,$$

$$A_4 = c_{21}\gamma^*\left(\frac{2xL_1}{r^2} + \gamma^*L_2\right) + \frac{2c_{22}}{r^2}\left[\gamma^*xL_1 + L_2(1 - \frac{2x^2}{r^2})\right] - c_{23}L_4,$$
$$A_5 = \frac{2xL_4}{r^2}, \quad A_6 = -\gamma^*L_4,$$

$$A_7 = \frac{2c_{25}}{r^2}\left\{\left[L_5(1 - \frac{2x^2}{r^2}) - \gamma^*xL_6\right] - \gamma^*\left(-\gamma^*L_5 + \frac{2xL_6}{r^2}\right)\right\} + \frac{2c_{26}}{r^2}\left[L_5\left(1 - \frac{2x^2}{r^2}\right) - \gamma^*xL_6\right],$$

$$A_8 = \frac{2c_{25}}{r^2}\left\{\left[L_5(1 - \frac{2x^2}{r^2}) - \gamma^*xL_6\right] - \gamma^*\left(-\gamma^*L_5 + \frac{2xL_6}{r^2}\right)\right\} - c_{26}\gamma^*\left(-\gamma^*L_5 + \frac{2xL_6}{r^2}\right),$$

$$A_9 = c_{27}\gamma^*\left(\frac{2xL_5}{r^2} + \gamma^*L_6\right) + \frac{2c_{28}}{r^2}\left[\gamma^*xL_5 + L_6(1 - \frac{2x^2}{r^2})\right],$$
$$A_{10} = \frac{1}{(c_{29} - \frac{\varepsilon_1}{t_0})}\left\{-\gamma^*L_3 - \frac{c_{30}}{t_0}\left(-\gamma^*L_1 + \frac{2xL_2}{r^2}\right) - \frac{2c_{31}xL_7}{r^2}\right\},$$

$$A_{11} = \frac{1}{\left(c_{29} - \frac{\varepsilon_1}{t_0}\right)}\left\{\frac{2xL_3}{r^2} + \frac{c_{30}}{t_0}\left(\gamma^*L_2 + \frac{2xL_1}{r^2}\right) - c_{31}\gamma^*L_7\right\},$$

$$c_1 = \frac{(\lambda + \mu + p/2)}{(\mu + k^* - p/2)}, \quad c_2 = \frac{k^*}{(\mu + k^* - p/2)}, \quad c_3 = \frac{\rho c_0^2}{(\mu + k^* - p/2)},$$
$$c_4 = \frac{\rho c_0^2\sigma_0^2H_0^2\mu_0^3}{\gamma_1T_0\omega_1^{*3}(\mu + k^* - p/2)}, \quad c_5 = \frac{c_0^2\sigma_0H_0^2\mu_0^2}{\omega_1^*(\mu + k^* - p/2)},$$
$$c_6 = \frac{\rho k_0c_0^2}{\mu_0H_0(\mu + k^* - p/2)}, \quad c_7 = \frac{2k^*c_0^2}{\gamma\omega_1^{*2}}, \quad c_8 = \frac{(k^* - p)}{\gamma\omega_1^{*2}},$$
$$c_9 = \frac{\rho\mu_1c_0^2}{\gamma\gamma_1T_0}, \quad c_{10} = \frac{j\rho c_0^2}{\gamma}, \quad c_{11} = \frac{k_4 + k_5}{k_6},$$

$$c_{12} = \frac{\mu_1\gamma_1T_0}{\rho\omega_1^*k_6}, \quad c_{13} = \frac{k_2c_0^2}{k_6\omega_1^{*2}}, \quad c_{14} = \frac{bc_0^2}{k_6\omega_1^*}, \quad c_{15} = \frac{k_3T_0c_0^2}{k_6\omega_1^{*2}},$$
$$c_{16} = \frac{k\omega_1^*}{\rho C_ec_0^2}, \quad c_{17} = \frac{\gamma_1^2T_0}{\rho C_ec_0^2},$$

$$c_{18} = \frac{k_1\omega_1^*}{\rho C_ec_0^2T_0}, \quad c_{19} = \frac{\lambda}{\rho c_0^2}, \quad c_{20} = \frac{2\mu + k^*}{\rho c_0^2}, \quad c_{21} = \frac{(\mu + p/2)}{\rho c_0^2},$$
$$c_{22} = \frac{(\mu + k^* - p/2)}{\rho c_0^2}, \quad c_{23} = \frac{k^*}{\rho c_0^2},$$

$$c_{24} = \frac{\gamma\omega_1^{*2}}{\rho c_0^4}, \quad c_{25} = -k_4\mu\omega_1^*, \quad c_{26} = -(k_5 + k_6)\mu\omega_1^*,$$
$$c_{27} = -k_5\mu\omega_1^*, \quad c_{28} = -k_6\mu\omega_1^*, \quad c_{29} = \frac{\mu_0\sigma_0}{\omega_1^*},$$

$$c_{30} = \frac{\gamma_1T_0}{\rho}, \quad c_{31} = \frac{\gamma_1T_0k_0\omega_1^*}{\sigma_0\mu_0^2H_0^2}, \quad c_{32} = \frac{1}{c_0^2}, \quad \varepsilon_1 = \frac{c_0^2}{c^2},$$
$$c^2 = \frac{1}{\mu_0\varepsilon_0}, \quad n = 1, 2, 3, \ldots, 7.$$

Appendix B

$$\psi_1(x, y, t) = \sum_{n=1}^{7}R_n\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1L_1\exp(-\gamma^*y),$$

$$\psi_2(x, y, t) = \sum_{n=1}^{7}R_nB_{1n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1L_2\exp(-\gamma^*y),$$

$$q_1(x, y, t) = \sum_{n=1}^{7}R_nB_{4n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1L_5\exp(-\gamma^*y),$$

$$q_2(x, y, t) = \sum_{n=1}^{7}R_nB_{5n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1L_6\exp(-\gamma^*y),$$

$$m_{yz}(x, y, t) = \sum_{n=1}^{7}R_nB_{14n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1A_5\exp(-\gamma^*y),$$

$$m_{xz}(x, y, t) = \sum_{n=1}^{7}R_nB_{15n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1A_6\exp(-\gamma^*y),$$

$$q_{xx}(x, y, t) = \sum_{n=1}^{7}R_nB_{16n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1A_7\exp(-\gamma^*y),$$

$$E_x(x, y, t) = \sum_{n=1}^{7}R_nB_{19n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1A_{10}\exp(-\gamma^*y),$$

$$E_y(x, y, t) = \sum_{n=1}^{7}R_nB_{20n}\exp(-S_ny + \mathrm{i}(ax - \xi t)) + A_1Q_1A_{11}\exp(-\gamma^*y),$$

$$\sigma_{xz}(x, y, t) = \sigma_{yz}(x, y, t) = q_{xz}(x, y, t) = q_{yz}(x, y, t) = 0.$$

References

[1] Eringen, A.C. Foundation of micropolar thermoelasticity. Course of lectures 23, CISM Udine, Springer; 1970.
[2] Eringen AC. Microcontinuum field theory I: foundations and solids. Berlin: Springer-Verlag; 1999.
[3] Ezzat MA, Awad ES. Constitutive relations, uniqueness of solution, and thermal shock application in the linear theory of micropolar generalized thermoelasticity involving two temperatures. J Therm Stress 2010;33: 226–50.
[4] Ezzat MA, Awad ES. Micropolar generalized magneto-thermoelasticity with modified Ohm's and Fourier's laws. J Math Anal Appl 2009;353:99–113.
[5] Ezzat MA, Awad ES. Generalized magneto-thermoelasticity with modified Ohm's law. Mech Adv Mater Struct 2010;17:74–84.
[6] Grot RA. Thermodynamics of a continuum with microstructure. I. J Eng Sci 1969;7:801–14.
[7] Riha P. On the microcontinuum model of heat conduction in materials with inner structure. Int J Eng Sci 1976;14:529–35.
[8] Iesan D, Quintanilla R. On a theory of thermoelasticity with microtemperatures. J Therm Stress 2000;23:199–215.
[9] Iesan D. On a theory of micromorphic elastic solids with microtemperatures. J Therm Stress 2001;24:737–52.
[10] Casas PS, Quintanilla R. Exponential stability in thermoelasticity with microtemperatures. Int J Eng Sci 2005;43:33–47.
[11] Scalia A, Svandze M. On the representation of solutions of the theory of thermoelasticity with microtemperatures. J Therm Stress 2006;29:849–63.
[12] Iesan D. Thermoelasticity of bodies with microstructure and microtemperatures. Int J of Solids Struct 2007;44:8648–62.
[13] Ames K, Straughan B. Continuous dependence results for initially prestressed thermoelastic bodies. Int J Eng Sci 1999;30:7–13.
[14] Montanaro A. On singular surfaces in isotropic linear thermoelasticity with initial stress. J Acoust Soc Am 1999;106:1586–8.
[15] Sun Y, Fang D, Saka M, Soh AK. Laser-induced vibrations of micro-beams under different boundary conditions. Int J Solids Struct 2008;45:1993–2013.
[16] Othman MIA, Zidan MEM, Hilal MIM. The effect of initial stress on thermoelastic rotating medium with voids due to laser pulse heating with energy dissipation. J Therm Stress 2015;38:835–53.
[17] Othman MIA, Zidan MEM, Hilal MIM. The effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation. Can J Phys 2014;92(11):1359–71.
[18] Othman MIA, Tantawi RS, Hilal MIM. Rotation and modified Ohm's law influence on magneto-thermoelastic micropolar material with microtemperatures. Appl Math Comput 2016;276:468–80.
[19] Othman MIA, Tantawi RS, Hilal MIM. Hall current and gravity effect on magneto-micropolar thermoelastic medium with microtemperatures. J Therm Stress 2016;39:751–71.
[20] Eringen AC. Plane wave in non-local micropolar elasticity. Int J Eng Sci 1984;22:1113–21.