2018-07-01

# Principles of Productivity Revealed from Secondary Mathematics Teachers' Discussions Around the Productiveness of Teacher Moves in Response to Teachable Moments

Kylie Victoria Palsky
*Brigham Young University*

# Principles of Productivity Revealed from Secondary Mathematics Teachers' Discussions Around the Productiveness of Teacher Moves in Response to Teachable Moments

Kylie Victoria Palsky

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

Keith R. Leatham, Chair
Blake E. Peterson
Dawn Teuscher

Department of Mathematics Education

Brigham Young University

ABSTRACT

Principles of Productivity Revealed from Secondary Mathematics Teachers' Discussions Around the Productiveness of Teacher Moves in Response to Teachable Moments

Kylie Victoria Palsky
Department of Mathematics Education, BYU
Master of Arts

How do teachers talk about the productiveness of teacher's in-the-moment responses to student mathematical thinking? This is a question current research does not fully answer as most research on teacher moves is focused on what teacher moves researchers have noticed teachers do rather than on what teachers think about these teacher moves. To fill the gap in the research and to answer the question, a group of 13 teachers were given ten classroom situations to compare and contrast for productivity. I analyzed (a) the content of the teachers' discussions by drawing on Teacher Response Coding (TRC) language, and (b) the extent to which the teachers' discussions align with theorized productive responses to student mathematical thinking, or *building*. From the teachers' group conversations, I articulated *principles of productivity*—articulations of the main ideas and conclusions of the teachers' conversations with regards to productivity. Focusing on the principles of productivity, I highlighted what teacher moves the teachers said were productive or not productive with respect to teacher's in-the-moment responses to student mathematical thinking. In analyzing the list of unique principles of productivity, I noticed three main themes that the principles were focused around: student mathematics, teacher moves, and mathematics, which reflected some of the ideas in research for productive teacher moves. Additionally, I analyzed the principles for alignment with the practice of *building,* which led to the conclusion that the ideas of orchestrating discussion and making explicit are the most salient of the sub-practices of building to the teachers. These results based on teachers' discussions around the productivity of teacher moves can help inform teacher education and professional development.

Keywords: teacher discussions, teacher response, teachable moments, productive teacher moves, teacher moves, teacher turns

## ACKNOWLEDGMENTS

I would like to thank everyone who has helped in any way along the long path of my thesis. The most influential and helpful throughout the whole process has been my advisor Keith, who put up with me learning how to write a thesis. I also could not have done it without the continual support and cheerleading of my husband Josh Palsky. Thank you to the friends I made in the program who encouraged me along the way, to my committee for giving feedback for improvement, and to my mom for encouraging me to apply to graduate school in the first place. My thesis would not be where it got to without everyone’s help in whatever form it came.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

# CHAPTER ONE: RATIONALE

There is an expectation that teachers facilitate discourse within the mathematics classroom (National Council for Teachers of Mathematics, 2007, 2014). Mathematical discourse necessitates classroom discussion, including verbal, visual, and written communication, with the intent to purposefully exchange ideas (National Council for Teachers of Mathematics, 2014). The focus on discourse has been broad, such as the need to have good tasks for discussion (e.g., Stein, Engle, Smith, & Hughes, 2008). However, while there is research on what teachers should do to have productive classroom discourse, there is little research on what teachers think is productive to do. Looking more specifically at how teachers discuss the productivity of teacher moves, or what a teacher turn accomplishes (e.g., clarify, validate), can inform the educational community on how to help teachers' development since understanding what teachers think is key to knowing how to further their growth as teachers (e.g., Ball, 1996). If teacher educators do not know or understand what teachers' think surrounding the productive use of student thinking, then teacher educators cannot expect to expand on the teachers' thinking and be able to help teachers learn about and implement productive teaching. There is a need to look at research when planning and implementing a teacher education program as well as a professional development in order to have it be the most effective it can be (Guskey & Yoon, 2009; Guyton, 2000).

Although the study of what teachers think is productive is valuable to the teacher educator and professional development communities, the available research barely touches on what is productive in a classroom, let alone what teachers think is productive. Many articles (e.g. Kazemi, Lampert, & Franke, 2009; Lampert et al., 2013) advocate for productive discourse, but exactly what it means to productively use student thinking is not well defined and not yet commonly understood (Van Zoest, Peterson, Leatham, & Stockero, 2016). While some research articles (e.g., Tobin,1986; Jacobs & Ambrose, 2008) suggest theoretically effective teacher moves, there is a noticeable lack of what teacher moves teachers think are productive teacher moves.

In order to prompt more valuable teacher conversation, I wanted the teachers' discussions to be centered on the productivity of teacher moves that were responding to student thinking that was worth taking advantage of in some substantive way. My goal was to focus teachers' discussions around moments in a class when it was particularly productive to do a variety of teacher moves in response to student mathematical thinking. Thus, I studied teachers' discussions around the productivity of teacher moves that are in response to significant instances of student thinking, or *teachable moments*, which are instances of student thinking made public in the classroom that are significant and worth following-up on in the class, or instances that can be classified as MOSTs (Leatham, Peterson, Stockero, & Van Zoest, 2015).

# CHAPTER TWO: THEORETICAL FRAMEWORK

In this study I explore what teachers think about the productiveness of teacher moves by analyzing their discussion surrounding teacher responses in hypothetical, written class discussions where a teacher is responding to a *teachable moment*. In this chapter I outline my framework for looking at *teachable moments,* which consists of what a *teachable moment* is (MOST) and how to respond to it (*building)*. I also describe how I drew on Mitchel (1994) to look at what I call "principles of productivity," and how the Teacher Response Coding Framework (Peterson, Van Zoest, Rougée, Freeburn, Stockero, & Leatham, 2017) added to the language I used in articulating these principles.

## MOST Framework

The MOST Analytic Framework (see Appendix A), developed by Leatham et al. (2015), provides a series of criteria for determining whether a given instance of student thinking is a teachable moment, or a MOST—a Mathematically significant pedagogical Opportunity to build on Student Thinking. A MOST is "in-the-moment thinking [that] is the most productive to pursue" (Van Zoest, Stockero, Leatham, Peterson, Atanga, & Ochieng, 2017, p. 35). The framework for identifying a MOST can help teachers and researchers *notice* student thinking by providing guidelines for *attending to* and *interpreting* student thinking (Jacobs, Lamb, & Philipp, 2010; Van Zoest, Peterson, Leatham, & Stockero, 2017). An instance of student mathematical thinking is a MOST if the underlying mathematics (*mathematical point)* is appropriate for the class to consider (*appropriate mathematics)* and central to the goal for student learning (*central mathematics)*, if there is an intellectual need for students to make sense of the mathematics (*opening)*, and finally, if the pedagogical timing is right to do so (*timing)*. If student thinking satisfies all of these criteria, it is considered a MOST. This MOST is an instance of student thinking that a teacher should take advantage of in the moment, or in other words, *build* on.

**Building on MOSTs**

The MOST Analytic Framework helps to identify the student instances in the classroom that are the most productive for a teacher to pursue. A recent conjecture of how a teacher could productively respond to a teachable moment, or a MOST, is to *build* on that thinking (Van Zoest et al., 2016), which contributes to *deciding how to respond* to student thinking (Jacobs et al., 2010). *Building* is conceptualized as "several teacher moves woven together to engage students in the intellectual work of making connections between ideas and abstracting mathematical concepts from consideration of their peers' mathematical thinking" (Van Zoest et al., 2016, p.1284-1285). Throughout my study, I used *building* as the lens through which I viewed the productivity of teacher moves and resultant practices.

Productively *building* on a MOST requires the "combining and coordinating of teacher moves" (Van Zoest et al., 2016). Thus, the teaching practice of *building* is more than a single teacher move; it is a collection of teacher moves. In order for teachers to *build* on student thinking they must complete two prerequisite actions: (a) invite or allow students to share their thinking and (b) recognize the instance of student thinking as a MOST (Van Zoest et al. 2016). Once a MOST is on the table, a teacher should begin to *build*, a practice made up of four sub-practices (see Figure 1). The first sub-practice is to ensure that the student mathematics of the MOST, which is the object of consideration, is clear. This sub-practice requires the teacher to *make precise* the student thinking that the students are meant to consider, which includes clarifying unclear parts of the student thinking and making it clear what aspects of the student thinking the class should consider. The second sub-practice has the teacher turn the mathematical thinking to the students. The term *grapple toss* is used to describe this sub-practice since the teacher must "toss" the student thinking to other students to "grapple" with, or make sense of. The third sub-practice requires the teacher to orchestrate a whole-class discussion. In this context *orchestrate* is taken to mean "arrange or direct the elements of (a situation) to produce a desired effect, especially surreptitiously" ("Orchestrate", n.d.). The fourth sub-practice is facilitating the extraction and articulation of the important mathematics from the discussion, or in other words, to *make explicit* the mathematical idea behind the student thinking (Van Zoest et al., 2016). Although I did not study teaching practices, knowing what teacher sub-practices I viewed as productive gave me a lens through which to view teachers' discussions surrounding the productiveness of teacher moves, and I was also interested in the similarities and differences between *building* and teachers' varying views of what practices are productive.

**Sequence of Sub-practices of the Teaching Practice of Building on MOST's**

0) Invite or allow students to share their thinking
0.5) Recognize the instance of student thinking as a MOST
1) Make the object of consideration clear (make precise)
2) Turn the object of consideration over to the students with parameters that put them in a sense-making situation (grapple toss)
3) Orchestrate a whole-class discussion in which students collaboratively make sense of the object of consideration (orchestrate)
4) Facilitate the extraction and articulation of the mathematical point of the object of consideration (make explicit)

*Figure 1.* Building prototype: Current conception of the teaching practice of building (Van Zoest et al., 2016).

## Principles of Productivity

Since the purpose of my research is to better understand what teachers think about the productivity of teacher moves, as well as the sub-practices that are comprised of these teacher moves, it was expected that the content of the teachers' discussions would revolve around the productivity of teacher moves. I was interested in how teachers talked about the productivity of these teacher moves, so I inferred "principles of productivity" from teachers' discussions. The idea behind the principles of productivity is similar to that of "rules of practice," which are statements of what to do and how to do it in particular situations, and "practical principles," which are broader than rules of practice and use past experiences to influence present situations (Elbaz, 1981). These were ideas Mitchell (1994) drew on as he articulated teachers' implicit theories about questioning, which were, in part, statements about what a teacher said should and should not be done in a class as relating to teacher questions (Mitchell, 1994). This articulating of teachers' implicit theories is similar to how I articulated the principles of productivity. A principle of productivity was typically a statement of "it is productive to…" or "it is unproductive to…" followed by the teachers' ideas of productive and unproductive teacher moves. Thus, the principles of productivity helped me see, overall, the teacher moves the teachers were describing as productive or unproductive along with any explanation or elaboration as to why.

Teachers create their own terminology to describe the purposes they see for teacher moves (e.g., Mitchell, 1994). This terminology often differs from wording used in the research community because teachers do not necessarily share researchers' language to describe their teaching (Sahin, Bullock & Stables, 2002). Because of the differences in language among teachers and between teachers and researchers, I used the Teacher Response Coding (TRC)

(Peterson et al., 2017) to provide a common language for me to draw on as I articulated principles of productivity from the teachers' discussions.

## Research Questions

The purpose of my research is to better understand teachers' implicit theories surrounding the productiveness of teacher moves in response to MOSTs. To accomplish this purpose I answer the following research questions:

1) What principles of productivity emerge from teachers' discussion related to the productivity of teacher moves in response to MOSTs?
2) How do these principles of productivity align with the conceptualization of *building?*

# CHAPTER THREE: LITERATURE REVIEW

There are several different categories of teacher moves evident throughout the literature, such as anticipating student thinking (e.g., Stein, Engle, Smith, & Hughes 2008), eliciting student thinking (e.g., Jacobs & Ambrose, 2008), enacting classroom management (e.g., Emmer, Evertson, & Anderson, 1980), and responding to student thinking (e.g., Peterson et al., 2017). Even though research has identified categories of teacher moves, as argued previously, there is not much research on what teachers think about the productivity of these teacher moves. I begin this review of related literature by presenting research that provides insight into what pre-service and in-service teachers think about the productivity of different teacher moves. However, the limited information on what teachers think regarding productivity of teacher moves leads me to additionally present the productiveness of teacher moves as identified by research. The ideas teachers had regarding productivity and the ideas research suggests regarding productivity were helpful because teachers, specifically the ones in my study, thought about productivity in similar ways. To end I discuss what research has identified and defined regarding the actual teacher moves that teachers employ in their teaching, since the teachers in my study sometimes discussed teacher moves in a similar way.

## Teachers' Ideas on Productive Teacher Moves

While research does not present principles of productivity the way I will, research (Cakmak, 2009; Mitchell, 1994) does contain results that provide similar information (i.e., what teachers thought was productive or unproductive, and why) with regards to teacher questions and teacher moves that use student thinking.

## Teacher Questions

A study on pre-service teachers' thoughts about questions in effective teaching processes showed that pre-service teachers think that the top reasons for using questions is to motivate students, to get the students' attention, and to make students active (Cakmak, 2009). Similarly, in-service teachers thought that questions can serve purposes related to management (e.g., organize the class and focus their attention), instruction (e.g., encourage students to think), and sociality (e.g., help build relationship-building situations and maintain self-esteem) (Mitchell, 1994). These categories for both the pre-service and in-service teachers suggest that these teachers think the main purposes of questions is to accomplish such things as would likely fall under the broader categories of engagement and classroom norms.

Not only did Mitchell's (1994) study provide information on what teachers thought questions should accomplish, but his study of teachers' implicit beliefs related to questioning led to teacher statements about what was essentially productive and not productive to do in relation to asking questions in a class. Mitchell (1994) reported individual teachers' statements about productivity as rules and principles (similar to my notion of principles of productivity) such as the following: "don't spend too much time questioning an individual student" (p. 74), "don't always choose those [students] who… can express themselves well" (p. 74), "in a 40 minute lesson… have a fairly short questioning situation" (p. 74), evaluate students' answers "without putting the student down" if they are wrong (p. 75), and "avoid lengthy pauses while waiting for an answer if it can be seen that the student is getting embarrassed" (p. 77). One difference between my study and that of Mitchell (1994) is that Mitchell derived his implicit theories by studying individual teachers, whereas my principles of productivity are derived from teachers' conversations.

## Teacher Moves that Use Student Thinking

Researchers (e.g., Franke, Kazemi, & Batty, 2007; Van Zoest, Peterson, Leatham, & Stockero, 2016) claim that teachers using student thinking in their responses is key to aspects of effective mathematics instruction. Since researchers assert the need for teachers to use student thinking, it is valuable knowing how teachers view the productivity of teacher moves that use student mathematical thinking. There are two articles that report, although indirectly, on what teacher moves teachers think of as productive in focusing on student mathematics (Stockero, Leatham, Ochieng, Van Zoest, Peterson, 2018; Leatham, Van Zoest, Stockero, & Peterson, 2014).

From a study of teachers' orientations (Stockero et al., 2018) I was able to draw out teachers' ideas of the productivity of teacher moves that varied in their use of student thinking to *build*. Teachers have different orientations towards using student thinking as a resource, which influences a teacher's perceptions, interpretations, and reactions to instances in a classroom (Stockero et al., 2018). Teachers exhibiting high potential to support *building* orientations believe that teacher moves aligned with *building* are productive while teachers exhibiting hinder or really hinder orientations value teacher moves that are counter to *building* (e.g., teaching mathematics though step-by-step, highly scaffolded instruction). Stockero et al. (2018) articulated teachers' thinking-as-a-resource orientations in similar ways to implicit theories or principles of productivity. Thus, many of these orientations can be seen as statements reflecting teachers views on the productivity of teacher moves. For example, teachers with high potential orientations for using student thinking to build had orientations such as "it is important to find out what students are thinking/understanding by having the class comment on or ask questions of the student whose thinking has been shared" or "it is important to give the class a chance to resolve students' multiple solutions that have been made public before moving on with a lesson" (Stockero et al., 2018, p. 22). These are just a couple examples of statements of orientation related to productively using student thinking; other related orientations gave insight into the teachers' idea that it was productive to allow students to introduce new ideas, react to other student ideas, and explain their own thinking to others such that the teacher was not positioned as the sole mathematical authority and the class was making sense of student mathematics.

On the other hand, Stockero et al. (2018) reported some teacher orientations that would seem to greatly hinder their ability to productively build on student thinking. Examples of these orientations include "it is the teacher's responsibility to correct student mistakes and misconceptions as quickly as possible" and "students should share their ideas one at a time and the teacher should resolve each idea before another idea is shared" (Stockero et al., 2018, p. 23). The orientations at this level were teachers' ideas that it was productive to highly scaffold lessons, with the teacher explaining, evaluating, and correcting student ideas.

By interpreting the results of a similar study (Leatham et al., 2014) that analyzed teachers' perceptions of productive use of student thinking, I was able to identify teacher moves that teachers considered productive. Teachers' perceptions of use of student thinking were compared to the *building* framework to determine the productivity. Based on this comparison, the most productive teacher perceptions were the ones that aligned the most with the concepts and practices associated with *building*. Thus, the most productive perceptions were when teachers thought that it was productive to invite the class to consider the student mathematical thinking, allow the students to make sense of the mathematics, and orchestrate a discussion that leads to a mutual understanding of the student mathematical thinking. In strong contrast to the perceptions aligned with building are those that are not well aligned. These less productive perceptions belong to teachers who think it is productive to elicit student thinking for the purpose of engagement, use student thinking by only providing positive feedback and validation, and elicit student thinking that merely replaces what the teacher would have said. The various perceptions on the continuum of aligned to not well aligned with building varied in focus from requiring more engagement with the student mathematics to mere student engagement. The classification of teacher moves associated with differing levels of productivity, based on alignment with *building,* provided valuable insight into what the teachers in my study might have viewed as productive and unproductive teacher moves.

The conclusion of the studies on teachers' ideas of the prodcutivity of teacher moves that use student thinking is clear: productive responses to student thinking require teachers to have high-potential orientations and perceptions of the purpose and use of student thinking. While I did not specifically study orientations and perceptions, the results outlining the aspects of each orientation and perception were informative of what I might expect from teachers in my study. My adopted view of what it means for teacher moves to be productive aligns with high-potential orientations and values student thinking. Since the results of Stockero et al., (2018) and Leatham et al. (2014) demonstrate that teachers had varying orientations and perceptions, I expected the teachers in my study to have differing views of what is productive and to have varying opinions for the use of student mathematical thinking.

**Productivity of Teacher Response Patterns as Classified by Research**

Given that there is limited research on what teachers view as productive, I also explore what research has said is productive to do. In addition to the theoretical framework of *building* that I adopted as my view of productive teacher practice, research reports on the productivity of what has been observed in classrooms, namely the I-R-E (Mehan, 1979), IDE (Nathan, Eilam, & Kim, 2007), and "talk moves" (Chapin & O'Connor, 2007) patterns of discourse.

The Initiation-Response-Evaluation (I-R-E) pattern of discourse (Mehan, 1979) is a response pattern to which many teachers are naturally drawn (Cazden, 2001) and thus it is a common way that teachers respond to almost any type of student thinking (e.g., Edwards & Mercer, 1987; Westgate & Hughes, 1997). During an I-R-E sequence, the teacher *initiates* the exchange by asking what is typically a closed, obvious-answer question (Nathan, Kim, & Grant, 2018). This question elicits a *response* from a student, and the teacher then *evaluates* that response in a way that often terminates the interaction and limits further contribution (Westgate & Hughes, 1997; Jia, 2005). Because the I-R-E limits student contributions and fails to create discussion that supports engagement and conceptual understanding (Cullen, 2002; Nystrand, 1997; Wells & Arauz, 2006), researchers (e.g., Nunan, 1987; Nystrand, 1997) are critical of I-R-E exchanges. Thus, even though the I-R-E response pattern is common in classroom discourse, researchers argue that it is an unproductive discourse pattern.

In contrast to I-R-E, Nathan, Eilam, and Kim (2007) proposed the IDE sequence as a productive discourse pattern. The productive nature of the IDE is in part due to the open-ended *initiation,* which varies from the type of initiation in the I-R-E. The *demonstrations* aspect of the IDE, which is not limited to the single, obvious answer student responses by which I-R-E is characterized, elicits student responses that the teacher integrated into *evaluative* and *elaborative* (non-evaluative) statements that invited other students to participate and provide their alternative perspectives (Nathan et al., 2007). The IDE was seen as creating productive discourse because it allows discussion to be focused on students and allows the students to interact with one another to build on mathematically correct concepts (Nathan et al., 2007).

In addition, research has found that teachers and students repeatedly used five “talk moves” to focus discussion on important mathematics (Chapin, O’Connor, & Anderson, 2003; Michaels O’Connor, Hall, & Resnick, 2003). These talk moves (that researchers view as productive) are revoicing, asking students to repeat another student’s previously elicited reasoning, eliciting student reasoning, asking students to add on information to student ideas, and employing teacher wait time (Chapin & O’Connor, 2007; Michaels & O’Connor, 2015).

My research contributes to this literature related to teacher moves teachers think are productive, why they think this, and how teachers’ views of productive teacher moves align with *building*. My study was similar to those of Mitchell (1994), Stockero et al. (2018), and Leatham et al. (2014) because the teachers in my study could look at questions as teacher moves and I looked at how their ideas related to building. However, I provide a broader picture than Mitchell (1994) of what teachers think regarding the productivity of teacher moves (beyond just questions) as drawn from their discussions. Additionally, Leatham et al. (2014) and Stockero et al. (2018) looked at how teachers’ ideas of productivity aligned with building as a whole, so my research elaborates on theirs as it provides specific information about how the principles of productivity align with each prerequisite to and sub-practice of building, while also taking into account principles of productivity that may not be directly related to building.

# CHAPTER FOUR: METHODOLOGY

Teachers were invited to a professional development workshop and asked to discuss in small groups the productiveness of teacher moves in a set of given scenarios. These scenarios were designed with the *building* framework in mind to allow the researcher to compare what the teachers thought was productive to the theorized productive practice of *building.*

## Participants

The thirteen teachers involved in my study are middle and high school teachers from across the country. These teachers were chosen based on their desire to focus on and learn more about using student thinking in their classrooms. All of the teachers were invited to and engaged in a professional development workshop. Because the teachers are part of a professional development research study I refer to them from here on out as professional development teachers, or PD teachers, to make it clear when I am referring to them as opposed to the hypothetical teachers in the scenarios. The workshop was split into two days over which the PD teachers were introduced to the ideas of MOSTs and building on MOSTs, where the introduction of the latter occurred following the activity described below.

## Data Collection

The PD teachers were given ten lesson scenarios (see Appendix B) that were created using LessonSketch (www.lessonsketch.org). These scenarios ranged from four to five frames with varying student and teacher turns based on criteria identified below. Also, each lesson scenario begins with a MOST, thus narrowing the study to looking at productive *building* on MOSTs. The previous day the PD teachers watched and discussed a Counting Cubes lesson (see Appendix C) on which the lesson scenarios are based. The researchers created these scenarios such that every group of PD teachers was reacting to the same set of scenarios done by the same hypothetical scenario teacher in response to the same task (see Appendix C). The scenario teacher turns were chosen to include or exclude a variety of the component sub-practices of building as well as variations on those sub-practices (see Table 1).

Table 1

*The variation and/or existence of the sub-practices of building for each scenario 1-10*

| Scenario | To make precise | Grapple toss | Orchestrate | To make explicit |
|---|---|---|---|---|
| 1 | Already precise | Grapple tossed | Facilitated discussion | Made explicit |
| 2 | Not made precise | Tossed | Vaguely facilitated | Vaguely made explicit |
| 3 | Made precise | Grapple tossed | Facilitated discussion | Made explicit |
| 4 | Made precise | Grapple tossed | Not facilitated | Prematurely made explicit |
| 5 | Already precise | Tossed to individual student | Teacher-student discussion | Made explicit |
| 6 | Already precise | Teacher grapple | I-R-E | Made explicit |
| 7 | Made precise | Grapple tossed | Facilitated discussion | Not made explicit |
| 8 | Already precise | Grapple tossed | I-R-E | Made Explicit |
| 9 | Already precise | Grapple tossed | Facilitated discussion (MP: Define Variable), | Made explicit |
| 10 | Already precise | Grapple tossed | Facilitated discussion (MP: Difference between multiplication and repeated addition), | Made explicit |

Within the sub-practice category of *to make precise*, “already precise” means the scenario student mathematics did not need to be made precise by the scenario teacher since it was already clear what the scenario student’s mathematics was, “made precise” means the scenario teacher turn enacted the sub-practice of making precise, and “not made precise” means the scenario teacher did not make the scenario student’s mathematical thinking precise.

Within the sub-practice category of *Grapple Toss,* “grapple tossed” means the scenario teacher turn enacted this sub-practice as one of the scenario teacher turns, “tossed” means the scenario teacher tossed the mathematics to the scenario students, but not in a way that seemed to call for student sense making of that thinking, "tossed to individual student" means the scenario teacher only invited a single scenario student to consider the scenario student mathematical thinking, and "teacher grapple" means the scenario teacher did the grappling with the mathematics rather than inviting the scenario students to do so.

Within the sub-practice category of *Orchestrate*, "facilitated discussion" means the scenario teacher turn enacted the sub-practice of *orchestrate*, not necessarily related to the same mathematical point (MP), "vaguely facilitated" means the scenario teacher turn merely approximated the sub-practice of *orchestrate,* "not facilitated" means the scenario teacher turn did not enact the sub-practice of *orchestrate,* "teacher-student discussion" means the scenario teacher only had a discussion with one scenario student rather than a whole-class discussion, and "I-R-E" means the teacher implemented the discussion type of Initiate-Response-Evaluate (Mehan, 1979). The MP noted for two of the scenarios refers to the fact that the scenario teacher directed the discussion toward a MP that was different than the MP closest to the student thinking of the MOST.

Within the sub-practice category of *to make explicit,* "made explicit" means the scenario teacher turn enacted the sub-practice of making the scenario student mathematics explicit, "vaguely made explicit" means the scenario teacher did not fully make the scenario student mathematics explicit, "not made explicit" means the scenario teacher did not make the scenario student mathematics explicit, and "prematurely made explicit" means the scenario teacher made the mathematics explicit before it should have been.

The PD teachers were asked to record their observations about differences in the productiveness of the sequence of teacher moves in the ten scenarios. The specific prompt was as follows: “You will receive 10 comics that begin with a variation of student thinking that occurred during the Counting Cubes lesson. Use the Scenario Notes page to record your observations about the differences in the productiveness of the sequence of teacher moves in the scenario.” The PD teachers were divided into five groups (four pairs and one group of three). Each group discussion was individually video and audio recorded and photocopies were made of each PD teacher’s scenario notes.

The videos of each groups’ discussions were then linked to a timeline of StudioCode (SportsTec, 1997-2015) in order to code when PD teachers discussed specific scenarios. The scenario notes and the videoed discussions were then used to clarify the transcript and compile a spreadsheet where PD teacher turns within each groups’ discussion were labeled according to the order of the turn and the scenario(s) being discussed. These turns were then coded according to content of the turn relating to productivity. By labeling the PD teachers’ discussions by the scenario(s) they discuss, I was able to keep better track of which scenario teacher responses the PD teachers were discussing and when the discussion changed direction.

The PD teachers chosen for this study were asked to reflect on the responses of a scenario teacher in ten similar scenarios. By having all the participating PD teachers reflecting on the same lesson, and not their own lessons, the need to also analyze the contextual factors of each PD teacher’s classroom became unnecessary. Along with the common scenarios, the PD teachers were discussing the productivity of teacher moves within groups, which would have been difficult without a common scenario. Also, by providing the PD teachers with comic-strip-like lesson scenarios it made it easy for the PD teachers to identify which scenario teacher turns they were being asked to consider for the productivity of the teacher moves.

## Data Analysis

The analysis of the data surrounding PD teachers' conversations about the productivity of teacher moves is comprised of two main components: (a) the content of what the PD teachers said in their discussions, resulting in principles of productivity; and (b) the extent to which these principles of productivity align with building. The coding of these components is broken down into the five main steps laid out in Figure 2. I completed all five coding steps for a single group before moving on to the other groups. Analyzing each group in turn allowed me to be cognitively engaged with one particular group at a time through all the steps, which was helpful since each group had slightly different ideas and lines of reasoning in their discussions.

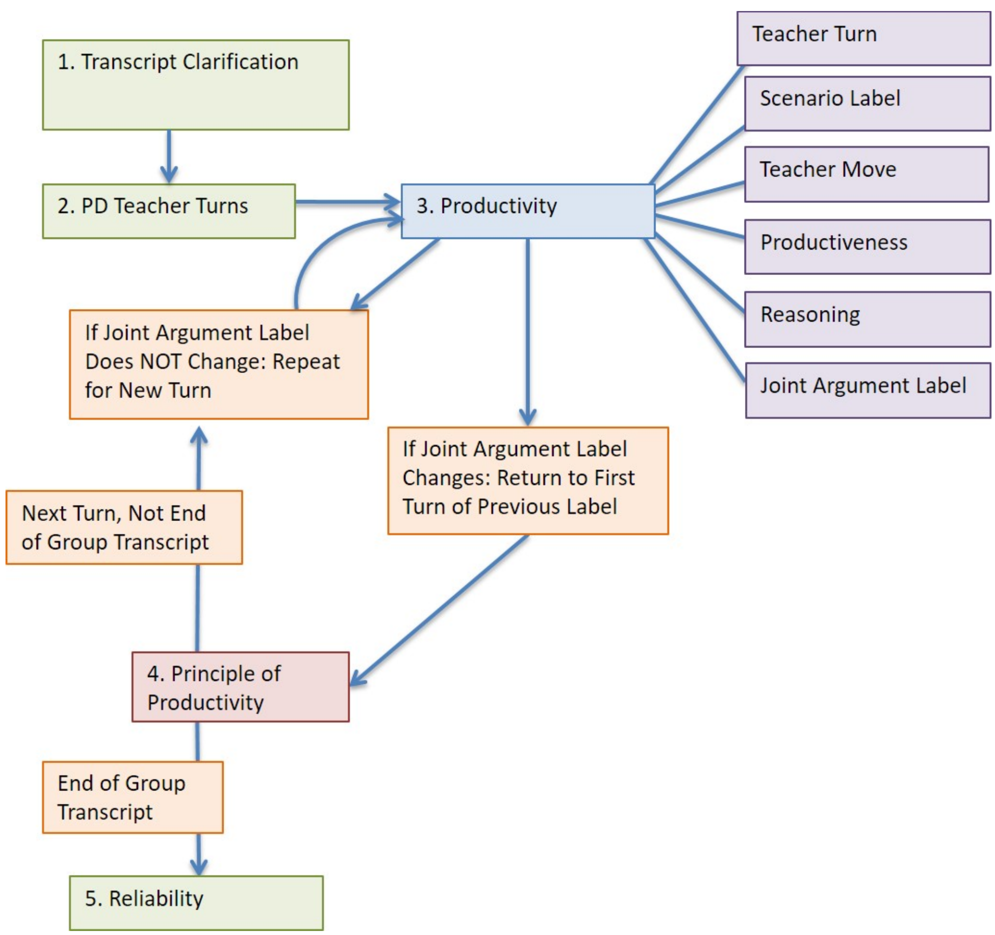



*Figure 2.* The steps of my coding process.

I organized and coded all my data within a document laid out as in Figure 3. The PD Teacher Turns column is where I put the clarified transcript (coding step 1), where each row represents a new PD teacher turn taken from the transcript of their discussions (coding step 2). The Scenario Label is where I marked which scenarios were discussed within each PD teacher turn. From there the categories capture coding steps three through five, where the resulting codes or sentences were put in the cells for the respective category (e.g., “What does the x+5 represent?” under Teacher Turn). If there was more than one code that belonged in a coding category for a PD teacher turn, then I created new lines of cells below for the same PD teacher turn where any new additional codes were coded. In the rest of this section I describe in detail what each coding step entails and then conclude by illustrating this process by applying the coding steps to an example.

| | | | Scenario Label | | | | | | | | | | Productivity | | | | | | | Principle |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | Reasoning for Productiveness | | | | |
| PD Teacher Turns | PD Teacher Turns Label | Joint Argument Label | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Turn being Discussed | Teacher Move | Productiveness | PD Teacher's Statement | Summary of Statement Reasoning | Statement Code | | Principle of productivity |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | |

*Figure 3.* The layout of my data input sheet.

**Description of Coding Steps**

In this section I detail what I did during each of the five steps of my coding process: transcript clarification, PD teacher turns, productivity, principle of productivity, and reliability.

**Step 1: Transcript clarification.** The first two steps, transcript clarification and PD teacher turns, prepared the way for me to code the data. Written transcripts of conversations are incomplete, as people gesture to things in their environment or use imprecise language (e.g., pronouns with unclear referents). As a result, since I coded the teacher turns from the transcripts, it was necessary to identify and clarify any references to scenarios or use of pronouns. When it was necessary to fill in missing words or to clarify referential terms (e.g., what *it* refers to), I recorded this information in single square brackets (as done by Herbel-Eisenmann & Otten, 2011). Whenever a PD teacher gestured or pointed, these actions were described in double square brackets (e.g., [[pointing to Scenario 5]]). In order to add these details to the transcript, I watched the video for gestures or references to specific scenarios, frames, or scenario turns, and referenced the PD teachers' scenario notes. I did this clarification within the original transcript files. Once the transcripts were clarified, I organized the PD teacher turns in a spreadsheet (see Figure 3).

**Step 2: PD Teacher turns.** Once the clarified transcripts were organized by turn in a spreadsheet, I identified which PD teacher turns would be coded. For instance, PD teacher turns such as "hmmm" or "yeah", when they served no furthering purpose for the conversation, were not coded. Each substantive turn served as my primary unit of analysis for coding step 3. The coding for each PD teacher turn was in the context of the PD teacher turns preceding it within the group conversation such that each PD teacher turn was seen as including the previous PD teacher turns. Additionally, I labeled each turn by the group the PD teacher turns were from as well as the number the turn was in the transcript so there was a system of keeping track of when in the conversation each turn took place. Each of these turns was labeled by a 1000's place number corresponding to a particular group, with the rest of the number indicating the turn number within the group (e.g., 1012 for the $12^{th}$ PD teacher turn of the first group I coded).

**Step 3: Productivity.** This step consisted of identifying the teacher turn that was the object of a PD teacher's conversation, as well as classifying the intended purpose of that response in conjunction with the respective productivity and reasoning. I gathered the information needed for this analysis in four steps: (a) identified the teacher turn being discussed, (b) classified the productivity of the teacher move, (c) captured the PD teacher's reasoning about the productivity of the teacher move, and (d) labeled the joint arguments.

First, I identified the teacher turn(s) being discussed by the PD teachers within a PD teacher turn and coded the teacher move (e.g., clarifying, validating) for each respective teacher turn based on how the PD teachers discussed each teacher turn. The teacher turn captured the literal speech or actions of either the scenario teacher (i.e., "What is similar or different between those examples?") or a PD teacher (e.g., when a PD teacher suggested an alternate scenario teacher turn). The teacher move referred to what the PD teachers thought the scenario or alternate scenario teacher turn would accomplish (e.g., clarifying, validating). Since each teacher turn could contain multiple moves, I coded and referred mainly to the teacher move classification for discussing and generalizing productivity. In order to code the teacher move, I drew on the language from the *building* and TRC frameworks (Table 2). For example, when the PD teachers discussed the need to clarify what a student said, I checked that they were discussing "clarifying" in the same way as the TRC defines it. If the PD teacher's conception of clarify matched the TRC's definition of clarifying, then I used the word "clarify" in my coding. By making sure my codes matched the framework language, when applicable, I was able to be more consistent throughout my codes. If the language in either of these frameworks did not accurately match what the teachers were saying, then I used an open coding scheme. For example, the PD teachers discussed how some questions scaffolded the students, an idea that the *building* and TRC framework do not capture, so I created a teacher move code called "Scaffolding."

Table 2

*Descriptions of Teacher Moves (Peterson et al., 2017)*

| Teacher Move | Description |
|---|---|
| **Adjourn** | The teacher either explicitly or implicitly indicates that the instance(s) will not be considered publicly at that time, but suggests the instance may be considered later. |
| **Allow** | The teacher invites or leaves space for students to respond to the instance. |
| **Check- in** | The teacher elicits students' self-assessment of their reaction to or understanding of the instance. |
| **Clarify** | The teacher seeks to make the instance precise. |
| **Collect** | The teacher requests or provides additional ideas, methods, or solutions. |
| **Connect** | The teacher asks for or makes a connection between or among representations, methods/strategies, solutions, or ideas that includes the instance. |
| **Correct** | The teacher describes or asks for a correct way of approaching, or thinking about, the instance. |
| **Develop** | The teacher provides or asks for an expansion of the instance that goes beyond a simple clarification. |
| **Dismiss** | The teacher either explicitly or implicitly indicates that the instance(s) will not be considered publicly. |
| **Evaluate** | The teacher asks for or provides a determination of the correctness of the instance. |
| **Justify** | The teacher asks for or provides a justification of the instance. |
| **Literal** | The teacher asks for or provides brief factual information related to the instance. |
| **Repeat** | The teacher (verbally or in writing) repeats or rephrases the instance without changing the meaning or asks a student to repeat the instance. |
| **Validate** | The teacher says something about the instance to affirm its value and/or encourage student participation (e.g., thank you, good). |

Second, I identified whether the PD teacher said the teacher move was productive or unproductive. I used the current PD teacher turn and the preceding PD teachers' discussion to infer the productiveness of a teacher move that was the object of the PD teacher's consideration.

If within a PD teacher turn the PD teacher did not imply a particular move was productive or unproductive, then the productiveness of the response was coded *cannot infer.*

Third, if the PD teachers gave any support or explanation for why a teacher move was more or less productive, then in order to capture and synthesize this reasoning I a) recorded the PD teacher's statements they gave as reasoning, b) captured the essence of that reasoning in a sentence or two, and c) captured the main ideas of their reasoning in one or more codes. Before I tried to summarize the reasoning, I recorded the PD teacher's literal statements they gave as the reason for the productiveness of a teacher move. From these statements, I was able to summarize the PD teacher's reasoning. To provide consistent language to synthesize what the PD teachers were saying, I again drew on the language of the TRC. Summarizing the PD teacher's reasoning for the productivity of a teacher move allowed me to synthesize what made the PD teachers view a particular move as more or less productive. Once I had my summary of the PD teacher's reasoning for the productiveness of a teacher move, I did an open coding on the summary to capture the essence of the PD teacher's reasoning. These codes allowed me to see what themes seemed to arise from the data. To simplify these codes, I tried to have the code itself be unbiased as to the productivity and added the tag "– Present" or "– Absent" as needed. For instance, if the PD teachers discussed how a teacher did not clarify student thinking I would apply the code "Clarify Student Thinking" and add the tag "-Absent".

Fourth, I identified and labeled the joint arguments. As I coded each PD teacher turn, I decided if the current PD teacher turn was contributing to and building on a joint argument formed through the PD teacher discussion. Each time a PD teacher turn built on the joint argument, the PD teacher turn received the same numerical label. If a PD teacher turn changed topics, then that turn and any subsequent turn that built on the same argument received the same new numerical label (e.g., if the first joint argument was labeled "1", the next joint argument would be labeled "2"). Each joint argument consisted of at least three PD teacher turns and every teacher turn was a part of a joint argument.

**Step 4: Principle of Productivity.** The final step within the coding of teacher turns was to articulate the principle(s) of productivity. A principle of productivity is an articulation, in general terms, of the main ideas and conclusions of a collection of PD teacher turns with regards to the productivity of teacher moves. I articulated the principle(s) of productivity at the end of a joint argument because a joint argument includes all the teacher turns in a sequence that are discussing the same related ideas, which would result in a principle of productivity around those related ideas. By waiting until the end of a joint argument to articulate the principle(s) of productivity, I was able to draw on many ideas that built off one another to articulate principle(s) of productivity that were as accurate to the PD teachers' discussions as possible. It was possible to have multiple principles of productivity within a joint argument because even though the conversation within a joint argument would be related, the reasons for productivity might have varied. I drew on all of the Productivity coding to capture as accurately as possible the PD teachers' general ideas of productivity. Capturing this content allowed me to account for the PD teachers' main ideas surrounding productivity, and helped me to look at how the PD teachers talked about different teacher moves being productive. In order to create a common language for these principles, I again drew on the teacher move language of the Teacher Response Coding (TRC) (Peterson et al., 2017) (recall Table 2). Once the principle(s) of productivity had been articulated for a given joint argument, I continued on to code the next PD teacher turn that was part of a new joint argument.

**Step 5: Reliability.** The final step within my coding process was checking the consistency and reliability of my codes. Once I finished coding all the PD teacher turns for a particular group, I compared the codes for the last several PD teacher turns in that group to the codes of the first several PD teacher turns within that group. In comparing the codes from when I started coding a group to how I was coding when I finished coding a group, I was checking whether or not I was consistent in the use of the codes. Thus, I compared these codes to check the consistency of my coding. If the codes were not consistent between the first and last codes, then I adjusted the codes that seemed to have strayed from the rest. Once my codes within a group were consistent (i.e., the same PD teacher turns at the beginning received the same codes as if it were coded at the end of the group's PD teacher turns), I moved on to code another group. I performed the same consistency check within that second group, and then performed a consistency check between the first and second groups' codes. To do this check between the groups, I compared the coding of the last several PD teacher turns in the second group to the coding of the first several PD teacher turns in the initial group. I continued this pattern of checking consistency within and between subsequent groups in order to establish consistency and reliability in my coding.

An example of when checking the reliability in my codes refined my coding relates to how I coded general teacher moves. I began by coding teacher moves that the PD teachers did not assign any particular purpose to as "generic". However, as I coded, this code developed into "General – 1 Move" and "General – Multi Move" to capture when teachers were talking generally about one move or generally about multiple moves at once. When I compared the last codes to the first codes in my spreadsheet, I realized that I needed to adjust the codes to be consistent. As a result, I recoded the original "generic" teacher move codes to the appropriate new general code.

**Sample Application of Coding Process**

In this section I illustrate how I applied steps one through four to the transcript of a group's discussion. (Step five is not modeled here as it is not applicable to such a small sample.**)** The following excerpt (Figure 4) is taken from one group's discussion at the end of a joint argument in which they compared the first two frames in Scenarios 1, 2, and 3 (see Appendix B) in order to evaluate the productivity of the teacher moves in Scenario 3.

> Sarah: Because if you look at the difference in scenario one and scenario two, this one, "Interesting, what's similar or different in the expressions?" And then this is where it fell apart. I think if they had understood more of what this represented from the first frame—
>
> Hillary: Yeah I think the teacher is assuming that they already get this.
>
> Sarah: Right, and in this class he asked it too and the kids understood it but not in this one. So in this one it was maintained without asking, just because of the group of kids, but in this one it wasn't maintained without asking it. Then the whole rest of the conversation fell apart.
>
> Hillary: Right. So here being able to clarify that the students really do understand what that represents then you are able to have this productive conversation.

*Figure 4. Transcript excerpt from one group's discussion.*

**Step 1: Transcript Clarification.** This first step of *transcript clarification* took place on the entire transcript for the group containing Hillary and Sarah, and I give the non-clarified excerpt (see Figure 4) as an example of what the original transcript looked like. The original transcript excerpt in Figure 4 contained several cases when the PD teachers used vague language (e.g., "it", "that"), as well as times where they either referenced or gestured to items not clear in the transcript (e.g., "this class," "this one"). In order to create a more actuate representation of the PD teachers' discussion (see Figure 5), I watched the video and looked at the PD teachers' scenario notes to infer the missing or unclear information.

[Turn 1] Sarah: Because if you look at the difference in scenario one [[points to Scenario 1]] and scenario two [[points to Scenario 2]], this one [[points to and makes a circle gesture around the teacher question in Frame 1 of Scenario 2]], "Interesting, what's similar or different in the expressions?" And then this [[points to Frame 2 in Scenario 2]] is where it [the discussion] fell apart. I think [[pointing to Frame 1 in Scenario 3]] if they [the students] had understood more of what this [the expression $x+5$] represented from the first frame [in Scenario 3]—

[Turn 2] Hillary: Yeah, I think the teacher is assuming [[points to Scenario 2]] that they [the students] already get this [[pointing to first frame discussion of the expression x+5 in Scenario 3]].

[Turn 3] Sarah: Right, and in this class [[gesturing to Scenario 1]] he asked it ["Interesting, what's similar or different in the expressions?"] too and the kids understood it ["Interesting, what's similar or different in the expressions?"], but not in this one [[points to Scenario 2]]. So in this one [[ gestures to Scenario 1]] it [a productive discussion] was maintained without asking ["what does the x+5 represent?" from Scenario 3], just because of the group of kids, but in this one [[gestures to Scenario 2]] it [a productive discussion] wasn't maintained without asking it ["what does the x+5 represent?" from Scenario 3]. Then the whole rest of the conversation fell apart [[makes a sweeping motion over Frame 2 through 5 of Scenario 2]].

[Turn 4] Hillary: Right. So here [[points to Frame 1 in Scenario 3]], being able to clarify that the students really do understand what [[points to discussion of x+5]] that [expression $x+5$] represents, then you are able to have [[makes a sweeping motion over Frame 2 through 5 of Scenario 3]] this productive conversation [in Scenario 3 from Frame 2 through Frame 5]."

*Figure 5.* Transcript excerpt from one group's discussion with clarifications.

**Step 2: Teacher turns.** I organized these PD teacher turns into the coding document and none of them were considered irrelevant. The group Sarah and Hillary were in was the first group I coded, and I labeled the group accordingly. I then labeled each turn by the number it was in the whole transcript (see Figure 6). The PD teacher turns in the example are the ending portion of a larger discussion leading up to the conclusion reached in these four PD teacher turns, so I drew on the preceding PD teacher turn coding as needed to provide context. For simplicity in describing my analysis of the sample excerpt, I refer to the PD teacher turns in the excerpt as PD Teacher Turns 1-4 rather than Turns 76-79. Also, because the clarification step made it clear which scenarios were being discussed during each turn, I was able mark the appropriate scenarios in the Scenario Label section.

| | | | Scenario Label | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PD Teacher Turns | PD Teacher Turns Label | Joint Argument Label | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Sarah: Because if you look at the difference in scenario one [[points to Scenario 1]] and scenario two [[points to Scenario 2]], this one [[points to and makes a circle gesture around the teacher question in Frame 1 of Scenario 2]], "Interesting, what's similar or different in the expressions?" And then this [[points to Frame 2 in Scenario 2]] is where it [the discussion] fell apart. I think [[pointing to Frame 1 in Scenario 3]] if they [the students] had understood more of what this [the expression $x+5$] represented from the first frame [in Scenario 3]— | 1076 | 5 | X | X | X | | | | | | | |
| Hillary: Yeah, I think the teacher is assuming [[points to Scenario 2]] that they [the students] already get this [[pointing to first frame discussion of the expression x+5 in Scenario 3]]. | 1077 | 5 | | X | X | | | | | | | |
| Sarah: Right, and in this class [[gesturing to Scenario 1]] he asked it ["Interesting, what's similar or different in the expressions?"] too and the kids understood it ["Interesting, what's similar or different in the expressions?"], but not in this one [[points to Scenario 2]]. So in this one [[ gestures to Scenario 1]] it [a productive discussion] was maintained without asking ["what does the x+5 represent?" from Scenario 3], just because of the group of kids, but in this one [[gestures to Scenario 2]] it [a productive discussion] wasn't maintained without asking it ["what does the x+5 represent?" from Scenario 3]. Then the whole rest of the conversation fell apart [[makes a sweeping motion over Frame 2 through 5 of Scenario 2]]. | 1078 | 5 | X | X | X | | | | | | | |
| Hillary: Right. So here [[points to Frame 1 in Scenario 3]], being able to clarify that the students really do understand what [[points to discussion of x+5]] that [expression $x+5$] represents, then you are able to have [[makes a sweeping motion over Frame 2 through 5 of Scenario 3]] this productive conversation [in Scneario 3 from Frame 2 through Frame 5]." | 1079 | 5 | X | | X | | | | | | | |

*Figure 6. Coding of Teacher Turn in the document with additional Joint Argument Label as it fits in the whole group transcript.*

**Step 3: Productivity.** For PD Teacher Turn 1, I first identified the teacher turn the PD teacher was discussing: "Interesting, what's similar or different in the expressions?" (referred to hereafter as "turn A") from scenarios 1, 2, and 3. The PD teacher was not giving insight into the nature of turn A other than that the scenario students were not ready for the question. Thus, turn

A was given a code “Generic – 1 Move” [1] because I could not infer what the PD teacher thought it should accomplish. Next, I inferred the productivity of turn A as *not productive.* Sarah explained why turn A was unproductive:

> And then this [Scenario 2, Frame 2] [[points to Scenario 2, Frame 2, the one after response (1)]] is where it [the discussion] fell apart. I think [[pointing to Scenario 3, Frame 1]] if they [the students] had understood more of what this [the expression $x$+5] represented from the first frame [in Scenario 3]—.

To synthesize, Sarah said the discussion “fell apart” after the scenario teacher asked “Interesting, what’s similar or different in the expressions?” because the scenario teacher assumed the scenario students understood the expression $x + 5$. I summarized this reasoning as, “Asking a follow-up question on a topic for which students are ill-prepared is unproductive”, which led to the reasoning code of “Student Preparedness - Absent”. This PD teacher turn built on the same joint argument as the PD teacher turns before it (that are part of the larger transcript, but not the excerpt), so this PD teacher turn received the Joint Argument Label was “5” (as it was part of the fifth joint argument in the group’s discussion, recall Figure 6).

For PD Teacher Turn 2, Hillary discussed the same turn A as in PD Teacher Turn 1: “Interesting, what’s similar or different in the expressions?” Since each PD teacher turn built off of one another and comprised the same joint argument, I could draw on conclusions made in previous PD teacher turns. In PD Teacher Turn 2, Hillary was building on the statements in PD Teacher Turn 1, so similarly, turn A was coded as “General –1 Move” for teacher move and as

[1] The distinction of “1 Move” for the general move is significant because teachers would also talk about a general sequence of moves instead of specifically one, which would be coded as “General – Multi Move”.

*not productive* for productivity. The entire PD teacher turn was Hillary's support for the unproductivity: "Yeah, I think the teacher is assuming [[points to Scenario 2]] that they [the students] already get this [what the x+5 represents] [[pointing to discussion of $x + 5$ in Scenario 3, Frame 1]]." To synthesize, Hillary agreed that turn A caused the discussion to fall apart because the scenario teacher was assuming the scenario students understood the expression $x + 5$. I summarized the PD teacher's reasoning as "Teacher assuming student understanding is unproductive", which I coded as "Clarify Student Understanding – Absent". Since this turn built on the argument started in PD Teacher Turn 1, this turn was also part of the fifth joint argument and received the same Joint Argument Label "5".

For PD Teacher Turn 3, Sarah discussed turn A from scenario 1, 2, and 3 and "What does the x+5 represent?" (referred to hereafter as "turn B") from Scenario 3. The PD teacher was not giving insight into the nature of turn A other than that the scenario teacher turn helped to think about the need to clarify what $x + 5$ represented; in other words, the PD teacher was considering turn A in the context of the absence or presence of turn B. While there was a lack of information on what purpose Sarah thought turn A served, she did discuss how turn B helped to clarify. Thus, turn A was again coded as "General –1 Move" and turn B was coded as "Clarify". Turn A was coded as *not productive* and turn B was coded as *productive.* In support of the codes for turns A and B, the PD teacher said,

> he asked it ["Interesting, what's similar or different in the expressions?"] too and the kids understood it ["Interesting, what's similar or different in the expressions?"], but not in this one [[points to Scenario 2]] …in this one [Scenario 2] [[gestures to Scenario 2]] it [a productive discussion] wasn't maintained without asking it ["what does the x+5

represent?" from Scenario 3] …Then the whole rest of the conversation fell apart [[makes a sweeping motion over Frame 2 through 5 of Scenario 2]].

To synthesize, Sarah said a productive discussion depended on what the scenario students understood; when the scenario students did not understand x+5, the scenario teacher needed to ask "What does x+5 represent?" before asking "Interesting, what's similar or different in the expressions?" in order for the discussion to not fall apart. I summarized this PD teacher's reasoning for turn A as "Teacher assuming student understanding is unproductive" with a code of "Clarify Student Understanding – Absent". For turn B the summary was "Asking a question to clarify student understanding is productive" with a code of "Clarify Student Understanding – Present". As this PD teacher turn built on the arguments in PD Teacher Turn 1 and 2, this PD teacher turn received the same Joint Argument Label "5" since it was also part of the fifth joint argument.

For PD Teacher Turn 4, Hillary discussed turn B: "What does the x+5 represent?". In this PD teacher turn, Hillary explicitly referred to the need to clarify what the scenario students understood as being the purpose for turn B. Thus, the teacher move was coded as "clarify". The teacher turn was coded *productive.* Hillary stated,

> being able to clarify that the students really do understand what [[points to discussion of $x + 5$]] that [expression $x + 5$] represents, then you are able to have [[makes a sweeping motion over Frame 2 through 5 of Scenario 3]] this productive conversation [in Scenario 3 from Frame 2 through Frame 5].

To synthesize, Hillary said that clarifying, rather than assuming, what the scenario students understood in relation to the x+5 allowed for a productive discussion. The summary of statement reasoning relating to turn B was the same as in PD Teacher Turn 3 ("Asking a question to clarify student understanding is productive") with the same code ("Clarify Student Understanding – Present"). This PD teacher turn continued to build on the arguments before it, which were part of the fifth joint argument, so this PD teacher turn also received the same Joint Argument Label "5". Thus, each PD teacher turn in this related conversation (consisting of the four example PD teacher turns, as well as the turns before it in the larger transcript) were given the same Joint Argument Label of "5". However, in the context of the entire transcript for this group, the next PD teacher turn began a new argument, which resulted in a new Joint Argument Label "6". Since the next PD teacher turn had a new joint argument label, I returned to the last PD teacher turn in the joint argument with label "5" and articulated the principle(s) of productivity for the fifth joint argument.

**Step 4: Principle of Productivity.** For this step, I articulated the principles of productivity for the PD teacher turns that were all building on the same argument (i.e., had *joint argument label* "5"). Since the principles of productivity are designed to capture the main ideas of productivity in a whole joint argument, the principles here were articulated in the context of the entire joint argument and only represent the principles from the excerpted portion. One of the principles of productivity for these PD teacher turns, with the same label and in the same group, was "It is productive to ask a clarifying question to allow the students to clarify their thinking (not what they understand, but what they have said)". The word "clarify" is maintained from their wording since the PD teachers used it to imply that the scenario teacher should have had a teacher move or moves focused on clarifying the scenario student contribution to ensure that the scenario teacher and the scenario class understood what the scenario student has said before moving on. This use of the word matches with the definition of *clarify* in the TRC. Another principle that arose from analysis of this excerpt was, "It is unproductive to assume student understanding”. To articulate this principle, I drew on the Summary of Statement Reasoning from Turn 2 and Turn 3 where the PD teachers talked about how the scenario teacher was assuming student understanding. The third principle that could be articulated from this joint argument came from a collection of turns only one of which (Turn 1) appears in this excerpt. This principle dealt with the PD teacher’s concern that the scenario teacher was asking a question about comparing expressions when the scenario students did not yet understand the expression itself: It is unproductive to ask a follow-up question on a topic for which students are ill-prepared.

**Analysis of Principles of Productivity**

Once I had coded each of the six groups in my study following the above steps, I needed to clean up the principles of productivity among all the groups since the principles were not yet unique enough to report. After the repeated and similar principles were condensed, I coded the principles according to their alignment with the prerequisites and sub-practices of building, and according to themes that arose from within the collection of principles.

**Clean-up of the principles of productivity.** I gathered the 148 principles of productivity from across all the groups into a list. Within this list I labeled each principle by their group number and gave them a consecutive number 1 through 148 in order to keep track of the principles as I worked with them. Having the principles together allowed me to look at similarities and differences between all the principles from across the groups. Whenever a set of principles was trying to communicate the same idea, I picked the principle that was worded most clearly, made sure it captured the essence of each of the principles, and copied the result onto the old principle. This process created a list of principles of productivity that contained repeated principles. In order to not lose the significance of an idea from the groups it arose in, but also to make the list of principles more manageable, I labeled the repeated principles according to the group it came from as well as the number of occurrences. For instance, Group 5 had a specific principle of productivity occur seven times throughout their discussion. Since the principle was not found in other groups, only one group number and corresponding number of occurrences was needed (see Figure 7).

| Principle of Productivity | G (group #) - (# of occurrences) |
|---|---|
| It is unproductive to introduce mathematical ideas without accompanying meaning and underlying reasoning. | G5-7 |

*Figure 7.* Cleaned up of the Repeated Principles of Productivity with Example.

Once I addressed explicit repetition in the principles, I looked for principles that articulated similar ideas without using identical language. In order to get a better grasp of what the PD teachers focused on with regards to productivity, I wanted to make sure that the principles portrayed unique ideas. To narrow down the principles to unique ideas, I began by sorting the principles into broad categories that resembled the Statement Codes used to capture the Summary of Statement Reasoning in the initial coding (Step 3) of the data. Then, within each category I looked for principles that were similar. To me, similarity meant that principles were trying to communicate the same ideas with insignificant word variances, but variances that kept the principles from being identified as repeats. If the word differences were not significant enough to change the meaning of the principles, then I chose the wording that was most accurate to the PD teachers' ideas and was worded more clearly. Then, using the same notation for repeated principles, these similar principles were collapsed as well. Sometimes similar principles within one category had variations that seemed important. In order to not lose these differences, but to clean up the wording, I made sure these similar principles had parallel wording and structure so I could be consistent and also notice differences easily.

As an example of collapsing similar principles, principle #54 from Group 4 and principle #133 from Group 6 had one phrase variation (underlined in Figure 8). To decide if this wording was significant, I went back to the original PD teacher turns where the discussion took place that resulted in each of these principles. Although Group 6 used the word "re-summarize" and Group 4 did not, Group 4's appreciation of the scenario teacher response had implied that the scenario teacher did more than just add the vocabulary. The scenario teacher did so while drawing on the scenario student's ideas from the summary, or in other words, re-summarizing the scenario student's ideas in the summary. In order to capture this idea of the scenario teacher "re-summarizing" what the scenario student had said with added vocabulary, these principles were combined into the following principle: "It is productive to have students summarize and connect ideas from a discussion and then for the teacher to just re-summarize and add vocabulary to the students' summary" with the corresponding information of occurrence, "G4-1, G6-1".

| | |
|---|---|
| #54 | It is productive for a teacher to have a student summarize ideas from a discussion and then for the teacher just to add the vocabulary to the student's summary. |
| #133 | It is productive for a teacher to have a student summarize ideas from a discussion, and then for the teacher just to <u>re-summarize and</u> add the vocabulary to the student's summary. |

*Figure 8.* Sample of Two Similar Principles of Productivity.

During the "clean up" phase for all the principles of productivity, I checked the wording of principles to make them clear and logically sound. This cleaned-up list of principles consisted of 65 principles in total. Although some of the principles in this list of 65 had similar ideas (as can be seen in my discussion of themes that arose from the principles in the next section), these ideas were different enough that they could not be combined while staying true to the PD teachers' original discussions and ideas of productivity. Thus, I did not combine the principles of productivity any further and the final list of 65 unique principles of productivity was what I used for further analysis.

**Coding the principles of productivity.** I coded this cleaned-up and simplified list of 65 principles for themes that I noticed within the principles and for alignment with the sub-practices of building. The themes stemmed from the initial categorizing of the principles that I used to help notice similar principles, and these categories were grouped into broader themes in order to try and capture each principle within a larger theme. I adapted questions from Stockero et al. (2018) (see Figure 9) in order to determine whether there was evidence that the PD teachers' ideas, captured in the principles of productivity, directly aligned with the sub-practices of *building*. Thus, a principle of productivity was said to align with a sub-practice if the answer to the question for that sub-practice in Figure 9 was "yes". If a principle of productivity did not contain the ideas necessary for alignment with any sub-practice of building (i.e., the answer to every question was "no"), the principle of productivity was said to not align with building. Saying that a principle of productivity did not align with building, however, is not to say that it misaligned with building. Principles of productivity could contain ideas important to building while not relating directly to a sub-practice of building. I recorded the coding of the list of principles of productivity for the theme(s) each principle encapsulated and for alignment with the sub-practices of *building* in a spreadsheet (Figure 10) since each principle could relate to multiple themes and/or multiple sub-practices of building.

| | | |
|---|---|---|
| **Sub-practices** | **Invite or Allow** | …do the teachers say or imply that the student mathematics of the scenario instance needs to be made public by inviting or allowing the student to share their thinking? |
| | **Recognize MOST** | … do the teachers say or imply that one should consider whether the student mathematics of the scenario instance would be classified as a MOST? |
| | **Make Precise** | ... do the teachers say or imply that one should consider whether the student mathematics of the scenario instance would need to be clarified? |
| | **Grapple Toss** | ... do the teachers say or imply that one should consider whether the student mathematics of the instance should be turned over to the class for consideration in a way that necessitates sense making of the idea? |
| | **Orchestrate** | ... do the teachers say or imply that class discussion should be directed towards making sense of the student mathematics of the instance? |
| | **Make Explicit** | ... do the teachers say or imply that the mathematical idea underlying the student mathematics of the instance would need to be made explicit? |

*Figure 9.* Guiding questions for determining the alignment of principles of productivity with the building-related constructs (adapted from Stockero et al., 2018).

| | | **Prerequistis and Sub-practices** | | | | | | | **Themes** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Principle of Productivity** | **Number of Group Occurrences** | Invite or Allow | Recognize MOST | Make Precise | Grapple Toss | Orchestrate | Make Explicit | Not Aligned with Sub-Practices | Norms | Focus on Student's Mathematics | Teacher Moves (Redirect, Follow-up, Elicit, Clarify, Questions) | Reasoning/ Meaning | Summary | Mathematics Focus (Math Goal/Resolve) | Enagement | Confusion/ Misconception | Other |
| summary is not new information. | | | | | | | X | | | | | | X | | | | |
| It is productive to summarize the discussion. | G4-1, G6-1 | | | | | | X | | | | | | X | | | | |
| It is productive to (pursue and) reach a mathematical goal of the task. (It is unproductive to not reach nor pursue a mathematical goal of the task.) | G2-2,G3-3, G5-4, G6-2 | | | | | | | X | | | | | | X | | | |
| It is productive to ask a clarifying question to allow students to clarify their thinking, especially when incomplete student thinking is on the table, in preparation for the class to consider that thinking. | G1-1, G3-1, G4-1, G5-1 | | | X | X | | | | | X | X | | | | | | |
| It is productive to have a class conversation that relates to or develops an idea that is central to the mathematical goal of the task. | G4-1, G5-1 | | | | | | | X | | | | | | X | | | |
| It is unproductive to ask a question with an obvious answer which doesn't lead the class toward the mathematical goal of the task or leads them to incorrect mathematics. | G1-1 | | | | | | | X | | | X | | | X | | X | |
| It is productive to have students summarize and connect ideas from a discussion and then for the teacher to just re-summarize and add terminology to the students' summary. | G1-1, G2-1, G4-1, G6-2 | | | | | | X | | | X | | | X | | | | |

*Figure 10.* Screenshot of Coding for Sub-practices of building and Themes.

The result of this analysis was a collection of principles each of which aligned with one or more of the themes. In my results I discuss the principles of productivity according to each theme, thus various aspects of a given principle may be discussed in relation to multiple themes. In order to present the list of principles of productivity in a manageable way, however, I placed each principle into a single theme. I sorted the principles based on which theme seemed to capture the essence of each principle the closest (see Appendix D). Since the theme of Focus on Student Mathematics was the broadest, it was the last theme into which the principles were sorted.

# CHAPTER FIVE: RESULTS

My first research question deals with what principles of productivity emerged from the PD teachers' discussion relating to the productivity of teacher moves in response to MOSTs. When taken as a whole, the collection of principles answers the first research question. The second research question deals specifically with how the principles of productivity align with the conceptualization of building. Almost half of the principles aligned with at least one sub-practice of building. To answer these questions, I first talk about the 65 principles of productivity with respect to the themes that emerged from the principles and then with respect to the framework of building I placed on them.

## Themes Emerging from Principles of Productivity

Three main themes described a majority of the principles of productivity[2]. The most common theme was a focus on student mathematical thinking. The next most common theme was about the productivity of a variety of teacher moves that cut across aspects of class discussion. The principles related to the third theme communicate the importance of considering the mathematics when considering the productivity of a teacher move.

### Focus on Student Mathematical Thinking

These principles of productivity (28 of 65 principles) articulate what a teacher should and should not do in order to effectively focus discussion on student mathematical thinking. One principle in particular captures the overall idea of this theme: It is productive to honor student thinking. In order to honor and focus on student thinking, these principles suggest that (1) the

[2] These most common themes had between 15 and 28 principles. The five other themes that emerged (Reasoning/Meaning, Misconception/Confusion, Engagement, Norms, Other) each captured 5 principles or fewer and thus are not discussed here.

ideas of discussion should come from the students, (2) students should be engaged in making sense of those ideas, and (3) the mathematics underlying the students' ideas should guide the discussion and summary.

**(1) Ideas come from the students.** According to the PD teachers, the discussion ideas should come from the students and not from the teacher. The students should be the ones to provide the mathematical ideas for discussion, and it is the teacher's duty to take the time to thoroughly understand the student thinking behind the shared ideas. The teacher should avoid providing their own interpretations of what a student meant to say and should instead ask clarifying questions that allow students to clarify their own thinking. Having the original ideas and clarifications come from the students is what focuses the discussion on student mathematical thinking. Furthermore, the teacher should not provide the connections between ideas. When a teacher does provide information, such as definitions or connecting ideas, they should draw on student thinking to connect the student mathematics to the definition or the idea that is being presented. Above all, the discussion and summary, including the resolution of ideas, should not be teacher-centered, or dominated by the teacher, since the focus should always be on the student mathematical thinking.

Additionally, the PD teachers felt that teachers should not limit the type of student thinking that could be brought out in the discussion. By only focusing on one solution, one student's idea, or only the right answer, the teacher unproductively limits the mathematics the students can explore. Also, limiting the ability of students to contribute ideas because a teacher cut off discussion to take over the thinking was similarly viewed as unproductive.

**(2) Students make sense of ideas.** Once student ideas are made public and clarified, the PD teachers expressed that the students should be the ones making sense of the mathematics.

Teachers should allow the students to explore, develop, and create their own understandings and meanings of the mathematics before giving them standardized methods and ideas, thus positioning students as the sense makers, with their thinking at the forefront. Teachers can maintain this positioning by not asking leading questions or jumping to ideas the students are not prepared for. These unproductive actions put the teacher in a sense-making situation rather than the students. This focus on student sense-making is also seen in the PD teachers viewing it as productive to have students (as opposed to the teacher) summarize the discussion. In this way the student thinking from class is prevalent and students are the ones connecting and making sense of the ideas as they summarize.

**(3) Students' mathematics guides the class.** The mathematics underlying students' shared ideas should guide the discussion and summary. The PD teachers talked about the importance of a discussion and a summary being focused on the mathematics that could be brought out from the student thinking. A teacher needs to decide which mathematics to pursue since that mathematics will affect the direction of the discussion and the content of the summary. This decision needs to balance attention to all student thinking while only pursuing student mathematics that is productive to pursue.

If a teacher summarizes a discussion or re-summarizes a student summary, the teacher's summary should refer to the student thinking and be focused on the related mathematics. The resolution of the mathematics, often within the summary at the end of a discussion, should be a resolution of the students' ideas. Many principles that focused on students' mathematics highlight the PD teachers' decisions that the discussion and the summary should be centered around and driven by student thinking and ideas.

In summary, the PD teachers' discussions led to many principles discussing the importance of focusing on students' mathematics, with detailed ideas of how to aid in that focus and what to avoid doing in order to keep that focus. The principles collectively provide a vision of what the PD teachers thought was productive with respect to focusing on student mathematics. From these principles I was able to synthesize the principles into what I will call a meta-principles, which captures the essence of all the PD teachers' ideas for this theme: It is productive to allow students to come up with the mathematical ideas in the discussion such that the students are able to make sense of these ideas and in such a way that the focus of the discussion and summary is the students' mathematical thinking (rather than the teacher's mathematical thinking).

**Teacher moves**

The principles (21 of 65 principles) around this theme give insight into what the PD teachers thought was productive and unproductive about a variety of teacher moves. I look at the most frequently discussed teacher moves of eliciting and following up on student thinking (6 of 21 teacher moves focused principles), and asking open (as opposed to closed) questions (4 of 21 teacher moves focused principles) followed by various other teacher moves the PD teachers mentioned.

The PD teachers talked about the need to both elicit and follow up on student thinking. A particular principle captures the related nature between these two teacher moves: It is productive when a teacher response elicits valuable student thinking and/or more information from the students. Thus the PD teachers felt that it is important to invite students to share their thinking and then utilize and follow-up on that student thinking. This follow-up move takes precedence over validation and can take the form of a generic follow-up move or having students expand on their original student turn. However, a teacher should not ask a follow-up question if the students do not yet understand the topic and are ill-prepared for the follow-up.

The PD teachers repeatedly talked about how unproductive it is to ask questions that merely elicit yes or no responses from students (referred to hereafter as yes/no questions). One principle combines this idea with the productiveness of open/close-ended questions: It is unproductive to ask yes/no questions (because the questions close the students up). A teacher should ask open-ended questions that let the thinking come from the students rather than the teacher. Thus, the PD teachers were concerned that yes/no questions were just a way for the teacher to tell students what they wanted them to think. An example of a question from the scenarios that sparked this concern was, “If you don't use the building number, wouldn't you have to work a lot harder?” This scenario teacher question is putting the scenario teacher’s ideas on the table, but in the form of a yes/no question.

The PD teachers’ discussions led to principles that help us see how they characterized a variety of other teacher moves: redirect, evaluate, and general questions. With respect to redirecting moves, the PD teachers discussed how a teacher could ask a question to get the class back on track, but if students were wrong, then the students should self-correct instead of a teacher redirecting. With respect to evaluative moves, the PD teachers argued that evaluative questions are productive when asked without harshness.  Two main evaluative questions they looked at when drawing this conclusion was a decidedly not harsh question, "Is there anything wrong with not using the building number?", and a harsh evaluative question, “If you don’t use the building number, wouldn’t you have to work a lot harder?”. They felt that the second question was calling out the students for doing the wrong thing versus the first question that was evaluative but less attacking. General questioning moves are productive when they lead to good, student-centered discussion, or when the question by itself or as a set-up to a sequence allows the class to explore a topic or idea in more depth and get needed information on the table for consideration. General questioning moves are unproductive when the question tries to draw out ideas a student has already covered, or when the question has an obvious answer or could lead the class toward incorrect mathematics instead of the mathematical goal of the task.

These principles of productivity related to teacher moves can be captured in the following meta-principle for this theme: It is productive when a teacher response elicits valuable student thinking and then follows-up on student thinking to allow students to share more information or expand on their responses to get more information from the students, and it is unproductive to ask close-ended questions that either state what the teacher wants the students to think or closes the students to further contributions.

**Mathematics Focus**

This final theme consists of principles (15 of 65 principles) that highlight the need to consider the underlying mathematical focus of classroom discussion. The principles of productivity bring out three main ideas within this theme: the mathematical goal of the task, the nature of the mathematics, and resolving the mathematics.

The PD teachers discussed the importance of having everything involved in the discussion lead to or be part of the mathematical goal of the task, as captured by this principle: It is productive to (pursue and) reach a mathematical goal of the task. (It is unproductive to not reach nor pursue a mathematical goal of the task.). The PD teachers stated that in order for a teacher response to truly be productive in a classroom, it had to help the class reach the understanding associated with the mathematical goal for the task. The student thinking and methods the students used to solve a task should be productively employed toward reaching the mathematical goal of that task. The ensuing class conversation, including teacher questions and summary, should be focused on the mathematical goal. Although the PD teachers felt that discussing classroom norms could be productive in general, it was not viewed productive with respect to the mathematical goals of a task. Thus, in the eyes of the PD teachers, everything in a classroom should be aimed at helping the class reach the mathematical goal of the task.

There were also principles that articulated the PD teachers' descriptions of the nature of the mathematics that underlies a productive class discussion. The overarching idea is that the mathematics the class discusses should come from and be brought out by the students' ideas rather than the teacher. This concept overlaps with the theme arising from the PD teachers' decision that it is important to focus on student mathematics in a variety of ways. The mathematics should arise from student's explorations and examining methods, procedures, connections between students' solutions, or explanations underlying the mathematics the students are using. There is a standardized mathematics language that the mathematical community uses, which the students should learn, but first they should be given the opportunity to come up with the mathematical ideas for discussion and then develop their own understanding of the mathematics. A focus on mathematics does not necessarily mean the class is focusing on the standard way to think about mathematics, but it is more of an attention to the origins and applicability of the mathematics being focused on. For instance, teachers and students should avoid introducing ideas that are not grade-level appropriate. Instead, the mathematics of discussion should be focused on the grade-level mathematical points student thinking could bring out.

The PD teachers discussed the importance of resolving student mathematics. This resolution could take place at the end of the discussion as a means of resolving the mathematics that was talked about in discussion. Not much was said about how to resolve the mathematics other than the need for a resolution and for the teacher not to dominate the resolution.

The PD teachers discussed mathematics a class should focus on and how the mathematics would affect a class discussion. The principles that arose from these ideas gives the broader vision of what a productive focus on mathematics looks like. I articulated these ideas in a meta-principle capturing the PD teachers' ideas about focusing on the mathematics: It is productive for the class discussion to be focused on pursuing and reaching a mathematical goal of the task while allowing the students to explore and develop the mathematical ideas leading to a resolution of the mathematics at the end of discussion.

**Principles of Productivity Related to the Sub-practices of Building**

The sub-practices of building are seen as a sequence of actions a teacher should theoretically do in order to build on a MOST. These sub-practices, along with the two prerequisites to building, are (0) Invite or Allow, (0.5) Recognize MOST, (1) Make Precise, (2) Grapple Toss, (3) Orchestrate, and (4) Make Explicit. I sorted the 65 principles of productivity according to how I saw the teachers' ideas, as represented in the principles of productivity, aligning with each of the prerequisites and sub-practices. The result was 32 principles of productivity that aligned with at least one sub-practice of building. The comics did not seem to elicit much discussion from the teachers about the sub-practices of Make Precise and Grapple Toss, as the principles only aligned with them a few times. Because these two sub-practices can occur within a single move, and due to the teachers' minimal awareness of them, I talk about these two sub-practices together followed by Orchestrate and then Make Explicit. The order I discuss the principles' alignment with building follows the order of the prerequisites and sub-practices of building.

**0) Invite or Allow and 0.5) Recognize MOST**

The first prerequisite to building is to invite or allow students to share their thinking. I determined whether principles aligned with this prerequisite by asking, “Do the teachers say or imply that the student mathematics of the scenario instance needs to be made public by inviting or allowing the student to share their thinking?” The PD teachers recognized the need to elicit and invite students to share their thinking. The three resulting principles reflect that the PD teachers thought it is unproductive to invite students to share their thinking and then just tell them the answers, it is productive to allow student thinking to be make public and then be discussed, and it was productive when a teacher turn elicits valuable student thinking from the students. Because these principles relate to the productivity of inviting students to share their thinking, these principles aligned with the initial “Invite or Allow” prerequisite to building.

The second prerequisite to building is recognizing the shared student thinking as a MOST. Not only did the PD teachers recognize the need to have students share their thinking, but they also discussed the importance of eliciting valuable student thinking or pursuing a MOST. There were only two of the 65 principles of productivity that answered the question, “Do the teachers say or imply that one should consider whether the student mathematics of the scenario instance would be classified as a MOST?” This prerequisite requires that teachers be aware that the student thinking that was shared is a MOST, or significant student thinking worth building on in discussion, in order for the teacher to productively build on the shared thinking. The PD teacher’s discussions included the decision that it is unproductive to pursue non-MOSTs. Several PD teachers discussed or made decisions about what student thinking was a MOST or not a MOST. For example, Group 1 asked, “Do you think this [student response “Didn’t the problem say that the variable needed to be the building number?”] would be the MOST right here [in Scenario 1, Frame 3]?" They later stated, "That's [student response: "In one they are added and in the other two they are multiplied" is] not a MOST". In deciding that pursuing a non-MOST was unproductive, the PD teachers demonstrated the importance they put on recognizing a MOST to pursue. Not all the PD teachers used the language of MOST, however, instead just referring to student thinking that was important to elicit and follow-up on. This recognition of the importance of eliciting valuable student thinking highlights the PD teacher's desire to identify valuable student thinking.

The collection of the principles of productivity that aligned with the prerequisites to building show what the PD teachers thought about the productivity of ideas that align with these prerequisites. The following meta-principle is an articulation of the ideas in this collection of principles: It is productive to invite students to share their thinking and then pursue that thinking if it is a MOST. Following the framework for *building,* a teacher can begin to build on that student thinking once student thinking has been made public and the teacher has determined that student thinking constitutes a MOST. This building process consists of the four main sub-practices of building. I now present the principles of productivity based on how the principles aligned with these sub-practices of building.

**1) Make Precise and 2) Grapple Toss Sub-Practices**

The first two sub-practices of building are Make Precise and Grapple Toss. The question I asked myself to determine if a principle of productivity aligned with the sub-practice of Make Precise was, "Do the teachers say or imply that one should consider whether the student mathematics of the scenario instance would need to be clarified?" and the question to determine alignment with Grapple Toss was, "Do the teachers say or imply that one should consider whether the student mathematics of the instance should be turned over to the class for consideration in a way that necessitates sense making of the idea?"

The first sub-practice, Make Precise, is concerned with making student mathematics clear and precise, as well as making the object of discussion clear. The principles of productivity that align with this sub-practice (3 of 65 principles) only align with one aspect of the sub-practice: clarifying student mathematics. The PD teachers discussed that it was unproductive to assume student understanding or what a student said. The principles are clear that there are multiple reasons why it is productive to ensure that student thinking is clarified, but they also describe who should do the clarifying. When students share their thinking, that thinking should be clarified in order for the teacher to more fully understand what the student meant, or what the student understands. Not only should the clarification help the teacher understand the student thinking, but the student mathematics should be clarified in order for the *class* to consider the student mathematics as well. This clarification is especially important when incomplete student thinking is on the table. The PD teachers discussed the importance of the teacher prompting the students for clarification, rather than the teacher attempting the clarifying themselves, in order to prepare the class to consider the student thinking.

The second sub-practice, Grapple Toss, is when the teacher turns the clarified student mathematics over to the class for them to make sense of it. The PD teachers recognized the need to clarify student thinking *before* it is tossed to the class for consideration. This act of letting the class consider the clarified student thinking is considered a Grapple Toss. There is only one principle out of the 65 principles that connects the idea of clarification with a class sense-making situation: It is productive to ask a clarifying question to allow students to clarify their thinking, especially when incomplete student thinking is on the table, in <u>preparation for the class to</u>

consider that thinking. This principle, present among four groups, connected the idea of a Make Precise to a Grapple Toss and also highlighted an important component of a Grapple Toss: the class making sense of the *clarified student mathematics.*

Together, the principles that aligned with these two sub-practices led to a meta-principle that captures the PD teachers' vision of a productive way to immediately follow-up on a MOST: It is productive to invite a student to clarify their thinking when it is incomplete or imprecise (rather than assuming that student's understanding) in preparation for the class to consider and explore that thinking.

**3) Orchestrate**

The third sub-practice, Orchestrate, involves orchestrating a whole-class discussion in which students collaboratively make sense of the object of consideration. In order to decide if a principle of productivity aligned with this sub-practice I asked the question, "Do the teachers say or imply that class discussion should be directed towards making sense of the student mathematics of the instance?" The PD teachers discussed ideas related to who was involved with class discussion and how they were involved in the class discussion, as well as what the class discussion was focused on. There were 10 principles of the 65 principles that highlighted these ideas. These principles that aligned closest with the sub-practice of Orchestrate reflect the teacher's attention to the importance of students being involved in making sense of mathematics. The principles I discuss here center on how students are engaged in the discussion as well as the mathematics the students are engaged with.

The PD teachers discussed the importance of students being engaged in class discussion as well as how they should be thus engaged. There was one principle in particular that was the most popular on the topic of student engagement: It is productive to have multiple students engaged in the mathematical conversation. (It is unproductive to engage only one student in class discussion). A discussion is thus productive if it involves multiple students. Furthermore, another principle states that a discussion becomes unproductive if a teacher cuts off such a multiple-student discussion. With respect to *how* students should be involved, the PD teachers indicated that the students should be focused on making sense of mathematics. A teacher should in no way take over the mathematical thinking in discussion such that there is no student ownership and the teacher is doing all the thinking. One way a teacher can take over student thinking is by leaping from one student idea to a conclusion. Students should be the ones providing the ideas for discussion, and hence, the ideas in the conclusion as well, which is talked about in the next sub-practice. The teachers should allow the students to be involved in the whole process of discussion such that the students are able to follow the discussion and be the ones providing ideas for discussion without the teacher taking over the discussion.

The PD teachers were also concerned with the nature of the mathematics the students are making sense of during class discussion. This mathematics should (a) arise from the comparing and contrasting of student solutions rather than from discussing just one student solution, and (b) be the driving force behind the discussion. These characteristics highlight the need for discussion to be centered around student mathematics. One principle in particular shows the PD teachers' concern that discussion should also be focused on *making sense* of the student mathematics: It is productive for the teacher to allow student thinking to be made public and then discussed in a way that adds meaning to the mathematics of the student's statement. The teachers understood an important aspect of a productively orchestrating discussion: the class sense-making with *student mathematics*.

The PD teachers thus discussed several aspects of orchestrating discussion including who is involved in the discussion and what students are making sense of in the discussion. The principles of productivity that aligned with the sub-practice of Orchestrate give insight into what the collection of PD teachers thought constituted productive teacher moves during the orchestration of discussion, as articulated in this meta-principle: It is productive to engage multiple students in discussion and ask these students to provide the various solutions, ideas, and information needed for making sense of the student thinking in discussion.

**4) Make Explicit**

The fourth sub-practice is characterized by the extraction and articulation of the mathematical point of the object of consideration. The question I asked to determine whether a principle aligned with this sub-practice was, “Do the teachers say or imply that the mathematical idea underlying the student mathematics of the instance would need to be made explicit?” This question led to 12 principles of the 65 principles aligning with the sub-practice of Make Explicit. These principles reflected how the PD teachers were attentive to the summaries at the end of a discussion. At the core of the principles that relate to this sub-practice is the idea that it is productive to summarize the discussion. The PD teachers state in this principle the importance of summarizing a discussion, and while this idea underlies the other principles related to this sub-practice, the other principles give insight into the nature of the summary. The other principles elaborate on two aspects of summaries that are critical to their productivity: who is involved in the summary and how they are involved, and the purpose of the summary.

With respect to who is involved in the summary and how they are involved, the PD teachers determined that it was most productive for students to be the ones to summarize discussions. The most common principle, as well as one other principle, articulated the student’s role to summarize, but the other principles were more concerned with the teacher's related role. Even though the PD teachers decided a student should summarize the discussion, they felt the need to articulate what a teacher summary should look like if a teacher did summarize the discussion. A teacher summary, meaning a summary the teacher gives, should refer to student thinking. Along with this requirement is the need for there to be student thinking to refer to. The teacher should not summarize a discussion if the students have not yet developed the ideas the teacher is trying to summarize. We see from these ideas about a productive summary that students or their thinking should in some way be involved in the summary such that the conclusion is not focused on the teacher. This concept of the student being involved in the summary process was a focus of a majority of the teachers, as well as how the roles of the teacher and student should interact: It is productive to have students summarize and connect ideas from a discussion and then for the teacher to just re-summarize and add terminology to the students' summary. The teacher should facilitate a student connecting and summarizing the mathematical ideas the class discusses, and then make sure that student's summary is clear while adding the vocabulary the class would not be familiar with yet. The PD teachers paid attention to who was involved in making explicit and what the teachers and students were doing as well.

With respect to the purpose of a summary, the PD teachers expressed the need for the summary to resolve the student mathematics from the discussion. This mathematical resolution, however, should not just focus on one idea when other ideas were also discussed and part of the goal of the task—it should be comprehensive with respect to the mathematical ideas that arose during the discussion. That said, the summary should not go beyond those ideas by introducing new mathematical concepts. Although the PD teachers did not view it as productive to introduce new mathematical ideas in the summary, they did think that it was productive to introduce terminology related to those ideas. To be productive, however, the teacher needs to provide meaning and context for that terminology.

The sub-practice of Make Explicit was the most common sub-practice that the principles of productivity related to as the teachers seemed to easily discuss the summary in the scenarios. The number of principles that related to this sub-practice paint a picture of what a productive articulation of ideas entails. Seeing the roles both the students and teacher play in a summary as well as what the summary should contain leads to a collective vision of what the PD teachers see as necessary for a productive summary and this meta-principle: It is productive to have students summarize and connect ideas from a discussion and then for the teacher to re-summarize the resolution, ensuring that all of the relevant mathematics is resolved, and add meaningful, contextualized terminology to the students' summaries as needed.

The meta-principles for each theme and sub-practice of building are collected in Figure 11. This set of meta-principles collectively captures what was most salient to the PD teachers in terms of the productivity of teacher moves in general and pertaining to the sub-practices of building in particular.

| | | |
|---|---|---|
| Themes | Focus on Student Mathematics | It is productive to allow students to come up with the mathematical ideas in the discussion such that the students are able to make sense of these ideas and in such a way that the focus of the discussion and summary is the students' mathematical thinking (rather than the teacher's mathematical thinking). |
| | Teacher Moves | It is productive when a teacher response elicits valuable student thinking and then follows-up on student thinking to allow students to share more information or expand on their responses to get more information from the students, and it is unproductive to ask close-ended questions that either state what the teacher wants the students to think or closes the students to further contributions. |
| | Mathematics Focus | It is productive for the class discussion to be focused on pursuing and reaching a mathematical goal of the task while allowing the students to explore and develop the mathematical ideas leading to a resolution of the mathematics at the end of discussion. |
| Sub-practices of Building | 0) Invite or Allow and 0.5) Recognize MOST | It is productive to invite a student to clarify their thinking when it is incomplete or imprecise (rather than assuming that student's understanding) in preparation for the class to consider and explore that thinking. |
| | 1) Make Precise and 2) Grapple Toss | It is productive to invite a student to clarify their thinking when it is incomplete or imprecise (rather than assuming that student's understanding) in preparation for the class to consider and explore that thinking. |
| | 3) Orchestrate | It is productive to engage multiple students in discussion and ask these students to provide the various solutions, ideas, and information needed for making sense of the student thinking in discussion. |
| | 4) Make Explicit | It is productive to have students summarize and connect ideas from a discussion and then for the teacher to re-summarize the resolution, ensuring that all of the relevant mathematics is resolved, and add meaningful, contextualized terminology to the students' summaries as needed. |

*Figure 11.* The articulated meta-principles drawn from the principles of productivity for each theme and sub-practice of building.

# CHAPTER SIX: DISCUSSION

The two focuses of my research were on what teachers thought were productive moves and to what extent those ideas were related to building. With regards to the first focus, I discuss the principles of productivity with respect to (a) what other research has found teachers think about the productivity of teacher moves and (b) what research has identified as productive teacher moves. With regards to the second focus, I discuss the extent to which the principles of productivity align with the sub-practices of *building.*

## Comparing the Principles of Productivity to the Literature on Teacher Moves

As I argued in my literature review, researchers have plenty to offer on their views of what is productive for a teacher to do in a classroom, but there is little research on what teachers think is productive to do. I begin by comparing my results to the results of these few studies. I then proceed to compare my results to what researchers suggest are productive teacher moves.

### Teachers' Ideas on Productive Teacher Moves

As outlined in the literature review, some research gives insight into teachers' ideas regarding the productivity of teacher moves related to both questioning and using student thinking. While the PD teachers in my study were not asked to look specifically at the productivity of teacher questions or teacher moves that use student thinking, they discussed similar ideas to what research has found teachers think regarding the productivity of these particular types of teacher moves.

**Teacher Questions.** Research suggests involving students by engaging them in a question-and-answer response pattern where the teacher asks a follow-up question that allows the student thinking to guide discussion or the teacher incorporates a student answer into the teacher's response question (Nystrand & Gamoran, 1991). As discussed in the teacher moves section of the results, the PD teachers had ideas about the productivity of a variety of different teacher questions. Studies on pre-service teachers (Cakmak, 2009) and in-service teachers (Mitchell, 1994) found that these groups of teachers thought the main purposes of questions were to accomplish such things as would likely fall under the broader categories of engagement and establishing and maintaining classroom norms. By contrast, the PD teachers characterized productive teacher questions as those that served the purpose of involving the students in open-ended, student-centered thinking and discussion. Thus, with regards to questioning, the teachers in Cakmak (2009) and Mitchell (1994) focused on student involvement for involvement's sake while the PD teachers focused on how productivity depended on the quality of student involvement.

The teachers in Mitchell (1994) also discussed what teachers should and should not do with relation to questioning, and these ideas were similar to how the PD teachers in my study sometimes discussed productivity of teacher moves, which included but was not limited to questioning. Both groups of teachers—the teachers in Mitchell (1994) and the PD teachers—discussed the need to engage students. While the teachers in Mitchell (1994) discussed this engagement in terms of questioning an individual student and the PD teachers talked more generally about the need to engage and involve more multiple students in discussion, the ideas were the same: it is unproductive when teacher moves involve only one student. Another commonality was the productiveness of evaluating student thinking without "putting the student down" (Mitchell, 1994, p. 75) or without "harshness" (PD teachers).

**Teacher Moves that Use Student Thinking.** The importance of using student thinking as recognized by research (e.g., Franke, Kazemi, & Batty, 2007; Van Zoest, Peterson, Leatham, & Stockero, 2016) is mirrored in the PD teachers' idea; both saw that a main purpose of teacher moves is to focus on student mathematics (e.g., Conner et al., 2014). Since the PD teachers were chosen based on their desire to focus on and learn more about student thinking in their classrooms, it is less surprising that a major theme arising from the PD teachers' discussion is the need to focus on student mathematics. The PD teachers thought (a) the ideas of discussion should come from the students, (b) students should be engaged in making sense of those ideas, and (c) the mathematics underlying the students' ideas should guide the discussion and summary. According to Leatham et al. (2014), these ideas about how to focus on student mathematical thinking align closely to the actions a teacher would make who had the most productive teacher perception for using student mathematical thinking. Additionally, the PD teachers' ideas of what is productive align with high- and medium- potential orientations (Stockero et al., 2018) based on their view that the student thinking is valuable and should guide the discussion rather than the teacher providing the ideas. Overall, my findings align with those of Stockero et al. (2018) and Leatham et al. (2014) since the way the PD teachers talked about the productivity of teacher moves is similar to what teachers thought were productive teacher moves as revealed in their orientations and perceptions.

Because of the research on teacher orientations (Stockero et al., 2018), I expected the PD teachers to have varying opinions for the use of student mathematical thinking. However, while the PD teachers' ideas of what was productive did vary between and within groups, their ideas regarding the productivity of focusing on student mathematical thinking were more or less consistent: student thinking should be the focus of and used in the teacher moves, the class discussion, and the summary.

**Productivity of Teacher Response Patterns as Classified by Research**

As mentioned in the literature review, research suggests that teachers often employ the Initiation-Response-Evaluation (I-R-E) discourse pattern (Mehan, 1979), where the teacher *initiates* the exchange by asking what is typically a closed, obvious-answer question (Nathan, Kim, & Grant, 2018) and then *evaluates* the student *response*, which often terminates the interaction and limits further contribution (Westgate & Hughes, 1997; Jia, 2005). Two of the scenarios were designed to illustrate the I-R-E teacher response pattern. The PD teachers attended to the unproductive nature of parts of the I-R-E sequence as they decided that questions are productive if they are open-ended rather than asking for an obvious answer, which is contrary to the typical aspects of *initiate.* The PD teachers' ideas of how student thinking should be elicited and what to do with student thinking that has been elicited match more with the IDE sequence of productive discourse (Nathan et al., 2007). The *initiation* part of IDE is intended to be open-ended followed by *demonstrations,* which are not limited to the single, obvious answer student responses that IRE is characterized by.

The *evaluative* and *elaborative* (non-evaluative) last step to the IDE pattern is more in line with what the PD teachers thought of as productive. The presence of the I-R-E in the scenarios helped elicit the PD teachers' comments on the *evaluate* portion of the I-R-E pattern in the scenarios. The *evaluate* part of the I-R-E pattern within the scenarios led the PD teachers to determine that an evaluative teacher question must be asked without harshness in order to be productive. The evaluation aspect of I-R-E is focused on the teacher evaluating the correctness of the student response (Wells & Arauz, 2006). However, the PD teachers determined that focusing only on right answers or how to get the right answer was unproductive. Similarly, the I-R-E pattern tends to limit student contributions (Westgate & Hughes, 1997; Jia, 2005), which is another thing the PD teachers decided was unproductive. The PD teachers were comfortable with the scenario teacher evaluating the scenario student turns as long as the evaluation was not attacking the student thinking, was not only focused on right answers, and did not limit student contributions. Thus the PD teachers' view of the I-R-E matched researchers' ideas that it is an unproductive discourse pattern (e.g., Nathan et al., 2007).

Additionally, the PD teachers agreed with research (Chapin & O'Connor, 2007; Michaels & O'Connor, 2015) on the productivity of talk moves. Comparing the talk moves that research (Chapin & O'Connor, 2007; Michaels & O'Connor, 2015) decided were productive to the principles of productivity, we see that the PD teachers also thought it was productive to elicit student thinking, respond to student thinking (e.g., follow-up on student thinking through a variety of teacher moves), comment on student thinking (e.g., provide terminology to student summaries and acknowledge all student thinking), and invite student responses (e.g., allow students to come up with the mathematical ideas in the discussion).

**Overall**

Overall, the results of this study align with what the limited current research has reported related to what teachers think about productive teacher moves. The PD teachers also tended to agree with what research has said about productive moves. There was, however, at least one idea about productivity that the PD teachers brought up that does not directly relate to current research on what teachers think or research has found regarding the productivity teacher moves: how students would recognize their ideas in teacher turns. While research (Peterson et al., 2017; Pierson, 2008) agrees that students recognizing student thinking is important, there are not results on the explicit productivity of teacher moves regarding this. The PD teachers were concerned when a student would not be able to recognize their ideas in a teacher response. An example the PD teachers noticed was when a teacher response would leap from a student response to a teacher response or conclusion when the class was not prepared for the teacher's ideas. There was one principle in particular that highlights these concerns with the teacher responses: It is unproductive for a teacher to jump to a conclusion that does not align with the student's ideas such that the student is unable to recognize their idea.

## Alignment with Building

The theoretical *building* framework, consisting of two prerequisite actions and four sub-practices, describes the practices a teacher should do to productively build on significant student thinking. For each prerequisite and sub-practice of building I was able to articulate an overarching principle of productivity. By looking at the overarching principles of productivity, the vision for each sub-practice of building that the building framework provides, and the design of the scenarios with respect to building, I explore the alignment of the PD teachers' ideas and building in the context of how the scenarios brought these ideas out. I also look at the principles' overall alignment with building.

### 0) Invite or Allow and 0.5) Recognize MOST

The two prerequisite actions to building are straightforward in what is important: (1) the teacher lets students share their ideas and (2) the teacher identifies a shared idea as a significant instance worth building on. The scenarios were designed to begin with a MOST, so the first prerequisite action is not present in the scenarios. Also, while the second prerequisite action is necessarily an action the scenario teacher does, the recognition of a MOST is less of a scenario teacher turn than it is the scenario teacher's thinking process. Thus, the fact that some of the PD teachers were aware of these ideas is important. The PD teachers paid attention to the need to draw out student thinking. Both the PD teachers and the mathematics education community (e.g., National Council of Teachers of Mathematics, 2014) have discussed the importance of eliciting student thinking. Some of the PD teachers also decided that elicited student thinking should only be pursued if it was a MOST. Research suggests the importance of noticing what is important when teaching (Leatham et al., 2015; van Es & Sherin, 2002). Many PD teachers discussed the need to elicit student thinking, but only a few mentioned a MOST or valuable student thinking as a reasoning for the productivity of a teacher move, which might be a result of the scenario design or the newness of MOSTs to the PD teachers.

**1) Make Precise and 2) Grapple Toss Sub-Practices**

Making the student mathematics of the MOST precise requires clarifying the student thinking so it is clear to the students of the class and also making obvious what student thinking the class is meant to consider. With regards to the clarifying aspect of the make precise, the PD teachers' comparisons between scenarios with and without a make-precise teacher turn led them to frequently discuss the need to make the student thinking clear before it could be considered. In talking about the importance of student thinking being clarified, they emphasized that a teacher should not assume what a student is thinking. This form of clarifying is in line with Leatham et al. (2014) where the teacher and students "come to a mutual understanding of what was said or meant" (p.78) instead of the teacher providing their own interpretation.

The second more subtle aspect of this sub-practice is the need to make obvious what the class is supposed to consider. The PD teachers did not mention the need to make sure the students in the class understood what student thinking and mathematics they were being asked to consider, just that the student thinking was clarified. The scenario teacher's question, "Interesting. What's similar or different in how the 5's are used in these three expressions?", along with the variations of this question, make it clear that the scenario teacher wants the class to consider the explanation of the 5 in the student thinking. In doing so, the scenario teacher made it clear what they wanted the class to consider from the student thinking. While the PD teachers often thought of this question as productive, they did not seem to hone in on the second aspect of making precise that the question exhibited.

The Grapple Toss is composed of *tossing* the student thinking that was made precise to the class in order for the students to *grapple* with the thinking of that MOST. The grapple toss variations in the scenarios prompted the PD teachers to discuss the need to engage the whole class instead of one student, let the students have the opportunity to make sense of the mathematics, or have the students be the mathematical thinkers instead of the teacher. The PD teachers were aware of the need to let the whole class engage in sense making, which is an essential part of a grapple toss.

Another important component of a grapple toss is for the teacher to toss and let the class consider, specifically, the student thinking of the MOST. The PD teachers discussed the importance of teachers focusing on student mathematics and considering mathematics, which focus holds many potential benefits for teachers and students (Chamberlin, 2003). While the language between the PD teachers and researchers varies, both were able to see the importance of building on (e.g. Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Stein & Lane, 1996), using (Peterson & Leatham, 2009), attending to (Chamberlin, 2003), and responding to student thinking (Pierson, 2008). PD teachers were aware of the need to not only focus on and respond to the student mathematics, but to let the students be the ones making sense of the student mathematics. However, only one principle dealt with the idea that the mathematics the students should be considering is the student mathematics that was just made precise (It is productive to ask a clarifying question to allow students to clarify their thinking, especially when incomplete student thinking is on the table, in preparation for the class to consider that thinking). This might seem like a trivial distinction that the PD teachers overall did not make but focusing on the student mathematical thinking of the MOST is essential to building. According to the theoretical building framework, a productive follow-up to a clarification of an initial MOST is to give the student mathematics over to the class to consider (Van Zoest et al., 2016). Most of the principles that talked about having the class sense-make and explore mathematics were about the mathematics in general rather than *student mathematics* in particular.

The comics were designed to draw out the ideas related to the sub-practices of Make Precise and Grapple Toss, and the PD teachers discussed many fundamental parts of these sub-practices. The one thing that was not as salient for most of the PD teachers was the need to focus on not just general student thinking, but on the student thinking that was made precise in order to be grapple tossed.

**3) Orchestrate**

The third sub-practice of Orchestrate requires the teacher to orchestrate a whole-class discussion. The definition for this sub-practice gives the least amount of detail as to what a teacher should do in order to productively orchestrate a discussion around the clarified and grapple-tossed MOST. One of the scenarios was designed specifically to have a discussion between the scenario teacher and one student. The PD teachers saw this type of discussion as unproductive, which led them to discuss the need for multiple students to be engaged in a discussion. This idea of engaging multiple students in discussion is reminiscent of the "whole-class" requirement for orchestrating. The other principles of productivity about a class discussion let me articulate the collective PD teachers' ideas of what a discussion should consist of: a teacher asking the engaged students to provide the various solutions, ideas, and information needed for making sense of the student thinking in discussion. The teacher asking students to share their ideas and then making sense of the ideas shared is similar to two of the five practices for facilitating mathematical discussions around cognitively demanding tasks: "(3) selecting particular students to present their mathematical responses …, and (5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas" (Stein et al., 2008, p. 321). All together the five practices suggest how to facilitate discussion in order to use student thinking (Stein et al., 2008), but the practices are in the context of a cognitively demanding task. A MOST, however, can occur in any setting (Leatham et al., 2015), so orchestrating needs to be able to take place in any context instead of requiring the anticipating, monitoring, selecting, and sequencing build up. An important aspect of the sub-practice of orchestrating as part of building is a discussion being based on the student mathematics of a MOST in whatever situation it arises.

An interesting aspect of the PD teacher's discussions and resultant principles relates to the mathematics they thought it would be productive to pursue in discussion. The discussion of two scenarios were directed toward a mathematical understanding that was the closest mathematics to the MOST. According to the MOST framework (Leatham et al., 2014), teachers should pursue mathematics that is appropriate for the class, central to the goals for the class, and closely related to the student mathematics that is under consideration. There were several scenarios designed where the scenario teacher pursued mathematics that did not fit these requirements. These scenario designs led to conflicting discussions among the groups, as some PD teachers thought that all the mathematics related to a student's statement should be covered in discussion while other teachers honed in on the in-appropriateness of the mathematics covered. The PD teachers were still new to the idea of a MOST, and more exposure and experience in identifying the important mathematics underlying a student's statement might have affected their views on the productivity of a discussion.

**4) Make Explicit**

The final sub-practice of building, Make Explicit, is the practice of facilitating the extraction and articulation of the important mathematical ideas from the discussion. The PD teachers discussed ideas related to making explicit more than any other sub-practice. During their discussions, the PD teachers even recognized the subtleties built into the scenarios of not making explicit, vaguely making explicit, and prematurely making explicit. The result was principles about the need to summarize or resolve mathematics instead of focusing on classroom norms or validations, criticisms of summaries that led to principles about what made other summaries better, and the need for students to understand something before a teacher jumped to a conclusion of it. Additionally, the PD teachers gave insight into what they thought was a productive way to make explicit: have a student summarize and then let the teacher re-summarize with contextualized terminology. The student summary should connect ideas from discussion and the teacher's re-summary should resolve the relevant mathematics while adding meaning and contextualized terminology. The PD teachers provided their interpretation of what the student and teacher roles should look like in the summary, which can provide insight into how teachers might interpret the extraction and articulation of making explicit within the practice of building.

**Overall Alignment with Building**

Overall, about half of the PD teachers' principles of productivity aligned with building. While the ideas of inviting/allowing students to share their thinking, recognizing a MOST, making precise, and grapple tossing were less common (1 to 3 principles each of the 65) among the principles of productivity, the ideas were still present. The teachers were either less aware of these aspects of building, the scenarios did not draw them out as the researchers expected, or the PD teachers chose not to discuss them, but these prerequisites and sub-practices of building were less salient to the PD teachers. On the other hand, the PD teachers frequently discussed (10 to 12 principles each of the 65) aspects of orchestrating a discussion and making the mathematics explicit. The concepts related to these sub-practices of building were ideas that either the PD teachers had already thought about or that were more apparent to the teachers through the presented scenario variations. Furthermore, while less than half of the overall collection of principles directly aligned with building, many of the other principles contained ideas central to building (e.g., focus on student mathematics, just without the subtle nature of which student mathematics is being focused on).

Building is not just one teacher move or teacher turn, it is a sequence. Despite the prompt to look specifically at the productivity of each "sequence of teacher moves," few principles (only 5 of the 65 principles) captured the importance of sequencing. When the PD teachers did discuss the productiveness of sequences of teacher moves they either referred to the productiveness of pairs of teacher moves or of general sequences of teacher moves. When teachers clearly discussed the sequencing of pairs of teacher moves, their discussions led to principles of productivity that connected these teacher moves. An example of such a principle of productivity is that it is productive for the teacher to allow student thinking to be made public and then discussed in a way that adds meaning to the mathematics of the student's statement. The first teacher move captures the prerequisite action of inviting or allowing student thinking to be made public. The PD teachers thought that once student thinking was made public that the next teacher move should allow for discussion to happen around the student thinking, which coordinates with the Orchestrate sub-practice of building. This principle of productivity connected teacher moves associated with a prerequisite to build and a sub-practice of building. Although rare, such principles demonstrate that the PD teachers were aware at some level of the importance of sequencing two or more teacher moves. In addition, discussions regarding sequencing were seldom specific and clear enough for me to infer their ideas related to productivity. The PD teachers' discussions often resulted in "General – Multi Move" teacher move codes where the PD teachers were discussing multiple teacher moves at once, but their discussions remained too broad and generic to conclude anything about the teacher moves.

# CHAPTER SEVEN: CONCLUSION

There is limited research that reports directly what mathematics teachers think about the productivity of teacher moves. Knowing what they think could help influence teacher educators' teaching and professional development materials. I began my research with several guiding frameworks in mind: the MOST framework (Leatham et al., 2015) for identifying significant moments of student thinking, and the *building* framework (Van Zoest et al., 2016) that outlines a theoretically productive practice. In order to help fill the gap in the research, I analyzed teachers' discussions at a professional development workshop through the lens of *building* as well as through emergent themes regarding productivity. I found that, in general, the PD teachers' ideas of productivity aligned with at least one sub-practice of building directly and also provided details on productive teacher moves. Their ideas also tended to agree with other teachers' and researchers' ideas regarding the productivity of teacher moves. Drawing from these results, I discuss the contributions and the implications of this research. I also discuss some of the limitations of my study.

## Contributions

There are two main contributions this study makes to the field of mathematics education. First, the framework of building is a theoretical framework that is still being studied. While there is evidence of teachers implementing some of the sub-practices, Van Zoest et al. (2017) have yet to observe teachers effectively coordinating all four of the sub-practices of building, so there is not much building occurring in classrooms. Researchers have observed teacher moves and reported some teachers' ideas of productive teacher moves, but have not studied teachers' thoughts relating to the sub-practices of building. My study provides insights into which aspects of building are or are not relatively natural for teachers to think about. For instance, the need to focus on student mathematical thinking was at the core of many of the PD teachers' discussions, and this is an essential part of productively responding to student thinking. Also, the principles of productivity aligned frequently (22 of 65 principles of productivity) with the Orchestrate and Make Explicit sub-practices. The PD teachers were aware of the productivity of orchestrating a discussion where students are the originators of ideas, and they were also aware that such discussions should have a conclusion that resolves the mathematics and provides needed terminology. These teachers were talking about, and therefore thinking about, these aspects and sub-practices of building that they value, and the frequency of these ideas shows how salient they are among at least the PD teachers.

On the other hand, some aspects of building were less salient to the PD teachers. For instance, the idea of recognizing valuable thinking and how it would influence a teacher move was something the PD teachers discussed less commonly (2 of 65 principles of productivity). Relatedly, while the PD teachers discussed the need to clarify student thinking (3 of 65 principles of productivity), they never discussed how the teacher moves should help the class know exactly what they should be considering from the MOST. Finally, while the PD teachers were aware of the need to include students in discussion and have them make sense of the mathematics, only one of the 65 principles clearly captured the importance of the class considering and staying focused on the mathematics directly related to the valuable student thinking, or the MOST. Knowing what building concepts teachers might already attend to, as well as those that they may not, can help guide teacher educators' efforts to prepare teachers to build on valuable student thinking.

Second, by comparing what teacher moves research has identified as productive moves in responses to student thinking to what the PD teachers think is productive or unproductive regarding teacher moves, I was able to see how closely related the thinking and findings of these two groups of people are. Research has been able to capture ideas important in the eyes of the researchers, and in some respects also important to teachers. The teachers studied in research, the PD teachers in my study, and researchers all agree that it is productive to focus on and use student thinking and mathematics in teacher moves. That said, teacher moves that are categorized as productive by researchers do not always catch the nuances that teachers attend to. The teachers in Mitchell (1994) and the PD teachers provided more specific ideas about why they think particular teacher moves are more or less productive. For instance, a teacher in Mitchell (1994) described why engaging only individual students in questioning was unproductive: "[because] others will switch off" (p. 74). This teacher provided their reason for why a teacher move, specifically a question in this case, should engage more than one student. An example of the PD teachers providing detailed information on what makes teacher moves related to a summary more productive is demonstrated in this principle of productivity: It is productive to have students summarize and connect ideas from a discussion and then for the teacher to just re-summarize and add terminology to the students' summary. The PD teachers were explicit about how a sequence of teacher moves should involve the students and the teachers to have a productive summary. Thus my study deepens the field's understanding of what teachers think are productive and unproductive ways to respond to student thinking.

## Implications

The results of this study have implications for both the preparation of pre-service teachers and the professional development of in-service teachers. For the benefit of current teachers, the results of current teachers' discussion around the productivity of teacher responses can help inform the preparing of professional development. High quality professional development for all teachers (e.g., No Child Left Behind Act [NCLB], 2002; The Teaching Commission, 2004) draws on current research on what teachers think (Guskey & Yoon, 2009). Those preparing professional development could look at the final list of principles of productivity (see Appendix D) or at the collection of meta-principles (recall Figure 11) to get a starting point to understand what the teachers at the professional development might already think. For example, if the goal of a professional development was to help teachers have better classroom discussions, knowing that teachers want to include students and their ideas in discussion could guide the professional development to focus on how to best include students and what specific teacher moves might help student thinking be the guiding force of a discussion.

For the benefit of future teachers, the results of my study could help inform the decisions of teacher educators in the development of teacher education curriculum. Since teacher education can be accountable for making connections between teacher performance and student learning, teacher education needs to be effective, which includes using research to inform all aspects of teacher education (Guyton, 2000). Applying similar strategies as I suggested for preparing for an effective professional development, a teacher educator can prepare for a teacher education class. Pre-service teachers might not have the same conceptions as current teachers, but the principles of productivity (see Appendix D) or the meta-principles (recall Figure 11) provide a starting point for teacher educators to adapt and learn from. One way to draw on what in-service teachers think regarding the productivity of teacher moves to influence teacher education lessons would be to take what the PD teachers said was productive and help the pre-service teachers learn how to implement productive teacher moves or help them understand why a move is unproductive. For instance, in-service teachers think a teacher move is valuable if it elicits valuable student thinking and/or more information from the students, and a teacher educator could help a pre-service teacher practice eliciting student thinking and probing for more information. Additionally, the PD teachers were concerned with how to clarify student thinking, especially if it was unclear. The MOST framework helps to outline how to articulate what a student is actually saying and how much is inferable, which teacher educators could use to help pre-service teachers understand when language is imprecise. Since the PD teachers warned against assuming what students say and understanding, it is important to know exactly what is inferable from a student statement, so teacher educators could help pre-service teachers learn how to respond to help clarify the student thinking.

On the other hand, knowing the practices that were less salient to current teachers could inform teacher educators on what productive practices might need to be taught to future teachers. Some aspects of productively building on student thinking, although present among the discussions of the PD teachers, were not as prevalent as aspects such as the need to elicit and clarify. For instance, the PD teachers seldom discussed the need to Make Precise what the students should consider or a focus on Grapple Tossing the student thinking of the MOST. Teacher educators could help future teachers understand that students should be involved not only in the mathematics of the lesson, but also in the important mathematics of another student. That coupled with a deeper understanding of how to make student thinking precise by clarifying as well as making it clear what aspects of the student mathematics the teacher wants the class to consider could theoretically help the future teachers' practices be more productive.

## Limitations

Further research involving more and a larger variety of teachers could provide more robust information on what teachers think are productive or unproductive teacher moves in a mathematics classroom. Since the 13 PD teachers were picked for their desire to focus on and learn more about student thinking in their classrooms, that area of results in particular might be unique to this small data set, and not entirely generalizable.

Also, I did not include interviews as part of my study. Because of the format of the data collection I was not able to ask teachers to expand on interesting or useful ideas, or ask a clarifying question. The PD teachers would often say things that were unclear or incomplete ideas that a clarifying or follow-up question could have enlightened to provide more complete data. Research that sought such clarification could give further insight into why teachers say what they say.

## Conclusion

While there is still plenty of research needed on what teachers think, this study has made a step toward better understanding teachers and their decisions. It has shown that the PD teachers' ideas of what makes teacher moves productive or unproductive have some alignment with *building*, a theoretically productive way for teachers to respond to significant instances of student thinking. The framework of building in this study provides a definition and vision of productivity that is lacking in most of the research, so it provided a benchmark by which to measure the actual productivity of teacher moves in response to student thinking. Additionally, the PD teachers had ideas, which did not directly relate to building, that gave insight into what were the main focuses of teachers' discussion around productive responses to student thinking: focusing on student thinking, the productivity of different teacher moves, and the mathematics guiding teacher moves.

In answer to my first research question, "What principles of productivity emerge from teachers' discussion related to the productivity of teacher moves in response to MOSTs?", I articulated 65 unique principles of productivity (see Appendix D), and then I presented the common ideas that arose from these 65 principles. In order to capture more succinctly the main ideas present in the 65 principles of productivity, I also synthesized these principles into meta-principles that articulated the productivity of teacher moves as drawn from the PD teachers' discussions. From my analysis of these principles I was able to (1) see what principles of productivity emerged from the PD teachers' discussions and (2) analyze the content of the principles to better understand the PD teachers' ideas regarding the productivity of teacher moves. These principles frequently focused on the productivity of teacher responses that focus on student mathematics. The PD teachers decided that teacher moves were productive if the teacher moves (1) allowed the ideas of discussion to come from the students, (2) let the students be the ones engaged in sense making of the student ideas, and (3) let the mathematics underlying the students' ideas guide the discussion and summary. Another area of focus the PD teachers were concerned with was the productivity of specific teacher moves. Productive teacher moves included eliciting and following up on student thinking (when certain criteria the PD teachers decided on was met), and asking open (as opposed to closed) questions. The third most common topic the teachers discussed was the mathematics of the class: the mathematical goal of the task, the nature of the mathematics, and resolving the mathematics. In essence, a productive teacher move would help the class pursue and reach the mathematical goal of the task, and then resolve the mathematics discussed while ensuring that the mathematics discussed was grade-level appropriate and based on the students' ideas.

In answer to my second research question, "How do these principles of productivity align with the conceptualization of *building*?", I discussed the extent to which the principles of productivity aligned with each prerequisite and sub-practice of *building.* I found that the PD teachers' ideas of productivity, as represented by the principles of productivity, directly aligned with *building* about half of the time. The PD teachers' discussions about the productivity of teacher moves in response to MOSTs aligned most frequently with the last two sub-practices of building: Orchestrate and Make Explicit. The concepts underlying a productive Orchestrate and Make Explicit were important to the teachers, as they frequently discussed the need to have a whole class discussion where students were the originators and sense-makers of mathematical ideas, and the need to have someone summarize and resolve the mathematics at the end of discussion. The other actions of building (Invite/Allow, Recognize MOST, Make Precise, and Grapple Toss) were not as salient to the teachers, or the PD teachers had ideas that were close to a sub-practice but were missing essential components (e.g., focusing on the student mathematics of the MOST). Additionally, the results of my study relating to alignment with *building* support what Van Zoest et al. (2017) noticed: teachers are not effectively coordinating all four of the sub-practices of building. The PD teachers rarely talked about sequences of teacher moves or how two moves were related, so while some principles captured the coordinating of two sub-practices, the PD teachers never discussed how to coordinate all four of the sub-practices.

Through my research I was able to articulate the PD teachers' ideas surrounding the productiveness of teacher moves. I found that the PD teachers mainly focused on the productiveness of moves regarding three ideas: focus on student mathematics, specific teacher moves, and mathematics focus. Also, many of the PD teachers' ideas of productivity directly related to the *building* prerequisites and sub-practices. My research provides insight into what teachers think regarding the productivity of teacher moves that are in response to a *teachable moment.* This information begins to fill in gaps in the mathematics education research and can help inform the education of future and current teachers.

## REFERENCES


Ball, D. L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *The Phi Delta Kappan, 77*, 500-508.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15. Retrieved from http://www.jstor.org/stable/3699979

Çakmak, M. (2009). Pre-service teachers' thoughts about teachers' questions in effective teaching process. *İlköğretim Online*, *8*(3), 666-675. Retrieved from http://dergipark.gov.tr/download/article-file/90827

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning.* Portsmouth, Netherlands: Heinemann Educational Books.

Chamberlin, M. (2003). Teachers investigations of students' work: Meeting the challenge of attending to students' thinking. In N. A. Pateman, B. J. Dougherty & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp 189-196). Honolulu, HI: Center for Research and Development Group, University of Hawai'i.

Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W. G. Martin, M. Strutchens, & P. Elliot (Eds.), *The Learning of Mathematics* (pp. 113- 139). Reston, VA: NCTM.

Chapin, S. H., O'Connor, C., & Anderson, N. (2003). *Classroom discussions: Using math talk to help students learn, grades 1-6.* Sausalito, CA: Math Solutions Publications.

Conner, A. M., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics, 86*, 401- 429.

Cullen, R. (2002). Supportive teacher talk: The importance of the F-move. *ELT Journal, 56*, 117-127.

Edwards, D., & Mercer, N. (1987). *Common knowledge: The development of understanding in the classroom*. London: Methuen.

Elbaz, F. (1981). The teacher's "practical knowledge": Report of a case study. *Curriculum Inquiry*, *11,* 43-71.

Emmer, E. T., Evertson, C. M., & Anderson, L. M. (1980). Effective classroom management at the beginning of the school year. *The Elementary School Journal*, *80*, 219-231.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 403-434.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 225-256). Charlotte, NC: Information Age.

Guskey, T. R., & Yoon, K. S. (2009). What works in professional development? *Phi Delta Kappan*, *90*, 495-500.

Guyton, E. M. (2000). Foreward: Research on effective models for teacher education. In D. J. McIntyre & D. M. Byrd (Eds.), *Research on effective models for teacher education* (pp. ix-xii). Thousand Oaks, CA: Corwin Press

Herbel-Eisenmann, B. A., & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education*, *42*, 451-485.

Jacobs, V. R., & Ambrose, R. C. (2008). Making the most of story problems. *Teaching Children Mathematics, 15*, 260-266.

Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*, 169-202. Retrieved from http://www.jstor.org/stable/20720130

Jia, A. W. (2005). Feedback in the IRF discourse model in foreign language classroom. *Sino-US English Teaching*, *2*(7), 25-29.

Kazemi, E., Lampert, M., & Franke, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia.* (Vol. 1, pp. 12-30). Palmerston North, New Zealand: MERGA.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., ... & Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education*, *64*, 226-243.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education, 46*, 88-124. doi: 10.5951/jresematheduc.46.1.0088

Leatham, K. R., Van Zoest, L. R., Stockero, S. L., & Peterson, B. E. (2014). Teachers' perceptions of productive use of student mathematical thinking. In P. Liljedahl, S. Oesterle, C. Nicol, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 4, pp. 73–80). Vancouver, Canada: PME.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom.* Cambridge, MA: Harvard University Press.

Michaels, S., & O'Connor, C. (2015). Conceptualizing talk moves as tools: Professional development approaches for academically productive discussion. In L. B. Resnick, C. Asterhan, & S. N. Clarke (Eds.), *Socializing intelligence through talk and dialogue,* (pp. 347-362). Washington DC: American Educational Research Association.

Michaels, S., O'Connor, C., Hall, M., & Resnick, R. (2003). *Accountable talk: Classroom conversation that works.* Pittsburgh, PA: Institute for Learning, University of Pittsburgh.

Mitchell, J. (1994). Teachers' implicit theories concerning questioning. *British Educational Research Journal*, *20*, 69-83.

National Council for Teachers of Mathematics. (2007). *Mathematics teaching today: Improving practice, improving student learning*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Nathan, M. J., Eilam, B., & Kim, S. (2007). To disagree, we must also agree: How intersubjectivity structures and perpetuates discourse in a mathematics classroom. *Journal of the Learning Sciences, 16*, 525-565.

Nathan, M. J., Kim, S., & Grant, T. S. (2018). *Tracking changes in classroom discourse structure using human pattern identification and computer based motif analysis.* Unpublished manuscript.

No Child Left Behind Act of 2001, P. L. 107-110, 20 U.S.C. § 6319 (2002).

Nunan, D. (1987). Communicative language teaching: Making it work. *ELT Journal, 42*, 136-145.

Nystrand, M. (1997). *Opening dialogue: Understanding the dynamics of language and learning in the English classroom.* New York: Teachers College Press.

Nystrand, M., & Gamoran, A. (1991). Instructional discourse, student engagement, and literature achievement. *Research in the Teaching of English, 25*, 261-290. Retrieved from http://www.jstor.org/stable/40171413

Orchestrate [Def. 2]. (n.d.). Oxford Dictionaries Online. Retrieved November 20, 2015, from http://www.oxforddictionaries.com/us/definition/american_english/orchestrate.

Peterson, B. E., & Leatham, K. R. (2009). Learning to use students' mathematical thinking to orchestrate a class discussion. In L. Knott (Ed.), *The role of mathematics discourse in producing leaders of discourse* (pp. 99-121). Charlotte, NC: Information Age Publishing.

Peterson, B. E., Van Zoest, L. R., Rougée, A. O. T., Freeburn, B., Stockero, S. L., & Leatham, K. R. (2017). Beyond the "move": A scheme for coding teachers' responses to student mathematical thinking. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st annual meeting of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 17-24). Singapore: PME.

Pierson, J. L. (2008). *The relationship between patterns of classroom discourse and mathematics learning.* (Doctoral dissertation). Retrieved from Texas ScholarWorks at UTAUS.

Sahin, C., Bullock, K., & Stables, A. (2002). Teachers' beliefs and practices in relation to their beliefs about questioning at key stage 2. *Educational Studies*, *28*, 371-384.

SportsTec (1997-2015). Studiocode [Computer software]. Camarillo, CA: Vitigal Pty Limited.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*, 313-340. doi: 10.1080/10986060802229675

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*, 50-80.

Stockero, S. L., Leatham, K. R., Ochieng, M. A., Van Zoest, L. R., & Peterson, B. E. (2018). *Teachers' orientations toward using student mathematical thinking as a resource during whole-class discussion.* Unpublished manuscript.

The Teaching Commission. (2004). *Teaching at risk: A call to action.* New York: The Teaching Commission, The CUNY Graduate Center.

Tobin, K. (1986). Effects of teacher wait time on discourse characteristics in mathematics and language arts classes. *American Educational Research Journal*, *23*, 191-200.

Van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, *10*, 571-596.

Van Zoest, L. R., Peterson, B. E., Leatham, K. R., & Stockero, S. L. (2017). *Collaborative research: Investigating productive use of high-leverage student mathematical thinking.* Unpublished manuscript.

Van Zoest, L. R., Peterson, B. E., Leatham, K. R., & Stockero, S. L. (2016). Conceptualizing the teaching practice of building on student mathematical thinking. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1281–1288). Tucson, AZ: University of Arizona.

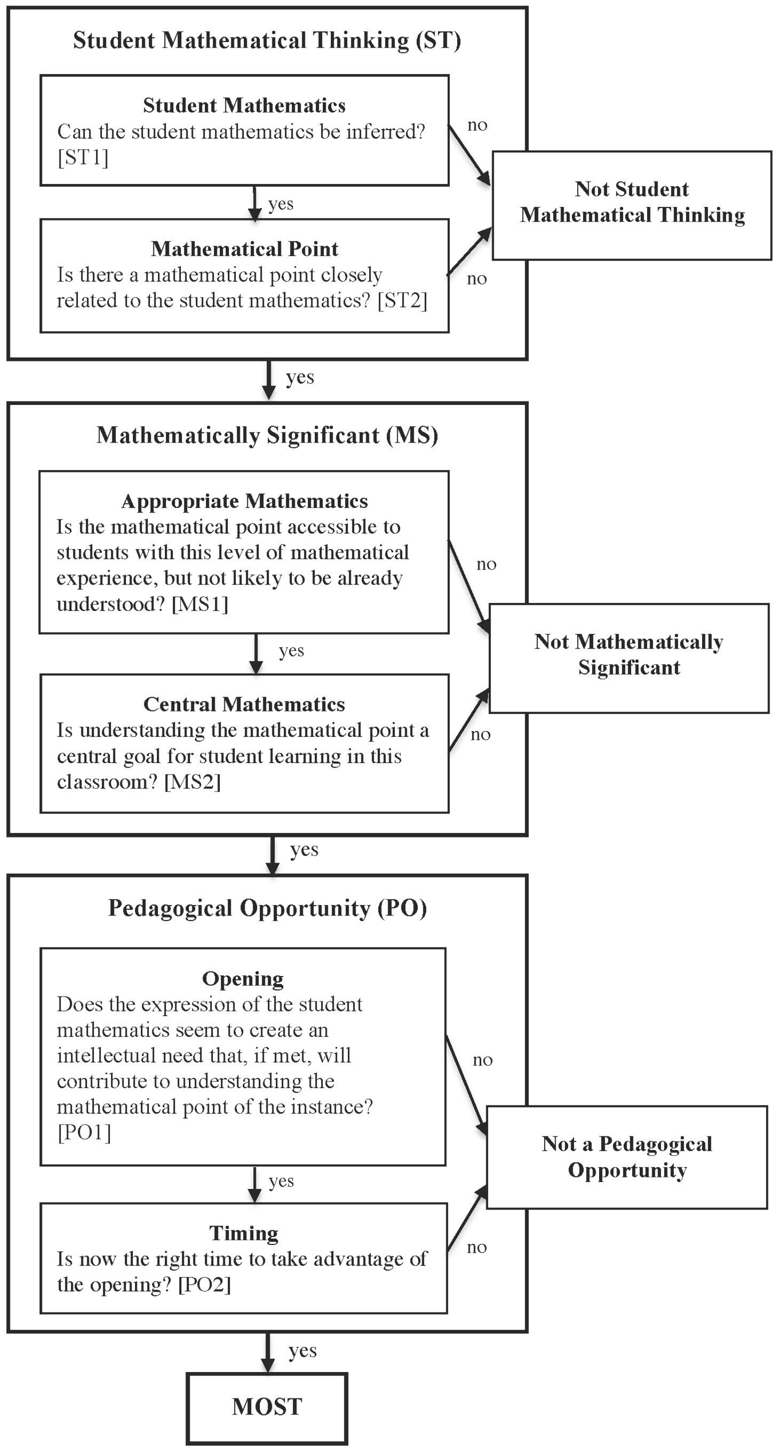
Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of instances of student mathematical thinking that are worth building on in whole-class discussion. *Mathematical Thinking and Learning, 19*, 33-54. doi: 10.1080/10986065.2017.1259786

Wells, G., & Arauz, R. M. (2006). Dialogue in the classroom. *Journal of the Learning Sciences, 15*, 379-428.

Westgate, D., & Hughes, M. (1997). Identifying "quality" in classroom talk: An enduring research task. *Language and Education, 11*, 125-139.

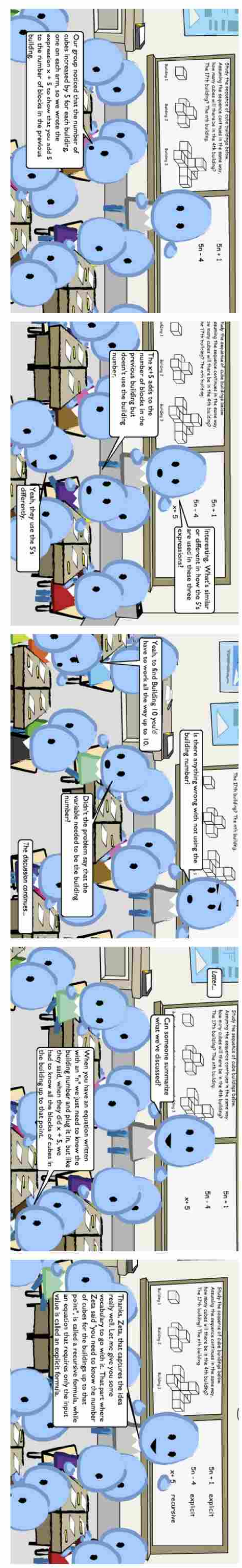
APPENDIX

## Appendix A

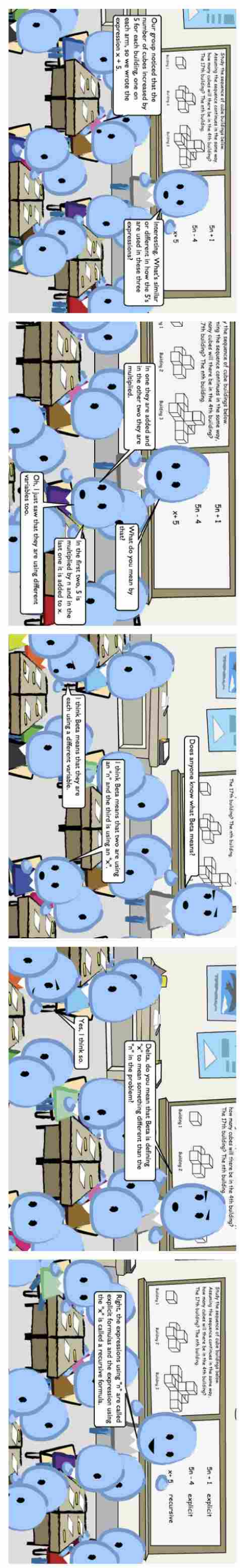


# Appendix B

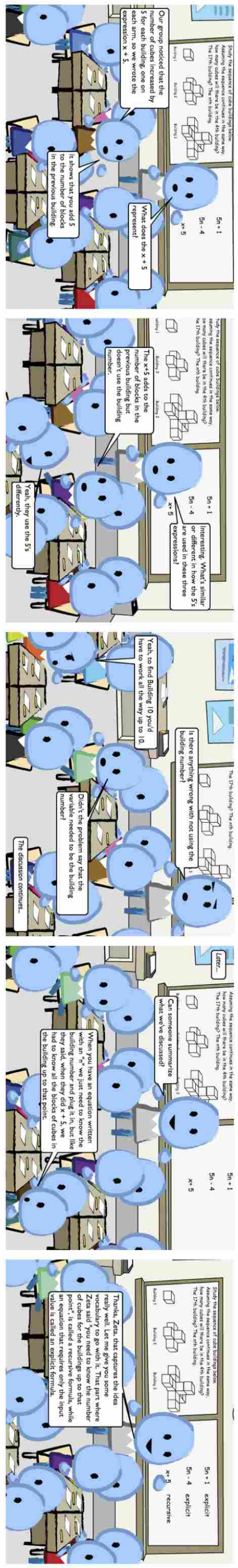
Scenario 1



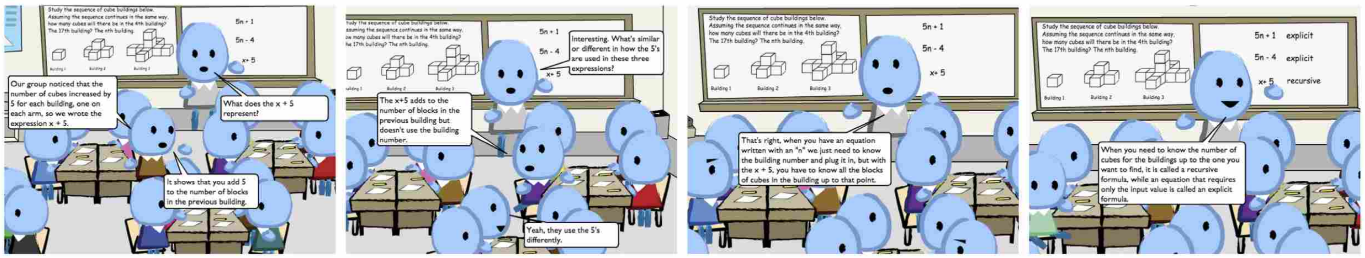
Scenario 2



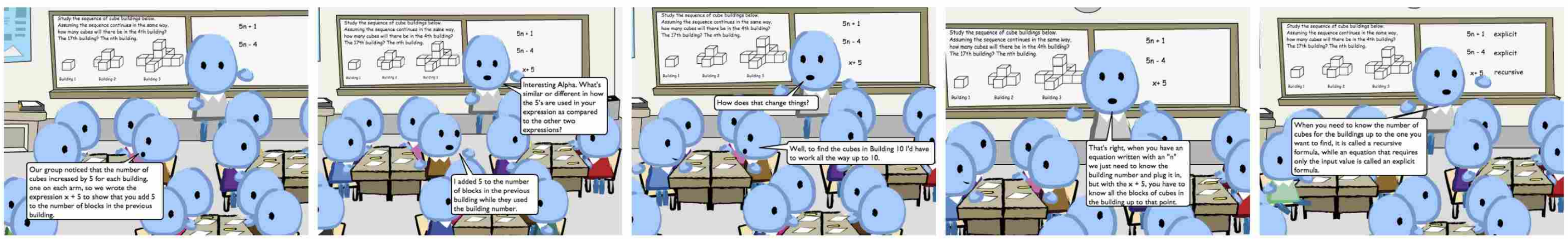
Scenario 3



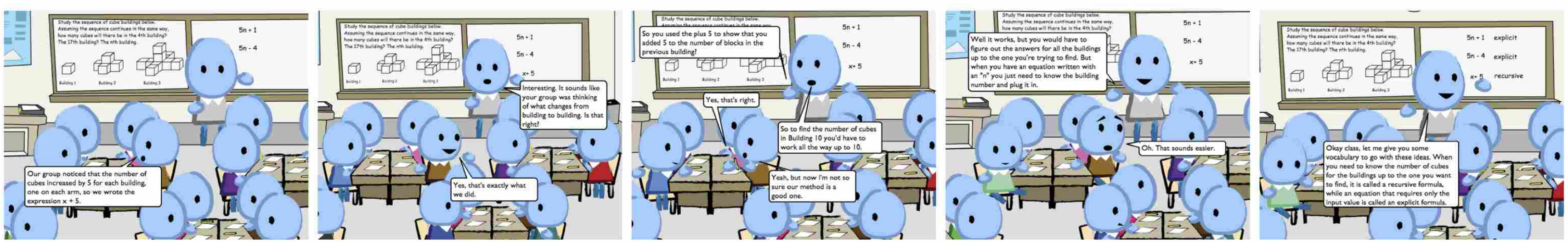
**Scenario 4**



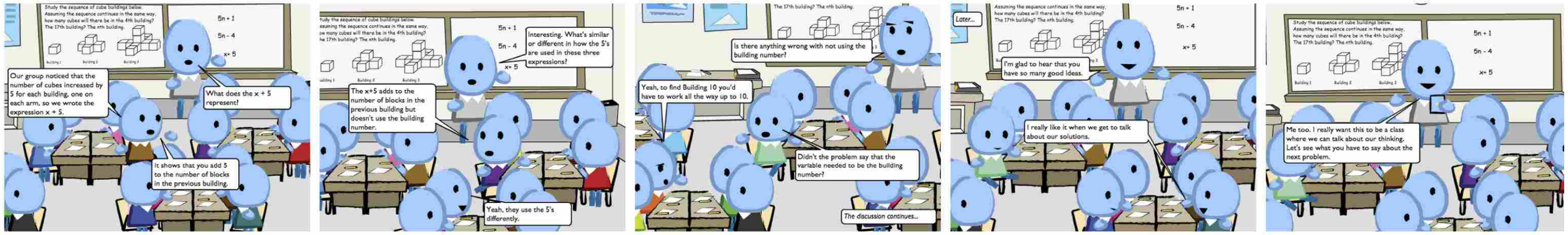

**Scenario 5**



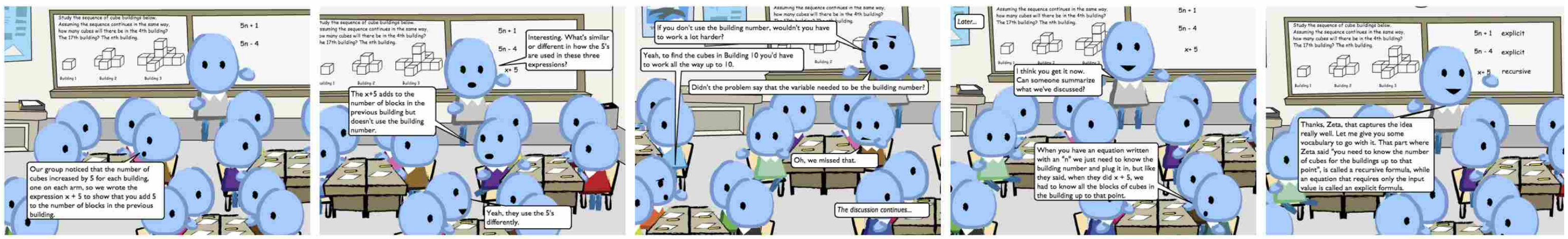

**Scenario 6**



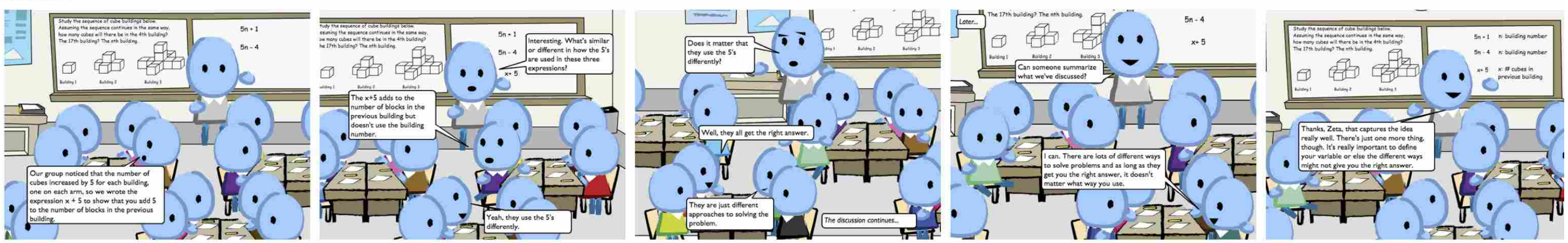
**Scenario 7**



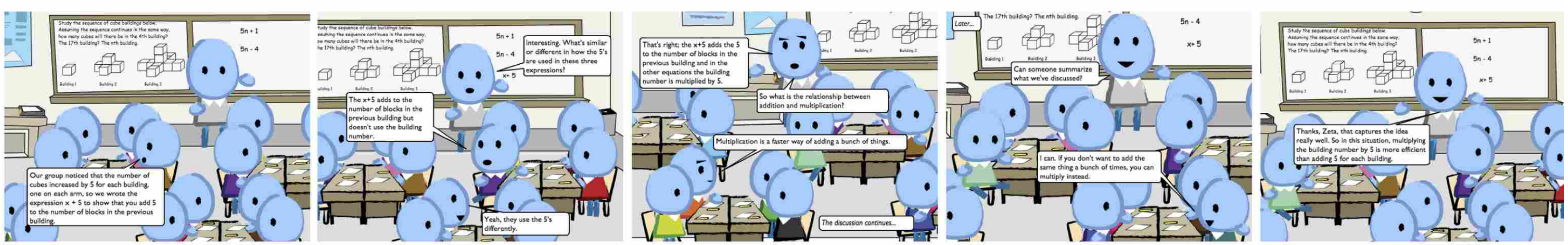

**Scenario 8**




**Scenario 9**



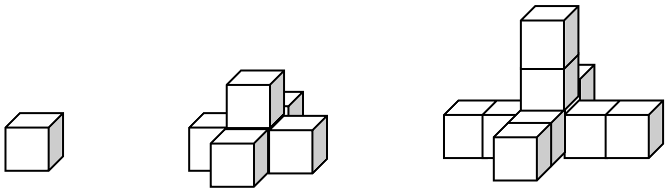

**Scenario 10**



**Appendix C**

Counting Cubes Task

Study the sequence of cube buildings below. Assuming the sequence continues in the same way, how many cubes will there be in the $4^{th}$ building? The $17^{th}$ building? The $n^{th}$ building.



Building 1 Building 2 Building 3

What are some ways that students might approach this problem (including formulas or expressions) that would likely lead them to a correct solution?

## Appendix D

| # | Theme | Principle of Productivity | Number of Group Occurrences |
|---|---|---|---|
| 1 | Summary | It is productive to have a student summarize at the end of discussion. | G5-1 |
| 2 | | It is productive to have students summarize and connect ideas from a discussion and then for the teacher to just re-summarize and add vocabulary to the students' summary. | G1-1, G2-1, G4-1, G6-2 |
| 3 | | It is productive to summarize the discussion. | G4-1, G6-1 |
| 4 | | It is unproductive for the teacher to focus on only one idea in the summary, when other ideas were also discussed and part of the goal of the task. | G1-1 |
| 5 | | It is unproductive for a teacher to summarize ideas before the students have been able to develop the ideas being summarized. | G1-2, G4-1 |
| 6 | | It is unproductive to validate the discussion without having a summary or mathematical resolve of the mathematics discussed. | G1-1, G2-1 |
| 7 | | It is unproductive for the teacher to give a summary without terminology or without providing meaning to any terminology given. | G1-1 |
| 8 | | It is productive for a teacher's summary of the discussion to refer to student thinking. | G4-1, G5-1 |
| 9 | | It is productive to have a summary that resolves the mathematics of discussion. (It is unproductive to conclude a discussion without resolving the mathematics.) | G3-1, G5-1, G6-1 |
| 10 | | It is productive for the teacher to make the objective of the discussion clear to the students such that the summary is not new information. | G1-1 |
| 11 | Mathematics Focus | Discussing class norms could be productive overall, yet not productive with respect to the mathematical goal of the task. | G4-2 |
| 12 | | It is productive to (pursue and) reach a mathematical goal of the task. (It is unproductive to not reach nor pursue a mathematical goal of the task.) | G2-2,G3-3, G5-4, G6-2 |
| 13 | | It is unproductive to ask a question with an obvious answer which doesn't lead the class toward the mathematical goal of the task or leads them to incorrect mathematics. | G1-1 |

| | | | |
|---|---|---|---|
| 14 | | It is unproductive to use student thinking (and the effort the students put into solving the task) just to get to an idea that is not the mathematical goal of the task. | G4-1 |
| 15 | | It is productive to have a class conversation that relates to or develops an idea that is central to the mathematical goal of the task. | G4-1, G5-1 |
| 16 | | It is unproductive for the teacher to dominate the resolution of mathematical ideas. | G2-1 |
| 17 | | It is productive to resolve students' ideas. | G6-1 |
| 18 | | It is productive to have a discussion that covers the mathematical points student thinking could bring out. | G6-1 |
| 19 | | It is productive to have students explore mathematics to develop their own meaning before giving them the standardized methods and ideas. | G1-1 |
| 20 | | It is productive to allow students to come up with the mathematical ideas in the discussion. (It is unproductive for the teacher to provide the ideas in the discussion.) | G4-1, G5-1, G6-2 |
| 21 | | It is unproductive to only focus on the right answer or how to get the right answer rather than methods, procedures, connections between students' solutions, or explanations. | G3-3 |
| 22 | | It is unproductive to introduce ideas that are not grade-level appropriate. | G4-1 |
| 23 | Reasoning/ Meaning | It is unproductive to introduce mathematical ideas without accompanying meaning and underlying reasoning. | G3-1, G4-1, G5-8 |
| 24 | | It is unproductive to introduce an idea without connecting the idea to the current context. | G6-1 |
| 25 | | It is unproductive to limit the discussion to one solution, possibly asking a student to only defend their answer, rather than comparing and contrasting the solutions available for discussion. | G1-1 |
| 26 | Teacher Moves (Redirect, Follow-up, Elicit, Clarify, Questions) | It is more productive to have students self-correct than for teachers to redirect the class when students are wrong. | G5-1 |
| 27 | | It is productive to ask redirecting questions if the class needs to be brought back on track. | G5-1 |
| 28 | | It is productive for a follow-up question to ask students to give more information that can lead to more discussion. | G2-1 |
| 29 | | It is productive to utilize and follow-up on student thinking. | G2-1, G6-7 |

| | | | |
|---|---|---|---|
| 30 | | It is unproductive to focus on validating discussion rather than utilizing and following-up on student thinking. | G6-1 |
| 31 | | It is productive to ask a question that either by itself or as a set-up to a sequence allows the class to explore a topic or idea in more depth and get needed information on the table for consideration. | G1-1 |
| 32 | | It is productive to have students expand on their thinking. | G5-1 |
| 33 | | It is productive when a teacher turn elicits valuable student thinking and/or more information from the students. | G2-1, G5-1 |
| 34 | | It is unproductive to invite students to share their thinking and then just tell them the answers. | G6-1 |
| 35 | | It is unproductive to assume student understanding. | G1-1, G4-1 |
| 36 | | It is unproductive for a teacher to clarify student thinking by rephrasing that thinking and asking for affirmation when the student could have clarified their own thinking. | G3-1 |
| 37 | | It is productive to ask a clarifying question to allow students to clarify their thinking, especially when incomplete student thinking is on the table, in preparation for the class to consider that thinking. | G1-1, G3-1, G4-1, G5-1 |
| 38 | | It is productive to ask a question that is more open and less invalidating (It is unproductive to ask a question that invalidates the student thinking.) | G4-2 |
| 39 | | It is productive to ask an evaluative question without harshness. (It is unproductive to ask a question that is harshly evaluative.) | G4-2 |
| 40 | | It is productive to ask open-ended questions. (It is unproductive to ask close-ended questions.) | G6-4 |
| 41 | | It is productive when teacher questions lead to good discussion that is not teacher-centered. | G1-1,G3-2 |
| 42 | | It is unproductive to ask a question when a student has already covered the ideas the question would have drawn out. | G2-1 |
| 43 | | It is unproductive to ask yes/no questions (because the questions close the students up). | G1-1, G4-2, G6-4 |
| 44 | | It is unproductive for a teacher to state what they want the students to think by putting the idea in the form of a yes/no question. | G1-1, G4-1 |
| 45 | | It is unproductive to ask a follow-up question on a topic for which students are ill-prepared. | G1-1 |

| | | | |
|---|---|---|---|
| 46 | Engagement | It is productive to have multiple students engaged in the mathematical conversation. (It is unproductive to engage only one student in class discussion.) | G2-1, G3-1, G4-2, G6-5 |
| 47 | | It is unproductive for the teacher to engage only one student in class discussion or cut off a discussion involving multiple students such that the teacher takes over the thinking. | G3-1 |
| 48 | Confusion/ Misconception | It is unproductive when a sequence of teacher moves adds to students' confusion. | G5-2 |
| 49 | | It is unproductive to allow misconceptions to continue throughout a discussion. | G5-1 |
| 50 | | It is unproductive to focus on an aspect of mathematics that can create a misconception. | G3-1 |
| 51 | Focus on Student's Mathematics | It is unproductive for the teacher to focus on how the class feels after a discussion rather than on the student thinking. | G5-1 |
| 52 | | It is productive for the teacher to allow student thinking to be made public and then discussed in a way that adds meaning to the mathematics of the student's statement. | G1-2 |
| 53 | | It is unproductive for a teacher to say an idea a student could have said (especially if it shortens the discussion). | G1-1, G3-1, G4-1, G5-1 |
| 54 | | It is unproductive for a discussion to be teacher-centered (focused on the teacher's ideas rather than on student thinking). | G3-1, G4-1, G5-3 |
| 55 | | It is unproductive for a discussion to be teacher-run where the teacher asks leading questions. | G4-1 |
| 56 | | It is productive to honor student thinking. | G2-1 |
| 57 | | It is unproductive for a teacher to jump to a conclusion that does not align with the student's ideas such that the student is unable to recognize their idea. | G2-1 |
| 58 | | It is unproductive to ignore student thinking. | G6-1 |
| 59 | | It is productive for the teacher to take the time to thoroughly understand the student thinking behind shared ideas. | G6-1 |
| 60 | | It is unproductive for the teacher to put words into a student's mouth (i.e., interject an idea that did not come from the students). | G2-1, G3-2, G4-1, G6-4 |
| 61 | | It is unproductive for the teacher to take over discussion, such that there is no student ownership and the teacher is doing all the thinking, by leaping from one student idea to a conclusion. | G2-1 |

| | | | |
|---|---|---|---|
| 62 | | It is unproductive to pursue non-MOSTs. | G1-1 |
| 63 | | It is unproductive for the teacher to make a leap to an idea when the class is not ready for it. | G2-1 |
| 64 | | It is unproductive to give definitions without drawing ideas from the students. | G6-1 |
| 65 | Other | The context of the teacher move affects the productivity of that teacher move. | G1-1 |