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The Principles of Effective Teaching Student Teachershave the Opportunity to Learn in an AlternativeStudent Teaching Structure

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ABSTRACT

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Research has shown that the focus of mathematics student teaching programs is typically classroom management and non-mathematics specific teaching strategies. However, the redesigned BYU student teaching structure has proven to help facilitate a greater focus on mathematics-specific pedagogy and student mathematics during post-lesson reflection meeting conversations. This study analyzed what specific principles of NCTM's standards of effective teaching were discussed in the reflection meetings of this redesigned structure. This study found that the student teachers extensively discussed seven of the eight principles NCTM considers to be necessary for effective mathematics teaching. Other pedagogical principles pertaining to student mathematical learning not included in NCTM's standards of effecting teaching were also discussed, as well as the student teachers' own understanding of mathematics. Behavior was discussed very little. This study also provides insights into how mathematics student teaching can be further restructured to assure that mathematics student teachers can leave their student teaching programs ready to implement the principles of effective teaching in their own classrooms.

Keywords: Conversations, Classroom Management, Mathematics, Effective Teaching, Student Teaching Structure, Reflection, Teacher Education, Opportunity to Learn

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE: RATIONALE

> There are two main arenas of knowledge that an apprentice cobbler needs to learn: (1) how to make shoes and (2) how to run a shoe store. Similarly, an apprentice teacher needs to learn (1) how to facilitate student learning and (2) how to run a classroom. Although in each case the apprentice needs to learn both aspects of the job, the former is far more important in general and, we would argue, should take precedence over the latter. What good is having a well-run shoe store, if you cannot make quality shoes? (Leatham &Peterson, 2010b, p. 100)

Behavior management and general pedagogical strategies are typically the things emphasized by cooperating teachers in the student teaching experience (Mitchell, Clarke, & Nuttall, 2007; Peterson & Williams, 2008; Moore, 2003; Borko & Mayfield, 1995; Bullough, Young, Erickson, Birrell, Clark, & Egan et al., 2002), and developing competency in these areas is often viewed by cooperating teachers (Leatham & Peterson, 2010a; Leatham & Peterson, 2010b), and sometimes even student teachers (Montecinos, Walker, Rittershaussen, Nunez, Contreras, & Solis, 2011), as the purpose of student teaching. This view of student teaching could be compared to a master cobbler who stresses the importance of learning to "run the shoe store." It is unfortunate that mathematics student teachers primarily have the opportunity to learn how to manage a classroom, because many mathematics education researchers and teachers believe student teaching is a critical aspect of pre-service teacher education (Zeichner, 2002; Clarke, Triggs, & Nielsen, W., 2013; Borker & Mayfield, 1995), and it is widely accepted as the most beneficial of all pre-service education (Mitchell, Clarke, & Nuttall, 2007; Wilson, Floden, & Ferrini-Mundy, 2001; Evertson, 1990; McIntyre, Byrd, & Foxx, 1996; Guyton & McIntyre 1990). The student teaching experience has the potential to instead provide student teachers with an opportunity to learn other important aspects specific to mathematics education pedagogy.

For example, NCTM's *Principles to Actions* (2014) outlines specific aspects of an "excellent mathematics education program" for schools of every level. They claim that such a program requires "effective teaching," where mathematics educators establish mathematics goals to focus learning, implement tasks that promote reasoning and problem solving, use and connect mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency through conceptual understanding, support productive struggle in learning mathematics, and finally, elicit and use evidence of student thinking (NCTM, 2014). No aspect of classroom management or discipline is mentioned, and each principle is specific to students' mathematics. Therefore in order for student teachers to prepare to become effective teachers, their student teaching experience would need to focus on these principles, rather than on student behavior, classroom management, or general pedagogy. A student teaching program focused on the principles of "effective teaching" could be compared to a master cobbler who instead values the apprentice's ability to "make quality shoes."

The mathematics student teaching program at Brigham Young University was redesigned in 2006 "to change the focus of student teaching away from students' behavior and onto students' mathematics" (Leatham & Peterson, 2013, p. 629), or in other words, to focus on "making quality shoes." Although redesigned several years before NCTM released *Principles to Actions* (NCTM, 2014), the purpose of the restructured BYU program and the standards in *Principles to Actions* (NCTM, 2014) both demonstrate a focus on students' mathematics. Leatham and Peterson (2013) found that when student teachers in this program commented on student mathematics during reflection meetings with their cooperating teachers, those same comments were almost never also coded as behavior comments (Leatham & Peterson, 2013). These findings indicate that if the student teaching structure is changed to direct student teachers' conversations towards student mathematics, there will be consequently less focus on behavior. A separate study about this same student teaching structure found that in 2006-2007, after the redesign, 27% of all comments between student teachers and cooperating teachers were specifically about teaching mathematics, compared to only 15% prior to the redesign. Comments about teaching mathematics *to students* increased from 6% to 20% (Franc, 2013).

The results published thus far (Franc, 2013; Leatham and Peterson, 2013) indicate that the Brigham Young University mathematics student teaching program seems to be structured in a way that encourages student teachers to discuss student mathematics more frequently than student teachers placed in a traditional program. Franc's (2013) results, however, are based on studying mostly short, informal conversations between student teachers and cooperating teachers, with a unit of analysis of 1-2 sentences. Leatham and Peterson (2013) studied longer, more formal conversations between the student teachers and cooperating teachers during formal reflection meetings, but again used a small unit of analysis of 1-2 sentences. Although we know these utterances contained more discussions of student mathematics than before the restructure, it is difficult to interpret these results in a way that helps us understand the bigger picture, or in other words the nature of the conversations student teachers had with their cooperating teachers as a whole. While the previous studies provide evidence that the student teachers in this program have an opportunity to learn about teaching mathematics to students and not just managing classrooms, whether or not they leave the program having had the opportunity to learn about the principles of "effective teaching" (NCTM, 2014) remains unknown. Therefore, the purpose of the current study is to understand exactly what principles of "effective teaching" (2014) student teachers in this restructured program have the opportunity to learn as they reflect on their practice through conversations with their cooperating teachers, This study will also look for evidence that these student teachers have internalized and generalized the principles they discussed in a way that suggests they might be successful in implementing the principles in their future classrooms.

CHAPTER TWO: THEORETICAL FRAMEWORK

The purpose of this study is to gain a better understanding of the content and nature of conversations within the BYU mathematics student teaching program between student teachers and their cooperating teachers as they reflect of their practice. Therefore, this chapter contains two sections, namely what student teachers should have the opportunity to learn, and what types of situations provide student teachers with an opportunity to learn. In the first section, I will outline literature on mathematics teaching and learning in general, and then present a framework of teaching standards published by NCTM that will provide the coding scheme for my data. In the second section I will build a definition of "opportunity to learn," and present research on reflection as a means of student teacher learning.

What Student Teachers Should Have the Opportunity to Learn

> Student teaching is teacher education when intending teachers are moved toward a practical understanding of the central tasks of teaching; when their dispositions and skills to extend and probe student learning are strengthened; when they learn to question what they see, believe and do; when they see the limits of justifying their decisions and actions in terms of "neat ideas" or classroom control; and when they see experience as a beginning rather than a culminating point in their learning. (Feiman-Nemser & Buchmann, 1987, p. 272)

In 2000, NCTM called for "a serious commitment to the development of students' understanding of mathematics" in mathematics education (NCTM, 2000, p. 18). The student teaching experience, as Feiman-Nemser and Buchmann (1987) suggest above, is truly teacher education only when student teachers have this commitment. Teaching is supporting student learning (Hiebert et al, 1996), so prospective teachers must learn to focus on student mathematical thinking and learning (Fieman-Nemser & Buchmann, 1987), and as Leatham and Peterson (2010a) state, learn to "anticipate, elicit, and use" it (p. 2). In 2014, the National Council of Teachers of Mathematics published Principles to Actions: Ensuring Mathematical

Success for All, a call to action for more excellent mathematics education. Needs in the following categories are addressed: teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism (NCTM, 2014). Because the purpose of this study is to better understand what principles of mathematics pedagogy student teachers have the opportunity to learn at BYU, the category of teaching and learning was most relevant framework for viewing my data.

> An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideals and reason mathematically. (NCTM, 2014, p. 7)

Effective teaching and learning includes eight aspects (NCTM, 2014). I will give the definition of each as defined by NCTM and then review what has been said about these principles elsewhere in the literature. These eight aspects will provide the lens through which I looked at the mathematics pedagogy that was discussed by the student teachers.

The descriptions I will provide of the principles of "effective teaching" (NCTM, 2014) help paint a picture of what they look like in the classroom and why they are important for teachers to learn. As I gathered information about the eight principles not only from *Principles to Action* (NCTM, 2014) but also from the literature on mathematics education, I was able to expound my understanding of each principle in a way that allowed me to better recognize the principle when it occurred in my data, assuring better accuracy in my coding.

Establishing Learning Goals to Focus Learning

> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. (NCTM, 2014, p. 12)

Hiebert et al. (2007) described a teaching process of 1) setting learning goals for students, precisely and explicitly; 2) implementing a teaching episode; 3) assessing whether or not the instruction facilitated students in achieving the goals; and 4) revising instruction in a way that does facilitate the desired student achievement. Hiebert et al. (2007) explain that without learning goals, there is almost no way of monitoring student mathematical learning. Setting goals simply "sets the stage for everything else" (p. 51), and thus student teachers should be given the opportunity to learn and experience what it means to go through this four step process.

Implementing Tasks that Promote Reasoning and Problem Solving

> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solutional strategies. (NCTM, 2014, p. 17).

Mathematical tasks play an important role in students' learning (Kilpatrick et al., 2001). Diezmann and Walters (2000) argued that learners need *challenging* tasks to both facilitate learning, and develop autonomy. Their review of the literature on challenging tasks shows that completing challenging tasks helps create intrinsic motivation and enhances self-efficacy and self-esteem. As stated by Kilpatrick et al. (2001), "students learn best when they are presented with academically challenging work that focuses on sense making and problem solving as well as skill building" (p. 335). Effective teachers, and thus student teachers, must learn how to both choose appropriate challenging tasks, as well as how to appropriately "scaffold" students through the process (NCTM, 2000; Kilpatrick et al., 2001, Diezmann & Walters, 2000).

Use and Connect Mathematical Representations

> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. (NCTM, 2014, p. 24)

Brenner et al. (1997) argues the importance of students being able to construct and move between multiple representations of algebraic problems. For example, when learning functions, they stress the importance of students' ability to move between the graphical, algebraic, tabular and verbal (Brenner et al., 1997). It is important that student teachers learn to facilitate this because traditional students typically perform well when using only symbolic representations in algebraic problems, but poorly when asked to draw conclusions from a word problem (Brenner et al., 1997).

Facilitating Meaningful Mathematical Discourse

> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. (NCTM, 2014, p. 29)

Smith and Stein (2011) published five suggested practices for teachers who wish to effectively of facilitate meaningful mathematical discourse in a classroom. The five practices are 1) anticipating student responses to challenging mathematical tasks prior to the lesson; 2) monitoring students' work on and engagement with the tasks; 3) selecting particular students to present their mathematical work; 4) sequencing the students responses that will be discussed in a specific order; and 5) connecting different students' responses and connecting the responses to key mathematical ideas. Although NCTM (2014) does not reference each of these five practices individually, they paint a picture of what a teacher who is engaging in facilitating mathematical discourse amongst students would be doing in a classroom and there are many overlapping ideas. Student teachers should have the opportunity to learn how to implement these five practices in a mathematical classroom.

Pose Purposeful Questions

> Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. (NCTM, 2014, p. 35)

Leahy, Siobhan, Lyon, Thompson, William, (2005) stress the importance of teachers carefully planning the questions they will use in class. Franke, Webb, Chan, Ing, Freund and Battey (2009) argue that teachers' questions have the ability to "scaffold students' engagement with the task, shape the nature of the classroom environment, and create opportunities for learning high-level mathematics" (p. 381). Finding from their study suggest that this scaffolding can be accomplished by providing sufficient wait time as well as pressing students for explanations and justifications. Student teaching could provide an opportunity for student teachers to learn how to scaffold and provide wait time in this way.

Build Procedural Fluency from Conceptual Understanding

> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. (NCTM, 2014, p. 42).

In *Principals to Actions* (NCTM, 2014), NCTM uses the terms "conceptual understanding" and "procedural fluency" from Kilpatrick et al.'s (2001) five strands of learning. Kilpatrick et al. (2001) describes conceptual understanding as the comprehension of mathematical concepts, operations, and relations. Rittle-Johnson, Siegler, and Alibali (2001) defined conceptual knowledge as "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain. This knowledge is flexible and not tied to specific problem types and is therefore generalizable" (p. 346).

Procedure fluency is having "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001, p. 5). Siegler and Alibali (2001) go on to define procedural knowledge as "the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and therefore is not widely generalizable (p. 346). Student teachers must learn to facilitate student conceptual and procedural knowledge of mathematical concepts, so teachers must help students learn both what to do, and why (Skemp, 2006).

Support Productive Struggle in Learning Mathematics

> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. (NCTM, 2014, p. 48)

Teachers must allow students to "grapple with mathematical ideas and relationships" (NCTM, 2014, p. 25). Hiebert et al. (2007) provides an extensive description of the benefits of student mathematical struggle, if done correctly. They point out that productive struggle does not mean pointless frustration that comes from poorly designed tasks. The tasks must remain in a student's zone of proximal development, allowing students to wrestle through ideas that are within reach, comprehensible, but not yet formed. This struggle allows students to go through a process of wanting to make sense of situations, connect them with what they already know, and restructure accordingly, allowing an overall deep understanding of content (Hiebert et al., 2007). During this struggle, teachers must decide when to prompt and when to step back (Kilpatrick et al., 2001), a difficult task that should be given attention in pre-service programs.

Elicit and Use Evidence of Student Thinking

> Effective teaching of mathematics uses evidence of student thinking to assess the progress toward mathematical understandings and to adjust instruction continually in ways that support and extend learning. (NCTM, 2014, p. 53)

Teachers should continuously be looking for evidence of student learning, and using the evidence to revise their instruction in a way that would not have been possible otherwise (Leahy, Siobhan, Lyon, Thompson, William, 2005). This involves more than just noticing whether student answers are right or wrong (Crespo, 2000). It involves knowing what mathematical history students are coming (Clements & Sarama, 2004) from and having well defined goals.

Situations that Provide Student Teachers an Opportunity to Learn

As I examined what student teachers in the BYU student teaching program had the opportunity to learn, the eight aspects of "effective teaching" described above provided an excellent framework for examining the content of their conversations. However, I wanted to gather further evidence that the student teachers were accepting of these pedagogical ideas they discussed, leaving these conversations having acknowledged that they had learned something new. I will explain how I decided that there was sufficient evidence that the student teachers had "taken up" an idea, followed by describing situations that provide an opportunity for student teachers to learn.

Opportunity to Learn

Although the student teachers were likely exposed to many things throughout their entire student teaching experience, the focus of this study is only on the content student teachers interacted with in the context of reflection meetings. I found "opportunity to learn" as the best way to describe how content is made available for student teachers to learn as they interact with it through practicum observation, teaching, discussion, and reflection (Leatham & Peterson, 2013).The National Research Council (2001) define "opportunity to learn" as "circumstances that allow students to engage in and spend time on academic tasks" (p. 333). Although never defined or discussed in the literature as pertaining to student teachers rather than students, we can extend this definition to student teachers. We will define opportunity to learn as circumstances that allow student teachers to engage in, spent time on, and reflect on pedagogical tasks. I add the concept of reflection to our definition as reflection on lessons taught and learned can result in a significant portion of the content student teachers have the opportunity to learn. This idea will be further discussed in the next section. Just as opportunity to learn for students is influenced by both the teacher and the curriculum (Hiebert and Grouws, 2007), I argue the opportunities student teachers have to learn are influenced by the cooperating teacher, university supervisor, and fellow student teachers with whom they converse and the things they choose to emphasize when conversing with the student teachers. This influenced my decision to examine the formal conversations between student teachers, cooperating teacher, and university supervisor, known as "reflection meetings," as this time spent conversing influences what the student teachers have the opportunity to learn.

Reflection

Richert (1992) recognizes that many teacher education programs have been created to promote the reflective practice. Koerner, Rust, and Baumgartner (2002) stressed the importance of cooperating teacher and student teacher communication. A specific type of communication between cooperating teacher and student teacher, one that cooperating teachers should view as essential to facilitate, is the reflective practice (Stegman, 2007), where "teachers look back on the teaching and learning that has occurred as a means of making sense of their actions and learning from their experiences" (Richert, 1992, p. 172). Chalies, Ria, Bertone, Trohel, & Durand (2004) argued that cooperating teachers are an aid to this reflective practice, and Clarke, Triggs, & Nielsen, (2013) stated that those who support a reflective disposition are the most effective. This is consistent with Borko and Mayfield's (1995) findings that student teachers thought the cooperating teachers who held the longest formal reflection conferences with the most specific feedback had the most influence upon their learning.

Reflection often occurs in the student teaching experience in post-lesson meetings between cooperating teacher and student teacher, sometimes called "post lesson interviews (Clarke, Triggs, & Nielsen, 2013), "formal conferences" (Borko & Mayfield, 1995) or "reflection meetings" (Peterson & Leatham, 2013). Each of these types of conversations are "explicit occasions for reflection and collaboration" and "represent much of what [student teachers] have the opportunity to learn" (Peterson & Leatham, 2013, p. 629). They are a time for reflection, and contribute to student teachers' experience, identity, and construction of new knowledge (Chalies, Ria, Bertone, Trohel, & Durand, 2004). For the duration of this study, I will focus on student teacher and cooperating teacher communication in the context of these formal reflection meetings only. I do not underestimate the value of other conversations between dyads, but rather acknowledge that these meetings represent a significant portion of conversations and of what the student teachers had the opportunity to learn, and were thus the focus of data collection in my study.

The literature suggests that these "reflection meetings", as I will refer to them, can provide an opportunity for *new* knowledge construction for student teachers. As Richert (1992) states, it is a time of reenactment and reconstruction of what happened in the classroom in an effort to make sense of it. This represents a much different view than cooperating teachers and student teachers in the literature who believe that student teachers learn primarily from "experience" (Peterson & Williams, 2005; Leatham & Peterson, 2010b; Borko & Mayfield, 1995). The reflective practice supports the notion that teaching experience is the beginning point of learning, not the culminating point (Borko & Mayfield, 1995; Feiman-Nemser & Buchmann, 1987). These reflection meetings thus provide the context in which the conversations between student teachers and their cooperating teachers and university supervisors that I analyzed took place.

Student Teacher "Take Up" of New Ideas

Up to this point, I have explained how the eight principles from the Principles to Action (NCTM, 2014) provided the lens through which I examined what the student teachers had an "opportunity to learn" during their reflections meetings. Now I will discuss how I looked for evidence that the content was actually "taken up" during these opportunities to learn.

Ward and McCotter (2004) studied the nature and quality of student teacher personal reflection in response to lessons plans they had implemented, and used the wide range of the types of reflection they observed to develop a rubric for future evaluation of the quality of student teacher reflection. They found one aspect of personal reflection that should be evaluated for quality as "change" (the extent to which the student teachers are going to do something with what they learned). The extent of which the student teachers were likely to change based on what they'd learned was evaluated on a spectrum consisting of four levels: routine, technical, dialogic, and transformative—routine being the lowest level of reflection and transformative being the highest. In analyzing the "change" of the student teachers, the dialogic level of reflection represented student teachers who "develop new insights about teaching or learners" that have the potential to lead to a change of practice (p. 250). Furthermore, transformative reflection showed a complete reframing of perspective leading to a change of practice. They found this level of reflection did not surface often (Ward & McCotter, 2004).

Stegman (2007) similarly studied the content of student teacher reflection with the use of guided questions. Reflection content was placed in one of four categories similar to those of

Ward and McCotter (2004). Results showed that the quality of the student teachers' reflection seemed to improve over the course of the semester.

Neither Ward and McCotter nor Stegman, however, studied the quality of reflection within *conversation* with other student teachers, cooperating teacher, and university supervisors. Only written reflection was studied, and whether or not the student teachers were specifically mathematics teachers is unclear from the research. An adapted form of the framework developed by Ward and McCotter (2004) (to be described later in my methodology) was used in this study to evaluate to what extent the pedagogical principles of "effective teaching" (NCTM, 2014) were "taken up" as they discussed the principles during their formal reflection meetings. The idea of "take up" can be compared to what Ward and McCotter (2004) describe constitutes the two deeper levels of reflection, namely dialogic and transformative, where new insights are developed and lead to a fundamental change in practice. If a student teacher reaches the dialogic or transformative level of reflection, we will take this as evidence that the student teacher has "taken up" the principles of "effective teaching" (2014) they've discussed and are likely to carry their new knowledge into their future teaching.

Summary

If student teaching is indeed the most beneficial aspect of teacher education (Mitchell, Clarke, & Nuttall, 2007; Wilson, Floden, & Ferrini-Mundy, 2001), then mathematics teacher education programs should strive to give student teachers the opportunity to learn how to be an "effective teacher" (NCTM, 2014) during their practicum, not just how to manage a classroom.

The purpose of this study is to examine what student teachers participating in the redesigned BYU mathematics student teaching program have the opportunity to learn, as well as how accepting they are of these ideas. The NCTM (2014) principles of effective mathematics teaching provide a suitable framework for coding the student teachers' conversations with their cooperating teacher to find exactly what pedagogical concepts are being discussed. Meanwhile, an adapted form of a rubric previously designed to evaluate the complexity of the ways student teachers reflect developed by Ward and McCotter (2004) will give insight into whether or not these student teachers took away something from ideas they discussed, perhaps leaving the BYU program committed to implementing the ideas in their future classrooms.

CHAPTER THREE: LITERATURE REVIEW

There are three main areas of research relevant to this study: 1) the role that cooperating teachers play in the learning of student teachers; 2) what student teachers typically talk about with their cooperating teachers and university supervisors, which is closely tied to what cooperating teachers value for their student teachers to learn ; and 3) the student teaching structure. The literature on the role cooperating teachers play in the learning of student teachers helps us understand the relevance of this study and why what student teachers discuss with their cooperating teachers is of great importance. Next, the purpose of this study is to analyze what student teachers in the BYU student teaching structure have the opportunity to learn as they converse with their cooperating teacher. Therefore it is essential to examine the literature on what we have found student teachers talk about with their cooperating teachers in the past in a traditionally designed student teacher structure, because what they talk about is what they have an opportunity to learn. Finally, since this study aims to learn what student teachers have the opportunity to learn when placed in a student teaching structure that has been redesigned, reviewing the research on the structure of a traditional program and highlighting the differences found in the BYU structure will help us understand why the student teachers in this program might discuss different pedagogical ideas than those of another structure.

The Role of Cooperating Teachers and University Supervisors

Clarke, Triggs, and Nielsen (2013) summarized three roles that a cooperating teacher might choose to play: classroom placeholder, supervisor of practica, and teacher educator. They lie on a spectrum from the least amount participation from cooperating teacher to the greatest. When a cooperating teacher is a classroom placeholder, as soon as the student teacher arrives, he or she “exchanges places” with the cooperating teacher, who “exits to the staffroom” for the rest of the practicum. Next on the continuum, a supervisor of practica expects that the student teacher has learned what they need to know about teaching during their university program, and then reports on how well the student teacher implements what they've learned in a classroom setting. This cooperating teacher offers only general, positive feedback. Clarke, Triggs, and Nielsen (2013) argue that this is the most commonly implemented cooperating teacher role. Third on the spectrum is a cooperating teacher as a teacher educator. This cooperating teacher is far more engaged than the previous two, acting like a "coach," who a) works closely with the learner in the immediacy of the action setting, b) encourages and elicits the meaning that the learner is making of his or her practice and c) judiciously provides guidance to facilitate the development of her or his repertoire (Clarke, Triggs, & Nielsen, 2013).

Bullough, Young, Erickson, Birrell, Clark, and Egan et al. (2002) found that student teachers mostly viewed their cooperating teacher as someone who provided a place for them to teach. This perception matches Clarke's description of a "classroom placeholder." Peterson and Leatham (2010a) argued that "cooperating teachers tend to see themselves more as an experienced colleague than as a teacher educator" (p. 225). This seems consistent with Clarke et al's (2013) "supervisor of practica" perception of a cooperating teacher. I liken Clarke's description of the "teacher educator" cooperating teacher to a "mentor." Mentors of beginning teachers are continually acting as a sponsor, encouraging self-reliance, encouraging true collegiality, encouraging reflection on practice, and providing timely and appropriate feedback, and evaluation (Peterson & Williams 1998). This seems clearly the most desirable role for a cooperating teacher to play, since, as Zeichner (2002) argues, "being a good cooperating teacher is more than providing access to a classroom or modeling a particular version of good practice. It involves active mentoring" (p. 59). However, it is unfortunate, as Zeichner (2002) also states, that often mentoring student teachers is clearly not valued as important by schools, or by universities.

It is reasonable to believe that the different roles cooperating teachers choose to fulfil will have a significant effect on what their student teachers will have the opportunity to learn. It seems that cooperating teachers are important in determining the quality of learning for their student teachers (Zeichner, 2002).

Because the literature shows that cooperating teachers focus on classroom management, we might predict that student teachers would in turn focus their thinking on management (Richert, 1992). Hawkey (1996) argued that if student teachers are unsure of themselves as teachers, they tend to conform to the environment in which they are placed. Peterson and Williams (2008) analyzed the results of a questionnaire given to sixteen individual members of eight dyads and audio recordings of meetings between the pairs. The authors looked for "core conversational themes" in what they talked about. One cooperating teacher believed that classroom management was the most important thing to learn about teaching, and that once it was successfully established everything else would fall into place. His student teacher had the same views about management. Although the authors couldn't provide evidence that this student teacher's view about management had *changed* because of his cooperating teacher, his response seemed peculiar because his university program did not focus on classroom management much at all. The pair also shared beliefs about mathematics, which they believed was not challenging at the junior high level, and not as important as learning to control the students. The pair mostly discussed classroom management, and fewer than 9% of utterances between them were about mathematics or about teaching a particular mathematics topic. This cooperating teachers' beliefs also seemed to dictate his mentoring style, which began as "one of dominance," where the student teacher would listen, and try to do it the cooperating teacher's way. This mirrors the cooperating teacher's beliefs about how students should be taught: the basics should come first. Finally, the researchers found that this student teachers' opportunity to strengthen his understanding of mathematics in and for teaching was very limited (Peterson & Williams, 2008).

In an analysis of a second pair, Peterson and Williams (2008) found that the cooperating teacher viewed the most important aspect of teaching as student "active participation" in mathematics. The student teacher shared this same view in her exit interview. The belief was consequently found to be one of the core conversational themes in their conversations together, along with mathematics. Unlike the other pair, this pair recognized the complexity of even low level mathematics such as dividing fractions. 24% of their utterances focused on teaching mathematics. This cooperating teacher's style of mentoring stressing student teacher's active participation in the learning process, just as he stressed this in junior high students' learning. Finally, in contrast to the first pair, the experience offered this student teacher "an opportunity to address her understanding of mathematics in and for teaching" (p. 474).

In summary, this study shows that there was a harmony between the beliefs of the cooperating teacher about mathematics, what they wanted their student teachers to learn, what they talked about together, and their mentoring style (Peterson & Williams, 2008). This provides us with further evidence that what student teachers have the opportunity to learn is directly influenced by their cooperating teachers.

Borko and Mayfield (1995) studied how the way cooperating teacher's viewed their roles affected the way they conversed with their student teachers. Some of the cooperating teachers believed they could play an active role in the learning of student teachers, while others did not. The cooperating teachers who didn't find significance in their role believed that student teachers learn primarily through experience (Borko & Mayfield, 1995), which is consistent with Peterson and Leatham's (2010b) findings. Borko and Mayfield (1995) summarize the effect of "triad" (cooperating teacher, university supervisor, and student teacher) beliefs on conversations:

> When triad members share a belief that teachers learn primarily through experience and practice, it becomes easy for cooperating teachers and university supervisors to offer few suggestions to student teachers and do little to challenge their ideas and practices, and for student teachers to pay only limited attention to feedback and suggestions and continue to teach in ways that maintain the status quo. (p. 516)

In contrast, the cooperating teachers who did believe they could play an active role in their student teachers' learning held longer and more frequent conferences, and provided more feedback. The student teachers found these cooperating teachers as more influential than those described above (Borko & Mayfield, 1995).

Borko and Mayfield (1995) also found that university supervisors believed student teachers learn through experience and practice. Some of the university supervisors had little knowledge about mathematics and mathematics pedagogy, and the authors attribute this as the reason the university supervisors provided little content-specific feedback. Interestingly, the student teachers reported little influence by university supervisors (Borko & Mayfield, 1995).

From this literature, is it is clear that the cooperating teacher, if acting as a mentor and teacher educator, can indeed influence the student teacher in both beliefs and conversation. This provides evidence that if the student teaching structure were redesigned to focus the cooperating teacher's emphasis and beliefs away from management and onto student mathematics, then perhaps the student teachers would follow their example.

What Student Teachers Do and Do Not Talk About with their Cooperating Teachers and University Supervisors

Because this study aims to find what student teachers in a specific program discuss with their cooperating teachers, and, as mentioned in the previous section, cooperating teachers play such a significant role in their student teachers' experience, it is relevant to examine what cooperating teachers are talking about with their student teachers in traditional programs.

What Student Teachers Talk About with Cooperating Teachers and University Supervisors

Historically, research has shown that what student teachers talk most about with their cooperating teachers, and also what cooperating teachers desire their student teachers to learn, is primarily how to manage a classroom. Tabachnick, Popkewitz, and Zeichner (1979) found that classroom management, procedural issues, and directions were the primary focus of cooperating teachers in their interactions with student teachers. Similarly, when studying what student teacher partnerships discussed in their planning meetings with their cooperating teachers, Bullough, Young, Erickson, Birrell, Clark, and Egan et al., (2002) found that "management and discipline were important concerns and topics of conversation [for the cooperating teachers]" (p. 75). Moore (2003) asked cooperating teachers what they were most concerned about with their pre-service teachers. 105 out of 136 responses (77%) mentioned time management, and 75 out of 136 responses (54%) mentioned classroom management, specifically managing group work (Moore, 2003). Peterson and Leatham (2010a) found that cooperating teachers believed that the purpose of student teaching is to interact with experienced teachers in *real* classrooms, and thus learn how to successfully manage those classrooms. They compare this to their commonly used analogy of "spending time in a real shoe store and learning how to run the shoe store" (Leatham & Peterson, 2010b). Similar results surface again and again:

> When the cooperating teachers were asked to rank the most important ideas they tried to convey to their student teachers, they indicated that preparation, classroom management, being flexible in the classroom, relationships, and caring were the most important, with preparation being the single most important issue across all school levels. (Mitchell, Clarke, & Nuttall, 2007, p. 13)

Meanwhile, there is little research on what student teachers talk about with their university supervisors, perhaps due to the fact that historically, university involvement in school practicum is relatively low (Zeichner, 2002). However, Borko and Mayfield (1995) found that, like with cooperating teachers, the student teachers most often discussed aspects of classroom management in their conferences with university supervisors.

What Student Teachers Don't Talk About with their Cooperating Teachers and University Supervisors

Coupled with these findings of a focus on classroom management comes a lack of focus on *mathematics* in mathematics education student teaching programs. In the study mentioned above, Peterson and Leatham (2010b) asked 45 cooperating teachers the question, 'Specific to teaching mathematics, what do you feel is the most significant contribution you make to the success of a student teacher?" 36 of the 45 cooperating teachers responded, and "despite the request for contributions specific to teaching mathematics, half of the 36 responses made no mention whatsoever of mathematics" (p. 110). Borko & Mayfield (2005) observed conferences between mathematics student teachers and their cooperating teachers and coded content using seven categories: pedagogy, mathematics, mathematics pedagogy, learners and learning, mathematics curriculum, learning to teach, and the profession of teaching. General, non-mathematics specific, pedagogical issues were discussed in eight of the nine conferences.

Classroom management was addressed in three. Conversations about students were primarily about the flow of the lesson, such as how to redirect students who were misbehaving or not paying attention. Borko and Mayfield (2005) concluded that the dyads "rarely engaged in discussions about their students' understanding or possible misunderstandings of particular topics" (p. 506). Mathematics and mathematics-specific pedagogy were discussed mainly at a superficial level. Cooperating teachers did not offer suggestions to the student teachers about how to focus their conceptual understanding of mathematics. Student teachers discussed mathematics and mathematics-specific pedagogy with their university supervisors in only 6 out of 12 conferences. Borko and Mayfield suggest this might be due to the fact that student teaching observation forms university supervisors were required to fill out did not focus on mathematics and mathematics-specific pedagogy, and were thus not what the student teachers were graded on (Borko & Mayfield, 2005).

Cooperating teachers have a strong influence on a student teacher's experience (Wilson, Floden, & Ferrini-Mundy, 2001; Peterson & Williams, 2008), and the literature shows that traditionally student teachers discuss classroom management with their cooperating teachers, and rarely engage in meaningful discussions of mathematics or students' understanding of mathematics (Borko & Mayfield, 2005; Tabachnick, Popkewitz, & Zeichner, 1979). This research, however, does not provide further insight into whether or not the student teacher structure can be redesigned in a way that somehow reverses these results, or in other words focuses conversations on mathematics and students' understanding of mathematics rather than on management.

The Student Teaching Structure

Because the context of this study of conversations between student teachers and their cooperating teacher takes place in an atypical student teaching structure, it is necessary to explore the literature on the traditional structure, as well as programs that have recently been redesigned. I will highlight commonalities found in restructured programs and the effects they've had on student teacher learning, followed by a description of the program within which this study takes place.

The Typical Structure

Borko & Mayfield's (1995) study on the roles of cooperating teachers and university supervisors in student teaching and Clarke, Trigg, and Nielsen's (2013) description of the common "supervisor of practica" role of cooperating teacher, help to paint a picture of the typical student teaching structure. Cooperating teacher and student teacher do not meet formally on a regular basis, and university supervisors have limited involvement. However, there is a recent effort to redesign student teaching programs to better match desired outcomes (Cochran-Smith, 1991; Leatham & Peterson, 2010a; Rodgers & Keil, 2007; Bullough, Young, Erickson, Birrell, Clark, & Egan et al., 2002; Brouwer & Korthagen, 2005; Zeichner, 2002; Levine & Trachtman, 1996).

Redesigned Structures

I will now discuss several suggestions from the literature about principles of an alternate student teaching structure that seem to be effective. These principles are more university involvement, student teachers in partnerships, and reflection-enhancing principles. Reviewing the literature on how student teaching structures have successfully been redesigned helps to provide insight into why some programs seem to foster a strong focus on classroom management and others instead foster a focus on student mathematics.

More university involvement. Feiman-Nemser and Buckmann (1987) called to attention the need of university supervisors to be actively present in student teaching, and Brouwer and Korthage (2005) stress the importance of regular contact between university supervisors and cooperating teachers.

> Without continuing communication between the university and the school-based teacher educators, the student teaching experience cannot be adapted to the needs of student teachers as individuals, their co-operating teachers or the children with whom they work. (Weiss & Weiss, 2001, p. 178)

The student teaching experience provides an opportune time for members of the teaching community, including experienced teachers, university faculty, and student teachers to come together and discuss the process of learning, each bringing a different facet and level of knowledge to the table (Moore, 2003). Even many cooperating teachers voice the need of more university involvement in school practicum (Mitchell, Clarke, & Nuttall, 2007). While Zeichner (2002) recognizes that "cooperating teachers and university instructors are often mutually ignorant of each other's work and the principles that underlie it," (p. 61) he points out that recently, there has been increase in university supervisor participation in school situations. The programs that offer student teachers regular collaboration with both cooperating teachers and university supervisors find that student teachers leave the programs having been offered consistency in the teachings of their university courses and the teachings of their cooperating teachers (Rodgers & Keil, 2007) as well as a motivation to engage in reforming education in their communities (Cochran-Smith, 1991).

Student teacher partnerships. Besides more university involvement, other changes to student teaching structures have been made. Bullough, Young, Erickson, Birrell, Clark, and Egan et al., (2002) studied differences in the experience of single placed student teachers versus that of student teachers in a partnership. While the lesson plans of single-placement student teachers were very much dictated by the guidelines laid out by their cooperating teachers, the partner-place preservice teachers have more control over how and what they would teach. They were given much more flexibility in lesson presentation than their single-placement counterparts. The partner teachers also spent 30% more time in planning, had more ideas to work with, and felt their lessons were richer than they would have been as a single student teacher. Finally, the partnerships were able to work more closely with students individually and in small groups (Bullough, Young, Erickson, Birrell, Clark, & Egan et al., 2002).

Reflection. As Richert (1992) well stated, "little to no attention has been given to the impact of specific program structures on the processes or content of beginning teachers' reflections" (p. 171) In a desire to add some insights, Richert (1992) constructed a study that would compare four different structure conditions and their effect upon the nature of student teacher reflection. In the first condition, student teachers had no partner and did not maintain a portfolio (a collection of the week's materials: lesson plans, handouts, overhead transparencies, examples of student work, examples of their responses to student work, text materials, etc). They were asked to maintain a personal journal as a means of recording their reflective thinking. Analysis showed these journal entries and the content of interviews with these student teachers to be very personal, consisting of the student teachers feelings and emotions at the end of the day. In the second condition, student teachers had a portfolio, but no partnership. Richert (1992) found through analysis of journals and interviews that the portfolio materials "reminded teachers of the content of their instruction," helping them to "reconstruct the past to understand it more fully and learn from those experiences" (p. 181). In the third condition, student teachers had a partner student teacher, but were not required to maintain a portfolio. "General pedagogy" was found to be the focus of these student teachers upon being interviewed by the researcher, consistent with the literature. These student teachers showed much concern for the learner (the students) and enjoyed having a partner to receive feedback from, similar to Bullough, Young, Erickson, Birrell, Clark, and Egan et al., (2002) study mentioned previously. Reflection of the partnerships was found to be deeper, more thorough, and clearer. Finally, the fourth condition type, where student teachers were in partnerships and were required to maintain a portfolio, exhibited a significant focus on content-specific pedagogy in their journals and interviews. 57% of reflection content was content-specific pedagogy, compared to 24%, 26% and 28% in conditions one, two, and three, respectively. This study elicits the effect of both partnerships and reflection-enhancing materials on what student teachers have the opportunity to learn. It is clear that the materials present during reflection (i.e. the portfolio) can have a significant effect on what is in turn discussed, consistent with Borko and Mayfield's (1995) findings on university supervisors who let their evaluation form dictate what they discussed with student teachers. The fact that the student teachers in condition four discussed content-specific pedagogy seems promising, since, as Peterson and Williams (2008) argue, "It is unfortunate that so little mathematical discussion occurs among student teachers and their cooperating teachers, particularly since the student teaching experience seems to be an ideal site to address mathematical knowledge for teaching" (p. 463). Restructuring student teaching to include a reflection element seems to be one way to help take the focus of student teaching away from classroom management and onto content-specific pedagogy.

The BYU structure

I will now describe one final student teaching structure that has shown positive results. Prior to being redesigned, Brigham Young University had followed the traditional model, with one student teacher and one cooperating teacher. The cooperating teacher determined when and how much the student teacher taught, as well as when and for how long the pair would meet formally. Leatham and Peterson (2010a), two professors in the BYU mathematics education department, found several problems with this traditional structure through analysis of their research on the program. The program elicited: 1) goals that were unclear, and not being accomplished; 2) a focus on student teaching survival and management skills; 3) student teacher 'focus on self'; 4) student teacher isolation; and 5) cooperating teachers who didn't view themselves as teacher educators. Note the consistency with the literature. These problems motivated Leatham and Peterson (2010a) to redesign the BYU student teaching structure in 2006. The primary belief behind the restructure was that "the purpose of student teaching is to learn how to anticipate, elicit, and use students' mathematics thinking (p. 231)." Throughout this new 14 week program, student teachers engage in many activities designed specifically to focus them on student mathematical thinking, such as keeping a journal, observing lessons of student teachers and writing follow up papers, and student interviews. For the first 4 weeks, student teachers teach only once a week and slowly transition into taking on a heavier load. Similar to the programs mentioned above, student teachers in this program are placed with a partner with whom they share a classroom, alternating between teaching and observing the other. Perhaps most unique to this program is the creation of "cluster groups" made up of two to three pairs of student teachers in nearby schools, a cooperating teacher, and a university supervisor. For each lesson taught in the first 4 weeks, other members of the cluster observe the lesson and then formally meet as a group to reflect on the lesson during what are called reflection meetings. The structure of the 14 week program and these meetings will be described in more detail in my methodology. Note that this program takes on all three principles mentioned in the literature above that seem to produce positive results.

For example, Franc (2013) studied the effect of this structure by comparing five minute or longer conversations (not including the post-lesson reflection meetings) that occurred between the student teachers and cooperating teachers in 1998 before it was redesigned and after it was redesigned. After analysis of 35 transcribed conversations, coded for mathematical, pedagogical, and student-related content, Franc (2013) found the following:

Table 1

Percentages of Pedagogy, Students, and Mathematics Codes

Topic	1998	2006-2007
Pedagogy	34%	19%
Students	7%	2%
Mathematics	3%	8%
Pedagogy Students	32%	16%
Pedagogy Mathematics	15%	27%
Students Mathematics	3%	7%
Pedagogy Students Mathematics	6%	20%

Note. Data taken from Franc, 2013

The new BYU structure fostered a decrease in general, non-content specific pedagogy, a decrease in pedagogical comments in relation to students (which are typically about behavior management (Leatham & Peterson, 2013), and an increase in all categories related to mathematics (Franc, 2013). Franc (2013) also approached this data from a framework of "ambitious teaching," the term being adopted from Kazemi, Franke, and Lampert (2009) and described as "student-centered teaching" with a focus specifically on "eliciting and using student mathematical thinking" (Franc, 2013, p. 8). Franc (2013) contrasts this type of teaching with "traditional" teaching, which is teacher-centered. From 1998 to 2006-2007, Franc found a significant increase in statements that aligned with ambitious teaching, and a decrease in statements that aligned with traditional teaching. The structure seems to support statements about pedagogy, students, and mathematics that directly support ambitious mathematics teaching (Franc, 2013).

Research Questions

This structure consists of strong university supervisor involvement, partner student teachers, and activities that foster cooperating teacher guided reflection, consistent with the successful structural principles highlighted in the literature. It has also proven to decrease focus on behavior, general pedagogy, and teacher-centered teaching and increase focus on facilitating student mathematics through student-centered teaching. However because Franc (2013) used a sentence by sentence unit of analysis, it is difficult to use the results to understand the nature of the conversations between the student teachers and their cooperating teacher as a whole. We have evidence that the student teachers in the BYU program had a greater opportunity to learn about eliciting and using student mathematical thinking, but do not know specifically what pedagogical principles the National Council of Teachers of Mathematics suggest lead to being "effective teacher" (2014) they have the opportunity to learn. It is for this reason that the current study will approach data from transcribed video recordings of post-lesson reflection meetings taking place in this restructured program from 2006-2007 with the following research questions:

(1) What principles of "effective teaching" (NCTM, 2014) did the student teachers have the opportunity to learn during the reflection meetings, and to what extent?

(2) To what extent did the student teachers "take up" the principles of "effective teaching?"

(3) How was the extent the student teachers "took up" the principles of "effective teaching" different when a US or CT is participating?

CHAPTER FOUR: METHODS

Context

The data for this study are transcribed video recordings of post-lesson reflection meetings that occurred in 2006 and 2007, the first two years after the restructure of the BYU student teaching program described in my literature review. Participants were video recorded during reflection meetings that occurred on the same day the lessons were taught. Participants in these meetings, referred to in the program as a "cluster group," consisted of student teacher pairs from nearby schools, a cooperating teacher, and 1-2 university supervisor(s). Data were collected from one cluster (3 pairs from 3 different schools) in 2006 and two clusters (4 pairs from 4 different schools) in 2007.

In the BYU restructured program, student teachers did not teach a lesson during the first two weeks, taught one lesson per week in weeks 3-5, and then took over approximately half of the cooperating teacher's load during weeks 6-14 as decided by the cooperating teacher and university supervisor. Once a week during weeks 1-5 and 14, the cluster group would come together to observe a lesson taught by a member of the cluster group (cooperating teacher during weeks 1-2, or both student teachers consecutively during weeks 3-5 and 14). During these lessons, observers were asked to move around the class freely, taking note of interesting student thinking. After school or during a prep period, the group would meet formally to reflect. A university supervisor or student teacher guided the discussion with the following questions: (a) What was the goal of your lesson? (b) How was your lesson designed to meet that goal? and (c) How do you feel the lesson played out? This was followed by a period of time designated for the student teachers to ask questions of each other about the lesson and the student thinking observed. Following the questioning period was a session of general comments by the student teachers about the lesson. In general, the cooperating teacher and university supervisor reserved their comments and questions until the end of the reflection meeting. However, there were many times the cooperating teacher or university supervisor would interject during the earlier phases of the reflection meeting. At the conclusion of each week, and thus after one full teach/observe/reflect cycle, student teachers were required to complete a reflection paper. The assigned topics for these papers varied from week to week (Leatham & Peterson 2010a).

A total of 37 reflection meeting conversations had been previously transcribed prior to this study. A subset of 14 reflection meetings was used in this study 6 meetings taken from the 2006 cluster group and 8 from 2007. The meetings took place during weeks 5 and 14, thus two meetings at each of the seven schools.

Analysis

I will now describe each pass I made through my data in an effort to answer my research questions listed above.

Pass 1

The unit of analysis for this study is conversation pieces formed by major topic changes. These pieces are referred to as "chunks" for convenience. My method for breaking up the reflection meeting conversations into smaller chunks was adapted from Ward and McCotter (2004), who also divided up written reflection of student teachers. The goal for the size of the chunks was larger than single sentences, as these had already been coded by Peterson and Leatham (2013) with Pedagogy, Mathematics, and Students codes. The purpose of this study is rather to better understand the nature of *entire conversations* between the cluster groups, so the chunks aimed to capture this instead. They could not, however, be so large that the topics of conversation within the chunk were no longer connected in any way. The length of chunks ranged from a few sentences to several pages of text.

The beginning of a chunk most often began with statements similar to "I have a comment" or "I have a question." If the comment or question seemed to spark interest in the group (as evidence by the fact that someone responds to it), then this was taken as evidence that a chunk had begun, and an opportunity to learn, as described in my theoretical framework, had arisen. In fact, a piece of reflective conversation was only a chunk *if* there was evidence of an opportunity to learn for everyone in the group who participated in the conversation. I decided that if someone posed a question or comment, and another member of the cluster group responded in some way, the fact that they responded could be taken as evidence that there was an opportunity to learn for him/her, and therefore an opportunity to learn for every member of the cluster group who participated in the conversation.

Some comments or questions stood alone and did not spark any interest or response from the group. These were not labeled as chunks, as they did not represent an "opportunity to learn." They were not taken up by any member of the group, so there is no evidence that anyone in the group had an opportunity to learn. These were labelled as "singletons." They were not discarded, however, but instead coded on their own and analyzed separately from the chunks.

The beginning of a chunk was sometimes more random, with no definitive beginning statement as mentioned above. In general, a new chunk had begun if there was no clear connection between the comments at hand and what came before them, and if there was never a return to the topic that came before them.

The end of a chunk, or change from one chunk to another, was most often clearly defined by phrases such as "are there any other questions?" or "are there any more comments? but also occurred when a new chunk randomly began as just mentioned. Note that these phrases such as "I have a question", "I have a comment", "are there any other questions", "are there any other comments", or any other statements about the reflection meeting itself were not included in the chunks. The chunks began just after this type of statement, or concluded just before. This was in an effort to keep the word counts (discussed later) as accurate as possible. Often, entire paragraphs or small conversations would transpire concerning only administrative aspects of the meeting (meeting time, bells ringing or announcements made during the meeting, etc.) These were consequently discarded.

I felt the structure of the post-lesson reflection meetings restricted the nature of the conversation in the first pages of text in every meeting. As mentioned above, three questions were asked at the beginning of every meeting: (a) What was the goal of your lesson? (b) How was your lesson designed to meet that goal? and (c) How do you feel the lesson played out? Because of the nature of the questions and the expectation that the two student teachers who taught the lesson answer, and then move on to the next question, initial analysis showed that next to none of these answers elicited a response from the group and sparked a discussion in a way that would define the beginning of a chunk. The structure seems to set the expectation that only the two student teachers answering these questions speak in this part of the meeting, and consequently these two student teachers were the only ones with an opportunity to learn during this portion of the meeting. Consequently I decided to eliminate this entire section in every meeting from my data, and instead only analyze the second and third sections, namely the questioning section and the comment section.

Pass 2

NCTM Codes. During my second pass through the data, once all the reflection meetings had been divided into smaller chunks, each chunk was coded according to the best fitting aspect/s of "effective teaching" as outlined in the theoretical framework. Codes were abbreviated as described in Figure 1.

Code	NCTM Effective Teaching Principle
EMG	Establishing Mathematics Goals to Focus Learning
IT	Implementing Tasks that Promote Reasoning and Problem Solving
UCR	Use and Connect Mathematical Representations
FMD	Facilitate Meaningful Mathematical Discourse
PPQ	Pose Purposeful Questions
PFCU	Build Procedural Fluency through Conceptual Understanding
SPS	Support Productive Struggle in Learning Mathematics
EUE	Elicit and Use Evidence of Student Thinking

Figure 1. List of NCTM Code Abbreviations

The amount of NCTM codes applied to a chunk ranged from one to at most three. To decide which NCTM code/s, if any, fit a particular chunk, I kept the book *Principals to Actions: Ensuring Mathematical Success for All* close by so as to constantly refer to NCTM's (2014) description of each principle of "effective teaching," as well as their description of what each principle looks like when being implemented in the classroom. Often, the sub-codes I describe later on were helpful in verifying that the right NCTM code was chosen. If no NCTM code fit, the chunks were set aside so new internal codes could later be developed in an effort to accommodate for any and all chunk content. The coding applied to these chunks set aside will be discussed in a following section.

Brief Descriptions. During this second pass though the data, it seemed beneficial to write a brief description of each chunk. This was basically one sentence describing the content of the chunk. These descriptions were helpful during my fourth pass through the data, as I will describe later. I established these brief descriptions early on in an effort to begin to familiarize myself with the content of chunks as soon as possible. Doing so made subsequent passes through the data more manageable as I had begun to memorize the main ideas in chunks early on.

Pass 3

Sub-codes. To more completely answer my first research question and thus better understand what it looks like when student teachers discuss these NCTM (2014) standards, I used a list of sub-codes taken from the NCTM's (2014) description of what the eight principles of "effective teaching" should look like. In *Principals to Actions: Ensuring Mathematical Success for All*, they describe what both teachers and students should be doing when enacting each of the eight principles (see Appendix A). The codes that portray what teachers should be doing and what students should be doing are separated by a line in the table.

During this pass, I reread the chunk, specifically looking for which of the sub-codes in Appendix A were discussed within the chunk. If only one sentence or one person's "turn" speaking was devoted to one of the sub-codes, that sub-code wasn't necessarily used. Again, just like the larger NCTM codes, the purpose of the sub-codes was to capture the content of the chunk *as a whole*.

I did several things to help maintain accuracy in my coding. First, I developed two analytical questions to ask myself as I chose my sub-codes: 1) which, if any, of these sub-codes do I personally seem to have the opportunity to learn from this chunk? And 2) would someone else pick these same sub-codes? After assigning sub-codes, I reread the chunk once more in an effort to make sure that all the big ideas discussed within the chunk were captured by the sub-codes I chose. As mentioned previously, if I felt the sub-codes disregarded a big idea discussed, the chunk was set aside for an internal code to be developed later on.

Pass 4

Internal codes. Another pass through the data was necessary in order to capture the ideas in the chunks that I had set aside because none of the existing codes and/or sub-codes fit. I first examined the chunks that seemed to clearly fit in one of the 8 categories of "effective teaching" (NCTM, 2014) but had ideas that could not be captured by the NCTM (2014) sub-codes from *Principals to Actions: Ensuring Mathematical Success for All.* I used the brief descriptions described above to organize these chunks into groups based on the fact that similar ideas were discussed. Once I was sure that the chunks were organized as well as possible based on similar content, I created my own description of a new pedagogical principle not discussed in *Principals to Actions: Ensuring Mathematical Success for All* and coded these chunks accordingly. In the results section, all internal codes are abbreviated with an "I".

Word Count. In addition to the NCTM and internal codes and sub-codes, in order to answer the aspect of my first research question that asks to what extent the NCTM standards are discussed, I also performed a word count of each chunk on this fourth pass through the data. It seemed that coding each chunk with NCTM codes and then simply counting how many chunks fell in each of the eight categories might misrepresent the data, as the length of chunks varied so greatly. One NCTM standard might be discussed for pages at a time, while another only for a few sentences. To account for these differences in the amount of words the cluster groups devoted to each principle, I labeled each chunk with a total word count, regardless of how many NCTM codes it had. If a chunk had multiple codes, the word count was applied to all of them. For example, if a chunk was coded as EGE #1 and IT #3, the word count was double counted in my analysis, once for each.

Pass 5

Student Teacher Acceptance of Ideas. The fifth and final pass through the data aimed to answer my second and third research questions. After understanding what ideas the student teachers discussed, I aimed to understand how accepting they were of these ideas or in other words how likely they were to leave the student teaching experience committed to these practices. Only a subset of all the available chunks were analyzed at this stage. Only chunks from the six 2006 reflection meetings were analyzed, and among those, only the chunks which contained discussion of the most common NCTM and internal codes and sub-codes. Because one of the purposes of this study was to understand the extent the student teachers had "taken up" the principles they discussed, I was most interested in the chunks in which they discussed the principles most prevalent in the data. I was less interested in the rest of the chunks because those principles already didn't represent a very strong opportunity for learning a principle simply because of their scarcity. The 2006 group was chosen for no other reason than picking a manageable subset of data for this study. My results will describe exactly which chunks were analyzed at this level. It was determined what chunks would undergo this analysis only after passes 1-4 had been completed. A total of 60 chunks were analyzed at this stage. To perform each stage of this analysis, I adapted the reflection rubric developed by Ward and McCotter (2004) mentioned previously in my theoretical framework. Each student teacher who participated in the conversational chunk, each individual comment, and each chunk as a whole was evaluated using the rubric. My adaptation of the Ward and McCotter (2004) spectrum is illustrated in Figure 2. My development and use of this framework will follow the figure.

	Routine	Technical	Dialogic	Transformative
Questions	No personal involvement in the analysis, as if it is done for its own sake.	Asking as if there is a personal stake in the question. Giving a reason for asking the question.	N/A	N/A
Comments	Trying to gain personal recognition. Possible resistance to an idea or to changing.	Statements of agreement with someone else's new idea. Personal response to the problem or situation, but no new insights or acknowledgement of need to change. Description of classroom happenings.	New insights are gained and vocalized. Epiphanies are acknowledged. Acknowledgement of a weakness that needs to be improved. Suggestions to others are given. Ideas are generalized principles non-specific to a certain class or student.	New insights are gained and committed to. Deciding to change practices in the future.
Answers	No connections made to personal practice or classroom application. Yes/no responses.	Description of classroom happenings. Answer is connected to personal practice or classroom application.	Answers to questions are not just descriptions of classroom happenings, but to bring a new idea to the table that hasn't yet been discussed. Answers refer to generalized principles none-specific to a certain class or student.	New insights are gained and committed to. Deciding to change practices in the future.

Figure 2. "Take up" Framework. Adapted from Ward and McCotter (2004).

Individual Comments. I began this stage of analysis by using the rubric in Figure 2 to code each individual question or comment from a student teacher. Each statement was coded as R, T, D, or TR for routine, technical, dialogic, or transformative, respectively. The rubric evolved and became more detailed throughout the analysis to accommodate for any and all types of statements. Early in analysis, I determined the necessity of expanding the rubric to include descriptions of questions, comments, and answers. Once the rubric was complete and seemed to consistently account for each statement I came across, I returned to the data once again and recoded to ensure further accuracy.

The Student Teacher Level. After coding each individual statement by student teachers with an R, T, D, or TR, I assigned each student teacher who participated in a given chunk a code based on the highest level they reached with their individual comments in that chunk. For example, if a student teacher made 3 routine statements, 2 technical statements, and 3 dialogic statements during a given chunk, that student teacher reached the dialogic level and was coded as such.

A Chunk's Overall Level. Finally, using both the codes from individual statements and the levels the student teachers reached within a chunk, I assigned each chunk a code of R, T, D, TR, or U for undeterminable. These codes were in an effort to capture the overall feel of a chunk based on the way student teachers were discussing the ideas within it. If a majority of student teachers reached the dialogic level for example, the chunk was coded with a D. If the student teachers varied in the level they reached, and no clear majority of statements were coded as one level, then the chunk was labeled as undeterminable.

CT and US involvement. Finally, during Pass 5 I noted the amount of times a cooperating teacher or university supervisor participated in a chunk. This was then used to evaluate the effect of a cooperating teacher on the levels reached by the student teachers in a chunk as well as the level assigned to the chunk overall. I looked for any patterns that existed in the coding of the individual comments, student teachers, and chunks overall in relation to whether or not a cooperating teacher or university supervisor was present in the conversation. Furthermore I took note of verbal moves made by cooperating teachers or university supervisors that seemed to lead to a higher level of reflection.

Validity Checks

I participated in two activities that helped assure accuracy, consistency, and duplicability in my coding with NCTM codes and sub-codes. First, after I had already "chunked" the data and while I was analyzing each chunk in an effort to apply NCTM codes and sub-codes, I had the opportunity to share nearly a fourth of my chunks with four other graduate student research assistants in the BYU Department of Mathematics Education to be coded by them using my methodology. Using my description of my coding methods, they analyzed nearly 50 chunks, after which we met as a group and checked to see if we had applied the codes in a similar way. This experience gave me the opportunity to adjust my codes as needed based on arguments from the group and to strengthen my methodology in such a way that made it more understandable, replicable, and easy to use.

Second, each time I came across a chunk that I wasn't entirely sure how to code, or I wasn't completely confident in the codes I had chosen, I consulted with my advisor. We discussed the content of the chunk and decided the appropriate codes together. This experience working with another experienced researcher helped ensure accuracy in my coding.

CHAPTER FIVE: RESULTS

This section will consist of three main parts, each focused on one of my three research questions. I will begin with a presentation of results from my analysis of what principles of NCTM's (2014) "effective teaching" model the student teaching discussed, or in other words had the opportunity to learn. This will be followed by a presentation of my findings of how well those principles were "taken up" by the student teachers. Finally, I will conclude with a presentation of my findings on how the cooperating teacher and university supervisors influenced the "take up."

What the Student Teachers had an Opportunity to Learn

A total of 209 reflection meeting conversation chunks summing to 91,703 words were coded for what principles of NCTM's "effecting teaching" model were present within the chunk. As described in my methodology, these 209 chunks make up the content of 14 reflection meetings (6 from 2006 and 8 from 2007) that took place during the 5th and 14th week of the BYU mathematics students teaching program. The average number of chunks per meeting was roughly 15, the maximum containing 25 and the minimum containing 8. As mentioned in my methods, new internal codes were also created and used for the main ideas present in chunks that did not fit into any NCTM code. Appendix B and Appendix C show the frequency of each NCTM sub-code, the total amount of words devoted to that sub-code, and finally the frequency of each sub-code per cluster group. For example, sub-code Establishing Learning Goals (ELG) #1 was discussed in seven different chunks. The word count of those seven chunks totaled to 5,430 words. Out of those seven chunks, five of them were when Group 1 was meeting (the one cluster group from 2006), while Group 2 and Group 3 (the two cluster groups from 2007) each discussed it once. The sub-codes are sorted according to the frequency under a given principle.

There were 44 NCTM sub-codes that appeared in the conversations between cluster groups and all 44 sub-codes are listed in Appendix B. In addition, six internal sub-codes were created under the category of Implementing Tasks, one was created under Use and Connect Mathematical Representations, and two were created under Building Procedural Fluency. Finally, 12 internal codes that did not fall under the eight NCTM principles were created and are listed in Appendix C. Table 2 presents the 15 NCTM sub-codes and internal sub-codes that were the most commonly discussed. They are first arranged into three broader categories I created which seemed to capture all fifteen sub-codes, and then are arranged by the frequency of the number of chunks in which they occurred.

Table 2

List of Most Common NCTM Sub-codes

Broader Category	NCTM	Sub-code	Usage	Word Count	Group 1	Group 2	Group 3
Mathematics	N/A	#I1 Seeking a personal understanding of the mathematics being taught.	9	6,807	3	1	5
Pre-Class Planning	IT Internal	#I1 Thinking about the numbers and questions they choose and how those will affect student thinking.	8	3,573	5	1	2
	ELG	#4 Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	8	3,466	4	2	2
	ELG	#1 Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.	7	5,430	5	1	1
	IT Internal	#I5 The creation or adaptation of tasks.	6	3,163	2	2	2
In-Class Moves	EUE	#3 Interpreting student thinking to assess mathematical understanding, reasoning, and methods.	22	19,825	10	6	6

EUE	#5 Reflecting on evidence of student learning to inform the planning of next instructional steps.	15	6,580	8	3	4
SPS	#2 Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.	13	7,587	8	2	3
UCMR	#5 Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.	10	3,875	8	1	1
PEDS	#I2 Managing behavior.	8	4,051	1	5	2
PPQ	#3 Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion.	8	3,785	5	1	2
PPQ	#2 Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.	7	4,056	3	1	3
FMMD	#1 Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	7	3,895	3	2	2
MP	#I1 Anticipating how the choice of vocabulary, notation, and definitions will affect student mathematical thinking.	6	2,026	4	1	1
O	#I1 Miscellaneous.	6	2,723	3	1	2

These 15 sub-codes not only had a frequency of greater than 5 and a word count greater than 2000, but also met a third important criterion. The third criterion for deciding which sub-codes were "most common" was whether it was a discussion topic for all three cluster groups. Some codes had high frequency and high word count but were not a point of discussion for all three groups. Those codes were not identified as common. I will now discuss specifically what the conversations looked like when they discussed these 15 most common items. This particular cut-off was chosen after the sub-codes were arranged by frequency and there seemed to be a clear and expansive break between chunks with a frequency of 5 and a word count of 2000 and those with lower. I considered the fact that all three cluster groups discussed a principle as significant because it allowed me to make claims about what not only one particular group of student teachers discussed in one year of the program, but instead the three entirely different groups of student teachers spanning two years discussed.

I have divided these most frequent topics of conversation into three main categories: mathematics, pre-lesson teacher planning, and in-class teacher moves.

Mathematics (9 chunks, 6807 words)

The student teachers devoted a significant amount of words conversing about mathematics not directly connected to students or teaching, but in an effort to develop their own understanding of a topic or receive an answer to a mathematical question. Although not included in NCTM's (2014) principles of "effective teaching," the student teachers engaged in 9 different conversations about mathematics, often for long periods of time, totaling nearly 7000 words. The following conversation is an example of what these conversations often looked like. Pseudonyms are used in place of the student teachers, cooperating teachers, and university supervisors' actual names throughout the duration of this work, and US and CT are used as abbreviations for university supervisor and cooperating teacher, respectively.

> Katie: *Okay. First, just a math question because I was confused...of what exactly a scalene triangle is. I got confused. It has to have an angle greater than 90? So a right triangle can't be scalene? I got confused. Because I think I heard both explanations in both classes, and I don't remember what it is.*
>
> Jane: *There's a right triangle that could be scalene. Here you go. [Draws example on the board].*
>
> Katie: *So the angle doesn't have to be greater than 90? Is it just all the sides are different lengths? And so you will have one that's 90 or more?*
>
> US Steve: *So are you referring to, in Jennifer's class, when one of the students said that it had to have...a scalene triangle had to have one angle greater than 90 degrees, and that was kind of accepted as part of the definition?*
>
> Katie: *And I didn't know which it was.*

Julie: *I couldn't remember if it was or not. I didn't want to lead them totally astray by saying one way or the other, so I just went with the student's definition. I should probably clarify that next time.*
Katie: *Just, because I know that some of them were...with their scalene triangles, were noticing that they were right triangles too, and then I was like, "Can a scalene be right?" I couldn't remember.*

Although the students in the classroom were briefly mentioned, the conversation begins with Katie's mathematical question, which eventually is answered by Jane, presumably with a visual representation. The crux of the conversation is not addressing a question about how to teach scalene triangles to students, but on the definition of a scalene triangle itself, and therefore could not receive any other NCTM (2014) code.

Pre-Lesson Teacher Planning

The student teachers engaged in a significant amount of conversation concerning two aspects of pre-lesson planning: establishing learning goals and implementing tasks. Note that these are the first two of the eight principles of "effective teaching" (NCTM, 2014).

Establishing Learning Goals. The student teachers discussed both the importance of having goals, as well as how to use those goals when teaching. Consider the following excerpt from a chunk:

Christina: *Was there much focus on the notation, or did you just want them to understand the idea of domain and range?*
Ashley & Jennifer: *Just the idea.*
Jennifer: *I mean for us, as long as they can explain it, perfectly fine. They don't have to do, okay well, negative two is less than or equal to x and all this other stuff. Well as long as they can say it's everything but this number, or non-negative numbers, or then perfect.*

Both Ashley and Jennifer demonstrate they have clear expectations of the mathematical understanding with which they hoped the students would leave the lesson. Consequently this chunk was given the NCTM sub-code, "Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit" (7 chunks, 5430 words).

The student teachers discussed using goals as a guide to decision-making in the following chunk:

> Emily: *I noticed once you got the table up on the board, neither of you really asked anyone if they had anything different. You just kind of accepted that as true. Did you...*
> Megan*: We did that on purpose. Well no, like she said if they had incorrect tables it would throw off the other part of the task. So during her lesson it was my job to make sure that everybody had correct tables. I mean it wasn't a matter of them understanding what they're doing, it was a matter of them not having correctly, so I'd just say, "Check this value." And they'd always fix it. At that point we knew that everyone, well hopefully we got to everybody, we knew that they had correct tables.*
> Holly: *And we didn't want that variety to be in there because that's…*
> Megan: *It didn't add to what we were trying to do.*
> Christina: *As a comment I think that helped get closer to your goal.*

Holly and Megan had mentioned earlier in the meeting that their main goals was writing equations to represent situations and their sub-goals were that the students be able to recognize patterns, realize repeated addition is multiplication, and understand the concept of equivalent expressions.

Because the student teachers are not discussing the value of having goals, but instead are using a goal as a justification for a teacher move, this chunk was given the NCTM sub-code "Using the mathematics goals to guide to lesson-planning and reflection and to make in-the-moment decisions during instruction" (8 Chunks, 3460 words).

Implementing Tasks. The other common topic of conversation that fell under the category of "pre-lesson planning" was task-design. The student teachers conversed about two principles of task-design that were too specific to be coded with an NCTM "Implementing Tasks" code, and were consequently labeled with internal codes of my creation. Consider the following example. The student teachers and cooperating teacher are discussing a task that aimed at helping the students see a repeating pattern and use this pattern to begin to develop an equation. I do not have access to the task and consequently the details of the task, other than it is evident the student teachers in this conversation are concerned with how the numbers they chose in the task affected the students' thinking.

> Megan: *And I kind of wished I picked a prime, well yeah, I'd pick a prime number because it was cool how a lot of kids were figuring out, "Oh I can do three times this and it works." But that didn't really lead to our purpose of our lesson and what we were trying to get at. So I would do…*
> CT Larson: *17, 23*
> Megan: *Yeah, like a prime number instead to see how they could, they could still add numbers to get there, but they couldn't multiply like they were.*

This chunk was coded as "Thinking about the numbers and questions they choose and how those will affect student thinking" (8 Chunks, 3573 Words). The student teachers conversed similarly to this excerpt (which was roughly the first half of a chunk) on eight different occasions.

In six chunks, the student teachers discussed the creation or adaptation process of designing tasks (6 Chunks, 3163 Words).

> US Karl: *I was really impressed how you took this problem out of the book and made a task out of it. I thought it was a great task, and it seemed to me that the reason you chose it is because you felt you saw that you could add those other, kind-of earlier questions and get them to do some things that would lead them up to the thing that you wanted them to do and I'm assuming this came from the homework from the end of the section that this material's in right?*
> Jennifer: *Basically we took one homework question…*
> Ashley: *From the review section*
> Jennifer: *and added like all the questions that were after like all the other questions and combined them all into one idea.*
> US Karl: *Great strategy. Easily generalizes to other text books, particularly traditional type textbooks that tend to have lots of practice problems in them and word problems closer to the end. It's an easy way to turn it around and take one of those and make a task out of it. I think that was nice.*

Five out of these six chunks similarly contained discussion of the adaptation or gathering of outside materials to create a mathematical task, while the sixth chunk was a dialogue of the problems we face when creating mathematical tasks. Again, because the idea of task-creation strategies was more specific than the NCTM sub-codes, and did not fit well under any of them, it warranted a new internal code.

Summary. The student teachers discussed two principles of pre-class planning, namely the first two principles of "effective teaching": Establishing Learning Goals and Implementing Tasks (NCTM, 2014). When the student teachers discussed establishing learning goals, they talked about both the importance of having goals, as well as how to use those goals when teaching. The student teachers did not heavily discuss implementing mathematical tasks in the way outlined by NCTM, 2014, but rather conversed about two principles of task design that warranted the creation of new internal codes: thinking about the numbers we choose in tasks, and the creation or adaptation process of task design.

In-Class Teacher Moves

In addition to the above mentioned categories of conversation concerning pre-class planning and mathematics, the student teachers frequently discussed things that happened during class time, namely eliciting and using evidence of student thinking, supporting productive struggle, engaging students in discourse, using and connecting mathematical representations, asking questions, definitions/vocabulary/notation choices, and managing student behavior.

Eliciting Student Thinking. The single most significantly discussed principle of "effective teaching" (NCTM, 2014) overall was "Interpret student thinking to assess mathematical understanding, reasoning, and methods" (22 Chunks, 19825 Words). The student teachers conversed about interpreting student thinking in three ways: 1) They grappled with interesting mathematical choices students made in an effort to understand what the students were doing and why they were doing it; 2) They discussed whether or not they thought the students understood a certain topic; and finally 3) They discussed student thinking that was impressive in some way.

Several chunks contained evidence of student teachers trying to uncover the reasons behind students' mathematical decision making, such as in the chunk containing this comment:

> Emily: *I had an interesting observation as I was watching these two girls in your class, Jennifer. Every time they tried to figure it out using the equation, they would set the entire equation equal to zero and try to find where it was equal to zero. Why would they do that?*

The student teachers then proceed to discuss the reasoning behind these students' decision to set the equations equal to zero. The question is later answered by a university supervisor who sheds light on why they might have been choosing to set them equation to zero.

The student teachers often evaluated whether or not the students actually understood a topic, as in the conversation that followed this opening statement:

> Emily: *Do you think they really understand what real numbers means?*

The comment launches the student teachers into a discussion of what evidence they had or didn't have of whether the students understood the meaning of real numbers.

Finally, the student teachers discussed student mathematical thinking that was impressive in some way. The following example was taken from a chunk within which the student teachers were discussing student understanding of radians.

> Jennifer: *I actually had one who looked…because it's a 90 degree angle. It's on a clock…they're supposed to say how far the end of the hand on the clock goes in fifteen minutes. And I had one student who looked and figured out that it was about one and a half radians to ninety degrees and multiplied it by the radius. That was pretty cool.*

Using Student Thinking. The second most common sub-code overall was "Reflecting on evidence of student learning to inform the planning of next instructional steps" (15 Chunks, 6580 Words). The cluster groups discussed this principle of "effective teaching" (NCTM, 2014) in three ways, the difference lying in the amount of time between the observed student thinking and the use of it: 1) using student thinking instantly; 2) using student thinking observed in a previous class in the current class, and 3) using student thinking in future lessons as a full-time teacher.

Often the student teachers would discuss making use of student thinking almost instantly, as in this case:

> Katie: *But, other than that everyone seemed to get it and most people... that was another thing, and, four, I wanted, I picked a number that if they rounded up the answer would be wrong because it doesn't make sense in the context and I thought maybe that would be something that we could bring up. But walking around it seemed like most people got it. So, I didn't want to just talk again about things they already understood. Or at least, I thought they understood. And then, so that's why I just let them go to their homework and if they had questions...*

More frequently, however, the student teachers discussed using evidence of student thinking in a previous class period or in a class taught by another student teacher to influence the way the lesson was taught a second time.

> US Steve*: So, the question that didn't get asked that I'd like to ask is, it seemed like the summary discussion the second time was very, very different from the summary discussion in first period, so what happened in between first and third that caused you to change your conversation?*
>
> Katie: *I don't know. I felt like in first period we were just talking about things they already knew. But, they even kept saying "This is easy. We know it." There were comments like that. So, I thought maybe I'd move on to something that they didn't know.*

A third type of discussion arose where the student teachers conversed about using what they learned about student thinking in a future lesson when they become in-service teachers.

> US Steve: *I do have one more. Someday you're going to have a class of your own. And you're going to be teaching section 3.1 again of this chapter and you're going to go to your file and you're going to pull out candy bar rates, and how will it look different the next time you teach it? What would you change in this task now that you've used it once?*
>
> Katie: *Um... Well...*
>
> US Steve: *And maybe I can open that up to everybody.*
>
> Katie: *Yeah! Let's open it up to everybody.*

US Steve: *You all were here participating in the lesson, so knowing what you saw in the homework as we were walking around, and the questions that students had, and Katie's goals that she had which were to introduce function notation and find output values and input values given the other one, both from tables and graphs and I suppose equations, what might you change to get those goals?*

Julie: *There were a couple questions on the homework about the graphs, and I had a few people who were a little confused about like, what f(0) was. And so maybe if you kind of talked a little bit more using the graph the students had made, saying you know "What, how can we find f(0)? What is f(0)?" And maybe that might have helped them a little bit more.*

Supporting Productive Struggle. Second only to their discussions about interpreting student thinking were discussions on "Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them" (13 Chunks, 7587 Words). On 13 different occasions the student teachers discussed aspects of this sub-code including how grouping and pairing affects student struggle, the importance of giving students time to struggle, and the dilemma of how much to tell students. In this excerpt, taken from the middle of a chunk, the student teachers mention both allowing the time to struggle, and keeping the students alone versus in partners to facilitate struggle:

US Steve: *So I guess I'm wondering about the working alone verses working together from the beginning—just, what you were thinking when you put that in your plan?*

Jane: *I wanted to see what they would come up with by themselves, because I know like that sometimes when I work in partners, I just kind of say, "What'd you get?" And then just kind of copy them. And I even saw a few of those…a few of the kids doing that.*

Julie: *I just wanted them to be able to have time to think about it.*

Facilitating Meaningful Mathematical Discourse. The next topic of conversation that fell under the larger category of "in class teacher moves" is facilitating meaningful mathematical discourse. Only one principle of facilitating meaningful mathematical discourse was discussed heavily by all three cluster groups. The groups talked about "Engaging students in purposeful sharing of mathematical ideas and reasoning" (7 Chunks, 3895 Words). They discussed engaging students in sharing their mathematics both at the board, and at their desks with their neighbors. Consider this example from a chunk with this code:

> Ashley: *Megan, what have you tried as far as getting your class to talk? Have you tried anything or…*
> Megan: *What haven't I tried? We've done a lot of stuff. We do a lot of, well personally I try and make it very acceptable for people to come to the board. If I ask people to come to the board, like if I tell them their work is right they'll go to the board. They know it's right. They don't have to go if it's wrong.*

This sub-code clearly fits the chunk containing this excerpt because it begins with a question specifically addressing the sharing of student mathematical thinking through student discourse.

Use and Connect Mathematical Representations. The one principle of Use and Connect Mathematical Representations that stood out in the data was "Focus students on the structure or feature of mathematical ideas" (10 Chunks, 3875 Words). The cluster groups talked about helping students see purpose in different mathematical structures of equivalent expressions, helping students know what to look for when working with different mathematical forms, and finally how to help students use the rules and properties they already know to solve new problems. An example of this second mentioned item is an excerpt from a chunk where the group discussed how to help students see the benefits of keeping square-roots instead of decimals:

> Ashley: *Megan I thought you did a nice job of pointing out, this is a very specific comment, that the square root of 5 is more accurate than the decimal approximation of it.*
> Megan: *Well they're getting really confused. Like they don't get, they're not okay with leaving it as a square root. They hate that and so I'm trying to help them become okay with that.*
> Jennifer: *We noticed that in our class they do that too.*
> Megan: *They want the decimals.*
> Jennifer: *Yeah.*

Pose Purposeful Questions. NCTM (2014) provides a description of three different types of questions teachers can ask their students, each providing a sub-code for this study. Although the student teachers addressed each type of question in their conversation at least 7 times, only two of the three types of questions were discussed by all three cluster groups: "Ask questions that probe thinking and require explanation and justification" (8 Chunks 5029 Words), and "Ask questions that make mathematics more visible and accessible" (8 Chunks, 3085).

When the student teachers discussed asking questions that probe thinking and require explanation and justification, they talked heavily about asking the students to explain how they got their answer rather than immediately telling the students if they were right or wrong.

In this chunk, coded as "Ask questions that make mathematics more visible and accessible," the student teachers, university supervisor, and cooperating teacher discussed what questions to ask to help students generalize:

> US Brad: *Well I guess the other variation is, "Okay with this equation, if you had a group of 77 people come how would you figure out the profit?"*
> Megan: *That's a good idea.*
> US Brad: *And, "How would you use this equation to figure out the profit?"*
> CT Larson: *You use a specific value again.*
> US Brad: *To force them to have to realize that R and E doesn't help them. They need to have n in there in order for it to be useful. That might be another way to deal with that one.*

Mathematics Pedagogy. The student teachers discussed one principle of mathematics pedagogy that was not specific to task design or any of the other 8 "effective teaching" (2014) principles, but that was still related to "in-class teacher moves." There were six separate instances of the cluster groups conversing on teacher choices to use certain definitions, vocabulary, or notation during the lesson taught, as in the following case (6 Chunks, 2026 Words):

Megan: *I have kind of a weird question that's mostly for my benefit for our teaching in our class. We decided not to write the little positive sign every time it's a positive number. And so our kids, when they do 8 – 5 they think in the caldron that you're talking about cold cubes, 5 cold cubes. So I was wondering when you stop writing that positive sign, if you guys thought about it or not.*

Emily: *Yeah, and their quizzes today didn't have those. And so they've been doing them on their quizzes so I don't know if that messed them up or not. Have we always been doing the positive signs?*

Christina: *I think so far we have.*

Jennifer: *You kind of alternated today. It wasn't always consistent today.*

Emily: *I think because the students come up and they write it without it.*

Christina: *And if there were questions, like after the students would write it and I caught it then I would go up and I'd be like, "Oh this is a positive one."*

Emily: *I think it was important today just so they could find that rule. But yeah after today, like tomorrow's lesson they don't use the little positive signs.*

Thus a new internal code was created under the larger category of "mathematics pedagogy" to capture these instances.

Behavior (8 chunks, 4051 words). Finally, eight chunks were coded as discussions about behavior. Three out of these eight were not solely coded for behavior, but were given another NCTM sub-code in addition to behavior, meaning that managing students was only one, but not the only, major theme of the conversation.

Summary. In addition to discussing mathematics and pre-class planning, the student teachers talked about pedagogical principles taking place during class time, such as eliciting and using evidence of student thinking, supporting productive struggle, engaging students in discourse, use and connect mathematical representations, asking questions, definitions/vocabulary/notation choice, and managing student behavior. Taken together with the two principles mentioned above in the pre-class planning section, it is clear the student teachers heavily discussed aspects of seven of the eight principles of “effective teaching” (NCTM, 2014).

Singletons

In addition to coding all chunks, as mentioned in my methodology, I also wanted to look at the individual comments from student teachers, cooperating teachers, and university supervisors that were not “taken up” by the group and were left without any response. It seemed that discarding these might disregard any insights I might find from looking at commonalities amongst them.

A total of 59 comments of this type were analyzed, ranging from a couple of sentences to a full page of text. Initial analysis attempts using the NCTM code and sub-code framework proved unsuccessful because many of the comments quickly moved through several topics, were too short to fairly assign an principle of “effective teaching,” or contained material that didn’t match any preexisting codes. Instead, I wrote a short description of each comment and then grouped them according to similarities. The results are shown in Table 3.

Table 3

Singleton Count

Description	Quantity
ST compliments another ST	1
US compliments ST	10
CT compliments ST	3
Total Compliments	26
ST points out interesting student thinking or student understanding	5
CT points out interesting student thinking or student understanding	4
Total student thinking	9
ST gives advice	4
US gives advice	11
CT gives advice	1
Total advice	16
CT running through a long list of unrelated items from their notes	2
US running through a long list of unrelated items from their notes	1
Total Long Unrelated List	3
Other	5
Total	59

Compliments. Nearly half of the singleton comments consisted of a member of the cluster group complimenting one or both student teachers on something they did during the lesson, as in the following example:

> Christina: *I think you introduced your tables really well though. Like telling them to look for patterns rather than, "Here just fill this out." But, you gave them a good heads up with that.*

Christina compliments Megan and Holly on their ability to introduce the students to a task in which they were required to fill out a mathematical table. No one else in the cluster group responded to this compliment.

Interesting student thinking. On nine occasions, a student teacher or cooperating teacher pointed out something interesting they saw a student do during class, or discussed the students' understanding of something. In the following comment, Emily describes her observation of student understanding of the difference between square roots and squaring.

Emily: *I thought just vocab a lot of times it seemed like they got mixed up between square root and square. Like they would say square root when they meant square. I thought that was kind of interesting. And one kid said that the square root of just 2 was 1. And so they still don't quite seem to have those things solidified yet.*

Advice. In 16 of these comments, someone in the cluster group gave a piece of advice or suggestion to one of the student teachers who taught the lesson, such as something they could have done better, or something they should do in a future lesson. Consider the following example:

US Brad: *Can I, I just want to make two comments. The first comment, with regard to the proof issue, I think one thing that can be done, I don't know that you have to prove it but I think it is valuable to help the students recognize that what they have done does not constitute a proof. To help them realize, "okay we've done some examples, we see a pattern, but do we know it always works?" Well we think it does, but we really haven't proven it. Just help them recognize what constitutes a proof and what does not constitute a proof so that even if you don't prove it you could at least have told them, you know, "We think it works and you can trust that some other guy has proven it, or girl, has proven it and we're going to use it. But in this case it may be, "Have we proven it? Well we've seen a lot of examples, we feel pretty confident. Let's see if we can use it, but we're going to come back and we're going to see if we can really, are we sure it works for everything? And we'll visit that at another time." So I think that's one way of dealing with that proof to help them see that many examples does not constitute a proof. I think that's important.*

The student teachers had previously discussed how to help the students understand the necessity of proofs earlier in this cluster meeting. This university supervisor returns to the topic and gives his input. Following this comment, he moves onto another unrelated comment that begins a separate chunk.

In summary, the most common types of comments that did not result in a discussion amongst the group and thus stood alone were compliments or advice given to the student teachers who taught the lesson and pointing out interesting student thinking or understanding.

Summary

My first research question asked exactly what the student teachers in the redesigned BYU student teaching program have the opportunity to learn as they reflect on lessons taught during a reflection meeting with their cluster group. The student teachers in 2006-2007 discussed all eight principles that of teaching that the National Council of Teachers of Mathematics (2014) suggest are part of being an "effective teacher," as shown in Table 2, and heavily discussed seven out of the eight principles, the exception being "Build procedural fluency through conceptual understanding." Further synthesis of these results and why the student teachers discussed certain principles more than others will follow in the discussion and implications section.

How the Student Teachers Talked About these Common Sub-codes

In addition to quantifying what the student teaching clusters talked about most often, my second research question required that I also look at whether or not the NCTM (2014) "effective teaching" principles they were discussing were "taken up." As explained in my methodology, only the chunks from the 2006 cluster group were analyzed at this level, and only the subset of chunks in which the topic was one of the most frequent 15 codes discussed above was analyzed.

Individual Comments

Each individual comment by a student teacher in a total of 60 chunks was coded with an R, T, D, or TR as described in my methodology. Table 4 shows these results.

Table 4

Individual Comments Count

Code	Number of Comments	% of comments
Routine	115	26.93%
Technical	213	49.88%
Dialogic	95	22.25%
Transformative	4	.91%

Most of the comments by the student teachers were coded as technical. They were primarily comprised of descriptions of classroom happenings, like the following statement from Megan, where she describes how she makes her classroom a safe place for sharing at the board:

> *We do a lot of, well personally I try and make it very acceptable for people to come to the board. If I ask people to come to the board, like if I tell them their work is right they'll go to the board. They know it's right. They don't have to go if it's wrong.*

The second largest percentage of comments were routine. These comments were mostly short, with the speaker providing no personal connection to the question or comment. The following excerpt is part of a larger conversation about a task where students must write an equation based on a set of data. Each statement from these student teachers was coded as routine.

> Jennifer: *Did you ever think about having like an initial cost idea, like a startup cost?*
> Megan: *Like a y-intercept?*
> Jennifer: *Yeah*
> Megan: *Not really.*
> Jennifer: *Have you dealt with that at all yet?*
> CT Larson: *That's part of the problem later.*
> Ashley: *It comes later. I see.*

The students or classroom are not mentioned, and no conclusions are drawn. The opportunity for the student teachers to take something away from this conversation to apply in their future teaching is low.

The percentage of dialogic comments was a close third. In these comments, a student teacher typically reached a new conclusion or insight and shared it with a group. The following statement from Jennifer was coded as dialogic. This conversation was about student struggles with finding the domain of an equation.

> Jennifer: *So I think that was just a problematic thing for them as well, well what does it mean for a thing to be real. And I think in their minds, well, two is real, its right there. And it has nothing to do with the square root of a negative.*

This comment not only directly references the students and their ideas (automatically taking it to at least the technical level) but Jennifer also presents this new idea that the students might actually be struggling with the concept of "real."

Analysis of all individual comments from the six student teachers in 2006 shows that that majority were of a technical nature, meaning that the student teachers were discussing specific events that occurred in the classroom that day, without necessarily offering any new insights. However 22.25% of comments did receive a dialogic code, where the student teachers offered a new idea to the group, often more generalized to teaching and not just pertaining to a particular student or class. Finally, a student teacher only verbally committed to implementing a new idea in the future on four occasions, resulting in a transformative code.

Student Teacher Level

In addition to coding each student teacher comment, I also noted the highest level of reflection reached by each student teacher who took part in the chunk. Table 5 shows these results.

Table 5

Student Teacher Level Count

	Emily	Jennifer	Megan	Holly	Christina	Ashley
Total Chunks Participated In	29/60	39/60	33/60	18/60	22/60	28/60
Routine	6	7	5	5	2	10
Technical	12	13	13	9	10	9
Dialogic	11	17	13	4	9	9
Transformative	0	2	2	0	1	0

With the exception of Jennifer, and perhaps Holly, there was no clear distinction of what level the student teachers would most often reach in a chunk. It is difficult to make a claim about the tendencies of a student teacher because only 60 out of 209 chunks were analyzed in this way. However, we can note that just as seen with the individual comments, the transformative level was a rare occurrence, and only achieved by half of the student teachers. Also, not every student teacher participated in all 60 chunks, Jennifer participating the most at 39, and we only have evidence that the student teachers had an opportunity to learn in the chunks in which they participated.

I also investigated any differences in the level a student teacher would reach in a given chunk based on whether or not they were the one who taught that day's lesson, or just an observer.

Table 6

Student Teacher Subject/Observer Count

		Routine	Technical	Dialogic	Transformative	Total
Emily	Subject	0	6	7	0	13
	Observer	6	6	4	0	16
Jennifer	Subject	2	7	6	2	17
	Observer	5	6	11	0	22
Megan	Subject	1	11	7	2	21
	Observer	4	2	6	0	12
Holly	Subject	4	8	1	0	13
	Observer	1	1	3	0	5
Christina	Subject	1	7	4	1	13
	Observer	1	3	5	0	9
Ashley	Subject	3	8	5	0	16
	Observer	7	1	4	0	12

Four of the student teachers participated in more chunks where they had taught the lesson at hand, while Emily and Jennifer participated more when they were a class observer. Five out of the six student teachers reached the technical level more often when they were the subject than the observer. No other clear distinctions can be made.

The Chunks Overall

Finally, the following codes were used to try to capture the feeling of the chunk in its entirety. I used the coding of individual comments, as well as the highest levels the student teachers reached, in order to label each chunk with an overall code. For example, if the majority of student teachers within a chunk only reached a technical level of reflection, then the chunk was labelled as technical. If there was no clear majority of levels reached by the student teachers, then the individual comments were counted and used to determine the level of the chunk overall. Finally, if neither the student teachers' levels nor the individual comments showed a trend, I looked for whether or not a resolution was reached by the end of the chunk. For example, dialogic codes were assigned to chunks where the members of the conversation reached some sort of resolution, where the technical and routine, lower level chunks were unproductive, cyclical, or simply descriptive of a classroom lesson. Often there was a great variety in the levels reached by the student teachers and the individual comments, it was unclear if the group reached a resolution or not, and the code "undetermined" was used. Descriptions of the differently types of coded chunks follow Table 7.

Table 7

Chunks Overall Count

Level	# of chunks out of 60
Routine	3
Technical	22
Dialogic	20
Transformative	1
Undetermined	14

Routine. Only three chunks were coded as routine. These were rare occurrences of a small chunk where a student teacher gave only short statements in reply to a university supervisor who was leading the conversation. The substance of these chunks is maintained only by the university supervisor, so the chunks were labeled as routine.

Technical vs. Dialogic. There is a stark contrast between the chunks that reached the dialogic level overall when compared to those that only reached technical. Consider the following chunk, coded as technical, where the student teachers discuss a task requiring students to find the domain of different equations:

Emily: *I had an interesting observation as I was watching these two girls in your class, Jennifer. Every time they tried to figure it out using the equation, they would set the entire equation equal to zero and try to find where it was equal to zero. Why would they do that?*

Jennifer: *I don't know.*

Christina: *Wait, say it again.*

Emily: *They were setting y equal to zero and trying to find where their equation equaled zero. Every single one of them, every equation they did it like that and I don't' know why. And they couldn't figure it out, so then they went to their graphing calculator and they got the right answer from that.*

Christina: *I wonder if its just an indication of [announcement over the intercom]*

Craig: *What's the question?*

Ashley: *She said that a group of girls set every equation equal to zero to try and figure out*

Emily: *So yeah, I wasn't sure why they were doing that. But they were talking about, oh yeah you have to find where your y intercept is, no you have to find where your x intercept is. They were just like, I don't know, it was kind of interesting. Um, and then they got their right answer from their graphing calculator. And [US Brad] came and asked them on that g of x where it's everything but the negative two, he asked them, well what does that mean. And they were like oh it's all real numbers even though it was everything but negative two. And he was like, so why didn't you just write all real numbers and they were like oh we should have. And he's like don't! don't write it.*

US Brad: *Because they had written x less than negative two and x greater than negative two, but their notation was kind of funky and I didn't want to say, well what does this mean and they said all real numbers. And I said, now don't erase, but as a result of this, why didn't you just write all real numbers? Any how.*

Jennifer: *Because negative two is obviously not real.*

Emily: *So anyways. There was just some interesting thinking going on there with that group.*

This chunk begins with a compelling question about the reasoning behind student mathematical thinking. While the student teachers seemed intrigued by the question, the conversation never moves past describing what the students did to any presentation from the student teachers of possible reasoning behind the students' decision to set the equation equal to zero. Thus this chunk never reached the dialogic level, and the likelihood that the student teachers were able to take away any new insights from this conversation to apply in their future teaching is low.

The concluding remarks of this next conversation differ significantly. Here the student teachers are again discussing a lesson where the task required students to find the domain of different equations.

> Christina: *Was there much focus on the notation, or did you just want them to understand the idea of domain and range.*
> Ashley & Jennifer: *Just the idea.*
> Jennifer: *I mean for us, as long as they can explain it, perfectly fine. They don't have to do, okay well, negative two is less than or equal to x and all this other stuff. Well as long as they can say its everything but this number, or non-negative numbers, or then perfect.*
> Emily: *Sometimes I wonder about that, because just because they can explain it in their own words which is wonderful, but what about like on a standardized test, like where all of a sudden they see this well x is less than or equal to a negative two. How do they know that that is everything less than, like I don't know.*
> Jennifer: *I think that as soon as they can explain it, and if they're given a standardized test like that with choices I think that they can, from their understanding of it, they can do it. Writing it themselves is the issue I think.*
> Christina: *Or maybe that's for another lesson, like showing them bit by bit.*
> Ashley: *Yeah. And that to me is almost just notation too. I mean, they've seen less than or greater than before. And so, the one thing that we were worried about is like the real number symbol, or set notation where you have the bracket for when its included and the parenthesis. We saw that taught today, but we specifically didn't bring that up, because if students are trying to think about that and trying to get the concept of domain at the same time. No se. I do not think so.*

In this chunk, Emily poses a question she has about students' ability to choose the correct answer on a test if the notation is different than what they're used to seeing in class. Not only do Jennifer, Christina, and Ashley each attempt to help alleviate Emily's worries with reassuring ideas of why the students might indeed be successful on a test, but their comments also take a step back from the students and classroom at hand and become more general. Their comments don't mention specific students or happenings in that particular class, and represent a higher likelihood of these student teachers having the opportunity to take away something from this conversation that could be applied later in their teaching. Thus this chunk steps past the technical level into the dialogic.

Transformative. Only one chunk could be labelled as transformative. This conversation was between one student teacher and cooperating teacher, and was very short. The student teacher comments on how she wishes that there was enough time every day to teach a lesson a second time after teaching it once, correcting all the mistakes made and capitalizing on the strengths so the second time would be better. The chunk was labelled as transformative because the student teacher specifically verbally mentions putting new ideas into practice at a future time.

Undetermined. Finally, 14 chunks were coded as undetermined. After evaluating the codes of the individual comments, and the level each student teacher reached, often no conclusion could be drawn as to where the chunk ended up as a whole. One student teacher might reach a dialogic level in the same chunk where the other student teachers never progress past routine. I was careful to only label chunks as dialogic if I felt like the majority of the members of the conversation had the opportunity to take away something new. These undetermined chunks contained too many varying codes to draw any clear conclusions about the conversation as a whole.

Summary

My second research question asked how accepting the student teachers were of the ideas they discussed, and thus how likely they were to take away the learning to be later applied in their future classrooms. It was apparent that the individual comments were usually technical in nature, mostly concerning descriptions specific to the lesson taught that day. The conversations as a whole were nearly equally technical and dialogic, the technical chunks ending unresolved and the dialogic chunks reaching some kind of conclusion.

Cooperating Teacher/ University Supervisor Influence

My final research question aimed at finding if the participation of a cooperating teacher or university supervisor influenced that level of reflection the student teachers would reach in a chunk. Of the 60 chunks analyzed at this level, a cooperating teacher, university supervisor, or both participated in 38 chunks.

13 of these 38 chunks were labelled as dialogic overall, and one was labelled as transformative, leaving 24 chunks as either technical, routine, or undetermined. I will present four chunks in which a university supervisor or cooperating teacher seem to influence the direction the chunk takes. I will begin with two examples of chunks where a cooperating teacher or university supervisor positively influenced the reflection, thus leading to a dialogic code of the chunk overall. I will conclude with two examples where a university supervisor didn't positively influence the reflection, and the chunk was labelled as technical overall.

Examples where Cooperating Teacher and University Supervisor Positively Influence the Reflection

In the following chunk, a cooperating teacher opens with a question, echoed by a university supervisor, aiming to get the student teachers thinking about what types of questions can be asked to help students start to generalize a pattern they have observed to all possible cases.

> CT Larson: *What kinds of questions do you pose for kids so that they can start to formulate…*
> US Brad: *So they can generalize.*
> CT Larson: *And generalize you know, the equations.*
> US Emily: *Well I like how they had them first write it out in words before they wrote it out in variables. I mean that helped a lot of them and a lot of them skipped that step,*

but for the people that needed it they were able to put it into words because then they knew how to put variables in there. So that was one way that I thought that was good.

Jennifer: *I thought the pattern question was huge like as a class. You know, "Look for patterns. What patterns do you see? If I were to give you any random number could you find the profit?" Or stuff like that.*

Emily: *A lot of them do it, but they don't know where they did it or what they did.*

If you just ask them what did you do? What does that mean? When you get this number what does it represent? You know, if can help them break apart all of the things that they're getting out of the calculator.

CT Larson: *Okay, just something to think about because as long as you're attempting to implement worthwhile mathematical tasks you're going to have kids that can do a piece and they're going to be stuck. So you're going to have to try and think of the question because you can try and tell them, but then you caught in kind of a lurch because they know some stuff, but then they're being told some stuff. And in their mind it's like, "Okay you brought me here, but now you're pushing me back. Even though in telling them you're trying to lean them forward, they'll take as though, "Now you're telling. You led me and now you're [leans backward]." Do you understand what I'm saying? Think of a question that will bridge the gap for them. It's not, I don't know, because there were some students that were definitely like, "I don't know where." Like in Holly's class Zach was like, he started goofing with the boy because he was lost. And I was trying to, but if you know. And that's why I was asking you because I was starting to get a little frustrated with him because he was like [looks confused]. And I tried to ask questions. But I think always try and think about that. And sometimes in my experience it might be two years from now that you're like okay this is the question you ask when you come across it.*

Two student teachers strongly engage with the question, both reaching a dialogic level of reflection. The cooperating teacher specifically addresses the question to the student teachers, implying that he desires to hear their opinion on the matter. Cooperating teacher Larson gives his input on answering the question at the conclusion of the chunk, but only after giving the student teachers a chance to answer the question themselves.

Another university supervisor's participation in this second example is slightly different.

US Karl: *Okay, we don't have to spend this long on this part. I'm just going to throw this one out because it's related to these same homework problems. What's the difference between n and –n?*

Jennifer: *A certain value verses the opposite of that value?*

US Karl: *That's exactly right and the homework problem, where was the homework?*

Christina: *34*

Jennifer: *I think it might have been implying that the n value is positive and so the negative n would mean it's a negative value, meaning you owe money for…*

US Karl: *Yeah, - n + (-150) = -350. That was the sentence.*

Jennifer: *So you know, don't you know that that n value has to be, well n by itself has to be positive. So therefore the – n has to be negative.*

US Karl: *That's right. So the unknown has to be the thing that you would solve for, in this case it ends up being, it needs to be a positive number. And that's in contrast to the previous one where they put a plus in front of the n. And you know that in this case they put a plus there to tell you that it's a positive. See that's where this is a little bit confusing. That plus sign in front of the n tells you that n is positive. And that negative sign in front of the n tells you that the number you're looking for is negative. I mean they're saying you're going to put a negative number in this spot and the way I'm telling you is I'm putting a negative in front of the n. And I don't like that. I don't think this is good notation. It could really get in the way. So you may not have even focused on that. So you just ignored it anyway. Even had you focused on that it might have been a wise idea to, this is not a good principle to try to teach. I don't know what.*

Ashley: *Because if you solve that equation n would be positive.*

Emily: *When we had a lot of those on the homework a couple of days ago they had like – a + b = such and such. And they had to find two values for a and b that worked. And a lot of them, even though they already had the negative before the b they put that b equals a negative number. You know, just because they know it's supposed to be a negative in that spot. So we just kind of let it slide. "Good you know it's a negative." You know. But yeah that was something I thought about.*

Jennifer: *And the question also has a lot to do with that too because if the negative is there you're unknown has to fit that.*

US Karl: *Right. And yet clearly from the context what they want you to do is have a negative number plus a negative number = another negative number. So what they really want you to be thinking about is, "come up with another situation where you have some unknown value that's got to be negative." Get that to a negative number and get another negative. Notation wise it isn't optimal in my opinion. I think really what they want is just an n there with no plus or minus and then you need to decide that it needs to be a negative number if you solved it. [to Molly] What were you going to say?*

Molly: *I agree, but I was just trying to think of a situation where they would put that there. And then if you're talking about the caldron it does make sense because the way that the kids think about it is as cubes first and the kind of cubes second. So they're going to think, "Okay what cubes do I have? What kind of cubes are they?" So that's the only way I could explain it.*

US Karl: *Yeah, I think that helps maybe understand a little bit why they're doing that.*

Molly: *Because I'll have them, they'll say, "the answer's eight." And I'll say, "Is it eight?" And then they'll look back and say, "cold cubes, it's negative eight." So that's maybe why they're doing that.*

US Karl: *Okay, well thanks for have that little discussion.*

The university supervisor begins with a question directed at the student teachers, similar to the previous chunk. The student teachers remain engaged throughout the chunk, even after the university supervisor speaks a second and third time. At one point, the university supervisor directly asks Molly, "What were you going to say?" allowing her to further engage.

Examples where University Supervisors do not Positively Influence the Reflection

In the next two examples a university supervisor begins a chunk, and the student teachers never reach a dialogic level, engage deeply, or speak very much at all. In this third example, university supervisor Karl compliments the student teachers on their students' improved ability to explain how they know their answer is correct. The compliment does not seem to spark a discussion amongst the student teachers.

> US Karl: *Um, similar to the last couple days, as was mentioned, excellent questions I saw coming out. Lots of why and how do you know that and it's clear that students have been…they're used to being asked that question. Some of them may not like it, but some of them I think do, I mean they're willing to do it, and so they compared to the last time I was here there's a stark contrast. When you ask "how do you know," they, your students know, many of them know how to answer that question.*
> Ashley: *or at least they try to.*
> US Karl: *or at least they try, and so I think that's huge progress*
> Jennifer: *Lacey, it's like, "Can I just finish writing and sit down?"*
> Ashley: *She doesn't like to explain.*
> Jennifer: *No she doesn't.*

In this final example, university supervisor Brad presents an alternative to the student teachers for how the mathematical topic that day could have been introduced.

> *US Brad: And now the one thing, one question I, it's a question, a wondering. I noticed when Jennifer, when you came around, and it gets back to this idea of "what does it mean to work?" I noticed when you [Jennifer] came around kind of after you got them going, you were asking a lot of the groups, "Okay, what values of x can you put in the function and have it make sense?" Okay, I wondered if maybe that question could have been posed as part of your launch at the beginning. Instead of having to go around and do it with each group to kind of say, "Okay when we say what is the domain what do we really mean?" Well we're trying to find….But you're still then, you're with that balance of how much do you say without giving it away what you want them to wrestle with. So that's the tradeoff there. But I heard you say it to three or four different groups and I thought, "I wonder if that could have been said up front before you were going around to the individual groups. And the wording of that, I don't know. I don't know that I mean "make sense." Yeah "work" yeah. I wonder if you say, I'll just throw this out and this may not work. What values of x can you put in*

the function and have the output be a real number? But then you got to deal with the real number thing. I don't know. I don't know if you even want to go there. But that's a possible variation. Because then you're not tipping your hand and saying, "Look for the undefined points." You're saying, "What values of x can you put in the function and have the output be a real number?"

Jennifer: Or "can you still do it?"

Ashley: Mine was "and have there be an output."

US Brad: Oh yeah. Right, but it's still kind of related to, I wonder if that conversation as a whole class at the beginning. Maybe, maybe not. I'm not sure.

ST Stevens: I don't know if this would work either. You probably heard me saying this to some kids. I don't know if it was bad or not. Something I started using was I tried to build a little bit on the discussion about the $\frac{1}{x}2$ *and said, "Okay now look at this. Notice that when we did that we had this dilemma right, with 1/0. We didn't know what to do with that. It's like the calculator doesn't know what to do with it. You don't know what to do with it. So we're looking for values of x that have that sort of characteristic where you go, 'what do I do now?'" I don't know.*

US Brad: *Yeah and that's where you're tipping your hand toward looking for undefined points is what you really want to do and the question is how much of that do you want to do up front and how much of that do you want to them to discover. So I'm wondering if you pose it as, "For what values of x do you get the output that is a real number?" Where you're not saying, "Look for the undefined number", but you're saying, "look for the points where it works." Then they have to figure out where the undefined points are. I don't know. Anyhow, so that's a possible variation on that.*

The university supervisor poses the alternative, and two student teachers offer their suggestions, but he does not build on their ideas or push for more. The student teachers are never directly asked a question or encouraged to engage. While the university supervisor concludes with his own insights, as in the very first example above where cooperating teacher Larson closes the chunk with his comments, in this example the student teachers either are not given or do not take the opportunity to engage in a discussion before the university supervisor concludes with his thoughts.

Summary

The results of this study did not seem to show a definitive nor predictable connection between the participation of a cooperating teacher or university supervisor and the level of reflection achieved overall within a chunk. In some cases the students would engage with a question or comment by a cooperating teacher or university supervisor, while other times they would not. The direction the chunk takes seems to vary depending on certain choices the university supervisor and cooperating teacher make in their discussion. However, these the various excerpts with cooperating teacher and/or university supervisor participation do provide some insights into what types of moves by cooperating teachers and university supervisors seem to help or hinder the direction of the student teachers' reflection. This will be discussed in the discussion chapter.

CHAPTER SIX: DISCUSSION AND IMPLICATIONS

In this section, I will first discuss the results above in an effort to better illuminate exactly what the student teachers talked about. I will then discuss the results on *how* they reflected on these principles they discussed. I will also provide insights throughout into why the results might have occurred in this way. I will conclude with a section on what these results imply for mathematics student teaching programs and future research.

Discussion

What the Student Teachers Talked About and Why

The NCTM principles. One goal of this study was to find exactly what student teachers in the mathematics student teaching program at Brigham Young University have the opportunity to learn. Recall that the program was redesigned in 2006, years before the National Council of Teachers of Mathematics released their newest standards for mathematics education in *Principles to Action* (2014). However, the BYU program was redesigned within a mindset similar to that of Principles to Action (2014) because both the program and the Principles to Action document are based on the previous NCTM standards documents (NCTM, 1989, 1991, 2000), aiming to focus the student teachers participating in the program on student mathematics (Leatham & Peterson, 2013). Each of the eight principles NCTM (2014) believes are necessary for "effective" mathematics teaching (establishing mathematics goals to focus learning, implementing tasks that promote reasoning and problem solving, using and connecting mathematical representations, facilitating meaningful mathematical discourse, posing purposeful questions, building procedural fluency through conceptual understanding, supporting productive struggle in learning mathematics, and eliciting and using evidence of student thinking) place the focus of teaching on the students and their mathematical understanding, and thus provide an ideal framework for examining what the student teachers in the BYU program discussed.

Using the framework of these eight principles of “effective teaching,” I found that in 2006 and 2007, the student teachers at BYU discussed aspects of all of the eight principles on multiple occasions during the reflection meetings with their cluster groups. Even more impressive is that of the 15 most commonly discussed sub-codes, 9 of these sub-codes can directly be found in *Principles to Action* (NCTM, 2014). In fact, 7 out of 8 of the principles of “effective teaching” (NCTM, 2014) are accounted for in the most commonly discussed sub-codes, the only exception being sub-codes under “building procedural fluency through conceptual understanding.” Thus we have evidence that these student teachers had the opportunity to learn a wide variety of pedagogical principles that directly influence student mathematical thinking. These student teachers had the opportunity to learn how to “make quality shoes” (Leatham &Peterson, 2010b, p. 100), just as the program intended. These results did not occur simply because my choice of framework demanded that I fit each conversation into these principles of “effective teaching” as evidenced by the fact that, as mentioned in my methodology and results, if a topic did not fit into one of the eight principles, then a new internal code was created to accommodate it, and internal codes were created on several occasions.

As mentioned in my theoretical framework, *Principles to Actions* (NCTM, 2014) provides a description of what each principle of “effective teaching” should look like when enacted in a mathematics classroom. They provide a description of what teachers should be doing when carrying out that principle, as well as what students should be doing, and these provided the sub-codes for my study. Close observation of the resulting 15 most commonly discussed sub-codes and the infrequently discussed sub-codes (listed in Appendix B) when compared to all the possible sub-codes that *could* have been discussed (listed in Appendix A) reveals that the sub-codes concerning what students should be doing were almost never discussed. The student teachers were much more focused on the pedagogical sub-codes that concerned things *they* should be doing.

The literature suggests that beginning teachers tend to focus on themselves (Feiman-Nemser & Buchmann, 1987; Hawkey, 1996). Ward and McCotter (2004), when studying the written reflection of pre-service teachers, found student teachers to be more focused on themselves than on the students. However, while the student teachers in this literature are focused on their image and survival in the classroom, the student teachers in the present study are focused on themselves in a very different way. A close look at the sub-codes pertaining to what NCTM (2014) believes teachers should be doing when enacting a principle of "effective teaching" reveals that the sub-codes only describe what teachers should be doing *as pertaining to facilitating student mathematical learning.* Consider, for example, the sub-code "advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking" which falls under the larger principle of "pose purposeful questions." Although when the student teachers discussed this sub-code they were indeed discussing their own actions of asking questions, the quality of their questions was discussed only in reference to the extent they helped advance student thinking. Next consider two of the sub-codes under the same principle that NCTM (2014) describes as things students should be doing: "expecting to be asked to explain, clarify, and elaborate their thinking" and "thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly." These sub-codes, as well as those of the other principles, describe an aspect of student action that is first, extremely specific. Second, these sub-codes also describe internal thinking of the students, something that I argue would be difficult for these student teachers to have enough access to in order to be able to discuss it with their cluster group. I believe the specificity of the sub-codes pertaining to student actions, and the fact they describe what should be occurring inside the minds of the students contributed to my results containing such infrequent discussions on these sub-codes. In conclusion, unlike the literature on the tendency for beginning teachers to focus on their own image and survival in the classroom, the student teachers in this present study had an opportunity to learn how the principles of effective teaching, although primarily teacher pedagogical moves, can be used to facilitate student mathematical learning.

Recall from the results chapter that only two internal codes I created and none of the NCTM sub-codes under "implementing tasks that promote reasoning and problem solving" made the list of the 15 most frequently discussed sub-codes. The two internal codes discussed very frequently and by all three cluster groups were "thinking about the numbers and questions they choose and how those will affect student thinking" and "the creation or adaptation of tasks." Although not mentioned in *Principles to Action* (NCTM, 2014), because these were pedagogical principles are specific to implementing tasks, I placed them under that principle. While it may seem the principle of "implementing tasks" was not discussed frequently by the student teachers, a closer look at how the principle was discussed reveals that this was not necessarily the case. Table 8, extracted from Appendix B, shows the frequency of sub-codes in this principle.

Table 8

Frequency of IT Sub-codes

NCTM	Sub-code	Usage	Word Count	Group 1	Group 2	Group 3
IT	#5 Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	5	5307	4	1	
	#4 Supporting students in exploring tasks without taking over student thinking.	3	743	3		
	#2 Selecting tasks that provide multiple entry points through the use of varied tools and representations.	3	725	3		
	#1 Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.	3	628	1		2
	#7 Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.	2	771	1		1
	#3 posing tasks on a regular basis that require a high level of cognitive demand.	1	240	1		
	#6 Persevering in exploring and reasoning through tasks.	1	94	1		

Although these sub-codes were not discussed heavily enough to be included in the list of most common sub-codes, the student teachers discussed 7 out of the 9 possible sub-codes (all 9 can be found in Appendix A), and although not discussed by all three cluster groups or in more than 5 chunks, the sub-code #5 by far exceeded the word count criteria of 2000. In other words, the student teachers did discuss the elements of implementing tasks, though not as extensively as some of the other principles. Since the student teachers discussed "implementing tasks that promote reasoning and problem solving" in 18 chunks, nearing 10% of total chunks, and also discussed the two internal codes I created in another 14 chunks, (thus 15% total) I argue that the student teachers had an opportunity to gain a reasonable understanding of this principle.

A similar argument cannot be made about "build procedural fluency through conceptual understanding." Table 9 provides the sub-codes and internal sub-codes for this principle.

Table 9

Frequency of BPFCU Sub-codes

NCTM	Sub-code	Usage	Word Count	Group 1	Group 2	Group 3
BPFCU	#2 Asking students to discuss and explain why the procedures that they are using work to solve particular problems.	3	2647	2		1
	#1 Providing students with opportunities to use their own reasoning strategies and methods for solving problems.	2	1861	1		1
	#7 Determining whether specific approaches generalize to a broad class of problems.	1	436	1		
BPFCU Internal	#I1 Spending enough time to let students develop a deep understanding.	3	1194	1	1	1
	#12 Students are comparing and contrast the effectiveness of different strategies.	1	193	1		

Only 3 of the 9 possible sub-codes from *Principles to Action* (NCTM, 2014) were discussed, and neither internal code met my criteria for the most frequent sub-codes. Although my results do not provide much insight into why this principle as a whole was not heavily discussed, I can provide observations about individual sub-codes and possible reasons for their scarcity. First, sub-code #1 "providing students with opportunities to use their own reasoning strategies and methods for solving problems" is almost identical to IT sub-code #5 listed in Table 8 above. The chunks concerning this pedagogical principle were coded under IT rather than

BPFCU because the content of the chunk was more directly related to the task itself than to the strategies of students, and thus this sub-code might have been scarce simply because of its similarity to another sub-code elsewhere. Second, sub-codes #3 and #4 (see Appendix A) reference extremely specific teacher moves concerning using student-generated procedures during class, perhaps contributing to these sub-codes never being discussed. Also, the concept of using student-generated procedures is very closely related to the sub-code "reflecting on evidence of student learning to inform the planning of next instructional steps" under the principle "elicit and use evidence of student thinking," which, recall, was the second most commonly discussed sub-code overall. So again, perhaps the ideas captured by this principle were not discussed heavily because similar sub-codes existed elsewhere under the other 7 principles. Regardless, the essence of this principle, namely the use of students' procedures to help develop a conceptual understanding, does not seem to be captured in what these student teachers had the opportunity to learn.

It's interesting to consider whether or not the essence of the other principles was captured in the discussions of these student teachers. I used three criteria in my evaluation. First, I considered whether or not the ideas discussed most often by the student teachers captured NCTM's (2014) brief description of that principle in *Principles to Actions* (NCTM, 2014). Second, if the first criteria was not met, I looked at the sub-codes that almost made the list of the 15 most common sub-codes and considered if *these* gave any further evidence that the student teachers discussed the big ideas in NCTM's (2014) description of the principle. Finally, I also considered the vastness of sub-codes discussed by the student teachers, or in other words how many of the possible sub-codes they discussed. Using these criteria, I found that what the student teachers discussed seemed to successfully capture what NCTM might consider the core of each of the other principles. For example,

Consider NCTM's description of the principle "Establish Mathematics Goals to Focus Learning."

> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. (NCTM, 2014, p. 12)

As mentioned in my results, the student teachers discussed both establishing clear goals and using those goals. Since the student teachers left these reflection meeting having discussed two large ideas pertaining to establishing learning goals and also mentioned in NCTM's (2014) description of the principle above, I argue that the student teachers had an opportunity to gain a strong understanding of the essence of this principle. As I continued to evaluate whether or not the big ideas mentioned in the descriptions NCTM (2014) provides of the principle were captured by the sub-codes the student teachers heavily discussed, I found the essence of the principles "Use and Connect Mathematical Representations", "Facilitating Meaningful Mathematical Discourse," "Pose Purposeful Questions," "Support Productive Struggle," and "Elicit and Use Evidence of Student Thinking" to all be similarly captured in the student teachers' discussions. To provide an example, consider the description of the principle "facilitating meaningful mathematic discourse."

> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. (NCTM, 2014, p. 29)

The student teachers heavily discussed "engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations," and also discussed "selecting and sequencing student approaches and solutions strategies for whole-class analysis and discussion" in 9 different chunks, though this sub-code did not meet my criteria because it was not discussed by all three of the cluster groups or for 3000 words. Because the student teachers discussed, though not heavily, 7 of the 8 sub-codes under this principle (see Appendix B), and the two heavily discussed sub-codes just mention together capture NCTM's (2014) description of what this principle looks like, we have evidence that the student teachers had the opportunity to learn the crux of this principle.

Similar arguments can be made about each of the other principles, excluding "build procedural fluency through conceptual understanding," as mentioned above. In conclusion, the student teachers had the opportunity to learn the essence of 7 of the 8 principles of "effective teaching" (2014).

In conclusion, when considering whether or not the student teachers had the opportunity to learn the crux of each principle of "effective teaching," we find that 7 of the 8 principles were discussed extensively enough to suggest the student teachers had the opportunity to learn their essence.

One final insight we can extract from the results pertains to the immense amount of chunks and words devoted to student mathematical thinking. Recall that the two single most commonly discussed items (namely "interpreting student thinking to assess mathematical understanding, reasoning, and methods" as well as "reflecting on evidence of student learning to inform the planning of next instructional steps") fell under the principle "elicit and use student mathematical thinking" and together totaled to 37 chunks, or roughly 18% of the chunks. Considering that the goal of this program is to specifically focus the student teachers on student mathematical thinking (Leatham & Peterson, 2013), this finding shows that the program is indeed succeeding in what it was intended to do. The structure of the program itself is clearly contributing to the focus of student teachers. In addition, when considering the internal codes I created that met the criteria of the 15 most common sub-codes, though not specifically mentioned in *Principles to Action* (2014), the sub-codes "thinking about the numbers and questions they choose and how those will affect student thinking," and "anticipating how the choice of vocabulary, notation, and definitions will affect student mathematical thinking" also directly reference student mathematical thinking, and are therefore consistent with both the principles in *Principles to Action* (NCTM, 2014) and the intent of the restructured BYU program.

Behavior. It is significant that the student teachers discussed behavior very little, especially with the abundance of literature suggesting that student teachers and cooperating teachers primarily talk about behavior management (Mitchell, Clarke, & Nuttall, 2007; Peterson & Williams, 2008; Moore, 2003; Borko & Mayfield, 1995) Of the 209 chunks analyzed, only 8 chunks were given a behavior code, and 3 of these 8 were given another non-behavior code in addition to the behavior code. Thus behavior represented less than 4% of the chunks. The findings from this BYU program differ significantly from what is found in the literature with traditionally structured programs.

Mathematics. Finally, it's important to notice that one of the 15 most commonly discussed items was mathematics. The student teachers discussed solely their own understanding of mathematics, not in relation to the students, on 9 occasions. These discussions totaled to 6,807 words, the 3rd highest word count of any sub-code, meaning these conversations were often very long. Ball (1990) argues that we want students to develop "power and control" in mathematics, "validate their own answers," and "make conjectures, justify their claims, and "engage in a mathematical argument" (pp. 457-458). She claims that in order to help students achieve this kind of learning, teachers must have a deep understanding of mathematics themselves. Thus a teacher should have mathematical understanding deep enough that they can explain a topic "appropriately" and in "multiple ways" (p. 457). The fact that the student teachers in this program engage in deep discussions of their own understanding of mathematics is promising, and suggests that they have the opportunity to begin to develop the deep understanding of mathematics that Ball (1990) recommends. This contrasts with the literature on traditional programs that suggests that student teachers do not typically discuss mathematics with their cooperating teachers (Borko and Mayfield, 2005) and cooperating teachers do not believe mathematics is important for student teachers to learn in student teaching (Peterson and Leatham, 2010b).

How the Student Teachers Reflected and Why

The results presented in the previous chapter show a strong preference of the student teachers to make individual comments of a technical nature rather than dialogic. The coding of chunks overall revealed nearly an equal number of chunks remaining at the technical level as progressing to a dialogic level. The technical chunks never moved past the members of the cluster group describing various classroom happenings they observed. The dialogic chunks progressed to a resolution, where some or many members of the group provided new insights into teaching not specific to the students or classroom at hand but applicable enough to mathematics teaching in general that it is reasonable to assume the student teachers could take away this new knowledge and apply it in their future teaching career. I argue that the strong presence of technical comments and chunks can be attributed to two causes, each inherent in the structure of the program itself. First, we must recall that the members of the cluster group who were not teaching the lesson were asked to walk freely about the room, taking note of interesting student mathematical thinking. It is reasonable that in order to even progress to a dialogic level of reflection during a reflection meeting, it would be necessary that a cluster group member would first begin a discussion by pointing out something they observed during class as written down in their notes. Because this would result in a technical code, the presence of many technical codes is expected, and not necessarily a negative result. These comments have the potential to spark interest in the group and open an opportunity for discussion that although began with a comment or question specific to a student or lesson, could potentially result in the take-away of a general principle of mathematics teaching. Second, as described in my methods, at the beginning of each reflection meeting, a member of the cluster group asks the student teachers who taught the lesson three questions: 1) What was the goal of your lesson?; 2) How was your lesson designed to meet that goal?; and 3) How do you feel the lesson played out? This aspect of the structure also has the potential to encourage members of the group to make comments of a technical nature. Although the few paragraphs containing the answers to these questions at the beginning of each meeting were excluded from the data and not coded, these questions set a certain precedence, beginning each meeting with an immediate focus on what was observed, and creating an environment of discussion extremely hospitable to technical comments.

Neither of these structural aspects of the BYU mathematics student teaching program are bad or unnecessary, as they lay the ground work for discussions to begin that are relevant to students' mathematics, and are clearly successful in leading the conversations away from classroom management and onto the principles of "effective teaching" (NCTM, 2014). However, what is lacking in this student teaching program is any structure element aimed to help them take these conversations that begin with descriptions of things observed during class and help move them to a deeper, more dialogic level. The presence of a cooperating teacher and university supervisor have the potential to remedy this, as will be discussed in the implications.

Another purpose of this study was to understand how the way in which the student teachers discussed the principles of “effective teaching” (NCTM, 2014) differed when a cooperating teacher or university supervisor participated in the conversation. When posing this research question prior to analyzing the data, I expected that the cooperating teacher and university supervisor would have a clear and positive influence on the level of reflection reached in a given chunk in which they participated. However, the results above showed that while this was *sometimes* the case, the cooperating teacher and university supervisor in many cases did not necessarily help lead the reflection to a deeper level. These results allow us to conclude that the mere presence of a cooperating teacher or university supervisor does not automatically lead to more dialogic conversations. Although the purpose of this study and the coding methods I carried out were not aimed to analyze exactly what the cooperating teacher and university supervisor said to help or hinder the level of reflection, the results above do hint at a few insights.

First, analysis of the singletons revealed that on many occasions the cooperating teacher or university supervisor would point out something interesting they observed, give advice, or read off a long list of unrelated items from their notes. Although we cannot conclude that doing any of these automatically hinders the opportunity for learning to occur and a conversation chunk to begin, analysis of the ending of each singleton revealed that the cooperating teachers and university supervisors did not open up the discussion to the group with a question or specifically invite a student teacher to participate. Each singleton concludes in a very “matter of fact” fashion.

Second, the results from analyzing the level of reflection assigned to a chunk overall in relation to whether or not a cooperating teacher or university supervisor participated showed a few trends. The chunks that were coded as dialogic and had cooperating teacher or university supervisor participation almost always began with the cooperating teacher or university supervisor asking a question of the student teachers, or at least explicitly directing the conversation back to the student teachers mid-chunk. The conversations that remained at technical and had cooperating teacher or university supervisor participation did not contain any evidence of the cooperating teacher or university supervisor explicitly asking the student teachers a question or encouraging them to participate. Also, in these chunks the cooperating teacher and/or university supervisor did most of the talking, thus dominating the conversation. Again, the structure is lacking an element designed to encourage cooperating teachers and university supervisors in consistently having a positive effect on the level of reflection during these reflection meetings. Suggestions are provided in the implications section.

Conclusions and Answering the Research Questions

I will now synthesize the ideas presented in the discussion section in an effort to explicitly answer my three research questions.

Research Question #1: What did the Student Teachers have the Opportunity to Learn? Prior to this study, we knew that the mathematics student teaching program at Brigham Young University, purposely structured to focus student teachers on students' mathematics (Leatham & Peterson, 2013), was successful in helping the student teachers talk about students' mathematics, and rarely on behavior (Leatham & Peterson, 2013; Franc, 2013). The results of this study not only echo these positive results, but also provide new conclusions. First, we not only have evidence that the individual comments from student teachers, their cooperating teachers, and their university supervisors are focused on students' mathematics, but we now better understand that the nature of entire conversations between these cluster groups are also focused on students' mathematics, as the principles of "effective teaching" (2014) and many of the internal codes I created are focused on students' mathematics. Second, previous coding of this reflection meeting data revealed the number of statements from the participants that were focused on pedagogy, students, and mathematics in general had increased after the program was restructure when compared with before. With the use of the "effective teaching" (NCTM, 2014) framework, we now have a much more specific understanding of what the student teachers in this program are discussing. We have evidence that the student teachers had the opportunity to learn the essence of 7 of the 8 principles of "effective teaching."

Research Question #2: To what Extent were the Topics of Conversation "Taken Up? In addition to evaluating what principles of "effective teaching" (2014) the student teachers in the BYU structure had the opportunity to learn, another purpose of this study was to evaluate the evidence we have that the student teachers "took up" the principles they discussed in such a way that they left the program ready to implement the principles in their own classrooms. In order to measure the level of "take up," I evaluated the depth of the student teachers' reflection using a rubric ranging through routine, technical, dialogic, and transformative. Most of the individual comments were technical in nature, suggesting a low chance of "take up." When considering the chunks as a whole, there were nearly an equal number of technical chunks (primarily descriptions of classroom happenings and thus low level of "take up") and dialogic (generalizations of and new insights about the principles and thus high level of "take up"). As discussed above, the structural elements of the program contribute to these findings, and the program lacks any element aimed at helping the discussions move past the technical level and into the dialogic.

Research Question #3: What was the Influence of the Coopering Teacher and University Supervisor on "Take-up?" The final purpose of this study was to understand how the level of "take up" reached by the student teachers was different when a cooperating teacher or university supervisor participated in the conversation. Although there was no clear and consistent relationship between the presence of the cooperating teacher and university supervisor and the "take up" level reached by the student teachers in the chunk, it was clear that some of their actions hindered the "take up" (i.e. speaking matter-of-factly when concluding their remarks, failing to explicitly turn the conversation back to the student teachers, and dominating the conversations), while other actions seemed to facilitate the "take up" (i.e. explicitly inviting a student teacher's input and asking the student teachers a question). Again the structure of the BYU program is lacking the elements necessary to assist these cooperating teachers and university supervisors in facilitating "take up."

Implications

In this section, I will use the discussion of results above to provide implications for future research and the future of student teaching.

The student teachers in the BYU mathematics student teaching program had an extensive opportunity to learn about seven of the eight principles of "effective teaching" (NCTM, 2014). The student teachers also had the opportunity to learn several pedagogical principles pertaining to student mathematical learning not included in *Principles to Action* (2014), as well as expand their own understanding of mathematics. These results are particularly important to mathematics teacher educators, as they show that a mathematics student teaching program can indeed be restructured in a way that encourages student teachers and their cooperating teachers and university supervisors to discuss what the National Council of Teachers of Mathematics considers to be the necessary principles of effective mathematics teaching (NCTM, 2014). The results the also speak to the powerful content of NCTM's Principles to Actions (2014), as the pedagogical principles NCTM consider to be most important in a mathematics classroom are naturally being discussed by student teachers, cooperating teachers, and university supervisors when placed in a situation where the focus is intended to be students' mathematical thinking.

As mentioned in the discussion section, the results of this study suggested an equally strong tendency of these student teachers to discuss the principles of "effective teaching" (NCTM, 2014) in a technical way as in a dialogic. Just as some of the structural principles in the BYU mathematics student teaching program seem to foster a focus on descriptions of classroom happenings and consequently primarily a technical level of reflection, the results imply that the structure could be further enhanced in a way that could help push the reflection conversations towards a more dialogic or transformative level. And although there was no clear and distinct connection between the presence of a cooperating teacher and university supervisor and the level of reflection reached by the student teachers, or in other words the "take up" of the principles the student teachers discussed, the insights provided by the data into what the cooperating teachers and university supervisors did to help or hinder the reflection, as mentioned in my discussion, provide us with possible suggestions to how the structure would need to be redesigned in such a way that the cooperating teacher and university supervisor could better help conversations move past a technical level and into dialogic or transformative. Therefore, I suggest that mathematics student teaching programs could be restructured in four ways: 1) to encourage the cooperating teacher and university supervisor to continually advance the student teachers' reflection by pushing their ideas past specific instances and towards general principles of mathematics teaching; and 2) to encourage the student teachers themselves to extract general principles of mathematics teaching from their own observations; 3) to encourage the cooperating teacher and university supervisor to continually turn the conversation back to the student teachers, explicitly asking them questions and inviting their participation; and finally 4) to discourage the cooperating teachers and university supervisors from dominating the conversation with their own ideas and from running through a long list of unrelated comments about the lesson without giving the student teachers a chance to voice their input. The literature on the powerful role of cooperating teachers in shaping the beliefs and learning of their student teachers suggests that these outcomes are entirely within reach.

The results of this study suggest that changes like these might increase the opportunity for student teachers in this program to take away from their student teaching experience a deep, meaningful, and applicable understanding of the principles of "effective teaching" (NCTM, 2014). Future research related to this study could possibly examine if a student teaching structure can be further changed (for example to meet the four suggestions I made in this section and the previous) to assure these principles of "effective teaching" (NCTM, 2014) are not only being discussed as pertaining to specific students during a specific lesson on a given day, but are also being "taken up" in a way that assures the student teachers can leave the program ready to apply the principles in their own classrooms.

REFERENCES

Borko, H., & Mayfield, V. (1995). The roles of the cooperating teacher and university supervisor in learning to teach. *Teaching and Teacher Education*, 11, 501–518.

Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B. S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal*, 34(4), 663-689.

Brookhart, S., Andolina, M., Zuza, M., & Furman, R. (2004). Minute math: An action research study of student self-assessment. *Educational Studies in Mathematics*, 57(2), 213-227.

Brouwer, N., & Korthagen, F. (2005). Can teacher education make a difference? *American Educational Research Journal*, 42, 153–224.

Bullough, R. V., Jr., Young, J., Erickson, L., Birrell, J. R., Clark, D. C., Egan, M. W., et al. (2002). Rethinking field experience: Partnership teaching versus single-placement teaching. *Journal of Teacher Education*, 53(1), 68–80.

Carpenter, T. P., et al. (1996). Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction. *The Elementary School Journal*, 97(1), 3-20.

Chalies, S., Ria, L., Bertone, S., Trohel, J., & Durand, M. (2004). Interactions between preservice and cooperating teachers and knowledge construction during post-lesson interviews. *Teaching and Teacher Education*, 20, 765-781.

Clarke, A., Triggs, V., & Nielsen, W. (2014). Cooperating teacher participation in teacher education: A review of the literature. *Review of Educational Research*, 84(2), 163-202.

Clements, D. H., and Sarama, J. (2004). Learning Trajectories in Mathematics Education. *Mathematical Thinking and Learning,* 6(2), 81-89.

Cochran-Smith, M. (1991). Reinventing student teaching. *Journal of Teacher Education*, 42, 104-118.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education*, 3(2), 155-181.

Diezmann, C. M., & Watters, J. J. (2000). Catering for mathematically gifted elementary students: Learning from challenging tasks. *Gifted Child Today*, 23(4), 14-19, 52.

Evertson, C. M. (1990). Bridging knowledge and action through clinical experiences. In D. D. Dill (Ed.), *What Teachers Need to Know* (pp. 94-109). San Francisco: Jossey-Bass.

Feiman‐Nemser, S., & Buchmann, M. (1987). When is student teaching teacher education? *Teaching and Teacher Education*, 3(4), 255‐273.

Franc, N. (2013). *Comparing Two Different Student Teaching Structures by Analyzing Conversations Between Student Teachers and Their Cooperating Teachers* (Master's thesis).

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into Practice*, 40(2), 102–109.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Gresalfi, M. S. (2009). Taking up opportunities to learn: Constructing dispositions in mathematics classrooms. *Journal of the Learning Sciences*, 18, 327-369.

Guyton, E., & McIntyre, D. J. (1990). Student teaching and school experiences. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 514–534). New York: Macmillan.

Hawkey, K. (1996). Image and the pressure to conform in learning to teach. *Teaching and Teacher Education*, 12, 99–108.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, 25(4), 12-21.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second Handbook of Research on Mathematics Teaching and Learning*, *1*, 371-404.

Hiebert, J., Morris A. K., Berk D., and Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education,* 58(1), 47–61.

Kazemi, E., Franke, M. L., & Lampert, M. (2009, July). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell & T. Burgess (Eds.), *Crossing Divides, 32 Annual Conference of The Mathematics* (Vol. 1, pp. 11-29). Palmerston North, New Zealand: Education Research Group of Australasia.

Kilpatrick, J., et al. (2001). *Adding it Up: Helping Children Learn Mathematics*, National Academy Press.

Koedinger, K. R., Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *The Journal of the Learning Sciences* 13(2), 129-164.

Koerner, M., Rust, R. O., & Baumgartner, F. (2002). Exploring roles in student teaching placements. *Teacher Education Quarterly*, Spring, 35–58.

Leahy, Siobhan, Lyon, C., Thompson, M., William, D. (2005). Classroom assessment: Minute by minute, day by day. *Educational Leadership*, 3(63), 18-24.

Leatham, K. R., & Peterson, B. E. (2010a). Purposefully designing student teaching to focus on students' mathematical thinking. In J. Luebeck & J. W. Lott (Eds.), *Mathematics Teaching: Putting Research into Practice at all Levels* (pp. 225-239). San Diego, CA: Association of Mathematics Teacher Educators.

Leatham, K. R., & Peterson, B. E. (2010b). Secondary mathematics cooperating teachers' perceptions of the purpose of student teaching. *Journal of Mathematics Teacher Education*, 13, 99-119.

Leatham, K. R. & Peterson, B. E. (2013). Talking about Pedagogy, Students and Mathematics. In Martinez, M. & Castro Superfine, A (Eds.), *Proceedings of the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 629-636). Chicago, IL: University of Illinois at Chicago.

Levine, M., & Trachtman, R. (1997). *Making Professional Development Schools Work: Politics, Practice, and Policy. The Series on School Reform*. Teachers College Press, 1234 Amsterdam Avenue, New York, NY 10027

McIntyre, D. J., Byrd, D. M., & Foxx, S. M. (1996). Field and laboratory experiences. In J. Sikula (Ed.), *Handbook of Research on Teacher Education* (2nd ed., pp. 171–193). New York: Simon & Schuster Macmillan.

Mitchell, J., Clarke, A., & Nuttall, J. (2007). Cooperating teachers' perspectives under scrutiny: A comparative analysis of Australia and Canada. *Asia-Pacific Journal of Teacher Education*, 35, 5–25.

Montecinos, C., Walker, H., Rittershaussen, S., Nunez, C., Contreras, I., & Solis, M. C. (2011). Defining content for field-based coursework: Contrasting the perspectives of secondary preservice teachers and their preparation curricula. *Teaching and Teacher Education*, 27, 278-288.

Moore, R. (2003). Reexamining the field experiences of preservice teachers. *Journal of Teacher Education*, 54, 31–42.

National Council of Teachers of Mathematics (2014). *Principals to Actions: Ensuring Mathematical Success for All*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Peterson, B. E., & Williams, S. R. (2008). Learning mathematics for teaching in the student teaching experience: Two contrasting cases. *Journal of Mathematics Teacher Education*, 11, 459-478.

Richert, A. E. (1992). The content of student teachers' reflections within different structures for facilitating the reflective process. In T. Russel (Ed.), *Teachers and Teaching: From Classroom to Reflection* (pp. 168-187). USA: Falmer Press.

Rigelman, N. R. (2007). Fostering Mathematical Thinking and Problem Solving: The Teacher's Role. *Teaching Children Mathematics*, 13(6), p.308.

Rittle-Johnson, B., Siegler, R., & Alibali, M. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93, 346–362.

Rodgers, A., & Keil, V. L. (2007). Restructuring a traditional student teacher supervision model: Fostering enhanced professional development and mentoring within a professional development school context. *Teaching and Teacher Education*, 23, 63-80.

Sandoval, W. A., Deneroff, V., & Franke, M. L., (2002). Teaching, as learning, as inquiry: Moving beyond activity in the analysis of teaching practice. Paper presented at the Annual Meeting of the American Educational Research Association New Orleans, LA.

Shimizu, Y. (1999). Aspects of mathematics teacher education in Japan: Focusing on teachers' roles. *Journal of Mathematics Teacher Education*, 2(1), 107–116.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Skemp, R. R. (1978/2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School*. 12(2), 88-95.

Smith, M. S. & M. K. Stein (2011). *Five Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: SAGE Publications.

Stegman, S. F. (2007). An exploration of reflective dialogue between student teachers in music and their cooperating teachers. *Journal of Research in Music Education*, 55, 65–82.

Tabachnick, B. R., Popkewitz, T. S., & Zeichner, K. M. (1979). Teacher education and the professional perspectives of student teachers. *Interchange*, 10(4), 12-29.

Ward, J. R., & McCotter, S. S. (2004). Reflection as a visible outcome for preservice teachers. *Teaching and Teacher Education*, 20, 243-257.

Weiss, E. M., & Weiss, S. (2001). Doing reflective supervision with student teachers in a professional development school culture. *Reflective Practice*, 2, 125–154.

Wilson, S. M. , Floden, R. E., & Ferrini-Mundy, J. (2001). *Teacher Preparation Research: Current Knowledge, Gaps, and Recommendations*. Seattle, WA: University of Washington Center for the Study of Teaching and Policy.

Zeichner, K. (2002). Beyond traditional structures of student teaching. *Teacher Education Quarterly*, 29, 59-64.

APPENDIX A

List of NCTM Codes Sub-codes

NCTM Category	Sub-code Number	Sub-code
Establish Learning Goals	#1	Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.
	#2	Identifying how the goals fit within a mathematics learning progression.
	#3	Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.
	#4	Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.
	#5	Engaging in discussions of mathematical purpose and goals related to their current work in the mathematics classroom.
	#6	Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices.
	#7	Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going.
	#8	Assessing and monitoring their own understanding and progress toward the mathematics learning goals.
Implementing Tasks that Promote Reasoning and Problem Solving	#1	Motivating students'' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.
	#2	Selecting tasks that provide multiple entry points through the use of varied tools and representations.
	#3	Posing tasks on a regular basis that require a high level of cognitive demand.
	#4	Supporting students in exploring tasks without taking over student thinking.
	#5	Encouraging students to use varied approaches and strategies to make sense of and solve tasks.
	#6	Persevering in exploring and reasoning through tasks.
	#7	Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.
	#8	Using tools and representations as needed to support their thinking and problem solving.
	#9	Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

Use and Connect Mathematical Representations	#1	Selecting tasks that allow students to decide which representations to use in making sense of the problems.
	#2	Allocating substantial instruction time for students to use, discuss, and make connections among representations.
	#3	Introducing forms of representations that can be useful to students.
	#4	Asking students to make math drawings or use other visual supports to explain and justify their reasoning.
	#5	Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of representations.
	#6	Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.
	#7	Using multiple forms of representations to make sense of and understand mathematics.
	#8	Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.
	#9	Making choices about which forms of representations to use as tools for solving problems.
	#10	Sketching diagrams to make sense of problem situations.
	#11	Contextualizing mathematical ideas by connecting them to real-world situations.
	#12	Considering the advantages or suitability of using various representations when solving problems.
Facilitating Meaningful Mathematical Discourse	#1	Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.
	#2	Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.
	#3	Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.
	#4	Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.
	#5	Presentation and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.
	#6	Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.
	#7	Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.
	#8	Identifying how different approaches to solving a task are the same and how they are different.

Pose Purposeful Questions	#1	Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.
	#2	Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.
	#3	Asking intentional question that make the mathematics more visible and accessible for student examination and discussion.
	#4	Allowing sufficient wait time so that more students can formulate and offer responses.
	#5	Expecting to be asked to explain, clarify, and elaborate of their thinking.
	#6	Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly.
	#7	Reflecting on and justifying their reasoning, not simply providing answers.
	#8	Listening to, commenting on, and questioning the contribution of their classmates.
Build Procedural Fluency from Conceptual Understanding	#1	Providing students with opportunities to use their own reasoning strategies and methods for solving problems.
	#2	Asking students to discuss and explain why the procedures that they are using work to solve particular problems.
	#3	Connecting student-generating strategies and methods to more efficient procedures as appropriate.
	#4	Using visual models to support students' understanding of general methods.
	#5	Making sure that they understand and can explain the mathematical basis for the procedures that they are using.
	#6	Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.
	#7	Determining whether specific approaches generalize to a broad class of problems.
	#8	Providing students with opportunities for distributed practice of procedures.
	#9	Striving to use procedures appropriately and efficiently.
Support Productive Struggle in Learning Mathematics	#1	Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.
	#2	Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.
	#3	Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.
	#4	Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.

	#5	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
	#6	Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
	#7	Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
	#8	Helping one another without telling their classmates what the answer is or how to solve the problem.
Elicit and Use Evidence of Student Thinking	#1	Identifying what counts as evidence of student progress toward mathematics learning goals.
	#2	Eliciting and gathering evidence of student understanding at strategic points during instruction.
	#3	Interpreting student thinking to assess mathematical understanding, reasoning, and methods.
	#4	Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.
	#5	Reflecting on evidence of student learning to inform the planning of next instructional steps.
	#6	Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse.
	#7	Reflecting on mistakes and misconceptions to improve their mathematical understanding.
	#8	Asking questions, responding to, and giving suggestions to support the learning of their classmates.
	#9	Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.

Note. Sub-codes taken directly from NCTM, 2014, pp. 16-56.

APPENDIX B

List of NCTM Codes and Usage

NCTM	Sub-code	Usage	Word Count	Group 1	Group 2	Group 3
ELG	**#4 Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.**	**8**	**3466**	**4**	**2**	**2**
	#1 Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.	**7**	**5430**	**5**	**1**	**1**
	#5 Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom.	1	1165	1		
IT	#5 Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	5	5307	4	1	
	#1 Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.	3	628	1		2
	#2 Selecting tasks that provide multiple entry points through the use of varied tools and representations.	3	725	3		
	#4 Supporting students in exploring tasks without taking over student thinking.	3	743	3		
	#7 Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.	2	771	1		1
	#3 posing tasks on a regular basis that require a high level of cognitive demand.	1	240	1		
	#6 Persevering in exploring and reasoning through tasks.	1	94	1		
IT Internal	**#I1 Thinking about the numbers and questions they choose and how those will affect student thinking.**	**8**	**3573**	**5**	**1**	**2**
	#I5 The creation or adaptation of tasks.	**6**	**3163**	**2**	**2**	**2**
	#I3 Connecting the task to a real-world context.	5	1877	3	2	
	#I2 Being aware of and prepared for students who finish early.	3	2172	1		2

	#I4 Giving explicit instructions during the launch of a task.	2	457	1		1
	#I6 Students are using a variety of strategies.	1	170	1		
UCMR	**#5 Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.**	**10**	**3875**	**8**	**1**	**1**
	#3 Introducing forms of representations that can be useful to students.	6	4157	5		1
	#1 Selecting tasks that allow students to decide which representations to use in making sense of the problems.	3	1328	2	1	
	#4 Asking students to make math drawings or use other visuals supports to explain and justify their reasoning.	2	1456	1	1	
	#2 Allocating substantial instruction time for students to use, discuss, and make connections among representations.	1	108	1		
	#6 Designing ways to elicit and assess students' abilities to use representations to solve problems.	1	374	1		
	#7 Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.	1	184	1		
	#10 Sketching diagrams to make sense of problem situations.	1	1133	1		
	#11 Contextualizing mathematical ideas by connecting them to real-world situations.	1	111	1		
UCMR Internal	#I1 Considering advantages and disadvantages of different representations.	2	2250	1	1	
FMMD	#2 Selecting and sequencing student approaches and solutions strategies for whole-class analysis and discussion.	9	2372	7	2	
	#1 Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	**7**	**3895**	**3**	**2**	**2**
	#7 Seeking to understand the approaches use by peers by asking clarifying questions, trying out others' strategies and describing the approaches used by others.	3	2009	1		2

	#3 Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.	3	1721	2		1
	#5 Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.	2	1909			2
	#4 Ensuring progress towards mathematical goals by making explicit connections to student approaches and reasoning.	1	108	1		
	#6 Listening carefully to and critiquing the reasoning of peers, using examples to support to counterexamples to refute arguments.	1	247	1		
PPQ	#1 Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.	8	5029	4		4
	#3 Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion.	**8**	**3785**	**5**	**1**	**2**
	#2 Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.	**7**	**4056**	**3**	**1**	**3**
	#5 Expecting to be asked to explain, clarify, and elaborate on their thinking.	1	131	1		
BPFCU	#2 Asking students to discuss and explain why the procedures that they are using work to solve particular problems.	3	2647	2		1
	#1 Providing students with opportunities to use their own reasoning strategies and methods for solving problems.	2	1861	1		1
	#7 Determining whether specific approaches generalize to a broad class of problems.	1	436	1		
BPFCU Internal	#I1 Spending enough time to let students develop a deep understanding.	3	1194	1	1	1
	#12 Students are comparing and contrast the effectiveness of different strategies.	1	193	1		
SPS	**#2 Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.**	**13**	**7587**	**8**	**2**	**3**
	#1 Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.	6	1389	4	1	1

	#4 Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	3	1079	2	1	
	#5 Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.	1	844	1		
	#3 Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.	1	844	1		
	#6 Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.	1	436	1		
EUE	**#3 Interpreting student thinking to assess mathematical understanding, reasoning, and methods.**	**22**	**19825**	**10**	**6**	**6**
	#5 Reflecting on evidence of student learning to inform the planning of next instructional steps.	**15**	**6580**	**8**	**3**	**4**
	#2 eliciting and gathering evidence of student understanding at strategic points during instruction.	6	3124	4		2
	#1 identifying what counts as evidence of student progress towards mathematics learning goals.	2	1172	2		
	#9 Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.	1	844	1		

Note. Sub-codes are abbreviated version of the ones from NCTM, 2014, pp. 16-56. Full codes can be found in Appendix A. Most common 15 sub-codes are bolded.

APPENDIX C

List of Internal Codes (Non-NCTM) and Usage

Student Mathematics (SM)	#I2 Considering students' past knowledge.	6	948	1	3	2
	#I1 Anticipating student thinking.	4	474	2	1	1
Mathematics Pedagogy (MP)	**#I1 Anticipating how the choice of vocabulary, notation, and definitions will affect student mathematical thinking.**	**6**	**2026**	**4**	**1**	**1**
	#I2 Considering how pairing and grouping will affect student thinking.	2	921		1	1
Mathematics (MATH)	**#I1 Seeking a personal understanding of the mathematics being taught.**	**9**	**6807**	**3**	**1**	**5**
Pedagogy of Students (PEDS)	**#I2 Managing behavior.**	**8**	**4051**	**1**	**5**	**2**
	#I1 Class engagement and participation.	5	1264	3	2	
Pedagogy (PEDA)	#I1 Board planning and getting big ideas on the board.	4	809	3		1
	#I2 Grading.	2	1990	1		1
	#I3 Planning and time management.	2	219	1	1	
Other (0)	#I2 None.	9	1657	3	6	1
	#I1 Miscellaneous.	**6**	**2723**	**3**	**1**	**2**