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Wire segmenting for buffer insertion based on RSTP-MSP

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Abstract

This paper presents an approximation algorithm for simultaneously constructing a rectilinear Steiner tree and buffer insertion points into the tree. The objective of the algorithm is to divide each wire into multiple smaller segments and minimize the number of the buffer insertion points (Steiner points) which are only located at the end of each segment. We show that (a) the Steiner ratio is $\frac{1}{3}$, that is, the rectilinear minimum spanning tree yields a polynomial-time approximation with a performance ratio exactly 3; (b) there exists a polynomial-time approximation with a performance ratio 2.

Keywords: VLSI; Wire segment; Buffer insertion; Rectilinear Steiner tree; Minimum spanning tree; Approximation algorithm

1. Introduction

In VLSI technology, some gates on a chip often have to drive many sinks or long wires resulting in propagation delay. Three techniques are commonly applied to reduce such delay: gate sizing, wire sizing, and buffer insertion. This paper focuses on the last technique. Buffers can not only decouple a large load that is off the critical path but also directly reduce the ARC delay of a long wire.

Berman et al. [2] showed that simultaneously constructing a tree and inserting buffers at the internal nodes of the tree is NP-Complete. The authors of [8, 9] proposed a

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method of buffer insertion as follows: first, find a best location for a single buffer, and then recursively apply the algorithm. But the buffers' sizes are assumed to be variable. Ginneken [4] gave an optimal dynamic programming algorithm that inserts at most one buffer into a single wire. Okamoto et al. [11] extended Ginneken's method so that it can simultaneously construct a Steiner tree and insert buffers while keeping the restriction that at most one buffer may be placed on each wire. Alpert et al. [1] proposed a method that arbitrarily divides each wire into segments and eliminates at most one buffer per wire restriction. But their work is based on the assumption that the routing tree topology and buffer locations for each wire are provided.

Our work is to provide a polynomial-time approximation algorithm that can simultaneously build the routing tree topology (a rectilinear Steiner tree) and segment each wire under the assumption that the length-bound of segment is given. The goal is to minimize the number of buffer insertion points (Steiner points). In this paper, we show that the rectilinear minimum spanning tree-based algorithm has a performance ratio of exactly 3. We also present a new polynomial-time approximation with a performance ratio of at most 2.

2. Preliminary

For a given set of terminals V in the rectilinear plane, a *rectilinear Steiner tree* [5–7] is a tree spanning a superset X of V. Points in $X\setminus V$ are called Steiner points [5–7]. The rectilinear length of an edge between two points v_1 and v_2 is defined by $|v_1v_2| = |x_1-x_2|+|y_1-y_2|$, where (x_i, y_i) are the coordinates of v_i for $i = 1,2$. The length of a tree is the sum of its edges' lengths. The *rectilinear Steiner minimal tree problem* (RSTP) is to find a rectilinear Steiner tree of minimum length. An edge between two points of X may consist of a sequence of alternating vertical and horizontal lines. Turning points on such edges are called *corner*s. Given a constant $R > 0$, the *rectilinear Steiner tree problem with minimal number of Steiner points* (RSTP-MSP) asks for a rectilinear Steiner tree such that each edge has a length not more than R and the number of Steiner points is minimized.

The optimal Steiner trees for RSTP and RSTP-MSP may have different structures. In the classical RSTP [5], every vertex has degree less than five and Steiner points have degree either three or four. In RSTP-MSP, however, Steiner points may have degree two. For example, when V contains two terminals v_1 and v_2 with $|v_1v_2| > R$, the optimal Steiner tree is a path containing $\lceil |v_1v_2|/R - 1\rceil$ Steiner points whose degrees are two; Moreover, some vertices may have degree as large as eight. Consider nine terminals (white nodes) in Fig. 1(a). The optimal Steiner tree for RSTP in Fig. 1(b) includes two Steiner points (black nodes) and has length of $6R$. Two optimal Steiner trees for RSTP-MSP in Fig. 1(c, d) both include no Steiner point and have length of $8R$; Note in the first tree every terminal has degree less than three while in the second tree one terminal has degree eight. However, we can show the following lemma.

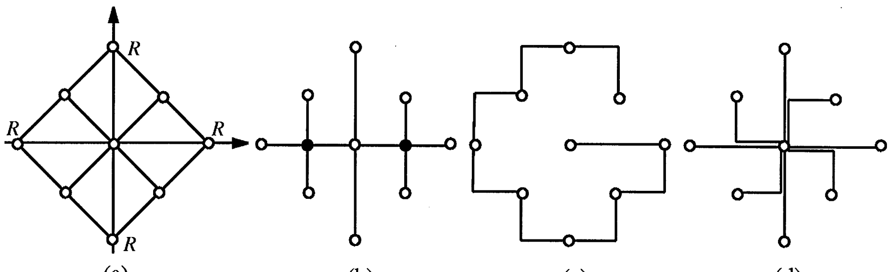

Fig. 1. An example: (a) nine terminals; (b) the optimal Steiner tree of RSTP; (c) and (d) the optimal Steiner trees of RSTP-MSP.

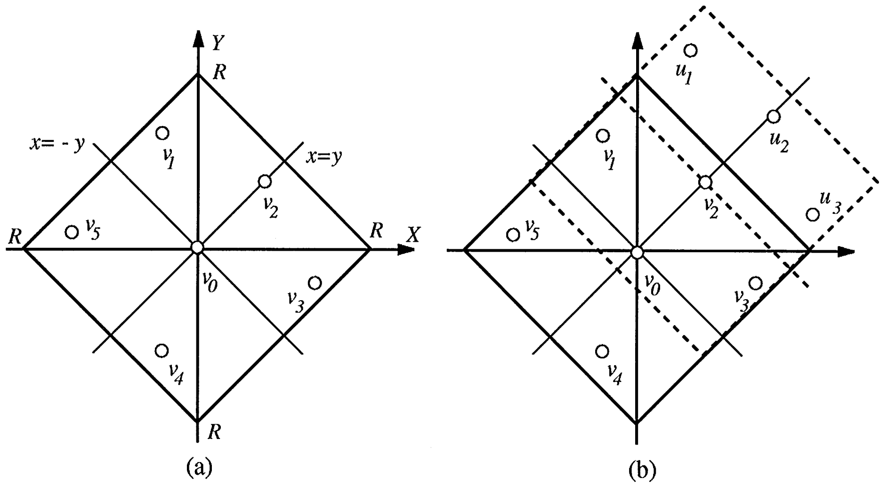

Fig. 2. Proof of Lemma 1.

Lemma 1. *There exists a shortest optimal Steiner tree of RSTP-MSP such that every vertex in the tree has degree at most four.*

Proof. Suppose, by contradiction, that every shortest optimal Steiner tree has a vertex whose degree is more than four. Then let T be such a tree with minimal number of vertices that have degrees more than four, and let vertex v_0 in T be adjacent to five vertices v_i, $i=1,2,\ldots,5$. We assume, without loss of generality, that $v_0=(0,0)$. Now consider v_0's neighborhood of radius R, which is a square. Note lines $x=y$ and $x=-y$ partition the square into four small squares. See Fig. 2.

Suppose vertices v_1 and v_2 are in the same small square, say, the one formed by lines $-x\leqslant y$ and $x\leqslant y$. If neither v_1 nor v_2 is on one of these two lines, then it can be verified that $|v_1v_2|<\max\{|v_1v_0|,|v_2v_0|\}$. Thus, replacing the longer edge of v_1v_0 and v_2v_0 with edge v_1v_2 will produce a Steiner tree whose length is shorter than T, contradicting that T is the shortest Steiner tree. Therefore, we can assume that vertices

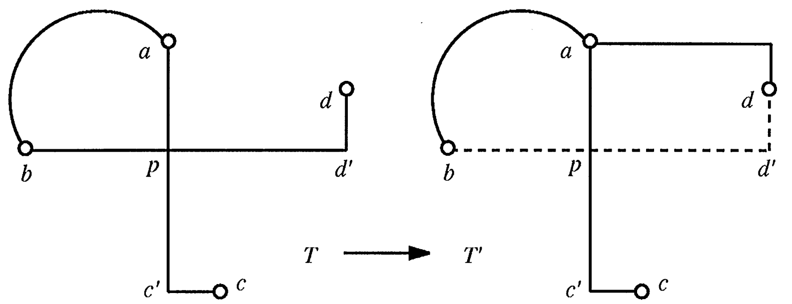

Fig. 3. Proof of Lemma 2.

v_1, v_2 and v_3 are in two small squares formed by $-x \leqslant y$ and v_2 is on line $x = y$. See Fig. 2(a). We now consider two cases separately.

Case 1: v_2 has degree less than four. Then substituting edge v_1v_0 with edge v_1v_2 will produce a shortest Steiner tree that has less number of vertices that have degree at least five than T, contradicting that T has minimal number of such vertices.

Case 2: v_2 has degree four (or bigger). Then v_2 is adjacent to three vertices u_1, u_2, and u_3 besides vertex v_0. If there exists a vertex u_i that is in the two small squares of v_2's neighborhood formed by the line in the direction of $x = -y$ and passing vertex v_2, then either $|v_1u_i| < |v_1v_0|$ or $|v_3u_i| < |v_3v_0|$. In either case, a shorter Steiner tree can be constructed by replacing edge v_1v_0 with edge v_1u_i or edge v_3v_0 with edge v_3u_i, contradicting that T is the shortest one. Hence, we can assume that u_i for $1 \leqslant i \leqslant 3$ are all in the other two small squares while u_2 in line $x = y$. See Fig. 2(b).

By repeating the above argument at vertex u_2 and so on, in the end we will reach Case 1 and find a contraction. The proof is then complete. □

Lemma 2. *There exists a shortest optimal Steiner tree for RSTP-MSP such that no two edges cross each other.*

Proof. Suppose that T is a shortest optimal Steiner tree that includes two edges ac and bd crossing at point p, and ac and bd may have corners at points c' and d', respectively. See Fig. 3.

We can assume that $|ap| \leqslant |pc'|$, $|bp| \leqslant |pd'|$ and $|ap| \leqslant |bp|$. Then we have

$$|ad| = |ap| + |pd'| \pm |d'd| \leqslant |bp| + |pd'| + |d'd| = |bd|,$$

$$|ab| = |ap| + |pb| \leqslant |bp| + |pd'| + |d'd| = |bd|.$$

Therefore, replacing edge ad with edge ab if there is a path in T between vertices b and a or c, and with edge ab if there is a path in T between vertices d and a or c, can make a shortest Steiner tree T'. Moreover, the number of crossings in T' is less than that in T. By repeating this operation we can obtain a shortest optimal Steiner tree without any crossing. □

In fact, by applying the combination of the arguments in the proofs of Lemmas 1 and 2, we are able to prove that there exists a shortest optimal Steiner tree for RSTP-MSP such that every vertex in the tree has degree at most four and there is no two edges crossing each other. Hence in the rest of the paper, we consider only such a kind of shortest optimal Steiner trees for RSTP-MSP.

A *rectilinear spanning tree* is a tree interconnecting the given terminals with edges only between a pair of terminals. The *rectilinear minimum spanning tree* (RMST) is a rectilinear spanning tree with minimum length. Note a rectilinear spanning tree (even if it is a RMST) may not be a feasible solution for RSTP-MSP, since some edges may be too long (their lengths exceed R). To make it feasible, we can break any edge longer than R into several small segments with lengths at most R by adding some Steiner points in the edge. The resulting tree will be called a *steinerized* rectilinear spanning tree. The steinerized RMST has the following interesting property.

Lemma 3. *Every steinerized RMST has the minimum number of Steiner points among the steinerized rectilinear spanning trees.*

Proof. Note that every RMST can be obtained from a rectilinear spanning tree by a sequence of operations that each replaces an edge by another shorter edge. Since the shorter edge needs less number of Steiner points than the longer edge when rectilinear spanning tree is steinerized. This leads to the conclusion. □

3. Steinerized rectilinear spanning tree

In this section we will show the following result.

Theorem 1. *The steinerized rectilinear minimum spanning tree is a polynomial-time approximation for RSTP-MSP with a performance ratio of exactly* 3.

To obtain the lower bound of performance ratio, consider a simple example in Fig. 4. There are four terminals (white nodes) such that every pair of them has length of $2L$, where $L=(1-\varepsilon)R$ and ε is a very small positive real number. Fig. 4(a) illustrates the optimal Steiner tree for RSTP-MSP that has only one Steiner point (black node). Fig. 4(b) illustrates the steinerized RMST that has three Steiner points. This implies that the performance ratio of steinerized RMST is at least three.

Note that every leaf in a shortest optimal Steiner tree is a terminal. A rectilinear Steiner tree is called *full* if every terminal is a leaf. When a rectilinear Steiner tree is not full, we can always find a terminal with degree more than one where we can break the tree into parts. In such a way, every rectilinear Steiner tree can be broken into several small full rectilinear Steiner trees, which are called *full components* of the rectilinear Steiner tree.

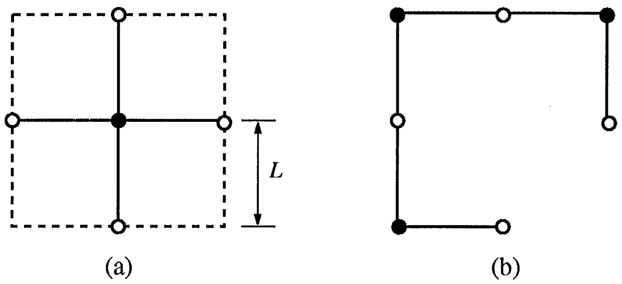

Fig. 4. The lower bound: (a) the optimal solution of RSTP-MSP; and (b) the steinerized RMST.

To obtain the upper bound of performance ratio, we need to study the properties of rectilinear convex path between two terminals in the shortest optimal full tree for RSTP-MSP. A path $q_1q_2\dots q_m$ is called a rectilinear *convex path* if one places a coordinate axis at any point on the path then at least one of the quadrants does not contain any point in the path. Note that angles can be defined along the convex path. The angles of 180° and 270° will play an important role in the proof of Theorem 1. For simplicity, such angles are called *big* angles.

Let T denote a shortest optimal full tree for RSTP-MSP.

Lemma 4. *Let $q_1q_2\dots q_m$ be a rectilinear convex path between terminals q_1 and q_m in T and $m\geqslant 2$. Suppose there are t_1 angles of* 180° *and t_2 angles of* 270° *among $m-2$ angles $\angle q_1q_2q_3$, $\angle q_2q_3q_4,\dots,\angle q_{m-2}q_{m-1}q_m$. Then overall, it is true that $|q_1q_m|\leqslant(t_1+2t_2+2)R$.*

Proof. First, we define a staircase to be a continuous path of alternating vertical lines and horizontal lines such that their projections on the vertical and horizontal axes have no overlapping intervals.

Then we prove the lemma by induction on m. For $m\leqslant 3$, it is true, since $|q_1,q_3|=2R\leqslant(t_1+2t_2+2)R$. Now, suppose $m\geqslant 4$. Consider the rectilinear convex hull H of points $q_1,q_2,\dots,q_m$. If at least one of q_1 and q_m does not lie on the boundary of H, then by the induction hypothesis, any rectilinear distance between two vertices of the convex hull H is at least $(t_1+2t_2+2)R$. Therefore, $|q_1q_m|\leqslant(t_1+2t_2+2)R$.

Next, we may assume that both q_1 and q_m lie on the boundary of H. Then the whole path $q_1q_2\dots q_m$ lies on the boundary of H (see Fig. 5(c)). Since there are overlapping intervals if a rectilinear convex path is not a staircase, we can always obtain $|q_1q_m|\leqslant(t_1+2t_2+2)R$ by induction hypothesis. So it suffices to show that this inequality is true for staircase (see Fig. 5(a, b)). We consider the following cases:

Case 1: When the rectilinear convex path is straight. Clearly, we have $|q_1q_m|=(t_1+1)R<(t_1+2)R\leqslant(t_1+2t_2+2)R$.

Case 2: When the rectilinear convex path is a L-shaped path. As shown in Fig. 5(a), the turning point of the L-shaped path may be a Steiner point with degree 2, 3 or 4.

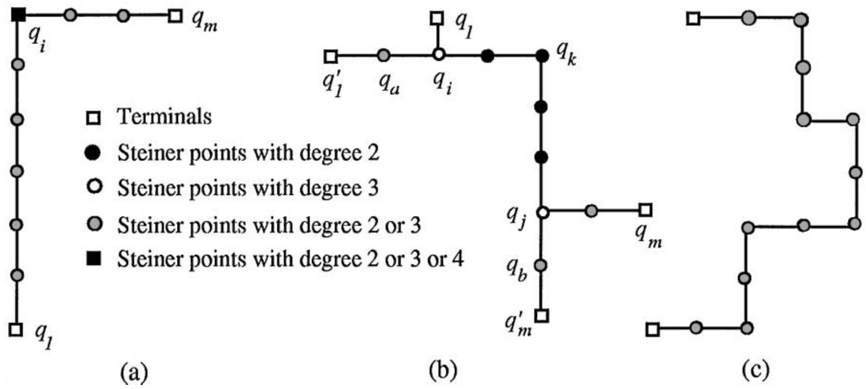

Fig. 5. Proof of Lemma 4: (a) L-shaped path; (b) Staircase; and (c) whole path.

It can also just be a corner. For the latter we have $|q_1q_m| = (t_1+1)R$ from Case 1. If the turning point is a Steiner point, say q_i, then we have $|q_1q_m| = (t_1+2)R \leqslant (t_1+2t_2+2)R$.

Case 3: The other possible shapes of rectilinear convex paths are those like $q'_1 \ldots q_m$ and $q_1 \ldots q_m$ as shown in Fig. 5(b) (may be in other directions). Otherwise, we can have a shorter tree by flipping some corner or Steiner point contradicting that T is the shortest optimal tree for RSTP-MSP. For those convex paths like $q'_1 \cdots q_m$, it is easy to show that $|q'_1q_m| \leqslant (t_1+2t_2+2)R$ holds. For those like $q_1 \ldots q_m$, when the turning point of path $q_i \ldots q_j$ is a Steiner point q_k shown in Fig 5(b), it can be verified that $|q_1q_m| \leqslant (t_1+2t_2+2)R$ also holds. When the turning point is a corner, $|q_1q_m| = (t_1+3)R$ and $t_2 = 0$ will be obtained. This violates the inequality $|q_1q_m| \leqslant (t_1+2t_2+2)R$. However, since there must exist another rectilinear convex path $q''_1 \ldots q''_n$ which shares $q_i \cdots q_j$ with the convex path $q_1 \ldots q_m$ and it is not difficult to show that $|q''_1q''_n| \leqslant (t''_1+2t''_2+1)R$ is true, where t''_1 is the number of angles of 180° and t''_2 is the number of angles of 270° among $n-2$ angles along this convex path. Thus, we can charge cost 1 from $|q_1q_m|$ to $|q''_1q''_n|$. Therefore, overall we have $|q_1q_m| \leqslant (t_1+2t_2+2)R$. The proof is then complete. □

Lemma 5. *In the shortest optimal tree for RSTP-MSP, there are at most two big angles at a terminal with degree two, one big angle at a vertex with degree three, and no big angle with degree four.*

Proof. The lemma follows from the definition of RSTP-MSP. □

Lemma 6. *Let T be a full rectilinear Steiner tree interconnecting n terminals, and s_i denote the number of Steiner points with degree i in T. Then $2s_4+s_3 = n-2$.*

Proof. Since T has $s_4+s_3+s_2+n-1$ edges, and the total degrees of vertices in T is $4s_4+3s_3+2s_2+n = 2(s_4+s_3+s_2+n-1)$. Hence, $2s_4+s_3 = n-2$. □

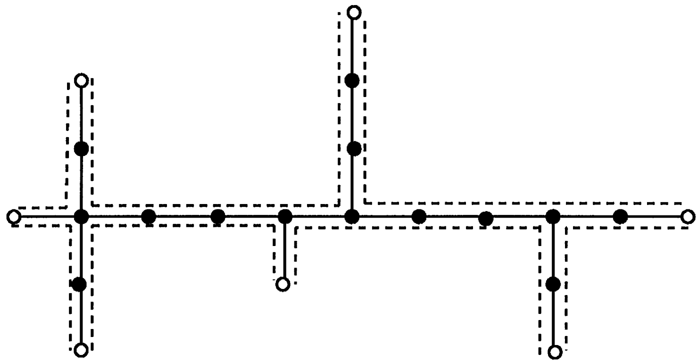

Fig. 6. Rectilinear convex paths having designated properties.

Given a shortest optimal Steiner tree for RSTP-MSP on n terminals, if it is a full rectilinear Steiner tree, then we can find a set of n rectilinear convex paths in the tree that satisfy the following three properties:

(1) each path connects two terminals,

(2) each terminal appears in exactly two paths,

(3) each angle at a Steiner point appears in the paths exactly once.

Fig. 6 shows an example. Terminals and Steiner points are marked by white and black nodes, respectively, and rectilinear convex paths by dash lines. Now, we are ready to show Theorem 1.

Proof of Theorem 1. Denote by $C(T^*)$ the number of Steiner points in T^*. First, we assume that T^* is a full rectilinear Steiner tree. Let s_i denote the number of Steiner points with degree i in T^*. Let s_2' denote the number of Steiner points of degree two with angle 180°, and s_2'' denote the number of Steiner points of degree two with angle 270°. Clearly, $s_2 = s_2' + s_2''$. By Lemma 6, $n = 2s_4 + s_3 + 2$. Consider a rectilinear spanning tree T_s consisting of $n-1$ edges each connecting two terminals at endpoints of a rectilinear convex path in T^*. By Lemma 4, each edge in T_s has length upper-bounded by $(t_1 + 2t_2 + 2)R$ where t_1 and t_2 are the numbers of big angles of 180° and 270° on the rectilinear convex path connecting two terminals, respectively. Hence we need at most $(t_1 + 2t_2 + 1)$ Steiner points to steinerize the edge. By Lemma 5, the rectilinear spanning tree T_s can be steinerized by at most $s_3 + 2s_2' + 2s_2'' + n - 1 = s_3 + 2s_2 + n - 1$ Steiner points. By Lemma 3, any steinerized MRST contains at most $s_3 + 2s_2 + n - 1$ Steiner points. Clearly,

$$s_3 + 2s_2 + n - 1 = 2s_4 + 2s_3 + 2s_2 + 1 = 2(s_4 + s_3 + s_2) + 1.$$

When $s_4 + s_3 + s_2 > 0$, we have $s_3 + 2s_2 + n - 1 \leqslant 3(s_4 + s_3 + s_2)$. When $s_4 + s_3 + s_2 = 0$, we have $T_s = T^*$. Therefore, in either case, every steinerized MRST contains at most $3(s_4 + s_3 + s_2)(= 3C(T^*))$ Steiner points.

Now suppose T^* is not a full rectilinear Steiner tree. Then T^* can be decomposed into several full components $T_1, T_2, \ldots, T_k$, each satisfying the above properties (1,2,3). Let $C(T_j)$ be the number of Steiner points in T_j. For each full component T_j, by the above argument, we know that the steinerized RMST on terminals in T_j contains at most $3C(T_j)$ Steiner points. Note that the union of steinerized RMSTs is a steinerized rectilinear spanning tree for all terminals. By Lemma 3, the number of Steiner points in T^* is at most $3\sum_{j=1}^{k} C(T_j) = 3C(T^*)$. The proof is then complete. □

4. 2-Approximation

Suppose T^* is a shortest optimal Steiner tree for RSTP-MSP that has k full components $T_1, T_2, \ldots, T_k$. In the proof of Theorem 1, we have showed that the steinerized RMST on terminals in T_j contains at most $2C(T_j) + 1$ Steiner points. In the following, we will study when this upper bound can be improved.

Lemma 7. *Let T'_j be the steinerized RMST on terminals in T_j. Then,*

(1) *T'_j contains at most $2C(T_j) + 1$ Steiner points.*

(2) *T'_j contains at most $2C(T_j)$ Steiner points when T_j contains a Steiner point with degree at most three.*

(3) *T'_j contains at most $2C(T_j)$ Steiner points when T'_j contains an edge between two terminals.*

Proof. Conclusions (1) and (3) follow immediately from the proof of Theorem 1. To show (2), let n_j be the number of terminals in T_j. Note that there are exactly n_j paths in the forms shown in Lemma 4 in T_j. Choose any $n_j - 1$ of them and connect two endpoints of each path. We will obtain a rectilinear spanning tree. Its steinerization is denoted by T_s. Now, assume u is a Steiner point with degree at most three. When there is a big angle at u, we choose $n_j - 1$ rectilinear convex paths not containing the big angle. When there is no big angle at u, we can choose any $n_j - 1$ rectilinear convex path. In such a way, we can obtain $C(T_s) \leqslant s_3 + 2s_2 - 1 + (n_j - 1) \leqslant 2(s_4 + s_3 + s_2) = 2C(T_j)$. □

In the following, we propose a Kruskal-type approximation algorithm.

Step 0: Given a set of n terminals,
    sort all $n(n-1)/2$ possible edges e_i between n terminals
    in increasing order of their length, $|e_1| \leqslant |e_2| \leqslant \ldots \leqslant |e_{n(n-1)/2}|$.
    $T_A := (X, \emptyset)$ and $i := 1$.
Step 1: **while** $|e_i| \leqslant R$ **do begin**
    if e_i connects two different connected components of T_A
    then put e_i into T_A.
    $i := i + 1$.
  end-while

Step 2: **for** three terminals a, b, c are in three connected components of T_A **do**
if there exists a point s within distance R from a, b, c,
then put the 3-star, consisting of three edges sa, sb, sc into T_A.
Step 3: **while** $i \leqslant n(n-1)/2$ **do begin**
if e_i connects two connected components of T_A,
then put e_i into T_A along with $\lceil |d_i|/R \rceil - 1$ Steiner points.
$i := i + 1$.
end-while
return T_A

Theorem 2. *Let $C(T_A)$ be the number of Steiner points in the approximation T_A produced by the proposed algorithm. Then $C(T_A) \leqslant 2C(T^*)$.*

Proof. Denote by $T^{(i)}$ the T_A at the beginning of Step i in the algorithm. Suppose $T^{(3)} - T^{(2)}$ contains m 3-stars. Then

$$C(T_A) \leqslant C(T_s) - m,$$

where $C(T_s)$ is the number of Steiner points in T_s which is a steinerized RMST on all given terminals. We construct a steinerized rectilinear spanning tree T as follows: Initially, put $T^{(2)}$ into T. For each full component T_j $(1 \leqslant j \leqslant k)$, add to T the steinerized rectilinear spanning tree H_j for terminals in T_j. If T has a cycle, then destroy the cycle by deleting some edges along with Steiner points of H_j. An important fact is that if H_j does not contain an edge between two terminals, then at least one Steiner point must be deleted when destroying a cycle in $H_j \cup T^{(2)}$. From this fact and Lemma 7, we have

$$C(T_s) \leqslant 2C(T^*) + h,$$

where h is the number of full components T_j's with properties that every Steiner point in T_j has degree four and $T_j \cup T^{(2)}$ has no cycle. Hence we have

$$C(T_A) \leqslant 2C(T^*) + h - m.$$

Now to prove the theorem it suffices to show $h \leqslant m$.

Suppose $T^{(2)}$ has p connected components. Then $T^{(3)}$ has $p - 2m$ connected components $C_1, C_2, \ldots, C_{p-2m}$. We now construct a graph H with vertex set consisting of n terminals and the edge set defined as follows: First, we put all edges of $T^{(2)}$ into H. Then consider every full component T_j $(1 \leqslant j \leqslant k)$ with properties that every Steiner point in T_j has degree four and $T_j \cup T^{(2)}$ has no cycle. If T_j has only one Steiner point, then this Steiner point connects four terminals which must lie in at most two C_i's. Hence, among them there are two pairs of terminals; each pair lies in the same C_i. Connect the two pairs with two edges and put the two edges into H. If T_j has at least two Steiner points, then there must exist at least two Steiner points each connecting three terminals. We can also find two pairs of terminals among them such that each pair lies in the same C_i. Connect the two pairs with two edges and put the two edges into H.

Clearly, H has at most $p - 2h$ connected components. Since every connected components of H is included in a C_i, we have $p - 2m \leqslant p - 2h$. Therefore, $h \leqslant m$. The proof is then complete. □

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