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A Multiobjective Optimization Method for Collaborative Products with Application to Engineering-Based Poverty Alleviation

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A thesis submitted to the faculty of
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in partial fulfillment of the requirements for the degree of

Master of Science

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ABSTRACT

A Multiobjective Optimization Method for Collaborative Products with Application to Engineering-Based Poverty Alleviation

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Collaborative products are created by combining components from two or more products to result in a new product that performs previously unattainable tasks. The resulting reduction in cost, weight, and size of a set of products needed to perform a set of functions makes collaborative products useful in the developing world. In this thesis, multiobjective optimization is used to design a set of products for optimal individual and collaborative performance. This is introduced through a nine step method which simultaneously optimizes multiple products both individually and collaboratively. The method searches through multiple complex design spaces while dealing with various trade-offs between products in order to optimize their collaborative performance. An example is provided to illustrate this method and demonstrate its usefulness in designing collaborative products for both the developed and developing world. We conclude that the presented method is a novel, useful approach for designing collaborative products while balancing the inherent trade-offs between the performance of collaborative products and the product sets used to create them.

Keywords: collaborative products, multiobjective optimization, poverty alleviation, product decomposition, reconfigurable products, modular products

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CHAPTER 1. INTRODUCTION AND BACKGROUND

1.1 Engineering-Based Poverty Alleviation

Poverty is a growing concern throughout the world. As reported by the World Bank in 2008, those in extreme poverty (1.6 billion people) live on less than $1.25 a day [2]. Groups working to assist these individuals to escape poverty have noted that one of the major challenges these people face is the inability to generate sufficient income [2]. As such, in recent years an engineering focus has developed that emphasizes sustainability in the developing world by designing products that generate income for the user [3, 4].

The action of using engineering principles and methods to assist in bringing individuals out of poverty is known as engineering-based poverty alleviation. Through the development and distribution of engineering-based poverty alleviation products, poverty-stricken individuals are able to benefit from improved health and education by increasing their income [3]. Various professional and educational organizations have contributed to advances in engineering-based poverty alleviation by creating products to benefit in areas such as irrigation, water purification, transportation, lighting, cooking, and health care [3–8]

Even though more than 20 million people have been brought out of poverty with such products, a large concern still exists for others due to the financial risks that are present when purchasing such products [3,4]. Thus, if financial risks can be lowered, the impact of these products will extend to more people who previously considered these products to be unaffordable. The goal of the method introduced herein is to design products that not only generate income, but also appeal to a greater number of individuals because of their affordability. Within the method, the research contributes to the field of numerical design optimization which allows the design of such products by dealing with trade-offs and meeting objectives to satisfy this goal.

1.2 Reconfigurable and Modular Products

There have been a few areas within product design research aimed at bringing individuals out of extreme poverty. Some of these include the design of reconfigurable products and modular products [1, 5, 9, 10]. Reconfigurable products are capable of changing their configuration to meet multiple functional requirements or a change in operating conditions [11]. This allows products to maintain a high level of performance and achieve a desired outcome within acceptable reconfiguration time and cost [12, 13]. Three main reasons that drive the need for reconfigurable systems include: (1) Multi-ability: the ability to change and reconfigure over time but not concurrently; (2) Evolution: the ability to modify over time for both planned and unknown configurations; and (3) Survivability: the ability to survive and demonstrate robustness despite potential failures in some components [13]. This thesis includes a combination of reconfigurable and modular products identified as collaborative products [1]. Collaborative products are further discussed in Section 1.3.

Modular-product design is essential in the design of collaborative products, as it involves the joining together of multiple products. In the literature, this type of design is known as *Type II modularity*. It is defined as the design of interfaces with modules that can only be attached to other specific modules through a unique interface, effectively reducing the complexity of the products [14, 15].

1.3 Collaborative Product Design

Collaborative products are created when physical components from two or more products are brought together to form another product capable of performing additional tasks [1]. Collaborative products have the potential to significantly influence the impact that income-generating products can have on poverty alleviation efforts by reducing the cost of a set of products capable of performing a specified set of tasks. This is accomplished by increasing the task-per-cost ratio of a set of products [1] so as to reduce the number of products needed to perform a set of tasks. It is this ability to perform a set of tasks with fewer products that effectively lowers the financial risk for the user and increases his or her likelihood of purchasing these products.

Aside from the developing world context, collaborative products can also apply in the *developed* world. Many individuals within the United States suffer from poverty, living in small dwellings with limited storage space [16]. Money is also limited for these individuals, and collaborative products are a way to help maximize available storage space while providing a set of product functions that are extremely affordable [1].

Other identified areas that could benefit from collaborative products in the developed world may include payload conscious industries [1]. Designing for these industries is unlike designing for the developing world since cost is not as large of a factor. A more important factor to consider would be a task-per-weight ratio to ensure weight is minimized as the number of tasks increases. The aerospace and backpacking industries are two examples where this ratio would be applicable.

Morrise et al. have developed a method for designing collaborative products, consisting of an eight-step process [1]. One simple example of a collaborative product developed by Morrise et al. using the eight-step process is a collaborative block plane [1]. This product is composed of a chisel and sanding block, which when combined together create the collaborative block plane. The combined state of the product is shown in Figure 1.1. This example is beneficial as it demonstrates an increased number of tasks at a lower cost. To further illustrate this, Morrise et al. completed a cost analysis of the collaborative block plane. For a comparable chisel, sanding block, and block plane the cost was approximately \$12, \$7, and \$22 respectively, resulting in a total cost of \$41. In comparison, the redesigned collaborative block plane cost approximately \$23, illustrating an increased task-per-cost ratio of 44%, demonstrating an increased benefit to individuals living in the developing world [1].

While the method presented by Morrise et. al serves as a basic foundation to the design of collaborative products, it leaves certain areas unexplored [1]. These areas include an ability to: (1) optimize the design of both the individual products *and* the collaborative product simultaneously, (2) seek to understand customer needs, (3) allow the designer to be more creative, and (4) minimize the number of product decompositions throughout the design process. In order to explore these areas and others, a new method has been developed which is presented in Chapters 2 and 3.

1.4 Multiobjective Optimization

The method presented in this thesis for designing collaborative products also involves many changing and competing needs that must be addressed to successfully design the product. One way to meet these demands and resolve the competing nature of both present and future needs

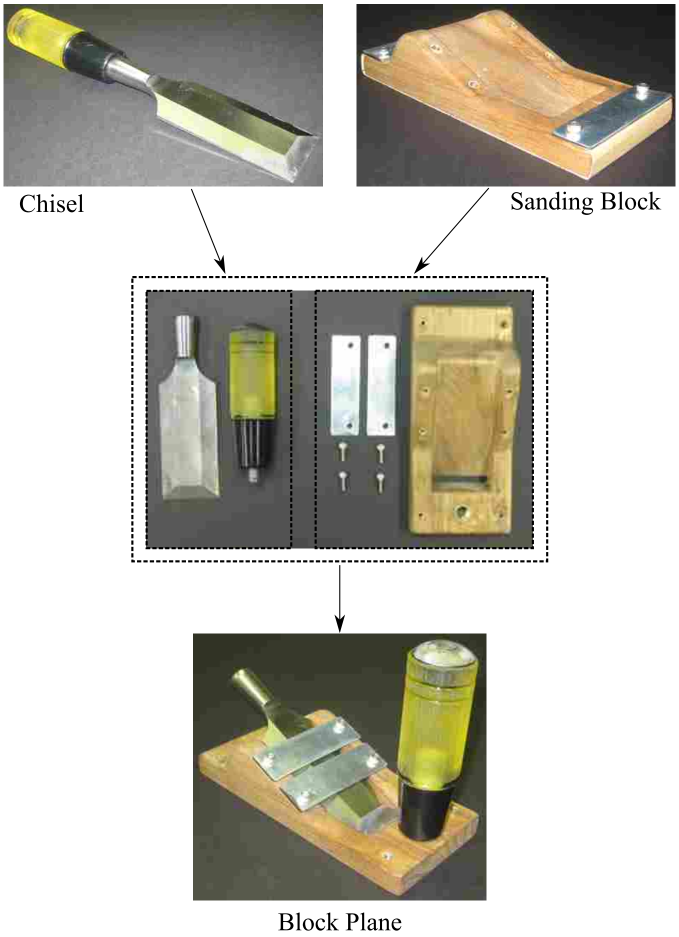


Figure 1.1: Collaborative block plane as presented by Morrise et al. [1]

of a set of products is through multiobjective optimization [17–19]. This technique serves as a fundamental foundation to the design method presented in this thesis. Multiobjective optimization characterizes the trade-offs between design objectives by identifying a Pareto frontier or a set of solutions understood to be *Pareto optimal* or *dominant* [20–22]. These Pareto solutions are of importance because they show that design objectives have been improved to their full potential without sacrificing the performance of objectives in other areas [5,17–19,23]. This important class of solutions was first identified by Edgeworth [24] with other early work in this area by Pareto [25] and Koopmans [20,26]

In order to define a Pareto solution two types of classes should be considered. They are Integrated Generating Choosing (IGC) [18] and Generate First-Choose Later (GFCL) [27–29]. Within IGC there are several approaches which are as follows: the Weighted Sum Method [30,31], the Compromise Programming Method [30], and the Physical Programming Method [32]. Within GFCL there are also many approaches which consist of: the Weighted Sum and Compromise Programming Approach [30, 31], the ε-Inequality Constraint method [33], the Normal Boundary Intersection Method [34], the Physical Programming Approach [18], the Normalized Normal Constraint Method [35, 36], and the Genetic Algorithm [37]. Each of these methods has a different approach used to formulate the multiobjective optimization problem. They also have different levels of effectiveness in being able to navigate the design space [32].

A general multiobjective optimization problem is composed of objectives, design variables, design parameters, and constraints [38]. The objectives take into account the trade-offs that will be addressed within the optimization problem and represent items that the designer desires to capture within a design (i.e., weight, cost, strength, performance). The objectives are composed of two elements, design variables and design parameters. The design variables define the design of a product (i.e., length, width, height) and are allowed to vary in order to meet the objectives desired in the optimization. The design parameters are treated as constants (i.e., material yield strength, modulus of elasticity) and are fixed values within the optimization problem. Constraints allow the designer to set bounds on a design to identify the design space of a selected product [38, 39].

A set of optimal solutions belonging to a Pareto frontier can be found through the following generic multiobjective optimization problem presented as *Problem 1* (P1):

$$\min_{x} \left\{\mu_1(x,p),\ \mu_2(x,p),\ \ldots,\ \mu_{n_\mu}(x,p)\right\} \quad (n_\mu \geq 2) \tag{1.1}$$

subject to:

$$g_q(x,p) \leq 0 \qquad \forall\, q \in \{1,\ldots,n_g\} \tag{1.2}$$

$$h_k(x,p) = 0 \qquad \forall\, k \in \{1,\ldots,n_h\} \tag{1.3}$$

$$x_{jl} \leq x_j \leq x_{ju} \qquad \forall\, j \in \{1,\ldots,n_x\} \tag{1.4}$$

where μ_i denotes the i-th generic design objective to be minimized; x is a vector of design variables; p is a vector of design parameters; x_u and x_l define the upper and lower bounds of the j-th design variable; g is a set of inequality constraints; and h is a set of equality constraints. Note that the objectives and constraints are functions of both x and p, and that the objectives will be minimized by changing the values of x.

Through the use of a multiobjective optimization problem, the designer can identify a feasible design space with a corresponding Pareto frontier. This allows the designer to manage the complex trade-offs and meet the desired objectives of a design which are in terms of the chosen design variables and parameters.

1.5 Chapter Summary

In this chapter the concerns of poverty alleviation have been discussed along with many of the current ways to address this issue. The literature suggests engineering-based poverty alleviation methods which serve as a foundation to the research discussed in this thesis. Such methods include modular, reconfigurable, and collaborative product design approaches. Multiobjective optimization theory has been presented as a way to address the many changing and competing needs often found in these product design approaches. These methods and theory function as a means to lower financial risk and bring individuals out of extreme poverty.

The remainder of this thesis is organized as follows: The theory for designing products for optimal individual and collaborative performance is found in Chapter 2. In Chapter 3, the design of a collaborative brick press demonstrates implementation of the presented method, followed by concluding remarks in Chapter 4.

CHAPTER 2. METHOD OF DESIGNING PRODUCTS FOR OPTIMAL COLLABORATIVE PERFORMANCE

This chapter presents a method that seeks to understand customer needs and meet them through the use of individual and collaborative products. The method consists of a nine-step process which can be abbreviated as follows: (1) Understand customer needs, (2) Identify a product that satisfies a need, (3) Decompose the identified product, (4) Use the decomposed components to satisfy additional needs, (5) Identify the product interfaces (6) Characterize the collaborative design space, (7) Define the areas of Pareto offset, (8) Identify the designs that fall within the offset areas, and (9) Identify the optimal product designs. It should be noted that Steps 1-5 serve as a means to prepare a collaborative product and its components for multiobjective optimization. Once the initial design work of the products has been carried out, the designer will then use the remaining Steps 6-9 to apply multiobjective optimization to the products in order to meet the desired objectives.

2.1 Step 1: Understand Broad Customer Needs

The method begins by understanding many broad customer needs that may exist in a society. One way to do this is to understand the culture of the society. This involves the study of groups and people as they go about their everyday lives. Research is carried out by immersing oneself in the culture and gathering information from individuals and potential customers of that society [40]. Other traditional methods used to gather this information include interviews, surveys, and observations [41,42]. By using one or multiple of these methods the designer is able to gather statements from the customer and translate them into customer needs. It is essential to have a clear understanding of the customer needs to determine how to best meet them.

One way to focus the efforts of gathering customer needs is to select and work within a need category. Examples of categories when designing for the developing world might include:

farming, hunting, tools, education, housing, cooking, health care, transportation, etc. The goal is to find an area that would benefit from a task-to-cost ratio increase. For individuals in the developing world, the financial risk is lowered as this ratio increases. If products can be combined to complete a greater number of valuable tasks, the user will benefit from a lower cost. The end result of completing this step is to come to know the customer on a deeper level in order to gain an understanding of what could be done to benefit their lives.

2.2 Step 2: Create/Select a Product that Satisfies One of the Broad Needs

After the customer needs have been sufficiently understood, the designer identifies a product that satisfies one of those needs. It can be a product that already exists in a society or one that is to be developed. Many design processes exist for creating new products, one of which consists of a five-step process [41]. The steps of this method are: explore, ideate and select, engineer, test and refine, and production ramp-up.

The explore step encompasses a wide range of activities including understanding the customer needs from step 1 and defining the problem to be solved. The ideate and select step allows the designer to formulate new ideas based upon customer needs, evaluate those ideas, narrow them down, and ultimately select the most promising concept for further development. During the engineering of an idea, detail design commences. The selected concept is proven from an engineering design standpoint by defining part geometry, material type, and manufacturing steps. The selected design is then tested for weaknesses and refined as necessary. Design changes are implemented as needed to ensure the product satisfies the key customer needs. Production ramp-up will likely take place at the end of the collaborative design process, rather than at this point in the method. It is a crucial step, but should be considered when all details of the collaborative product design have been established.

The resulting product from Step 2, whether newly designed or already existing, will serve as the starting point to the creation of a collaborative product. This product typically will have the following qualities: be comprised of multiple if not many components; is desirable but generally not purchased by a customer due to its high cost, weight, or size; and is generally used less frequently than typical, everyday products since the components of the product are unusable while reconfigured into the collaborative product state [1].

2.3 Step 3: Decompose the Selected Product into Components

Step 3 requires the designer to decompose the selected product into its individual components. This step is necessary to begin learning about what products will make up the collaborative product and be able to satisfy additional customer needs. Generally, the selected product is decomposed only into the components required to perform an intended function. In other words, the decomposition will not include secondary components such as fasteners [1].

This type of product is decomposed three ways: structurally, functionally, and by physical characteristics. From a structural standpoint, the product is decomposed where the resulting components make up the primary structure of the product. Functionally, the product is decomposed by identifying the primary function of each component identified in structural decomposition. Lastly, decomposition by physical characteristics is completed by identifying the relevant characteristics such as size, shape, and color of each component identified during structural decomposition.

An example of a bicycle wheel decomposition, provided by Morrise et al., helps to illustrate the decomposition process [1]. This example demonstrates the need for three types of decomposition and how each type brings clarity to the collaborative design process. See Figure 2.1 for the bicycle wheel decomposition based upon structural, functional, and physical characteristics. If only structural decomposition was carried out, then a bicycle wheel would be viewed based on its structure alone. In other words, a bicycle wheel would only relate to other wheels and would not have any known relationship based on function. Decomposition to this extent allows the designer to better understand the components and characteristics that a selected product contains.

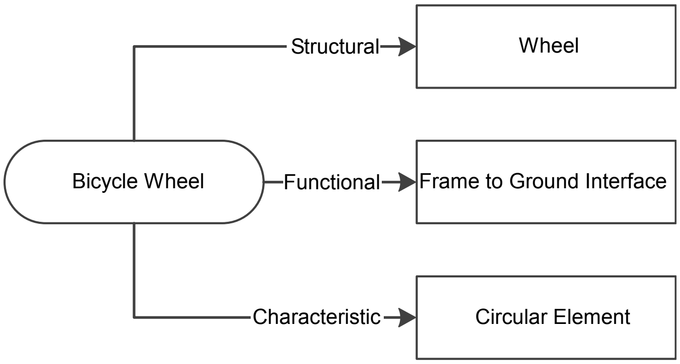


Figure 2.1: Bicycle wheel decomposition as presented by Morrise et al. [1]

2.4 Step 4: Determine What Other Products can be Created from the Components to Meet Different Broad Customer Needs, while if desired, Adding Missing Secondary Components

In this step, additional broad customer needs are met by determining what other products can be made from the decomposed product components. Needs are considered and thought is given to each decomposed product to determine how each need can best be met. The designer must be cautious of products that may require concurrent use since the collaborative product will require use of its components to function. Therefore, it may be best to select products that meet needs in different categories, activities, or seasons to prevent concurrent use. The designer may also desire to add secondary components to complete a design.

2.5 Step 5: Identify the Interfaces Between Components

Once all products have been chosen and the most important needs have been met, the designer must identify the interfaces between components. The addition of interfaces to the product may introduce weaknesses. However, it is because of these interfaces that the task per cost ratio is able to increase. As was stated in Sec. 1, this ratio is important to individuals in the developing world, as it defines the number of tasks a product can perform based on its cost. The higher this ratio is, the lower the financial risk will be for the end user. These interfaces are crucial to the functionality and reliability of the collaborative product as well as the safety of the user. They will determine how positive the user experience is and its usefulness as a collaborative product. A detailed process for designing interfaces will not be discussed in this thesis since sufficient methods already exist in the literature [43, 44].

2.6 Step 6: Characterize the Collaborative Design Space of the Product Set and Collaborative Product

This step begins with a knowledge of the product set and corresponding collaborative product that is desired. The feasible design space for each product is defined along with the corresponding Pareto frontiers (see Figure 2.2). Each feasible design space will consist of dozens, if not hundred of feasible designs. In selecting the design of a product that will be reconfigured for use in a collaborative product, the impact of design changes on the performance of the individual and collaborative products must be considered. In the context of multiobjective optimization, product performance is typically correlated to design objective values for each product. Recall that points along the Pareto frontier (graphically illustrated in Figure 2.2) represent the best possible trade-offs between the selected design objectives of each product. Although a design is located on the Pareto frontier of an individual product, the corresponding performance of the collaborative product, and the other products in the set, are not guaranteed to be Pareto optimal in each product's objective space. As such, the collaborative performance of a product is inversely correlated to the measured offset of a design from the corresponding Pareto frontier of that product. Building on this definition, the optimal collaborative performance of a product set is therefore obtained by maximizing the collaborative performance of each product simultaneously.

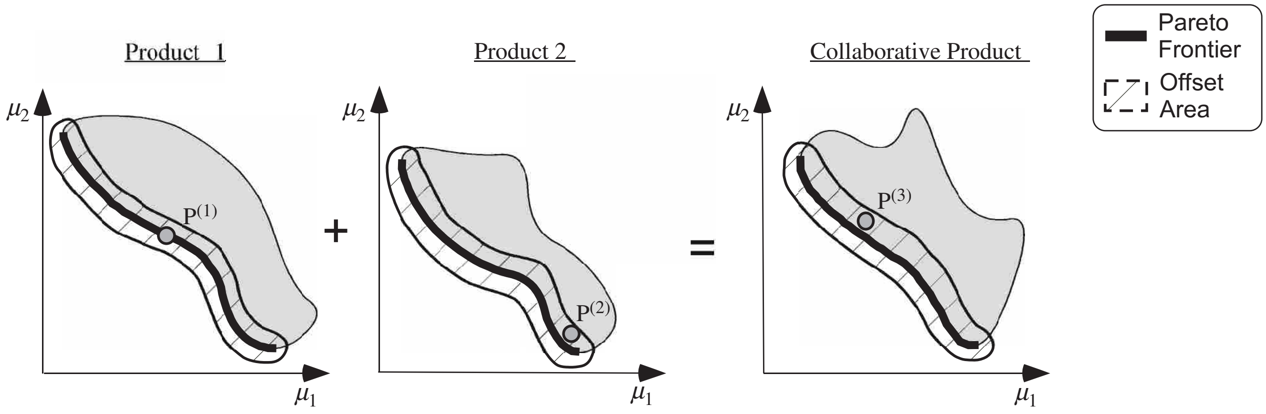


Figure 2.2: Graphical summary of the intent of the method presented in Chapter 2, illustrating the feasible bi-objective design spaces for a theoretical product set and corresponding collaborative product. The Pareto frontier (bold line) defines the most desirable set of solutions in each design space. The designs selected for each product are identified as points $P^{(1)}$, $P^{(2)}$, $P^{(3)}$. Note that the selected designs are within identified areas of acceptable Pareto offset.

Recognizing the inherent trade-offs and compromises in collaborative performance that must be explored, the purpose of Steps 6-9 is to implement an optimization-based approach to mitigating these trade-offs. Figure 2.2 graphically represents the intent of balancing these trade-offs using the method presented in this section for two products that are combined to create a third product. Although the presented method is not limited to the simple case presented in Figure 2.2, a limited number of products are used for simplicity of visualization purposes. From Figure 2.2 it can be observed that the presented optimization routines select designs for each product that fall within identified offset areas within each objective space. In order to enable the use of optimization methods to explore possible design solutions, objectives for each of the products in the set and the collaborative product are identified, and models of these objectives are created that incorporate the intended product interfaces. Using the developed models, the design space of each product is determined by a multiobjective optimization problem similar to (P1).

The optimization was carried out using the program, *Matlab*, which employed the active-set algorithm. This algorithm used an iterative method to solve a sequence of equality-constrained quadratic subproblems. The goal of the algorithm was to predict the active set or the set of constraints that are satisfied with equality for the solution of the problem [45].

To define each product and identify the variables that couple the design of each product in the set to the collaborative product, the design variables for each product are divided into three groups: interface variables (x_{I}), collaborative variables (x_{C}) and unshared (x_{U}) variables. The interface or platform variables are shared throughout the product set and define the connecting interface between each product. The collaborative variables are those connected to the elements of a product that are used to create the collaborative product. The unshared or unique variables are those connected to the elements of a product that are unique to each product in the product set. The characterization of the multiobjective design space for the *i*-th product in the set, and the collaborative product ($i = n_p + 1$), in terms of identifying the corresponding Pareto frontier (See Figure 2.2) is presented as *Problem 2* (P2):

$$\min_{\hat{x}^{(i)}} \left\{ \mu_1^{(i)}(\hat{x}^{(i)}, p^{(i)}),\ \ldots,\ \mu_{n_\mu^{(i)}}^{(i)}(\hat{x}^{(i)}, p^{(i)}) \right\} \quad (n_\mu^{(i)} \geq 2) \tag{2.1}$$

subject to:

$$g_{q^{(i)}}^{(i)}(\hat{x}^{(i)}, p^{(i)}) \leq 0 \qquad \forall\, q^{(i)} \in \{1, \ldots, n_g^{(i)}\} \tag{2.2}$$

$$h_{k^{(i)}}^{(i)}(\hat{x}^{(i)}, p^{(i)}) = 0 \qquad \forall\, k^{(i)} \in \{1, \ldots, n_h^{(i)}\} \tag{2.3}$$

$$\hat{x}_{jl}^{(i)} \leq \hat{x}_j^{(i)} \leq \hat{x}_{ju}^{(i)} \qquad \forall\, j \in \{1, \ldots, n_{\hat{x}}^{(i)}\} \tag{2.4}$$

$$\begin{aligned} \hat{x}^{(i)} = \ & \Big[x_{\mathrm{I},1},\ x_{\mathrm{I},2},\ \ldots,\ x_{\mathrm{I},n_{x_I}},\ x_{\mathrm{C},1}^{(i)},\ x_{\mathrm{C},2}^{(i)}, \\ & \quad \ldots,\ x_{\mathrm{C},n_{x_{\mathrm{C}}}^{(i)}}^{(i)},\ x_{\mathrm{U},1}^{(i)},\ x_{\mathrm{U},2}^{(i)},\ \ldots,\ x_{\mathrm{U},n_{x_{\mathrm{U}}}^{(i)}}^{(i)} \Big] \end{aligned} \tag{2.5}$$

$$x_{\mathrm{C}}^{(n_p+1)} = \left[x_{\mathrm{C},1}^{(i)},\ x_{\mathrm{C},2}^{(i)},\ \dots,\ x_{\mathrm{C},n_{x_\mathrm{C}}^{(i)}}^{(i)}\right]\ \forall i \in \{1,2,\dots,n_p\} \tag{2.6}$$

$$\hat{x}^{(n_p+1)} = \left[x_{\mathrm{U},1}^{(i)},\ x_{\mathrm{U},2}^{(i)},\ \dots,\ x_{\mathrm{U},n_{x_\mathrm{U}}^{(i)}}^{(i)}\right]\ \forall i \in \{1,2,\dots,n_p\} \tag{2.7}$$

where $\hat{x}^{(i)}$ is a vector of design variables containing the interface (x_I), collaborative (x_C), and unshared (x_U) variables for the i-th product in the set. The design parameters are also represented for the i-th product in the set by the term $p^{(i)}$. The Pareto frontier of each product is obtained by evaluating (P2) $\forall i \in \{1,2,\dots,n_p+1\}$.

It should be noted that in Eq. 2.5, the collaborative variables of the collaborative product ($i = n_p + 1$) encompass all collaborative variables from the identified product set. This coupling of the product set to the collaborative product design space is important since it illustrates to the designer the current collaborative nature of the product set.

2.7 Step 7: Define the Areas of Acceptable Pareto Offset

In looking at the formulation of (P2), the resulting Pareto frontier for each product represents the best possible solutions for each of the products without considering the interaction between each product. As the number of products being combined increases, it becomes less likely that the designs capable of creating a collaborative product all fall on the Pareto frontier of the corresponding product. This is because the number of objectives and constraints to be satisfied, along with the complexities of the interactions between the products, increases with each additional product. As more interactions and trade-offs become apparent, the harder it is to meet all of the demands between products. In order to facilitate the selection of designs that will minimize the offset from these Pareto frontiers of the entire product set, the next step in the method is to use these Pareto frontiers to define areas of acceptable Pareto offset for each product (see Figure 2.2).

Due to the large number of trade-offs that can exist between products this step is an essential part of the method to enable a designer to capture designs that may still be of interest. Even though designs will be captured that do not lie directly on the Pareto frontier, they are still important to note in order to better satisfy the chosen objectives of the multiobjective optimization problem.

This process is carried out by defining a single offset value (β) for each product that will limit subsequent optimization routines to only consider designs with offsets from the Pareto frontier that are less than β. In the case of a two-dimensional model, the values of β would be equivalent to defining a circle of radius β around each identified Pareto point from Step 1. In n-dimensional cases, the value of β represents the maximum allowable length of an n-dimensional vector between a design option and the closest Pareto point.

The β value is determined by the designer based upon the extent to which he or she wishes to limit the search space and focus optimization searches to the identified offset areas. This may require a trial and error approach where the designer will select a value, evaluate the multiobjective optimization problem, and evaluate the results to determine if a desired solution was reached. If a solution that satisfied the customer needs was not found, the designer may wish to increase or decrease the value of β and re-evaluate the optimization. As the designer increases or decreases the value of β, he or she will capture additional or fewer designs within the Pareto offset areas. This routine should be experimented with until a satisfactory design has been reached.

2.8 Step 8: Identify the Designs that Collaboratively Fall Within the Areas of Acceptable Pareto Offset

In order to identify these designs, a multi-dimensional design space is created using axes represented by the predicted Pareto offsets for each product in the set as well as the collaborative product. This design space represents a combination of feasible designs in terms of the individual products and the collaborative product. Each design is defined by calculating a value called an offset distance. This distance is the length from each design to the nearest point on the Pareto frontier. There will exist an offset distance value for every design found within the offset areas. Every design within the multi-dimensional offset design space is comprised of designs from each product in the product set and collaborative product. The multi-dimensional design space is composed of these new designs.

In the case illustrated in Figure 2.2, these offset points would be plotted in a three-dimensional area where each axis represents the offset values of that specific product. For example, Product 1 would correspond to offset axis 1 (O_1), Product 2 would correspond to offset axis 2 (O_2), etc. It is important to note that in Figure 2.2, only one design within each offset area of the many hundreds of potential designs has been represented ($P^{(i)}$). This is for simplicity of visualization purposes only. For an illustration of the three-dimensional plot containing all offset points, see Figure 2.3.

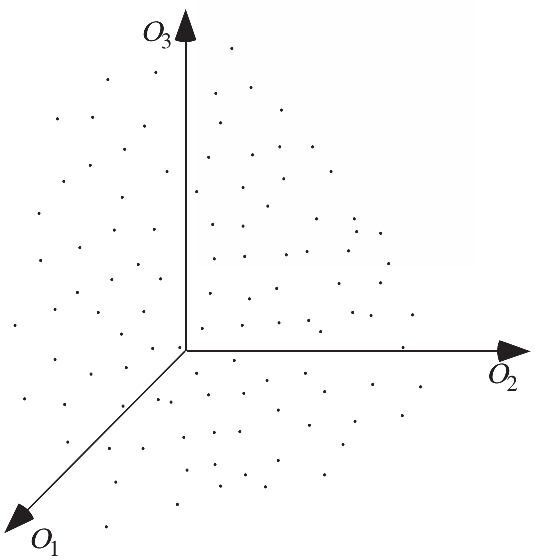


Figure 2.3: Graphical illustration of the offset points corresponding to Figure 2.2

From the offset points a three-dimensional Pareto surface will be constructed through an additional multiobjective optimization problem. This problem statement is presented as *Problem 3*(P3):

$$\min_{\hat{x}} \left\{ O^{(1)},\ O^{(2)},\ ...,\ O^{(n_p+1)} \right\} \tag{2.8}$$

subject to Equations 2.2–2.5 and:

$$O^{(i)}_{q^{(i)}} \leq \beta \qquad \forall\, q^{(i)} \in \{1,...,n_g^{(i)}\} \tag{2.9}$$

where $O^{(i)}$ is the n-dimensional offset length of a design of the i-th product from the corresponding Pareto frontier of that product. This additional constraint ensures that only the designs located within the offset areas will be identified since the offset distances must be less than or equal to β for each product.

The Pareto surface is constructed by adjusting the interface, collaborative, and adjustable variables. The interface and collaborative variables are shared between the optimized products and the collaborative product, while the adjustable variables are unique to each optimized product, but shared with the collaborative product. It should be noted that in cases were there are no more than two products being combined to create a collaborative product, the result of *(P3)* is a Pareto surface. For product sets greater than two, the graphical representation of this offset space can no longer be provided for all products simultaneously. Fortunately, a graphical representation is not necessary for this method to be useful.

2.9 Step 9: Identify/Select the Optimal Product Designs

Since the goal of the method is to select the optimal design of each product while balancing the trade-offs required to create the collaborative product, this final step of the method uses the results of (P3) to select a single set of product designs. Under ideal circumstances, the selected designs are represented by a single Pareto point on the Pareto frontier of each product (i.e., the offset of each product is zero). One method of accomplishing this selection is through the use of an aggregate objective function (J) that represents the preferences and needs of the designer. If an aggregate objective function is used, one way of reducing the computation expenses related to the optimization problem evaluations, would be to replace Eq. 2.8 with an equation of the form of Eq. 2.10.

$$\min_{\hat{x}} J(O^{(1)},\ O^{(2)},\ ...,\ O^{(n_p+1)}) \tag{2.10}$$

This aggregate objection function uses a weighted sum method to evaluate and select the final design. The weights within the function are determined by the designer depending on what objectives must be met to satisfy the customer needs.

At the conclusion of the design process presented in Chapter 2, the designer will have an understanding of the customer needs and a way to meet those needs with individual products and a collaborative product. Through the multiobjective optimization theory presented in Steps 6-9, the designer is able to simultaneously and numerically evaluate the performance of multiple designs in multiple design spaces. This allows the designer to optimize the products to ensure they operate efficiently in both the individual and collaborative product states to effectively lower the financial risk for the end user.

CHAPTER 3. CASE STUDY: COLLABORATIVE BRICK PRESS DESIGN

This section demonstrates the implementation of the method presented in Chapter 2 through the design of a collaborative brick press. The concept for a collaborative brick press has been provided by Morrise et al [1]. This design collaboratively uses the following six basic products to create the brick press: shovel, hoe, rake, water transportation roller, water pump, and a small cook stove. It is assumed these are potential products that a person living in poverty would be interested in purchasing as a way to improve his or her life situation. The ability to combine them together into an additional product would give individuals the potential to maximize their use and potentially increase their likelihood of purchasing these products. It should be noted that the intent of this example is not to show the feasibility and necessary logistics of implementing the collaborative brick press developed herein. Rather, the intent is to demonstrate the effectiveness of the method presented in Chapter 2 in identifying the optimal designs of a given collaborative product set.

The case study is useful in illustrating this method due to the following: (i) it solves a challenging engineering design problem, (ii) it shows the use of complex interfaces between products and how they are addressed, (iii) it incorporates the use of actual products used or found in developing countries, and (iv) it demonstrates the use of a multiobjective optimization problem to deal with competing objectives from each product. Figures 3.1 and 3.2 illustrate the conceptual design of each product and the interfaces required to create the collaborative brick press.

3.1 Case Study Step 1: Understand Broad Customer Needs

We begin this case study by understanding many broad customer needs. Since the nature of this thesis has strong application to individuals in the developing world we will understand needs related to this specific area. For this example, the selected customer needs include: cooking, home building, gathering food, transportation, and access to clean water. These needs were gathered

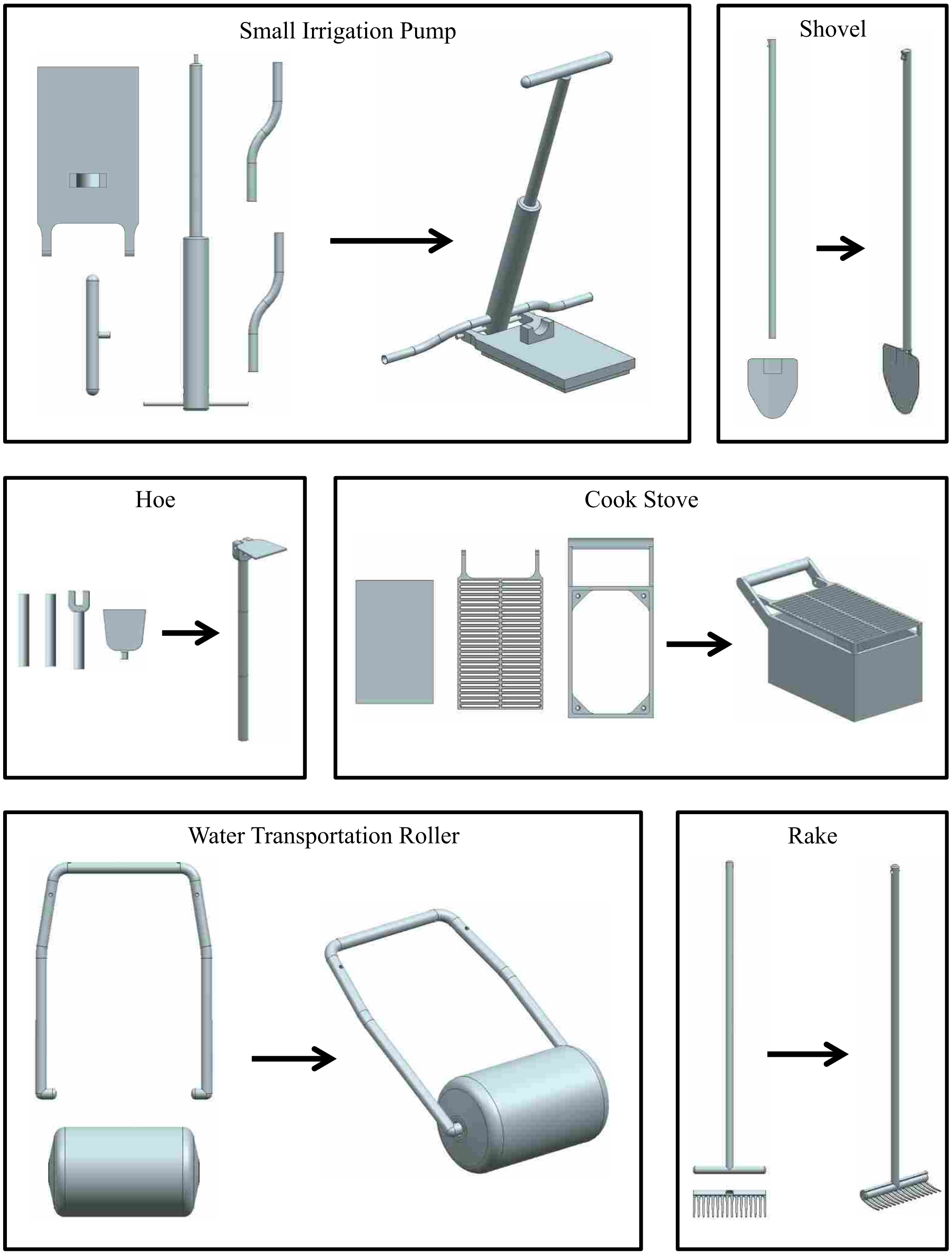


Figure 3.1: Decomposition of each product in the identified product set to create a brick press.

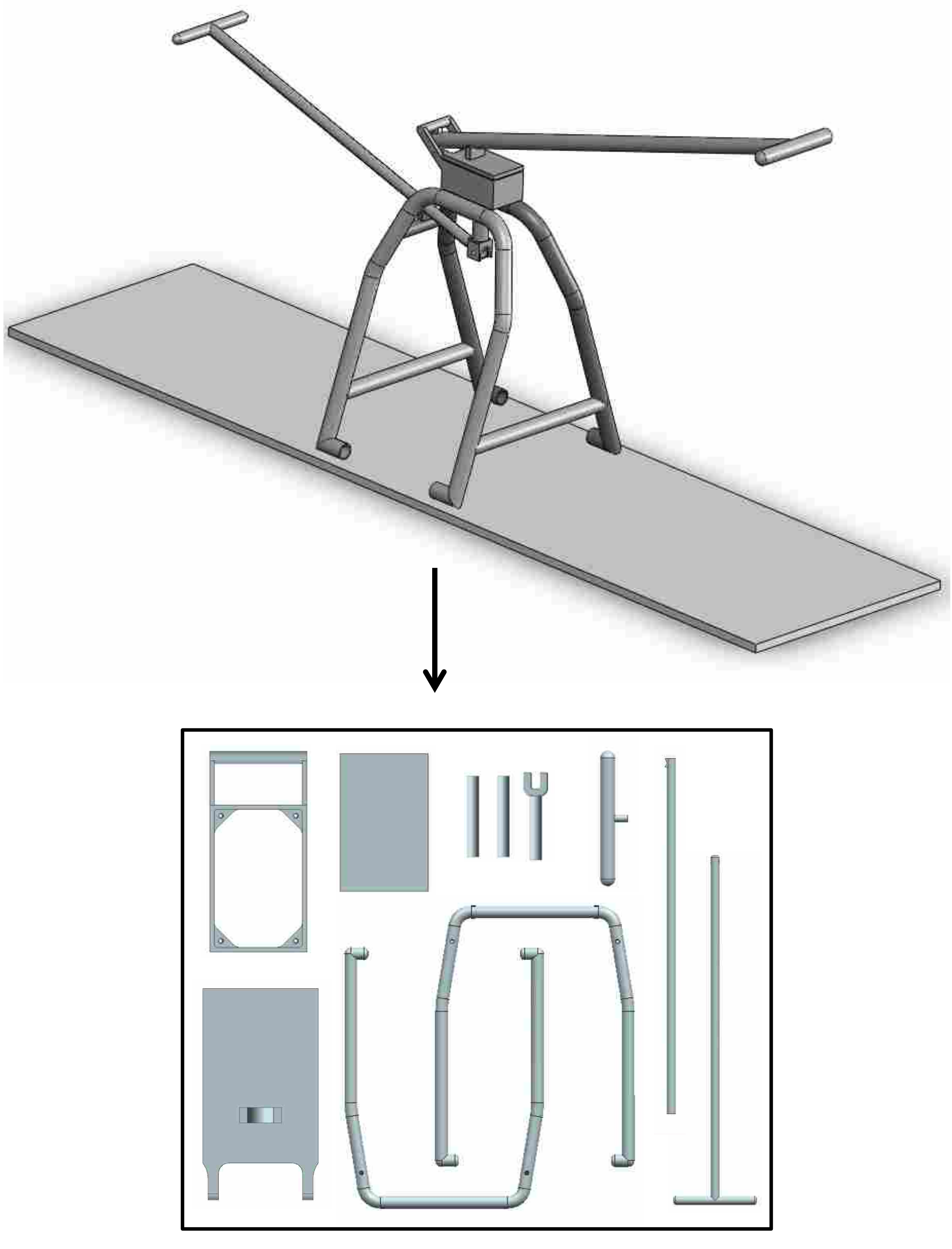


Figure 3.2: Illustration of the recombination of the components from the product set in Figure 3.1

through culture research of specific countries and observations and interviews of individuals who have lived abroad. Understanding these needs as a designer and knowing how to address them will benefit the lives of individuals in the developing world.

3.2 Case Study Step 2: Create/Select a Product that Satisfies One of the Broad Needs

In Step 1 a list of broad customer needs was gathered. From that list we will focus on the need of home building and select an existing brick press product able to meet the customer need. Brick presses are commonly found in the developing world as an effective way to construct sturdy, long-lasting homes. A brick press serves as an ideal collaborative product candidate since it contains a large number of components, is desirable but typically not purchased due to its high cost, and is used less frequently than other typical, everyday products.

3.3 Case Study Step 3: Decompose the Selected Product into Components

After the brick press has been selected as the product to satisfy the customer need, a decomposition process is carried out to determine the component make-up of the brick press. As is presented in Section 2.3, the product will be decomposed by structure, function, and characteristics. See Table 3.1 for the completed decomposition of the brick press.

Table 3.1: Brick press decomposition

Component	Structural	Functional	Characteristic
Press Mold	Mold	Hold Material	Rectangular Basin
Legs	Support Legs	Press to ground interface	Cylindrical Tubes
Long Posts	Long Handles	Leverage	Cylindrical Tubes
Handles	Short Handles	Human to press interface	Cylindrical Tubes
Mold Cover	Cover	Pressure Plate	Rectangular Plate
Eject Plate	Plate	Brick Ejector	Rectangular Plate

The decomposition allows the designer to easily see the make-up of the selected product and begin identifying components that can solve different broad customer needs.

Table 3.2: Other products created to fulfill different customer needs

Need	Component(s)	Product	Secondary Component(s)
Cooking	Press Mold, Eject Plate	Cook Stove	Cook Surface
Water Transportation	Legs	Water Roller	2 Water Barrels
Fresh Water	Mold Cover	Water Pump Base	Pump, Hoses
Farming	Long Post 1	Shovel	Blade
Farming	Long Post 2	Rake	Tines
Farming	Handles	Hoe	Blade

3.4 Case Study Step 4: Determine What Other Products can be Created from the Components to Fulfill Different Broad Customer Needs, while if desired, Adding Missing Secondary Components

During this step we review the other broad customer needs that were identified in Section 3.1. This is done to begin determining what other products can be created from the components to fulfill these needs. For this case study we will identify the components that make up the brick press and determine how these components might fulfill different broad customer needs. The identified needs and their corresponding products used to fulfill each need can be found in Table 3.2. Also note that necessary secondary components were added to complete the design of each product in the table.

3.5 Case Study Step 5: Identify Interfaces Between Components

To complete the collaborative design process, interfaces are added to ensure complete usability of the products. The brick press will experience large forces during operation and will therefore require interfaces that will ensure a robust design. It is important to identify interfaces that allow high functionality of the brick press in its collaborative state as well as in its individual state, but also achieve the lowest possible cost. As was stated in Chapter 2, these interface design methods exist in the literature [43, 44].

3.6 Case Study Step 6: Characterize the Collaborative Design Space of the Product Set and Collaborative Product

Once the collaborative product has been sufficiently developed, the designer next characterizes the collaborative design space of the six basic products as discussed in Step 6 of the presented

Table 3.3: Summary of the objectives that were selected for each product in the product set and collaborative product

	↑ / ↓	μ_1	↑ / ↓	μ_2
Shovel	↓	Stress (psi)	↓	Cost ($)
Rake	↓	Stress (psi)	↓	Cost ($)
Hoe	↓	Stress (psi)	↓	Cost ($)
Water Roller	↓	Stress (psi)	↓	Cost ($)
Cook Stove	↑	Cook Area (in^2)	↓	Cost ($)
Water Pump	↑	Flow Rate (L/s)	↓	Cost ($)
Brick Press	↓	Stress (psi)	↓	Cost ($)

method (see Section 2.6). This is carried out by constructing mathematical models of each product in the product set. In the case of our example, models were developed for each product, and are summarized in Appendix A. It is important to construct robust models that accurately represent each product to ensure that they hold up to the optimization under realistic conditions. Table 3.3 summarizes the objectives (↑ = maximize, ↓ = minimize) that were selected to characterize the performance of each product. Definitions of the objectives presented in Table 3.3 are as follows: (i) for the shovel, rake, and hoe the objective μ_1 represents the maximum bending stress in the product's handle; (ii) for the water roller and brick press, μ_1 represents the maximum bending, shear, and buckling stress that each product could experience; (iii) for the cook stove, μ_1 represents the available area for cooking food; (iv) for the water pump, μ_1 represents the rate at which the pump can pump water; and (v) the objective μ_2 represents the cost to purchase each product.

From the models and their corresponding functions, design variables, and design objectives we are able to construct a multiobjective optimization problem of the form of *(P2)* in Section 2.1. From this optimization problem, the design spaces for each product are defined with their corresponding Pareto frontiers (See Figure 3.3).

3.7 Case Study Step 7: Define the Areas of Acceptable Pareto Offset

In step 7 of the proposed method, the areas of acceptable Pareto offset will be defined. Since we have only two objectives for each product in the product set and collaborative product, the values of β are equivalent to defining a circle of radius β around each identified Pareto point from Step 1. For these two-dimensional cases, the value of β represents the maximum allowable length of a two-dimensional vector between a design option and the closest Pareto point. For our example, the β offset values were defined as shown in Table 3.4 for each product.

Table 3.4: Defined acceptable offset values (β) for the normalized objectives of each product

	β Value
Shovel	0.1
Rake	0.1
Hoe	0.1
Water Roller	0.1
Cook Stove	0.1
Water Pump	0.1
Brick Press	0.1

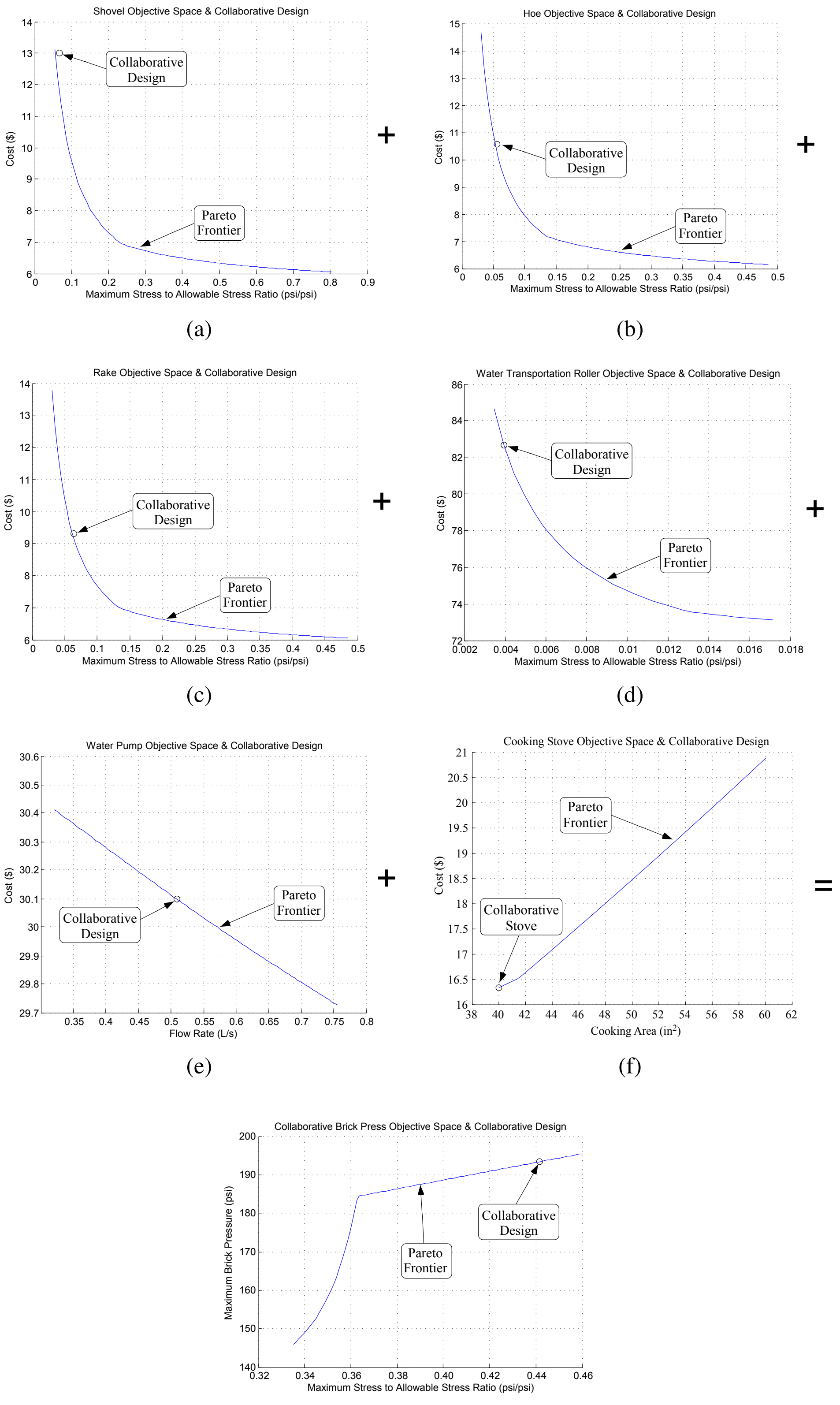
3.8 Case Study Step 8: Identify the Designs that Collaboratively Fall Within the Areas of Acceptable Pareto Offset

Once the offset areas were defined, the combinations of designs that fall in each offset area were identified using a multiobjective problem statement of the form of *(P3)* (see Section 2.3). Due to the number of products involved in this case study, a graphical representation of the results of evaluating this formulation is not meaningful, and is therefore not provided.

3.9 Case Study Step 9: Identify/Select the Optimal Product Designs

As was mentioned in Chapter 2, an aggregate objective function was used to select the optimal combination of product designs. For this example we used a weighted sum of offsets with all weights equal to one except for the brick press, which was equal to 10. The weights were selected with the goal of minimizing the offset of the collaborative product (brick press) from the corresponding Pareto frontier. The resulting design selection using these weights is illustrated in Figure 3.3.

From the results presented in Figure. 3.3 it can be observed that the identified design for each product is located on the Pareto frontier of the corresponding product objective space. Although the selected aggregate objective function and weights were successful in identifying de-



(a) (b) (c) (d) (e) (f) (g)

Figure 3.3: Graphical illustration of the Pareto frontiers for each product obtained through Step 6 of the method, and the optimal collaborative design of each product identified in Step 9 of the method.

signs on or near the Pareto frontier of each product, the majority of these designs are located near the boundaries of the Pareto frontiers. If solutions are more desirable in a particular region of the identified Pareto frontiers, additional constraints or alternative aggregate objective functions would need to be explored.

One of the benefits of the presented method is its ability to search through a complex design space. Under the presented method, many products are being simultaneously optimized not only on an individual level, but also on a collaborative level. Through the optimization, the collaborative performance is optimized while dealing with the various trade-offs between the products and the collaborative product.

From the case study, it can be illustrated that the task-per-cost ratio of the collaborative brick press has increased. More specifically, and assuming that the calculated total cost of all components making up the newly designed brick press are $160 and are capable of completing seven different tasks, the ratio will be 0.043. For comparison, a comparable brick press, cook stove, small irrigation pump, shovel, rake, hoe, and water transportation rollers approximately cost a total of $200 with a ratio of 0.030. This illustrates that the task-per-cost ratio has improved by 30% from 0.030 to 0.043 through the use of this method [1].

CHAPTER 4. CONCLUDING REMARKS

This thesis has presented a method for designing products for optimal collaborative performance with application to engineering-based poverty alleviation. The method builds upon and strengthens the existing process by Morrise et. al [1]. The new method seeks to accomplish several key points which will be discussed.

First, and perhaps the greatest contribution of this thesis to the design of collaborative products is the ability of the method to simultaneously optimize multiple products both individually and collaboratively. The method searches through multiple complex design spaces while dealing with the various trade-offs between the individual products and the collaborative product to optimize the collaborative performance of the products.

Second, from Step 1 of the method, customer needs are sought to be understood. The customer stands alongside the designer at the heart of the design process [46]. Therefore it is crucial to understand the customer needs from the beginning of the process. As was discussed in Chapter 2, customer needs are considered and addressed in Steps 1 and 4 of the revised method and the collaborative product is designed around those needs. We note that Morrise et. al assumed the needs were already known [1].

Third, the designer has a larger degree of creative freedom in the revised method. No longer must the designer select a single category to design within. Rather, the designer is able to explore *all* categories that may contain important customer needs and seek to understand them with greater creative freedom. The designer may also design outside of a particular category.

Lastly, the number of decompositions is minimized through the revised method. The collaborative design process is viewed from the reverse direction of that presented by Morrise et al. Rather than select a large number of products, decompose all of them, and then construct a collaborative product, we look at a single product that has potential to become a collaborative product.

From that single product, decomposition is carried out once to determine how the individual components can meet customer needs.

As described in the introduction, the task-per-cost ratio can be observed to more fully understand the potential impact a collaborative product may have on alleviating poverty. The method presented in this thesis is an optimization-based strategy for selecting designs of a given collaborative product set. The ability of this method to optimize based on objectives such as cost and task performance, enables the task-per-cost ratio of the product set to increase. As such, the resulting collaborative product would have a higher potential impact and application within the developing world. To illustrate application of this method, a collaborative brick press created by combining a shovel, hoe, rake, water transportation roller, water pump, and a small cook stove was provided.

From the case study, and the presented results, it is concluded that the presented method provides an effective tool for designing products for optimal collaborative performance. The potential that collaborative products can have on poverty alleviation by reducing the cost, weight, and size of a set of products was presented as motivation for this work. Opportunities for future work that builds on this method includes: (i) further verification of the revised method through additional case studies; (ii) exploring a dynamic offset area; and (iii) exploring methods of aggregation to select the final design.

Even though the presented brick press case study illustrates the effectiveness of the method presented in Chapter 2, it would be useful to explore additional examples to further verify the revised method. This could include simple or highly complex products found in both the developing *and* developed worlds. It may also be valuable to add additional design objectives and constraints to ensure that the identified product designs embody the goals of reducing the cost, weight, and size of a set of products.

Further research in how the offset area is defined in Step 7 would allow the designer to capture designs that could otherwise be overlooked. This could be done by exploring a dynamic offset area, rather than using the current scalar offset area. The dynamic area would be able to fluctuate and change shape to capture additional designs.

Additional methods of selecting the final design in Step 9 could also be explored. Currently a weighted sum method is used to complete this step, but other methods of aggregation could be used to identify and select the final design.

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APPENDIX A. MATHEMATICAL MODELS OF THE CASE STUDY

A.1 Shovel, Hoe, and Rake

It is assumed that the load (F) placed on the shovel is 50 lbs while the hoe and rake are at a load of 30 lbs, acting at the end of each beam. The reaction forces on the beam of the shovel are modeled at two different positions (L_1 and L_2) to represent where the hands of the customer are placed when using the shovel. The free-body diagram of this model is illustrated in Fig. A.1.

As models are constructed, the designer must determine which design variables and objectives should be considered in the design. These are based upon the product's intended function and the constraints that define the desired performance of the product. In the case of the shovel, hoe, and rake, the diameter (d), wall thickness (t), and length (l) of each handle are defined as design variables. The desired objectives of each design are minimizing the stress and total cost. The shared design parameters include the following: elastic modulus (30 x 10^6 psi); allowable stress (3 x 10^4 psi); allowable cost (\$15); additional cost of secondary components (\$5); deflection (1 in.); and cost-to-volume ratio (0.1065 $\$/in.^3$).

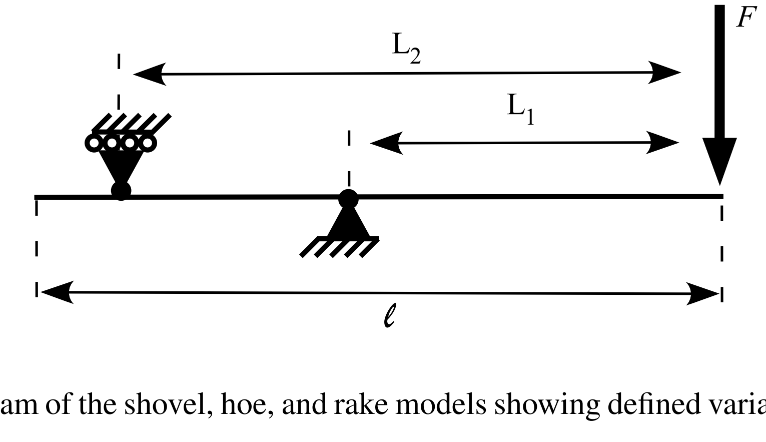

Figure A.1: Diagram of the shovel, hoe, and rake models showing defined variables and parameters

A.2 Water Transportation Roller

The water transportation roller model is illustrated in Fig. A.2. The design variables for the water transportation roller include the diameter (d) and wall thickness (t) of the handle used to pull or push the attached water barrel. The first desired objective is to minimize stress at two different locations: where the customer places his or her hands on the handle to move the water barrel and where the handle structure inserts into the water barrel. It is determined that these two areas would have the highest chance for failure within the structure. The second desired objective is to minimize the total cost of the water roller. The design parameters include the following: L_1 (10 in.); L_2 (30 in.); L_3 (6 in.); L_4 (6 in.); F (12 lbs); elastic modulus (30 x 10^6 psi); allowable stress (3 x 10^4 psi); deflection (1 in.); allowable cost (\$110); additional cost of secondary components (\$70); and cost-to-volume ratio (0.1065 $\$/in.^3$).

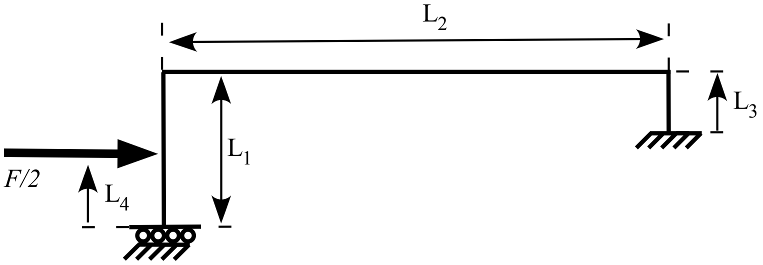

Figure A.2: Diagram of the water transportation roller model showing defined variables and parameters

A.3 Cook Stove

For the cook stove, a heat transfer model is developed to simulate the heat output from burning coals in the bottom of the stove. The heat flow is calculated using a view factor for two parallel surfaces. The design variables for the cook stove are determined to be the width (w), length (l), and height (h) of the stove cavity, making up its volume. The desired objectives include maximizing heat flow, while minimizing total cost of the stove. The design parameters include the following: an assumed emissivity value for burning wood (0.75); estimated temperature of the coals (1500 R); estimated temperature of the surface of the food being cooked (620 R); wall thickness (0.5 in.); allowable cost ($40); additional cost of secondary components ($10); cost-to-volume ratio (0.1065 $\$/in.^3$); and upper (1500 J/s) and lower (1000 J/s) limits of acceptable heat transfer flow rates. See Figure A.3 for an illustration of this model.

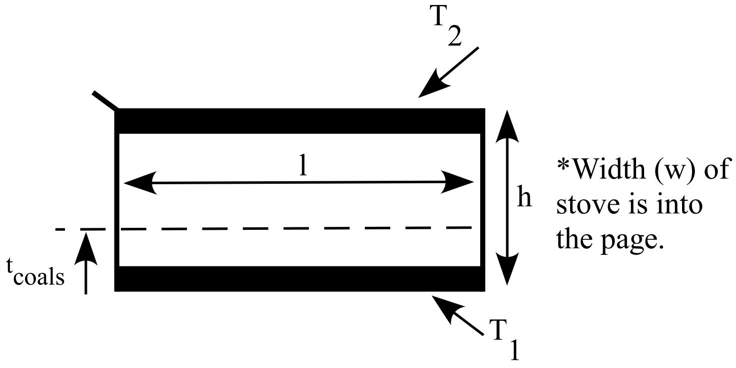

Figure A.3: Diagram of the cook stove model showing defined variables and parameters

A.4 Water Pump

The water pump model, found in Figure A.4, is based on a small irrigation pump commonly found in the developing world. The design variables of interest are the pump cylinder diameter (d_c) and area of the pump base plate (not illustrated). The base plate serves as a foundation to the pump and is where the customer can stand to provide support to the pump while in use. The chosen design objectives include maximizing water flow rate, while minimizing the total cost of the pump. The design parameters include the following: gravity constant (9.81 m/s^2); density of water (1000 kg/m^3); inner pipe diameter, d_p (0.0254 m); pipe length in, $l_{p,in}$ (3 m); pipe length out, $l_{p,out}$ (1 m); vertical distance from pump to source, z_{in} (-3 m), vertical distance from pump to outlet, z_{out} (1 m); force transmission efficiency of the pump (0.8%); force allowed by operator for hand actuation, F (600 N); operator stroke length, l_s; upper (0.3 s) and lower (6 s) bounds of operator stroke time; upper (0.32 L/s) and lower (0.76 L/s) bounds of flow rate; upper ($20) and lower ($45) bounds of pump cost; and cost-to-volume ratio (0.1065 $\$/in.^3$).

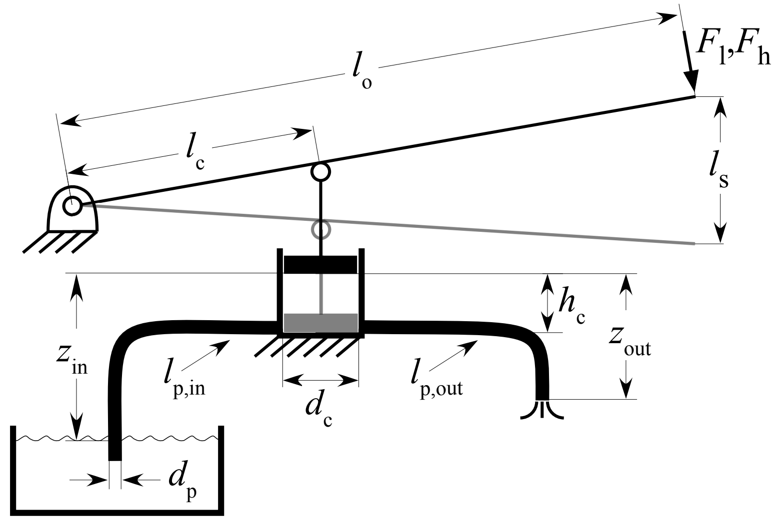

Figure A.4: Diagram of the pump model showing defined variables and parameters

A.5 Brick Press

The final product modeled in our case study consists of the collaborative brick press design. The force required to produce a brick of a certain size depends on the strength of the shovel handle, which is incorporated into the brick press as the handle to compress the brick. A second handle found within the press is that of the rake. The rake handle is used to eject the brick after it has been pressed and has the potential to also undergo large amounts of stress. Understanding that stress could be a factor in both of these products, a beam analysis model is used to optimize the stress in these two areas of the press.

An additional component of the collaborative product perceived to be of concern is the water transportation roller frame. This component makes up the legs of the brick press and must withstand the applied forces from the press when a brick is created. Realizing that stress would affect the frame, an additional beam analysis model is constructed to optimize the stress in the legs of the brick press.

A final component of the collaborative brick press found to be of concern is the small cook stove. More specifically, the cavity area of the stove where the brick is pressed. This area largely determines the required pressure to form a brick. The larger the area is, the higher the force required to produce a brick. While on the other hand, the smaller the area, the lower the required force will be. Knowing the effects that the area has on the required brick forming pressure, a model was created to optimize the area of the stove to ensure that the brick press could withstand the required forming pressure.

It should be noted that because the collaborative brick press incorporates all of the individual products into one design there are a large number of corresponding design variables in this model. These include: length (*l*), width (*w*), and height (*h*) of the brick press cavity as well as lengths, diameters, and wall thicknesses of the shovel, rake, hoe, and water transportation roller handles. The desired objectives include minimizing the required pressure to form a brick as well as minimizing the applied stress to identified areas of the brick press. Design parameters for the brick press include: L_1 (10 in.); L_2 (38 in.); L_3 (6 in.); wall thickness (0.5 in.); F_{comp} (300 lbs); F_{eject} (150 lbs); elastic modulus (30 x 10^6 psi); allowable stress (500 psi); and deflection (1 in.).

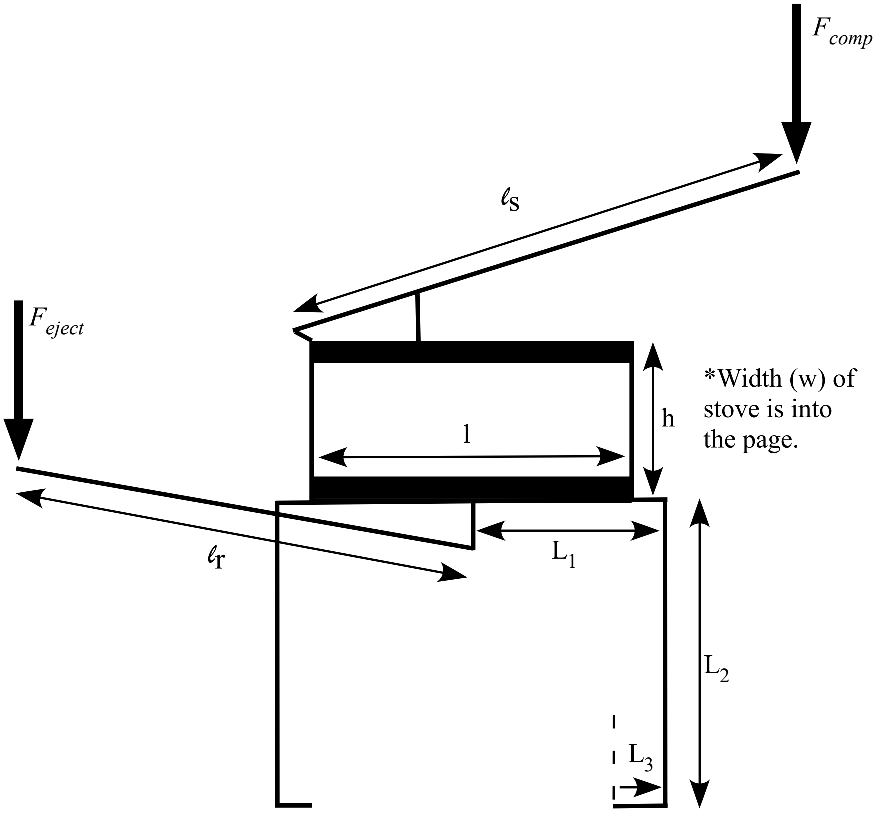

Figure A.5: Diagram of the brick press model showing defined variables and parameters