# Generation of multi-beam reflected from gradient-index metasurfaces 

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#### Abstract

Many recent metasurfaces, consisting of one or two-dimensional artificial structures with subwavelength unit cells, have been designed to manipulate optical waves by achieving the $2 \pi$ continuous phase gradients. In this paper, a method to transfer and control multi-beam reflections from a normal incident wave using a gradientindex metasurface is presented. By tuning properly the phase difference over the gradient-index metasurface, multi-beam anomalous reflections can be achieved, where the beam wavefronts are fully controlled by the refractive index distribution. The theoretical and simulated results show that some excellent multi-beam anomalous reflections are demonstrated by field distribution and radiation pattern of scattering field. Our method may help to offer a new design methodology for multi-beam steering in many interesting optics and microwave applications.


## Introduction

According to the fundamental generalized Snell's law [1], the gra-dient-index metasurfaces (GMSs) with the scattering of the gradientphase structures on the interface can manipulate the wavefronts of reflected, refracted, and diffracted waves. Using this concept, a method to transfer and control multi-beam reflections from a normal incident wave using a GMS composed of a one-dimensional (1-D) series of supercells is proposed. By tuning properly the phase difference over the GMS, multi-beam reflected from the GMS can be generated, where the beam directions are controlled by the refractive index distribution of the sub-supercells in the supercell. The reflected angles of multi-beam waves are theoretically predicted and confirmed using a commercial full-wave simulation tool, HFSS, based on finite element method for modeling 3-D structures.

## Theoretical analysis and simulated results

Following the fundamental generalized Snell's law, the $2 \pi$ continuous phase gradients generated by metasurfaces usually contribute to the generation of anomalous waves [1]. The relation between the incident angle $\theta_{i}$ and the anomalous reflection angle $\theta_{r}$ can then be expressed as follows
$\sin \theta_{r}=\sin \theta_{i} \pm \frac{\lambda_{0}}{2 \pi n_{i}} \frac{d \phi}{d x}=\sin \theta_{i} \pm \frac{\lambda_{0}}{2 \pi n_{i}} \frac{2 \pi}{L_{i}}=\sin \theta_{i} \pm \frac{\lambda_{0}}{n_{i} L_{i}}$
where $d \phi / d x$ indicates a phase gradient along the metasurface; $\theta_{r}$ and $\theta_{i}$
are the reflection angle and incidence angle, respectively; $n_{i}$ represents the refractive index of the incidence medium; and $\lambda_{0}$ represents the wavelength in free space. $L_{i}$ represents the periodic length of the supercell for the $2 \pi$ continuous phase gradient. When a normal plane wave ( $\theta_{i}=0$ ) propagates through the metasurface at the interesting wavelength $\lambda_{0}$, the arbitrary wavefront of reflections can be controlled by $L_{i}$ and $n_{i}$, as shown in Fig. 1. By extending this concept, we propose a method to create multi-beam reflections by applying multi-plane beams with different propagating angles. The multi-beam reflections can be achieved by designing several sub-supercells inside a supercell, where each sub-supercell with the $2 \pi$ continuous phase gradient and the specific length is designed to generate one reflected beam. Here, the top layer of a sub-supercell consists of dielectric slabs, where their refractive indices are assigned to achieve the $2 \pi$ continuous phase gradient over the sub-supercell. The second layer is a perfectly electric conductor (PEC) to support high-efficiency anomalous reflections [2]. The reflective properties of the GMSs can be analyzed by simulating the supercells using the master-slave (M-S) periodic condition and perfectly matched layers (PML) [3] in HFSS. To demonstrate the theoretical analysis of muti-beam anomalous reflections, three GMS models have been proposed and simulated at the operating wavelength of 850 nm . Figs. 1(b)-4(b) show the y-polarized electric field distributions, and the far-field responses of the reflected waves is presented in Figs. 1(c)-4(c). It has been shown that the desired beam directions of the multi-beam anomalous reflections can be achieved by designing the gradient metasurfaces carefully.

Their models can be extended to two-dimensional (2-D) GMSs by

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Fig. 1. Gradient-index metasurface for the one-beam anomalous reflection. (a) Schematic diagram and theoretical analysis of the GMS. (b) Simulated field distribution reflected from GMS. (c) Far-field radiation pattern of reflected wave.


Fig. 2. Gradient-index metasurface for the two-beam symmetric anomalous reflections. (a) Schematic diagram and theoretical analysis of the GMS. (b) Simulated field distribution reflected from GMS. (c) Far-field radiation pattern of reflected wave.


Fig. 3. Gradient-index metasurface for the two-beam asymmetric anomalous reflections (a) Schematic diagram and theoretical analysis of the GMS. (b) Simulated field distribution reflected from GMS. (c) Far-field radiation pattern of reflected wave.


Fig. 4. Gradient-index metasurface for the three-beam anomalous reflections (a) Schematic diagram and theoretical analysis of the GMS. (b) Simulated field distribution reflected from GMS. (c) Far-field radiation pattern of reflected wave.
constructing two periodic crossed sub-supercells with the $2 \pi$ continuous phase gradients along the x-axis and y-axis. According to the generalized 3D Snell's Law in Ref. [4], the directions of the beams are derived with different polarized incident waves in this case.

## Conclusions

In summary, a method to transfer and control multi-beam reflections from a normal incident wave using a GMS is presented. The GMS consists of the sub-supercells in a supercell which gives the $2 \pi$ continuous phase gradients. The sub-supercells composed of graded-index dielectric slabs above a perfectly electric conductor can manipulate the reflected wave independently by changing their lengths. The beam directions of the multi-beam reflection waves are theoretically predicted and confirmed by a full wave simulation tool based on finite element method. The theoretical analysis and simulated results show good agreements. This work provides a new design methodology for the multi-functional manipulation of optical and EM waves, which could be developed in many interesting optics and microwave applications.

## Acknowledgments

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. 2015R1A6A1A03031833).

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    https://doi.org/10.1016/j.rinp.2018.06.052
    Received 8 March 2018; Received in revised form 21 June 2018; Accepted 23 June 2018
    Available online 28 June 2018
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