Results in Physics 8 (2018) 7–15

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.journals.elsevier.com/results-in-physics

Eigenvalue approach to coupled thermoelasticity in a rotating isotropic medium

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ARTICLE INFO

Article history:
Received 7 May 2017
Received in revised form 10 September 2017
Accepted 12 September 2017
Available online 12 October 2017

Keywords:
Thermal stresses
Thermoelasticity
Energy dissipation
Rotation
Half-space

ABSTRACT

In this paper the linear theory of the thermoelasticity has been employed to study the effect of the rotation in a thermoelastic half-space containing heat source on the boundary of the half-space. It is assumed that the medium under consideration is traction free, homogeneous, isotropic, as well as without energy dissipation. The normal mode analysis has been applied in the basic equations of coupled thermoelasticity and finally the resulting equations are written in the form of a vector- matrix differential equation which is then solved by eigenvalue approach. Numerical results for the displacement components, stresses, and temperature are given and illustrated graphically. Comparison was made with the results obtained in the presence and absence of the rotation. The results indicate that the effect of rotation, non-dimensional thermal wave and time are very pronounced.

Introduction

During the past few decades, widespread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes such as those occurring in laser units, energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical behavior of materials and magnetic behavior of materials have been studied since the 19th century. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in Refs. [1–19]. Ailawalia and Narah [20] investigated the effect of rotation in a generalized thermoelastic medium with hydrostatic initial stress subjected to ramp-type heating and loading. Kumar and Devi [21] discussed Magneto thermoelastic with and without energy dissipation Half-Space in contact with Vacuum. He et al. [22] investigated the two-dimensional generalized thermoelastic diffusion problem for a half-space. The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen [23]. Zhu et al. [24] discussed the steady-state response of thermoelastic half-plane with voids subjected to a surface harmonic force and a thermal source. Deswal et al. [25] investigated the plane waves in a fractional order micropolar thermoelastic half-space. Singh [26] discussed the effect of hydrostatic initial stresses on waves in a thermoelastic solid half-space. A half-space problem in the theory of generalized thermoelastic diffusion has been studied by Sherief and Saleh [27]. Sharma and Bhargava [28] investigated the propagation of thermoelastic plane waves at an imperfect boundary of thermal conducting viscous liquid/generalized thermolastic solid. Sarkar and Lahiri [29] studied the three-dimensional thermoelastic problem for a half-space without energy dissipation. Quintanilla [30] investigated thermoelasticity without energy dissipation of materials with microstructure. Youssef [31], constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Atwa and Jahangir [32] have studied two-dimensional problem of generalized thermoelasticity to study the effect of rotation. Othman and Song [33] studied the effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation.

The solution of the present problem has been achieved in normal mode analysis and eigenvalue approach techniques to deter-

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https://doi.org/10.1016/j.rinp.2017.09.021
2211-3797/

mine deformation, stresses and temperatureform. The temperature, displacements and stresses are obtained in the physical domain. The effect of the rotation, time and non-dimensional thermal wave on the stresses, displacements and temperature distribution have been shown graphically. Comparisons are made with the results in the presence and absence of rotation of the thermoelastic half-space without energy dissipation.

Formulation of the problem

We consider a homogenous, isotropic thermoelastic half-space in two-dimensional space subjected to a time dependent heat source on the bounding plane. The medium is rotating uniformaly with respect to an inertia frame and constant rotating vector in an x, y, z rectangular Cartesian frame rotating with the medium is $\vec{\Omega} = \Omega \vec{n}$. The governing equations developed by Green and Naghdi [34], in the absence of heat sources or body forces.

The equations of motion in an isotropic thermoelastic medium are:

$$\rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right] = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial x \partial z} - \gamma\frac{\partial T}{\partial x} \tag{1}$$

$$\rho\left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w\right] = (\lambda + 2\mu)\frac{\partial^2 w}{\partial z^2} + \mu\frac{\partial^2 w}{\partial x^2}(\lambda + \mu)\frac{\partial^2 u}{\partial x \partial z} - \gamma\frac{\partial T}{\partial z} \tag{2}$$

The Temperature field $T(x, z, t)$ is assumed to satisfy the general heat conduction equation:

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_E \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 e}{\partial t^2} \tag{3}$$

The thermal stresses in an isotropic elastic solid subjected to plane strain in two dimensions are:

$$\begin{aligned}&\sigma_{xx} = 2\mu\frac{\partial u}{\partial x} + \lambda e - \gamma T, \sigma_{yy} = \lambda e - \gamma T,\\ &\sigma_{zz} = 2\mu\frac{\partial w}{\partial z} + \lambda e - \gamma T, \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right).\end{aligned} \tag{4}$$

where λ and μ are Lame's constant, ρ is the density, σ_{ij} are the components of the stress tensor, u, w are the components of the displacement vector, t is the time, T is the temperature, C_E is the specific heat, $\gamma = (3\lambda + 2\mu)\alpha_T$ where α_T is the coefficient of linear thermal expansion, K is the thermal diffusivity, characteristic of the theory, T_0 is the temperature of the medium such that $\left|\frac{T-T_0}{T_0}\right| \prec 1$.

$$e = \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right). \tag{5}$$

For convenience of the analysis, the following dimensionless quantities are introduced:

$$\begin{aligned}&(x', z') = \frac{1}{l}(x, z), \quad t' = \frac{C_1 t}{l}, \quad (u', w') = \frac{(\lambda + 2\mu)}{\gamma T_0 l}(u, w),\\ &T' = \frac{T}{T_0}, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\gamma T_0}, \quad C_T^2 = \frac{K}{\rho C_E C_1^2}, \quad \varepsilon_T = \frac{\gamma^2 T_0}{\rho C_E(\lambda + 2\mu)},\end{aligned} \tag{5a}$$

where l is the length and $C_T = \frac{C_3}{C_1}$ is the non dimensional thermal wave speed, $C_1^2 = \frac{\lambda + 2\mu}{\rho}$ is the longitudinal wave velocity, $C_3^2 = \frac{\mu}{\rho}$ is the shear wave velocity and ε_T is the thermoelastic coupling parameter. In view of this and quantities (5a), Eqs. (1)–(5) can be rewritten in the non-dimensional form as follows

$$\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial^2 u}{\partial z^2} + (1 - \beta)\frac{\partial^2 w}{\partial x \partial z} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} - \Omega^2 u \tag{6}$$

$$\frac{\partial^2 w}{\partial z^2} + \beta\frac{\partial^2 w}{\partial x^2} + (1 - \beta)\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial T}{\partial z} = \frac{\partial^2 w}{\partial t^2} - \Omega^2 w \tag{7}$$

$$C_T^2 \nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \varepsilon_T \frac{\partial^2 e}{\partial t^2}, \tag{8}$$

$$\begin{aligned}&\sigma_{xx} = 2\beta\frac{\partial u}{\partial x} + (1 - 2\beta)e - T\\ &\sigma_{yy} = (1 - 2\beta)e - T,\\ &\sigma_{zz} = 2\beta\frac{\partial w}{\partial z} + (1 - 2\beta)e - T,\\ &\tau_{xz} = \beta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\end{aligned} \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \quad \beta = \frac{\mu}{(\lambda + 2\mu)}$$

From Eqs. (5), (6) and (7), we get

$$\beta\nabla^2\frac{\partial u}{\partial x} + (1 - \beta)\frac{\partial^2 e}{\partial x^2} - \frac{\partial^2 T}{\partial x^2} = \frac{\partial^3 u}{\partial x \partial t^2} - \Omega^2\frac{\partial u}{\partial x} \tag{10}$$

$$\beta\nabla^2\frac{\partial w}{\partial z} + (1 - \beta)\frac{\partial^2 e}{\partial z^2} - \frac{\partial^2 T}{\partial z^2} = \frac{\partial^3 u}{\partial z \partial t^2} - \Omega^2\frac{\partial w}{\partial z} \tag{11}$$

Adding Eqs. (10) and (11) and using Eq. (5), we obtain

$$\nabla^2 e - \nabla^2 T = \frac{\partial^2 e}{\partial t^2} - \Omega^2 e \tag{12}$$

The mean value of the stresses is:

$$\sigma = \frac{\sigma_{xx} + \sigma_{zz}}{2} \tag{13}$$

Adding Eqs. (9) and (10) and using Eqs. (5)–(13), we obtain

$$\sigma = \alpha e - T \tag{14}$$

where

$$\alpha = \frac{(2 - 3\beta)}{2}$$

Eliminating the dilation e from Eqs. (8), (12) and (14), we get:

$$\nabla^2\sigma + (1 - \alpha)\nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \frac{\partial^2 \sigma}{\partial t^2} - \Omega^2(T + \sigma) \tag{15}$$

$$\nabla^2 T = \left(\frac{\alpha + \varepsilon_T}{\alpha C_T^2}\right)\frac{\partial^2 T}{\partial t} + \frac{\varepsilon_T}{\alpha C_T^2}\frac{\partial^2 \sigma}{\partial t^2} \tag{16}$$

Normal mode analysis

We apply the normal modes of the form:

$$(u, w, e, T, \sigma_{ij})(x, z, t) = (u^*, w^*, e^*, T^*, \sigma^*)(x)e^{(\omega t + ibx)} \tag{17}$$

where $i = \sqrt{-1}$, ω is the angular frequency and b is the wave numbers in the z-directions respectively. Using Eq. (17), we can obtain the following equations from Eqs. (15) and (16) respectively

$$\frac{d^2 T^*}{dx^2} = C_1 T^* + C_2 \sigma^* \tag{18}$$

$$\frac{d^2 \sigma^*}{dx^2} = D_1 T^* + D_2 \sigma^* \tag{19}$$

where

$$C_1=\frac{1}{\alpha C_T^2}[(\omega^2-\Omega^2)(\alpha+\varepsilon_T)+\alpha b^2C_T^2],$$
$$C_2=\frac{\omega^2\varepsilon_T}{\alpha C_T^2},$$
$$D_1=\frac{(\omega^2-\Omega^2)}{\alpha C_T^2}[\alpha C_T^2-(1-\alpha)(\alpha+\varepsilon_T)],$$
$$D_2=\frac{1}{\alpha C_T^2}[\alpha C_T^2(\omega^2-\Omega^2+b^2)-(\omega^2+\Omega^2)\varepsilon_T(1-\alpha)]. \tag{20}$$

Eqs. (18) and (19) can be written in the form of a vector-matrix differential equation as:

$$\frac{dV}{dx}=AV \tag{21}$$

where

$$V=\begin{pmatrix}T^*\\ \sigma^*\\ \frac{dT^*}{dx}\\ \frac{d\sigma^*}{dx}\end{pmatrix},\quad A=\begin{pmatrix}0&0&1&0\\0&0&0&1\\C_1&C_2&0&0\\D_1&D_2&0&0\end{pmatrix} \tag{22}$$

Solution of the vector-matrix equation

We find the solution of Eq. (21) by the following the method of eigenvalue approach as in following the solution methodology through eigenvalue approach as in Das and Bhakta [36].

The characteristic equation of a matrix A is of the form

$$\lambda^4-(C_1+D_2)\lambda^2+(C_1D_2-C_2D_1)=0 \tag{23}$$

The roots of the Eq. (23) are of the form

$$\lambda_i=\pm\lambda_1,\pm\lambda_2 \tag{24}$$

where

$$\lambda_i=\frac{(C_1+D_1)+(-1)^{i+1}\sqrt{((C_1+D_2)^2-4(C_1D_2-C_2D_1))}}{2},\quad i=1,2$$

The right eigenvector $\chi=[\chi_1,\chi_2,\chi_3,\chi_3]^T$ corresponding to eigenvalue λ can be considered as

$$\chi=\begin{pmatrix}(\lambda^2-D_2)\\ C_1\\ \lambda(\lambda^2-D_2)\\ \lambda C_1\end{pmatrix} \tag{25}$$

From Eq. (25), the eigenvector χ corresponding to the eigenvalue $\lambda=\lambda_i$ can easily be calculated. We use the following notations:

$$\chi_1=[\chi]_{\lambda=\lambda_1},\quad \chi_2=[\chi]_{\lambda=\lambda_1},\quad \chi_3=[\chi]_{\lambda=\lambda_2},\quad \chi_4=[\chi]_{\lambda=\lambda_2} \tag{26}$$

Assuming the regularity condition at infinity, the solution of Eq. (21) can be written as

$$V=A_1\chi_2e^{-\lambda_1^x}+A_2\chi_4e^{-\lambda_2^x}\quad (x\geqslant 0), \tag{27}$$

where A_1, A_2 are constants to be determined by the boundary condition of the problem. From Eqs. (14) and (22) on using Eqs. ((28)–(29), we can find the expression of $T^*(x)$, $\sigma^*(x)$ and $e^*(x)$ as follows:

$$T^*(x)=A_1(\lambda_1^2-D_2)e^{-\lambda_1^x}+A_2(\lambda_2^2-D_2)e^{-\lambda_2^x} \tag{28}$$

$$\sigma^*(x)=C_1(A_1e^{-\lambda_1^x}+A_2e^{-\lambda_2^x}) \tag{29}$$

$$e^*(x)=\frac{1}{\alpha}[A_1(\lambda_1^2+C_1-D_2)e^{-\lambda_1^x}+A_2(\lambda_2^2+C_1-D_2)e^{-\lambda_2^x}] \tag{30}$$

Application

It is clear, the considered model is associated with the following boundary conditions

(a) The boundary condition for traction freet in case of plane to the surface $x=0$ $\sigma(0,z,t)=\sigma_{xx}(0,z,t)=\sigma_{zz}(0,z,t)=0$ which gives

$$\sigma^*(x)=\sigma^*_{xx}(x)=\sigma^*_{zz}(x)=0\quad at\quad x=0 \tag{31}$$

(b) The thermal boundary condition is

$$q_n+VT=Q_0(0,z,t) \tag{32}$$

where q_n is the normal components of the heat flux vector and $Q_0(0,z,t)$ is the intensity of the applied heat sources. In order to use the thermal boundary condition (32), we use the generalized Fourier's law of heat conduction in the non-dimensional form, namely

$$q_n=-\frac{\partial T}{\partial n} \tag{33}$$

From Eqs. (31) and (32) and Eq. (17), we obtain

$$VT^*-\frac{dT^*}{dx}=Q_0\quad at\quad x=0 \tag{34}$$

Eqs. (28) and (29) with Eqs. (31) and (34) yield two non-homogeneous equations for two arbitrary constants A_1 and A_2, we get

$$A_1(\lambda_1+V)(\lambda_1^2-D_2)+A_2(\lambda_2+V)(\lambda_2^2-D_2)=Q_0$$
$$A_1+A_2=0 \tag{34a}$$

The constants A_1 and A_2 determined from Eqs. (34a) by using Cramer's rule we obtain

$$A_1=\frac{Q_0}{\Delta}\quad and\quad A_2=\frac{-Q_0}{\Delta} \tag{35}$$

where

$$\Delta=(\lambda_1-\lambda_2)[(\lambda_1+\lambda_2)(V+\lambda_1+\lambda_2)-\lambda_1\lambda_2-D_2]$$

From Eq. (10) and using Eq. (17), we get

$$\left[\frac{d^2}{dx^2}-\lambda_u^2\right]u^*=\eta_1e^{-\lambda_1^x}+\eta_2e^{-\lambda_2^x}, \tag{36}$$

where $\lambda_u^2=\left(b^2+\frac{\omega^2}{\beta}\right)$ and

$$\eta_i=\left(\frac{A_i\lambda_i}{\alpha\beta}\right)[(1-\beta)C_1+(1-\alpha-\beta)(\lambda_i^2-D_2)]\quad i=1,2$$

The solution of the ordinary differential Eq. (36) is

$$u^*(x)=A_3e^{-\lambda_u^x}+\frac{\eta_1}{(\lambda_1^2-\lambda_u^2)}e^{-\lambda_1^x}+\frac{\eta_2}{(\lambda_2^2-\lambda_u^2)}e^{-\lambda_2^x} \tag{37}$$

where $\lambda_1^2\neq\lambda_2^2\neq\lambda_u^2$ and A_3 is a constant From Eqs. (4) and (14) after using Eq. (17) we get

$$\sigma_{xx}=2\beta\frac{du^*(x)}{dx}+\left(\frac{1-2\beta}{\alpha}\right)\sigma^*(x)+\left(\frac{1-\alpha-2\beta}{\alpha}\right)T^*(x). \tag{37a}$$

From the boundary conditions (31) and using Eq. (37), we get

$$\frac{du^*(x)}{dx}=\left(\frac{\alpha+2\beta-1}{2\alpha\beta}\right)T^*(x)\quad at\quad x=0 \tag{38}$$

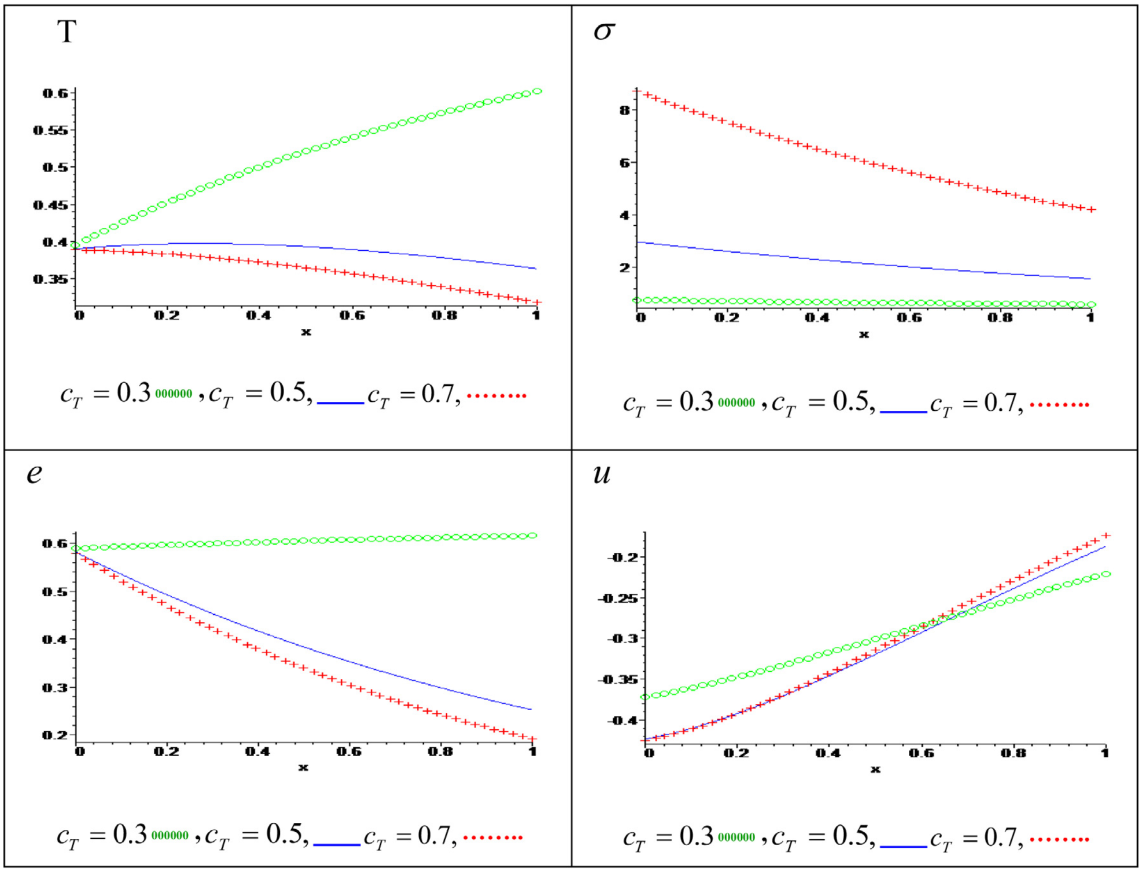

Fig. 1. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave c_T.

From Eqs. (28) and (37) and Eq. (38), we get

$$A_3 = \left[\frac{r^*(1-\alpha-2\beta)(\lambda_1^2-\lambda_2^2)}{\alpha\Delta\lambda_u} - \frac{\eta_1\lambda_1}{\lambda_u(\lambda_1^2-\lambda_u^2)} - \frac{\eta_2\lambda_2}{\lambda_u(\lambda_2^2-\lambda_u^2)}\right] \tag{39}$$

The dimensionless temperature T, normal stress σ, strain e and displacement u can be deduced from Eqs. (28)–(30) and Eq. (37) by using Eq. (17) as follows

$$T(x,z,t) = e^{\omega t}\cos(bz)[A_1(\lambda_1^2-D_2)e^{-\lambda_1^x} + A_2(\lambda_2^2-D_2)e^{-\lambda_2^x}, \tag{40}$$

$$\sigma(x,z,t) = C_1 e^{\omega t}\cos(bz)[A_1\lambda_1 e^{-\lambda_1^x} + A_2\lambda_2 e^{-\lambda_2^x}], \tag{41}$$

$$e(x,z,t) = \frac{e^{\omega t}\cos(bz)}{\alpha}\left[A_1(\lambda_1^2+C_1-D_2)e^{-\lambda_1^x} + A_2(\lambda_2^2+C_1-D_2)e^{-\lambda_2^x}\right] \tag{42}$$

$$u(x,z,t) = e^{\omega t}\cos(bz)\left[A_3 e^{-\lambda_u^x} + \frac{\eta_1}{(\lambda_1^2-\lambda_u^2)}e^{-\lambda_1^x} + \frac{\eta_2}{(\lambda_2^2-\lambda_u^2)}e^{-\lambda_2^x}.\right] \tag{43}$$

Special cases

(i) If $\Omega = 0$, in Eqs. (40)–(43), we obtain the corresponding expression for displacement, strain, principle stresses and temperature for isotropic thermoelastic solid without energy dissipation. The obtained results are similar to Sharma and Chouhan [35].

Numerical example and discussions

With an aim to illustrate the theoretical results obtained in the preceding section and to show the effect of rotation, we now present some numerical results. The numerical work has been carried out with the help of computer programming using the software Maple. Materia chosen for this purpose is magnesium crystal, the physical data for which is given as Following He et al. [22].

$$\varepsilon_T = 0.0168, \quad \alpha = 0.67, \quad \beta = 0:25, \quad \omega = 2,$$
$$a = 1.2, \quad b = 1.3,$$

$$V = 50, \quad Q_0 = 20, \quad C_T = 2.$$

The computations were carried out for:

Fig. 1 shows the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave c_T. It is observed that the temperature increases with increasing of the axial x at $c_T = 0.3$, while it decreases with increasing of the axial x at $c_T = 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $c_T = 0.3, 0.5, 0.7$, the strain increases with increasing of the axial x at $c_T = 0.3$, while it decreases with increasing of the axial x at $c_T = 0.5, 0.7$ and the displacement increases with increasing of axial x at $c_T = 0.3, 0.5, 0.7$.

Fig. 2: displays the variation of the temperature T, principle stress, strain e and displacement u with respect to the axial x for different values of the rotation Ω. It is observed that the temperature decreases with increasing of the axial x at $\Omega = 0.3, 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $\Omega = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial

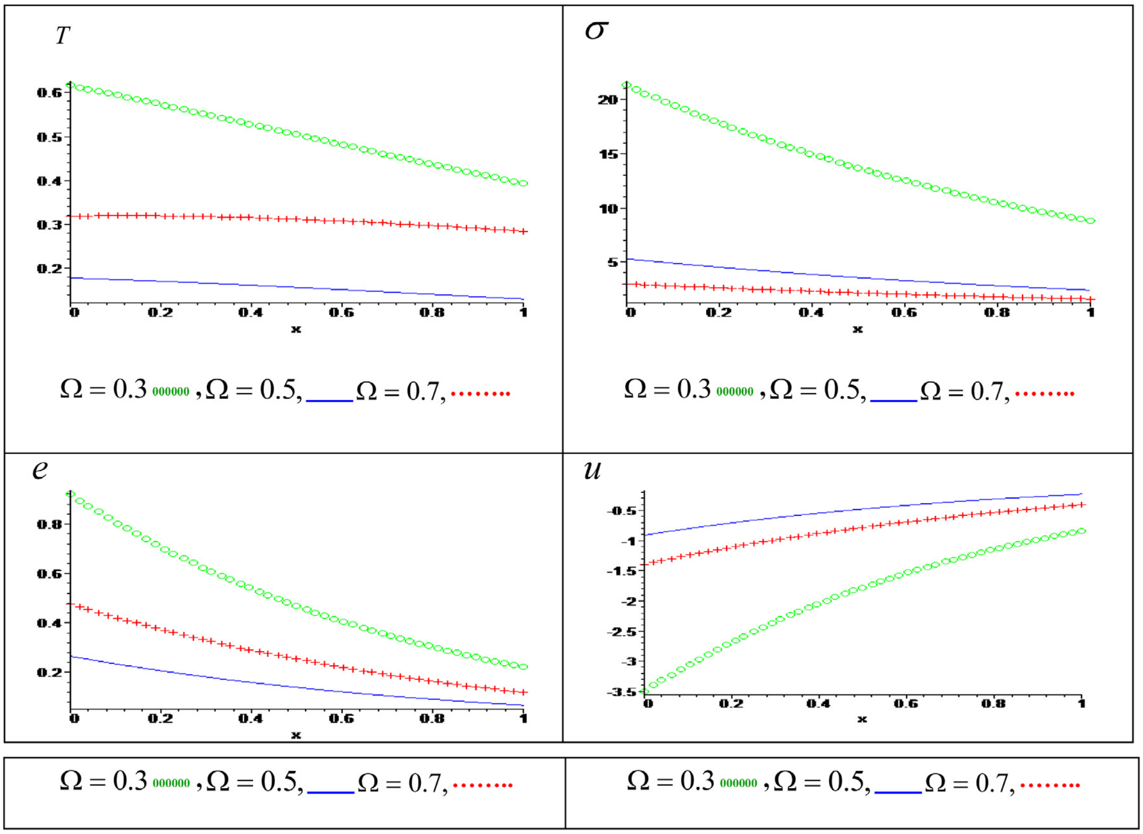

Fig. 2. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave Ω.

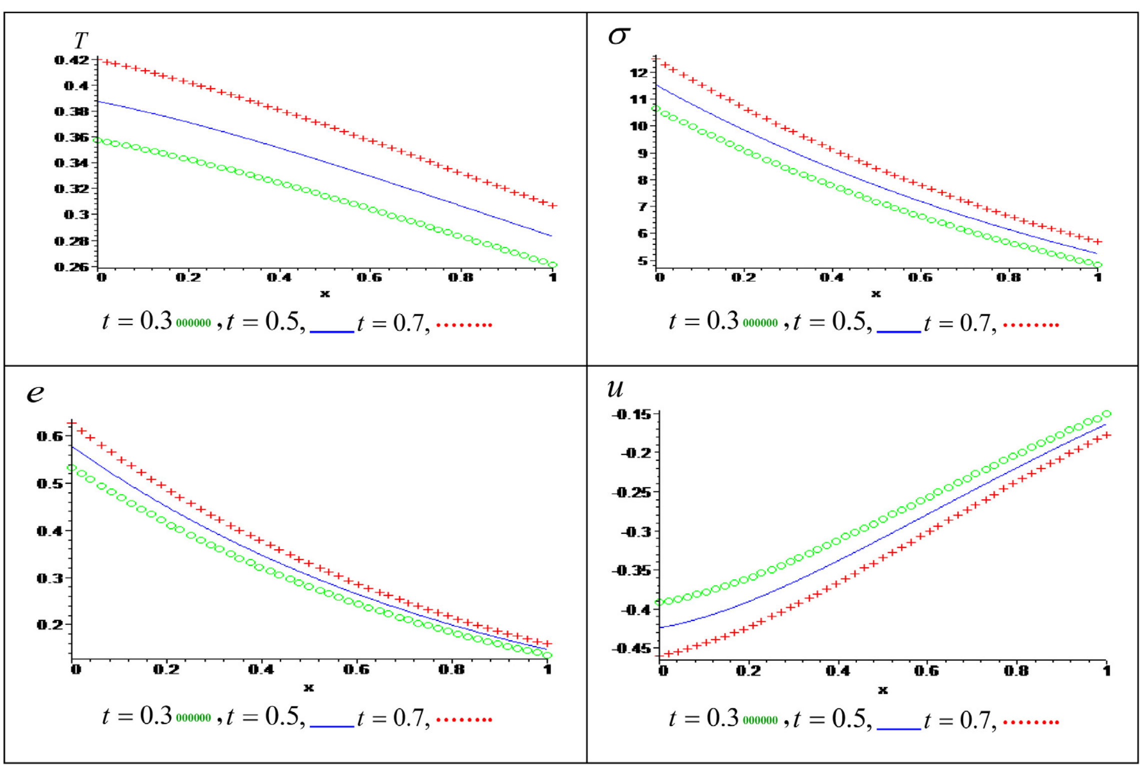

Fig. 3. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave t.

x at $\Omega = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $\Omega = 0.3, 0.5, 0.7$.

Fig. 3 explains the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x

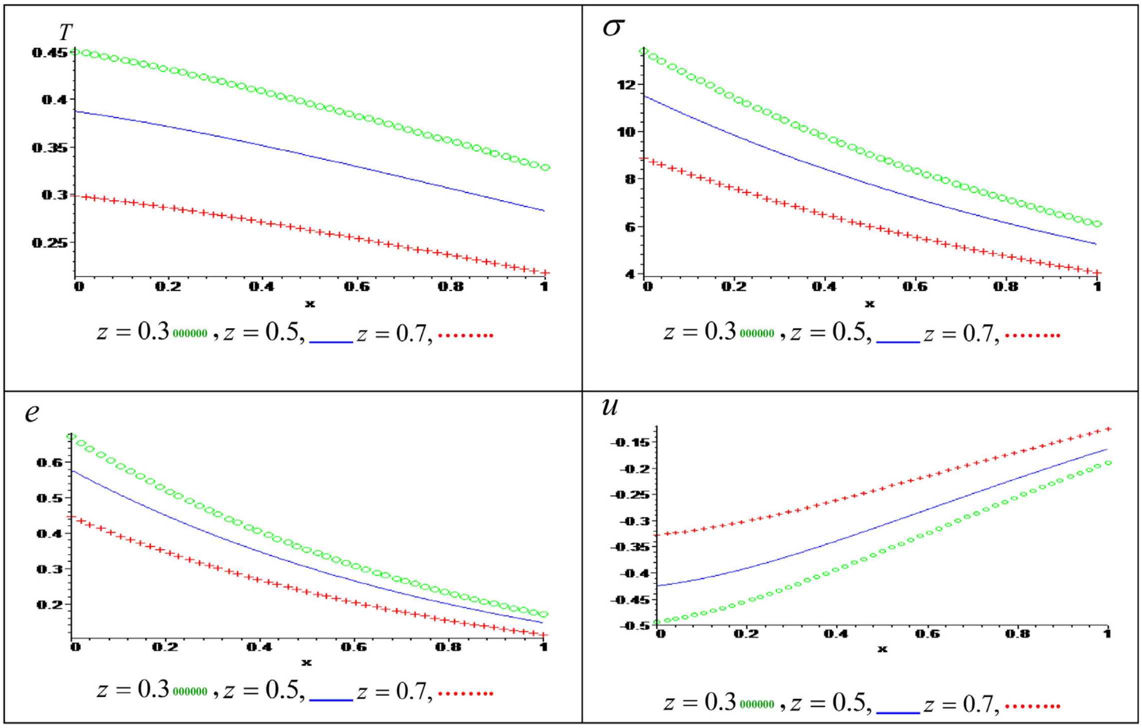

Fig. 4. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave z.

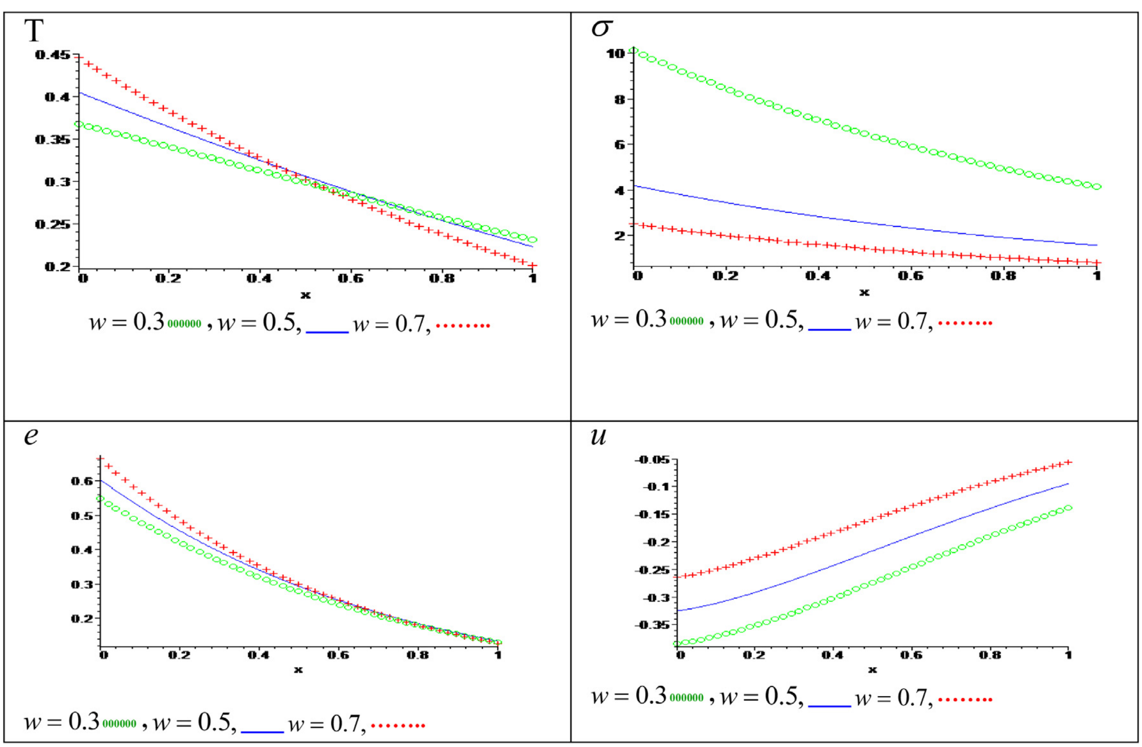

Fig. 5. Variation of the temperature T, principle stress σ, strain e and Displacement u with respect to the axial x for different values of the non-dimensional thermal wave w.

for different values of the time t. It is observed that the temperature decreases with increasing of the axial x at $t = 0.3, 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $t = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial x at $t = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $t = 0.3, 0.5, 0.7$.Fig. 4 shows the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the axial z. It is observed that the temperature decreases with increasing of the axial x at $z = 0.3, 0.5, 0.7$, the principle stress decreases with

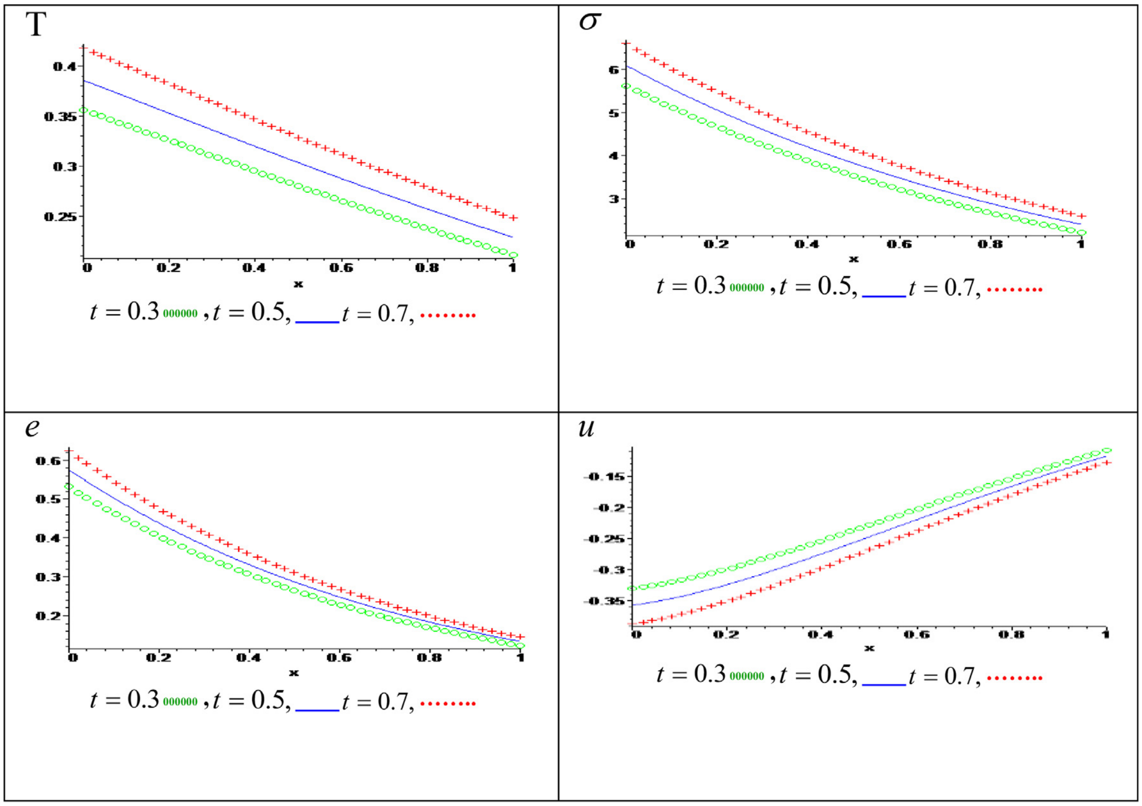

Fig. 6. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave t.

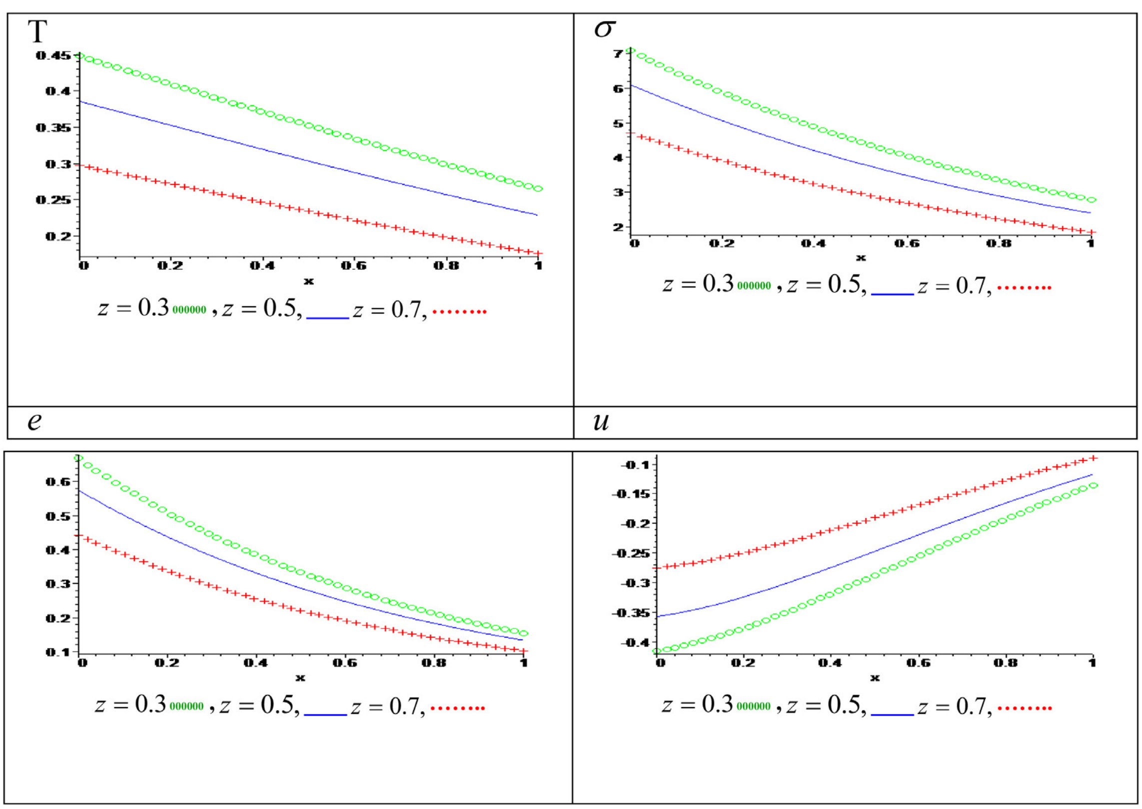

Fig. 7. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave z.

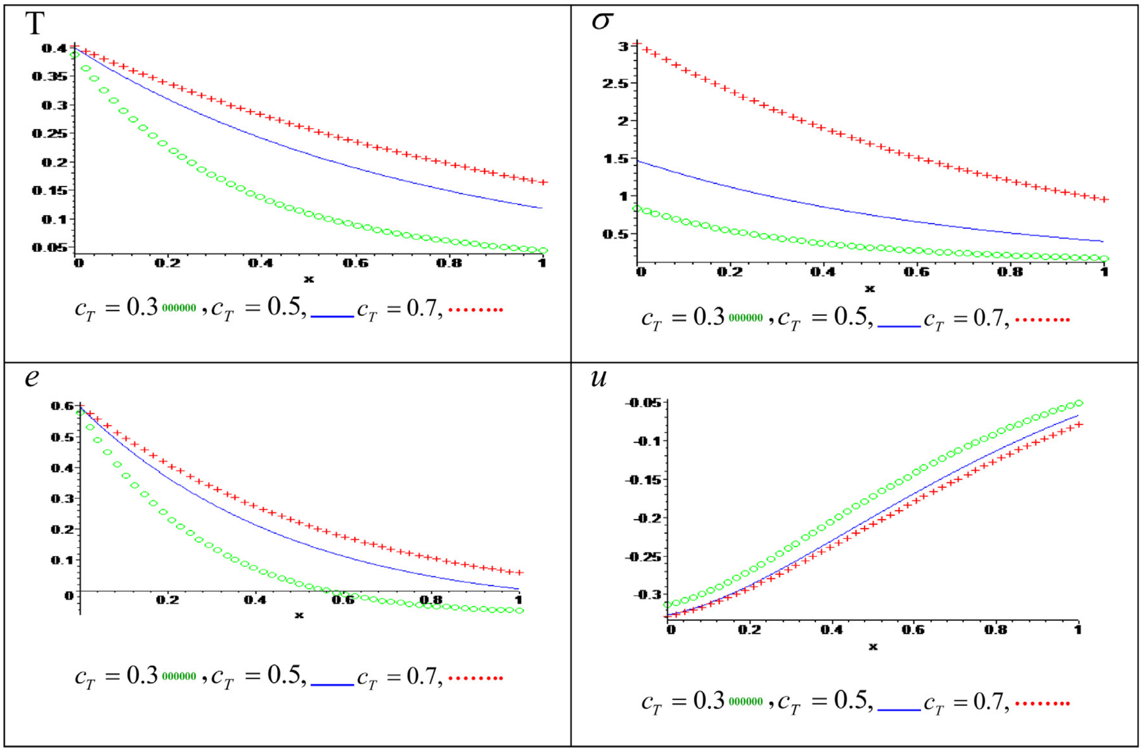

Fig. 8. Variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x for different values of the non-dimensional thermal wave c_T.

increasing of the axial x at $z = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial x at $z = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $t = 0.3, 0.5, 0.7$.

Fig. 5 shows the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x when vanishes the rotation Ω for different values of the frequency ω. It is observed that the temperature decreases with increasing of the axial x at $\omega = 0.3, 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $\omega = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial x at $\omega = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $\omega = 0.3, 0.5, 0.7$.

Fig. 6 shows the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x when vanishes the rotation Ω for different values of the time t. It is observed that the temperature decreases with increasing of the axial x at $t = 0.3, 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $t = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial x at $t = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $t = 0.3, 0.5, 0.7$.

Fig. 7 shows the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x when vanishes the axial z for different values of the time t. It is observed that the temperature decreases with increasing of the axial x at $z = 0.3, 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $z = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial x at $z = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $z = 0.3, 0.5, 0.7$.

Fig. 8 shows the variation of the temperature T, principle stress σ, strain e and displacement u with respect to the axial x when vanishes the rotation Ω for different values of the non-dimensional thermal wave c_T. It is observed that the temperature decreases with increasing of the axial x at $c_T = 0.3, 0.5, 0.7$, the principle stress decreases with increasing of the axial x at $c_T = 0.3, 0.5, 0.7$, the strain decreases with increasing of the axial x at $c_T = 0.3, 0.5, 0.7$ and the displacement increases with increasing of axial x at $c_T = 0.3, 0.5, 0.7$.

Conclusions

The analysis of graphs permits us some concluding remarks:

- The rotation plays a significant role in the distribution of all the physical quantities. The amplitude of all the physical quantities vary (increase or decrease) as rotation increases. Presence of rotation restricts the quantities to increase near the point of application of source as well as away from the source.
- The displacement component and stress components show an increase nature with increase or decrease amplitude with respect to x due to presence of rotation. The resulting quantities with and without rotation show opposite increase or decrease pattern in the form of waves. These trends obey elastic and thermoelastic properties of a solid under investigation.
- In absence of rotation, we observe the trends with increasing amplitudes in case of concentrated normal force/thermal point source. In case of uniformly distributed force/source, trends differ and some where it also results in non uniform pattern of graphs.
- The result provides a motivation to investigate conducting thermoelectric materials as a new class of applicable thermoelectric solids. The results presented in this paper should prove useful for researchers in material science, designers of new materials, physicists as well as for those working on the development of magnetothermoelasticity and in practical situations as in geophysics, optics, acoustics, geomagnetic and oil prospecting etc. The used methods in the present article is applicable to a wide range of problems in thermodynamics and thermoelasticity.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.rinp.2017.09.021.

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