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Rotational priority investigation in fuzzy analytic hierarchy process design: An empirical study on the marine engine selection problem

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ABSTRACT

The aim of this paper is to improve the applicability of the fuzzy-AHP method by using the rotational priority investigation (RPI) method. Despite its popularity and convenience, the AHP and fuzzy-AHP method is criticized by many scholars because of intransitivity and the rank reversal phenomenon. Experts may question the rational choice theory and cross priorities may indicate conflicting interactions. Also, the extraction of a number of alternatives may cause a different order of priorities. The rotational priority investigation method is proposed to investigate sub-group priorities and their corresponding rankings. Every rotation refers to the investigation of sub-group priority among the several rotations. An empirical study is presented by using the RPI method in the fuzzy AHP for the marine engine selection problem in the shipping industry.

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1. Introduction

The purpose of expert decision-making methods is to simplify and classify a decision- making problem and to make the best selection based on the interpretations of priorities and cost-benefit particulars. In the existing literature, there are several methods suited for decision-making problems and these methods improve the decision-making process by ensuring simplicity and functionality. In some cases, very complicated algorithms are proposed for complex decision-making problems. Often decision-making methods based on many assumptions and imperfections which may invalidate the process to some degree.

Among the variety of decision-making models, the Analytic Hierarchy Process (AHP) is the most cited and applied method in the literature. It has several benefits such as segmentation of the decision task, classification of the criteria and capability of using both linguistic and numerical information. Conversely, many scholars criticize such things as the setback of the rational choice theory, rank reversal and intransitivity.

The rational choice theory is an approach used in social sciences to understand human behavior and the particulars of their decisions. In the economics literature, Becker [1] is the first researcher to point out its importance and the theory is used in many scientific fields to explain decision characteristics. One of the basic principles of the theory is the transitivity

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of cross decisions. For example, if criteria *A* is 5 times more important than criteria *B* and criteria *B* is 4 times more important than criteria *C*, criteria *A* should be $5 \times 4 = 20$ times more important than *C* (also A > B > C). However, the Saaty's [2] scale limits priorities to a total of nine. Therefore, AHP does not ensure proof of the theory in some cases. The results may have different particulars under different sub-groups of alternatives.

Since scale inconsistency exists, the rank reversal phenomenon is expected in the case of addition and extraction of an alternative. The rotational priority investigation (RPI) is proposed to eliminate inconsistent results by calculating priorities of downsized elements of a decision matrix. In the RPI method, one alternative is extracted from a group of alternatives in every rotation and the final priority matrix is calculated for every combination of downsized groups. If the number of alternatives is *n*, then *n* number of combinations will be generated. The mean of the *n* priority matrices is considered the final priority matrix and the superior alternative is selected. The same procedure is proposed to be useful for the criteria matrix and the priorities of the criteria set may have critical changes which directly affect the final result of the analysis.

The remainder of this paper is organized as follows. Section 2 overviews the FAHP method, consistency control and describes the proposed method RPI. Section 3 presents an empirical work for the selection of a marine engine in a ship building project. Section 4 concludes the paper.

2. Criticism for AHP and FAHP method

2.1. Fuzzy sets and triangular numbers

Fuzzy set theory was first introduced by Zadeh [3] and it was developed based on the premise that the key elements in human thinking are not numbers, but linguistic terms of fuzzy sets. Fuzzy set theory has been widely applied to represent uncertain or flexible information in many different applications, such as selection problems, engineering design, and production management.

For numerical efficiency, trapezoidal or triangular fuzzy numbers are used to represent uncertain parameters. In this paper, a triangular fuzzy number is applied and defined as follows:

 \tilde{A} is a triangular fuzzy number which has three dimensions $\tilde{A} = (a, b, c)$, lower boundary, midpoint and upper boundary respectively (Fig. 1).

Center of gravity method, which is calculated by averaging *a*, *b* and *c*, is the most used method for defining the crisp result of a fuzzy set.

Definition 1. A fuzzy set \tilde{A} in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X is a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in \tilde{A} .

Definition 2. A fuzzy number is a fuzzy subset in the universe of discourse *X* that is both convex and normal.

Definition 3. A triangular fuzzy number denotes as $\tilde{A} = (a, b, c)$, where $a \le b \le c$ has the following triangular type membership function;

$$\mu_{\bar{A}}(x) = \begin{cases} 0, & x < a, \\ (x-a)/(m-a), & a \le x < b, \\ 1, & x = b, \\ (c-x)/(c-b), & b < x \le c, \\ 0, & c < x. \end{cases}$$
(1)



Fig. 1. A triangular fuzzy number Ã.

According to Zadeh [3], the fuzzy number addition operations and the fuzzy number multiplication operations of the triangular fuzzy numbers are expressed by standard fuzzy arithmetic operations.

2.2. Fuzzy analytic hierarchy process (FAHP)

The analytic hierarchy process (AHP), first proposed by Saaty [2], is a simple and convenient multi-criteria decision-making method. AHP is based on pair-wise comparison among attributes and alternatives. In the AHP method, the decision problem is divided into the following main steps [4]:

- Hierarchy structure for goal.
- Calculation of local weights.
- Assessment of priorities.

When the decision-maker has a vague and complex problem, the classical AHP method cannot be utilized because its numerical scale of judgments is limited to reflect the vagueness of human thinking [5]. Comparison ratios as linguistic terms with corresponding fuzzy sets are used to express comparison judgments of decision-makers (Fig. 2). In this paper, six fuzzy linguistic variables in Table 1 are applied to help the decision-maker to describe his/her subjective judgment for relative importance.

In the literature, there are many studies to improve the AHP method by fuzzy extension (FAHP) [6-12]. First, Laarhoven and Pedrycz [13] extend AHP to fuzzy hierarchical analysis by using triangular fuzzy numbers. Buckley [14] and Boender et al. [15] propose fuzzy sets for the assessment and analysis of pair-wise comparison. Chang [16] suggested the synthetic extent values of the pair-wise comparison for handling FAHP by using triangular fuzzy numbers.

The extent synthesis method is defined as follows:

Let $X = \{x_1, x_2, ..., x_n\}$ be an object set and $U = \{u_1, u_2, ..., u_m\}$ be a goal set. The extent analysis for each goal is performed under each object. Therefore, *m* extent analysis values for each object are indicated with the following parameters:

$$M_{g_i}^1, M_{g_i}^2, \dots, M_{g_i}^m, \quad i = 1, 2, \dots, n,$$
 (2)

where all the M_g^j (j = 1, 2, ..., m) are TFNs. The steps of Chang's extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the *i*th object is defined as

$$S_i \sum_{j=1}^m \mathcal{M}_{g_i}^j \otimes \left[\sum_{i=1}^n \sum_{j=1}^m \mathcal{M}_{g_i}^j \right]^{-1}.$$
(3)

To obtain $\sum_{j=1}^{m} M_{g_j}^{j}$, the fuzzy addition operation of *m* extent analysis values for a particular matrix is performed such as:

$$\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right).$$
(4)

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m}M_{g_{i}}^{j}\right]^{-1}$, the fuzzy addition operation of $M_{g_{i}}^{j}$ (j = 1, 2, ..., m) values is performed such as:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right),$$
(5)



Fig. 2. Fuzzy number of linguistic variable set.

Table 1Transformation for TFNs membership functions.

Fuzzy number	Linguistic scales	Membership function	Reciprocal
$egin{array}{c} ilde{A}_1 \ ilde{A}_2 \ ilde{A}_3 \ ilde{A}_4 \end{array}$	Equally important	(1, 1, 1)	(1, 1, 1)
	Slightly important	(1, 1, 3)	(1/3, 1, 1)
	Moderately important	(1, 3, 5)	(1/5, 1/3, 1)
	More important	(3, 5, 7)	(1/7, 1/5, 1/3)
$ ilde{A}_5 ilde{A}_6$	Strongly important	(5, 7, 9)	(1/9, 1/7, 1/5)
	Extremely important	(7, 9, 9)	(1/9, 1/9, 1/7)

and then the inverse of the vector in Eq. (5) is computed, such as:

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m}M_{g_{i}}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}u_{i}}, \frac{1}{\sum_{i=1}^{n}m_{i}}, \frac{1}{\sum_{i=1}^{n}l_{i}}\right).$$
(6)

Step 2: The degree of possibility of $M_2 = (l_2, m_2, u_2) \ge M_1 = (l_1, m_1, u_1)$ is defined as

$$V(M_{2} \ge M_{1}) = \sup_{y \ge x} \lfloor \min(\mu_{M_{1}}(x), \mu_{M_{2}}(y)) \rfloor,$$
(7)

and can be expressed as follows:

$$V(M_2 \ge M_1) = hgt(M_1 \cap M_2),$$

$$=\mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \ge m_1, \\ 0, & \text{if } l_1 \ge u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise. \end{cases}$$
(8)

Fig. 3 illustrates Eq. (8) where *d* is the ordinate of the highest intersection point *D* between μ_{M_1} and μ_{M_2} . To compare M_1 and M_2 , we need both the values of V ($M_1 \ge M_2$) and V ($M_2 \ge M_1$).

Step 3: The degree possibility for a convex fuzzy number to be greater than *k* convex fuzzy M_i (i = 1, 2, ..., k) numbers can be defined by

$$V(M \ge M_1, M_2, \dots, M_k) = V[(M \ge M_1) \text{ and } (M \ge M_2) \text{ and } \dots \text{ and } (M \ge M_k)] = \min V(M \ge M_i), i = 1, 2, 3, \dots, k.$$
 (9)

Assume that $d'(A_i) = \min V(S_i \ge S_k)$ for k = 1, 2, ..., n; $k \ne i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_n))^{I},$$
(10)

where A_i (i = 1, 2, ..., n) are n elements.

Step 4: Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \dots, d(A_n))^T,$$
 (11)

where *W* is a non-fuzzy number.



Fig. 3. The intersection between M_1 and M_2 .

2.3. Criticism of the AHP method

The most cited criticism of classical AHP is based on the rank reversal phenomenon in cases of inverted decision matrices and the change in the pattern of alternatives. Ishizaka and Labib [17] reviewed that issue and indicated that the structure of questions may influence the ranking order. The order of superiority can easily be determined by the question of positive superiority or negative superiority. The last rank selection of a positive superiority order is expected to be in the first rank of a negative superiority order. However, it is not unusual that AHP results violate this basic rule.

Roper-Lowe and Sharp [18] presented a similar situation in a computer operating system selection problem. Their major contribution is a new form of the AHP scale based on normalized priorities. In the normalized scale, the most superior selection is presented by unity. Holder [19] investigated the scaling problem in AHP and suggested non-linear form scales to ensure the principles of the rational choice theory. The multiplicative priority scale is proposed for improving such inconsistencies. However, there is no fully consistent scale yet. From the point of rational choices, scale limitation is the major problem and an unlimited scale is improper to be used for expert consultation. Therefore, the Saaty [20] scale is still the most applied method of assessment. Since the FAHP is derived from the conventional AHP method, identical shortcomings exist in fuzzy extended form.

3. Methodology

3.1. Rotational priority investigation (RPI)

The proposed model, rotational priority investigation (RPI), is designed for improving decision-making and consistency. One of the drawbacks of the AHP method (also FAHP) is decision variations on alternative extraction or addition. The RPI method originally develops an ex-post investigation by a rotational investigation of the performances of alternatives and criteria. The superiority of the RPI method is derived from three major improvements. Firstly, the classical extent FAHP model ignores several criteria because of the disjointed extent value sets. After using the RPI method, the priorities of elements are defined in different combinations of sub-groups and the weights are calculated in detail rather than ignoring any one. Secondly, the RPI method is useful under competitive alternatives which may induce the rank reversal phenomenon. In the case of the rank reversal phenomenon, the final decision is highly inconsistent and volatile. The RPI method ensures an additional search of the best solution by sub-group analysis and the final priorities are defined by investigating the superiority of alternatives in each rotation. Thirdly, the RPI method elicits the damped alternatives and improves the sensitivity of the cumulative result.

Let *n* number of elements exists. *n* iterations are performed and in each iteration one of the elements is extracted. One element is investigated n - 1 times and one of the iteration does not include that element. Final priorities are calculated by averaging the priorities calculated in every single rotation.

Let \tilde{A} is the $n \times n$ matrix of pairwise comparison which consists of pair-wise priority values, a_{ij} . The matrix is as follows:

	a _{n1}	<i>a</i> ",	a.,,,		a)	
	÷	÷	÷		:	
$\tilde{A} =$	<i>a</i> ₃₁	<i>a</i> ₃₂	<i>a</i> ₃₃		<i>a</i> _{3n}	
	<i>a</i> ₂₁	<i>a</i> ₂₂	<i>a</i> ₂₃		a_{2n}	
	(a_{11})	a_{12}	a_{13}	•••	a_{1n}	

The estimated relative weights of the elements of matrix \tilde{A} is defined by

$$\hat{A} * \hat{W} = \lambda_{\max} * \hat{W}, \tag{12}$$

where \hat{A} is the observed matrix of pairwise comparisons, λ_{max} is the largest eigenvalue of \hat{A} , and \hat{W} is its right eigenvector. \hat{W} constitutes the estimation of actual weights of elements, W where $W = (w_1, w_2, ..., w_n)^T$ is the vector of actual relative weights, and n is the number of elements. The result of rotational investigation is calculated by the solutions of sub-groups, s = (1, 2, ..., n) which refers to extraction of an element in each sub-group according to $i \land j \neq s$ as follows:

$$\tilde{A}_{1} = \begin{cases} a_{22} & a_{23} & \cdots & a_{2n} \\ a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n2} & a_{n3} & \cdots & a_{nn} \end{cases}, \quad W_{1} = (w_{2}, w_{3}, \dots, w_{n})^{T},$$

$$\tilde{A}_{2} = \begin{cases} a_{11} & a_{13} & \cdots & a_{1n} \\ a_{31} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n3} & \cdots & a_{nn} \end{cases}, \quad W_{2} = (w_{1}, w_{3}, \dots, w_{n})^{T},$$

$$\tilde{A}_{n} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1(n-1)} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2(n-1)} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3(n-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{(n-1)1} & a_{(n-1)2} & a_{(n-1)3} & \cdots & a_{(n-1)(n-1)} \end{pmatrix}, \quad W_{n} = (w_{1}, w_{2}, w_{3}, \dots, w_{n-1})^{T}.$$

Then, the final priorities are calculated by simple average as follows:

$$W_F = \left(\frac{\sum_{s=1}^{n} w_1}{n}, \frac{\sum_{s=1}^{n} w_2}{n}, \frac{\sum_{s=1}^{n} w_3}{n}, \dots, \frac{\sum_{s=1}^{n} w_n}{n}\right).$$
(13)

The RPI method is applied for both criteria level and alternatives level matrices. Therefore, criteria weights, alternative priorities and the final result will be affected from the proposed procedure.

3.2. Centric consistency index (CCI)

Saaty [20] first introduced the individual consistency index based on the eigenvalue method for the AHP method and it should be less than 0.1 for the acceptance of a matrix larger than 4 by 4. In the literature, however, a consistency control has not been proposed and applied to the FAHP method despite the fact that the consistency calculation is a critical step in the group decision-making method. In this study, the centric consistency index (CCI) [21,22] based on the fuzzy extended version of the geometric consistency index (GCI) [23] is proposed to calculate the consistency of each decision maker's matrixes. The calculation of the CCI method is as follows:

Let $A = (a_{Lij}, a_{Mij}, a_{Uij})_{n \times n}$ be a fuzzy judgment matrix, and let $w = [(w_{L1}, w_{M1}, w_{U1}), (w_{L2}, w_{M2}, w_{U2}), \dots, (w_{Ln}, w_{Mn}, w_{Un})]^T$ be the priority vector derived from A using the RGMM. The centric consistency index (CCI) is computed by

$$CCI(A) = \frac{2}{(n-1)(n-2)} \sum_{i < j} \left(\log\left(\frac{a_{Lij} + a_{Mij} + a_{Uij}}{3}\right) - \log\left(\frac{w_{Li} + w_{Mi} + w_{Ui}}{3}\right) + \log\left(\frac{w_{Lj} + w_{Mj} + w_{Uj}}{3}\right) \right)^2.$$
(14)

When CCI(A) = 0, we consider A fully consistent. Aguarón et al. [24] also provide the thresholds (\overline{GCI}) as $\overline{GCI} = 0.31$ for n = 3; $\overline{GCI} = 0.35$ for n = 4 and $\overline{GCI} = 0.37$ for n > 4. When $CCI(A) < \overline{GCI}$, it is considered that the matrix A is sufficiently consistent. Since the CCI is a fuzzy extended version of the GCI, thresholds remain identical.

4. The main diesel engine selection problem in the shipping industry

The main diesel engine selection is one of the major steps in ship building projects and it is determined by an agreement between the ship building company and the ship owner. However, the number of available diesel engines for larger size hulls is very limited due to the increasing number of new building projects and the limited manufacturing facilities for diesel engines by corporations. Therefore, ship owners are expected to make critical decisions about financial and technical particulars when purchasing a diesel engine. The main diesel engine of a merchant ship is operated until the end of the life of a ship, so its selection is made with longevity in mind.

The factors affecting diesel engine selection are classified into two categories: technical aspects and financial aspects. Recent literature has a number of studies that investigate the technical aspects of the diesel engine selection problem. However, technically superior, many alternatives can be rejected due to their financial drawbacks. The financial state of a project has particular importance for the sustainability and practicability of the project.

Criteria designation is a highly difficult problem since many products do not provide detailed information about their performance and particulars. Watson [25] indicated a number of criteria for marine engine selection, but these particulars are mostly based on the manufacturer's viewpoint. Moreover, there are some particulars which can be estimated previously such as vibration and noise. Even these particulars are subject to change in different hull designs and engine settlements. On the other hand, some indicators may provide information about the performance of the engine such as time used in the existing fleet, previous damage records, etc.

This paper investigates the marine diesel engine problem for the Panamax size (around 75.000 DWT) dry bulk carrier. Experts are required for corresponding research on the intended particulars of the hull.

The expert consultation process of the study is performed in two steps. In the first step, a group of technical experts/managers of selected shipping companies is required to define criteria for the intended problem which can be suitable for pairwise comparison. This step is based on interviews and six major criteria are defined for further analysis as follows:

Criterion 1. Power (KW) Criterion 2. Cost of purchase (price) Criterion 3. Fuel consumption Criterion 4. Maintenance (Reliability and maintainability) Criterion 5. Majority in existing merchant fleet

Criterion 6. Damage history of model

Power is one of the most important particulars of the main diesel engine selection problem. The power of the engine is based on ship tonnage, expected weather/sea conditions and estimated service speed. Although higher engine power brings stronger resistance to several navigational drawbacks, increasing power also requires higher capacity, increased fuel consumption and more manufacturing processes which increase the purchasing price. According to optimum shaft speed and expected working power, candidate engines will be selected in a feasible range. In the case of the intended Panamax size ship, six alternatives are selected from the two main manufacturing companies that have between 8.000 and 14.000 KW capacity engines. Alternatives of the empirical work are based on 5, 6 and 7 cylinder models manufactured by the companies.

Since one-tenth of the cost of a new building project is the cost of the main diesel engine, *cost of purchase* should be considered as one of the major indicators of the financial feasibility of a project. Another major indicator of financial feasibility is derived from the *fuel consumption* and its relative accuracy compared to generated power. Estimated fuel consumption (kg) per hour is calculated by using corresponding specific fuel oil consumption (SFOC) particulars. Estimated fuel consumption is based on factory trials, but its practical results will be changed due to deterioration speed and the use of the engine. Therefore, expert consultation is required rather than the direct use of quantitative measures. *Maintenance* attributes of an engine have two considerations. Firstly, how easily it can be performed and secondly, how much does it cost. Both considerations are subject to practical experience and expert consultation is expected to reflect such judgmental viewpoints.

Under the rational choice assumption, every ship owner conducts a similar selection process to decide on the proper diesel engine. Therefore, a short way of determining a diesel engine's popularity is by looking at the *majority of a diesel engine's use in an existing merchant marine fleet.* If a specific model and brand of diesel engine is frequently preferred by other shipping companies, this may denote the superiority of that model in overall circumstances. A similar indicator is frequency of technical damages in the history of the existing fleet which is installed with the intended model of engine. The *damage history of the model* may illustrate its structural and mechanical resistance in the practical life of the engine.

There are also many other variable conditions which may substantially change the engine selection process. Special discounts, financial supports and guarantees are some examples of such immeasurable and unpredictable factors. For a specific shipping company, the use of existing models of the diesel engines in the company fleet may be a strong incentive to choose identical products. These factors depend on each case and to ensure generality, this paper assumes a standard and common condition rather than a particular project.

5. The empirical work and results

The application of the RPI aided FAHP model is conducted for the main diesel engine selection problem in the ship building industry and the criteria set is defined as in Table 2. The candidate models of the project are gathered from their industrial use and Table 3 lists them with acronyms. Models are from two major companies and according to the number of cylinders, a total of six alternatives are selected for subsequent analysis.

Table 4 shows the particulars of the diesel engine models. Although a limited number of technical dimensions of the models are available from related websites, the performance results and practical efficiency can only be derived from experts who have practical or managerial level experience with these models.

According to the classical FAHP analysis, purchase cost and popular use in the existing world fleet are the most important criteria; power, fuel consumption, damage history and maintenance follow them respectively (see Table 5). Table 6 indicates the RPI-aided FAHP analysis. Under the RPI-aided process, the maintenance of the alternatives is found to be a quite weak

Table 2					
The criteria	for	the	marine	engine	selection.

Criterion of the shipping asset selection	The symbols of each criterion
Power	PW
Cost of purchasing	CP
Fuel consumption	FC
Maintenance	MN
Majority in existing merchant fleet	MF
Damage history of model	DH

Table 3

The alternatives for the marine engine selection.

Alternatives for the shipping asset selection	The symbols of each alternatives
German marine engine company 5 cylinder model	G5C
German marine engine company 6 cylinder model	G6C
German marine engine company 7 cylinder model	G7C
Finnish marine engine company 5 cylinder model	F5C
Finnish marine engine company 6 cylinder model	F6C
Finnish marine engine company 7 cylinder model	F7C

Table 4

Table 5

The features of the alternatives for the marine engine selection.

Model	Number of stroke	Weight (tons)	Dimension	Fuel consumption
G5C	2	324	$7122 \times 11,100 \times 3770$	2070.6
G6C	2	368	$8142 \times 11,100 \times 3770$	2484.7
G7C	2	238	$7624 \times 9335 \times 3150$	2033.5
F5C	2	124	$5582 \times 9282 \times 3150$	1724.8
F6C	2	124	$6462\times9282\times3150$	1769.4
F7C	4	428	$8810\times5410\times2300$	1725.8

The classical aggregated fuzzy judgment matrix for criteria of marine engine selection.

	PW	СР	FC	MN	MF	DH	Weight
PW	(1, 1, 1)	(0.13, 0.25, 0.56)	(0.76, 1.00, 1.00)	(1.00, 2.55, 4.78)	(0.33, 1.00, 1.00)	(1.24, 3.44, 5.32)	(0.18)
CP	(1.79, 4.00, 7.69)	(1, 1, 1)	(2.66, 4.75, 6.97)	(1.00, 2.58, 3.89)	(1.00, 1.00, 2.55)	(1.18, 3.32, 5.51)	(0.24)
FC	(1.00, 1.00, 1.32)	(0.14, 0.21, 0.38)	(1, 1, 1)	(1.24, 3.44, 5.32)	(0.20, 0.33, 1.00)	(1.00, 1.00, 3.00)	(0.15)
MN	(0.21, 0.39, 1.00)	(0.26, 0.39, 1.00)	(0.19, 0.29, 0.81)	(1, 1, 1)	(0.14, 0.20, 0.33)	(0.20, 0.33, 1.00)	(0.06)
MF	(1.00, 1.00, 3.03)	(0.39, 1.00, 1.00)	(1.00, 3.00, 5.00)	(3.00, 5.00, 7.00)	(1, 1, 1)	(0.22, 0.35, 0.97)	(0.20)
DH	(0.19, 0.29, 0.81)	(0.18, 0.30, 0.85)	(0.33, 1.00, 1.00)	(1.00, 3.00, 5.00)	(1.03, 2.86, 4.55)	(1, 1, 1)	(0.17)
CCI = ().10						

Table 6

The rotational matrix for criteria.

Criteria	R_1	<i>R</i> ₂	<i>R</i> ₃	<i>R</i> ₄	<i>R</i> ₅	<i>R</i> ₆	Weight
PW		0.25	0.23	0.19	0.24	0.16	0.18
СР	0.29		0.30	0.36	0.40	0.35	0.28
FC	0.19	0.21		0.11	0.20	0.17	0.15
MN	0.04	0.04	0.03		0.01	0.00	0.02
MF	0.26	0.27	0.23	0.30		0.31	0.23
DH	0.22	0.23	0.21	0.04	0.15		0.14

factor (just 0.02). After the classical FAHP analysis, figures of the priorities have similarity, but the last three criteria have important changes. For example, Maintenance has 0.06 priority weight in FAHP and it declines to 0.02 after the RPI process. Conversely, the popular use in the world fleet increases to 0.23 from 0.20 while the priority of damage history is diminished to 0.14 from 0.17.

Table 7 shows how priorities are spread over alternatives under each criterion. The German company ensures stronger and cheaper products. These models are also easy to repair and their reputation in existing fleets is rated over competitors. However, the Finnish company presents models that are less costly to operate due to their unique advantage of economic fuel consumption.

The RPI process for alternatives is presented in Table 8. Since there are six alternatives, the process is completed in six rotational investigations. *R* refers to every single rotation and the priorities of every rotation are indicated below. The rank reversal phenomenon exists in a number of rotational investigations. For example, the priorities of alternatives under maintenance criterion change when alternative F5C is extracted. G6C is superior to G7C, but it is reversed in the case of the extraction of F5C. A similar effect is recorded between G5C and G6C in the cost of purchasing criterion. A very considerable reverse effect is noted in the power criterion. In classical FAHP, F6C is removed from criteria since its priority weight is zero. However, it is superior to F7C in many rotations. Even by extraction of G7C, F7C is removed and F6C has 0.09 priority weight. These findings have two important outcomes. Firstly, rank reversal effect is confirmed again and it points out how results

Table /	Та	ble	: 7
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The classical aggregated fuzzy judgment matrix for alternatives of the marine engines under each criterion.

	G5C	G6C	G7C	F7C	F5C	F6C	Weight
PW G5C G6C G7C F7C F5C F6C CCI = 0.21	(1, 1, 1) (1.00, 1.00, 1.32) (0.68, 0.82, 0.99) (0.18, 0.30, 0.85) (0.14, 0.20, 0.33) (0.11, 0.14, 0.21)	$\begin{array}{c} (0.76, 1.00, 1.00) \\ (1, 1, 1) \\ (0.81, 0.88, 0.97) \\ (0.14, 0.20, 0.33) \\ (0.11, 0.13, 0.20) \\ (0.11, 0.16, 0.20) \end{array}$	(1.01, 1.22, 1.46) (1.03, 1.14, 1.23) (1, 1, 1) (1.03, 2.86, 4.55) (0.12, 0.18, 0.31) (0.11, 0.14, 0.20)	$\begin{array}{c} (1.18, 3.32, 5.51) \\ (3.00, 5.00, 7.00) \\ (0.22, 0.35, 0.97) \\ (1, 1, 1) \\ (1.00, 1.00, 1.32) \\ (0.62, 2.33, 2.86) \end{array}$	(3.00, 5.00, 7.00) (5.09, 7.42, 8.89) (3.27, 5.56, 8.01) (0.76, 1.00, 1.00) (1, 1, 1) (1.05, 3.33, 5.26)	(4.86, 6.94, 8.78) (4.97, 6.45, 8.81) (5.00, 7.00, 9.00) (0.35, 0.43, 1.62) (0.19, 0.30, 0.95) (1, 1, 1)	0.32 0.36 0.27 0.04 0.00 0.00
CP G5C G6C G7C F7C F5C F6C CCI = 0.13	$\begin{array}{c} (1,1,1)\\ (1.00,1.00,1.32)\\ (0.19,0.29,0.81)\\ (0.11,0.11,0.14)\\ (0.11,0.14,0.20)\\ (0.11,0.14,0.20)\end{array}$	$\begin{array}{c} (0.76,1.00,1.00)\\ (1,1,1)\\ (0.20,0.39,1.15)\\ (0.11,0.11,0.14)\\ (0.11,0.11,0.14)\\ (0.11,0.14,0.20) \end{array}$	(1.24, 3.44, 5.32) (0.87, 2.56, 4.98) (1, 1, 1) (0.13, 0.19, 0.33) (0.14, 0.20, 0.33) (0.14, 0.20, 0.33)	(7.00, 9.00, 9.00) (7.00, 9.00, 9.00) (3.02, 5.14, 7.66) (1, 1, 1) (0.33, 1.00, 1.00) (0.39, 1.00, 1.00)	(5.12, 7.11, 9.00) (7.20, 9.00, 9.00) (3.00, 5.00, 7.00) (1.00, 1.00, 3.00) (1, 1, 1) (0.33, 1.00, 1.00)	$\begin{array}{c} (5.00,7.00,9.00)\\ (5.00,7.00,9.00)\\ (3.00,5.00,7.00)\\ (1.00,1.00,2.55)\\ (1.00,1.00,3.00)\\ (1,1,1) \end{array}$	0.38 0.39 0.23 0.00 0.00 0.00
FC G5C G6C G7C F7C F5C F6C CCI = 0.10	(1, 1, 1) (1.00, 1.00, 1.00) (1.00, 1.00, 1.00) (1.00, 3.00, 5.00) (1.00, 3.00, 5.00) (3.00, 5.00, 7.00)	(1.00, 1.00, 1.00) (1, 1, 1) (1.00, 1.00, 1.00) (1.00, 1.00, 3.00) (1.00, 1.00, 3.00) (1.00, 1.00, 3.00)	(1.00, 1.00, 1.00) (1.00, 1.00, 1.00) (1, 1, 1) (3.00, 5.00, 7.00) (5.00, 7.00, 9.00) (5.00, 7.00, 9.00)	(0.44, 0.63, 1.00) (0.76, 1.00, 1.00) (0.14, 0.20, 0.33) (1, 1, 1) (1.00, 1.00, 3.00) (1.00, 1.00, 3.00)	(0.20, 0.33, 1.00) (0.42, 1.00, 1.00) (0.11, 0.14, 0.20) (0.33, 1.00, 1.00) (1, 1, 1) (1.00, 1.00, 3.00)	(0.18, 0.19, 0.33) (0.33, 1.00, 1.00) (0.11, 0.14, 0.20) (0.33, 1.00, 1.00) (0.33, 1.00, 1.00) (1, 1, 1)	0.00 0.01 0.00 0.29 0.33 0.36
MN G5C G6C G7C F7C F5C F6C CCI = 0.09	(1, 1, 1) (1.00, 1.00, 1.00) (1.00, 1.00, 1.00) (0.11, 0.11, 0.14) (0.14, 0.20, 0.33) (0.14, 0.20, 0.33)	(1.00, 1.00, 1.00) (1, 1, 1) (1.00, 1.00, 1.00) (0.11, 0.11, 0.14) (0.14, 0.20, 0.33) (0.14, 0.20, 0.33)	(1.00, 1.00, 1.00) (1.00, 1.00, 1.00) (1, 1, 1) (0.11, 0.14, 0.20) (0.20, 0.33, 1.00) (0.11, 0.14, 0.20)	(6.85, 8.45, 8.70) (6.85, 8.45, 8.70) (5.21, 7.86, 9.00) (1, 1, 1) (1.00, 3.00, 5.00) (1.00, 1.00, 3.00)	(3.00, 5.00, 7.00) (4.12, 5.63, 8.41) (1.00, 3.00, 5.00) (0.20, 0.33, 1.00) (1, 1, 1) (0.20, 0.33, 1.00)	(3.11, 5.46, 7.88) (3.00, 5.00, 7.00) (5.00, 7.00, 9.00) (0.76, 1.00, 1.00) (1.00, 3.00, 5.00) (1, 1, 1)	0.31 0.31 0.30 0.00 0.08 0.00
MF G5C G6C G7C F7C F5C F6C CCI = 0.29	(1, 1, 1) (1.00, 1.00, 1.00) (0.11, 0.14, 0.20) (0.14, 0.20, 0.33) (0.14, 0.20, 0.33) (0.11, 0.11, 0.14)	$\begin{array}{c} (1.00, 1.00, 1.00) \\ (1, 1, 1) \\ (0.14, 0.20, 0.33) \\ (0.11, 0.14, 0.20) \\ (0.14, 0.20, 0.33) \\ (0.11, 0.14, 0.20) \end{array}$	(4.78, 6.68, 8.74) (2.88, 4.52, 6.61) (1, 1, 1) (0.20, 0.33, 1.00) (0.20, 0.33, 1.00) (1.00, 1.00, 3.00)	(3.00, 5.00, 7.00) (4.77, 6.79, 8.84) (1.00, 3.00, 5.00) (1, 1, 1) (1.00, 1.00, 3.00) (5.00, 7.00, 9.00)	(3.00, 5.00, 7.00) (3.00, 5.00, 7.00) (1.00, 3.00, 5.00) (0.33, 1.00, 1.00) (1, 1, 1) (5.00, 7.00, 9.00)	(7.00, 9.00, 9.00) (5.12, 7.46, 9.00) (0.33, 1.00, 1.00) (0.14, 0.20, 0.33) (0.11, 0.14, 0.20) (1, 1, 1)	0.37 0.35 0.05 0.00 0.00 0.22
DH G5C G6C G7C F7C F5C F6C CCI = 0.23	(1, 1, 1) (1.00, 1.00, 1.00) (0.11, 0.14, 0.20) (0.11, 0.11, 0.14) (0.14, 0.20, 0.33) (0.11, 0.14, 0.20)	(1.00, 1.00, 1.00) (1, 1, 1) (0.14, 0.20, 0.33) (0.11, 0.14, 0.20) (0.11, 0.13, 0.20) (0.11, 0.14, 0.20)	(4.55, 6.78, 8.41) (2.55, 4.78, 6.97) (1, 1, 1) (0.14, 0.20, 0.33) (0.20, 0.33, 1.00) (0.20, 0.33, 1.00)	(8.11, 9.00, 9.00) (5.12, 7.46, 9.00) (3.00, 5.00, 7.00) (1, 1, 1) (0.14, 0.20, 0.33) (1.00, 1.00, 1.00)	(3.00, 5.00, 7.00) (3.00, 5.00, 7.00) (1.05, 3.14, 5.26) (3.00, 5.00, 7.00) (1, 1, 1) (0.33, 1.00, 1.00)	(5.00, 7.00, 9.00) (5.12, 7.46, 9.00) (1.00, 3.00, 5.00) (0.76, 1.00, 1.00) (1.00, 1.00, 3.00) (1, 1, 1)	0.44 0.40 0.16 0.00 0.00 0.00

may change in the classical FAHP. The rational choice assumption is clearly questionable. However, the RPI method provides an additional detailed investigation of relative priorities under the sub-groups and priority weights are revised. Although, other alternatives keep their relative positions in the pair-wise analysis under power of engine criterion, F7C reduces its contribution and F6C is in approximately the same priority level after the RPI process.

Tables 9 and 10 indicate the final results of the classical FAHP and the RPI-aided FAHP. In the classical form, the FAHP method recommends two major diesel engines which are G5C and G6C. The RPI-aided results indicate that G6C is slightly superior to G5C. Moreover, the F7C and F5C alternatives are reversed in the RPI process.

The interpretation of indifference in AHP applications is still an existing gap. It is obvious that a numerical value (e.g. 0.01) can be negligible for a particular example while it can be quite change-making for another application field. The presented empirical work is an illustration of "how the proposed approach works in practice". Therefore, we are not particularly interested in the example. In practice, however, several examples with different outcomes and a variety of different perceptions could be found. We are not able to judge whether a subjectively small change can produce great differences in the outcome. Rather than heuristically interpreting the difference, we left this dimension to the practical users who know well the meaningful difference value as a result of experience and professional insights. Therefore, we subjectively recognize the small

Table 8		
The netational	 6	- 1+ + +

The rotational matrix for alternatives under each criteria
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	<i>R</i> ₁	<i>R</i> ₂	<i>R</i> ₃	<i>R</i> ₄	R ₅	<i>R</i> ₆	Weight
PW							
G5C		0.38	0.42	0.32	0.32	0.30	0.29
G6C	0.50		0.49	0.36	0.34	0.39	0.34
G7C	0.37	0.34		0.32	0.24	0.22	0.25
F7C	0.04	0.13	0.00		0.10	0.09	0.06
F5C	0.00	0.00	0.00	0.00		0.00	0.00
F6C	0.09	0.15	0.09	0.00	0.00		0.06
CP							
C5C		0.61	0.48	0.38	0.30	0.30	0.37
650	0.63	0.01	0.40	0.30	0.35	0.55	0.37
G0C	0.05	0.30	0.52	0.33	0.38	0.41	0.33
G7C F7C	0.00	0.00	0.00	0.25	0.25	0.20	0.24
F5C	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FGC	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	0.00	0.00	0.00	0.00	0.00		0.00
FC							
G5C		0.00	0.07	0.00	0.00	0.00	0.02
G6C	0.03		0.12	0.00	0.00	0.03	0.03
G7C	0.00	0.00		0.00	0.00	0.00	0.00
F7C	0.29	0.29	0.25		0.44	0.44	0.29
F5C	0.34	0.34	0.26	0.48		0.50	0.32
F6C	0.34	0.37	0.29	0.52	0.56		0.35
MN							
G5C		0.43	0.44	0.28	0.33	0.35	0.31
G6C	0.44		0.45	0.28	0.32	0.37	0.31
G7C	0.42	0.42		0.27	0.35	0.29	0.29
F7C	0.00	0.00	0.00		0.00	0.00	0.00
F5C	0.14	0.15	0.11	0.17		0.00	0.09
F6C	0.00	0.00	0.00	0.00	0.00		0.00
ME							
MF		0.55	0.27	0.54	0.45	0.41	0.20
GSC	0.50	0.55	0.37	0.54	0.45	0.41	0.39
GBC	0.50	0.10	0.37	0.46	0.42	0.41	0.36
G/C	0.14	0.10	0.00	0.00	0.00	0.18	0.07
F/C	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FSC	0.00	0.00	0.00	0.00	0.12	0.00	0.00
FOC	0.36	0.35	0.26	0.00	0.13		0.18
DH							
G5C		0.72	0.51	0.44	0.50	0.46	0.44
G6C	0.59		0.49	0.41	0.43	0.40	0.39
G7C	0.33	0.28		0.15	0.07	0.14	0.16
F7C	0.07	0.00	0.00		0.00	0.00	0.01
F5C	0.00	0.00	0.00	0.00		0.00	0.00
F6C	0.00	0.00	0.00	0.00	0.00		0.00

Table 9

The final result for the classical FAHP method.

PW CP FC MN MF DH Weight of criteria Alternatives 0.18 0.24 0.15 0.06 0.20 0.17 G5C 0.32 0.38 0.00 0.31 0.37 0.44 0.32 G6C 0.36 0.39 0.01 0.31 0.35 0.40 0.32 G7C 0.27 0.23 0.00 0.30 0.05 0.16 0.16 F5C 0.00 0.00 0.33 0.08 0.00 0.00 0.06 F6C 0.00 0.00 0.36 0.00 0.22 0.00 0.10 F7C 0.04 0.00 0.29 0.00 0.00 0.05 0.16								
Weight of criteria 0.18 0.24 0.15 0.06 0.20 0.17 Alternatives		PW	СР	FC	MN	MF	DH	
Alternatives Alternative priority weight G5C 0.32 0.38 0.00 0.31 0.37 0.44 0.32 G6C 0.36 0.39 0.01 0.31 0.35 0.40 0.32 G7C 0.27 0.23 0.00 0.30 0.05 0.16 0.16 F5C 0.00 0.00 0.33 0.08 0.00 0.06 F6C 0.00 0.00 0.29 0.00 0.00 0.05 0.10 F7C 0.04 0.00 0.29 0.00 0.00 0.05 0.00	Weight of criteria	0.18	0.24	0.15	0.06	0.20	0.17	
C5C0.320.380.000.310.370.440.32C6C0.360.390.010.310.350.400.32G7C0.270.230.000.300.050.160.16F5C0.000.000.330.080.000.000.06F6C0.000.000.360.000.220.000.10F7C0.040.000.290.000.000.000.05	Alternatives							Alternative priority weight
G6C0.360.390.010.310.350.400.32G7C0.270.230.000.300.050.160.16F5C0.000.000.330.080.000.000.06F6C0.000.000.360.000.220.000.10F7C0.040.000.290.000.000.05	G5C	0.32	0.38	0.00	0.31	0.37	0.44	0.32
G7C0.270.230.000.300.050.160.16F5C0.000.000.330.080.000.000.06F6C0.000.000.360.000.220.000.10F7C0.040.000.290.000.000.05	G6C	0.36	0.39	0.01	0.31	0.35	0.40	0.32
F5C0.000.000.330.080.000.000.06F6C0.000.000.360.000.220.000.10F7C0.040.000.290.000.000.000.05	G7C	0.27	0.23	0.00	0.30	0.05	0.16	0.16
F6C 0.00 0.00 0.36 0.00 0.22 0.00 0.10 F7C 0.04 0.00 0.29 0.00 0.00 0.05	F5C	0.00	0.00	0.33	0.08	0.00	0.00	0.06
F7C 0.04 0.00 0.29 0.00 0.00 0.00 0.05	F6C	0.00	0.00	0.36	0.00	0.22	0.00	0.10
	F7C	0.04	0.00	0.29	0.00	0.00	0.00	0.05

numbers which have limitations on the assessment of practical meaning of significance or difference values. For example, in this paper, we deal with an engine selection for a conventional merchant ship. A small number may contribute to a significant financial gain (less fuel consumption, less maintenance cost, easy operation-less profession, etc.). Saving of a 100-liter of fuel oil (daily consumption is usually 10–15 metric tons) means approx. 150,000 USD less fuel cost in ten years (0.4 USD per liter). In addition, repair and maintenance costs and spare parts issues should be included. For this particular example, a 0.01 difference value may have a 150,000 USD value in business practice. Conversely, we do not deal with the practical valuations and leave it for the professionals to decide.

Table 10			
The final result	for the FAHP	method under	RAPI process.

	PW	СР	FC	MN	MF	DH	
Weight of criteria	0.18	0.28	0.15	0.02	0.23	0.14	
Alternatives							Alternative priority weight
G5C	0.29	0.37	0.02	0.31	0.39	0.44	0.31
G6C	0.34	0.39	0.03	0.31	0.36	0.39	0.32
G7C	0.25	0.24	0.00	0.29	0.07	0.16	0.16
F5C	0.00	0.00	0.32	0.09	0.00	0.00	0.06
F6C	0.06	0.00	0.35	0.00	0.18	0.00	0.05
F7C	0.06	0.00	0.29	0.00	0.00	0.01	0.10

6. Summary and conclusions

Several studies emphasized that the classical FAHP method has weaknesses in the use of scale values and a possible conflict with the rational choice theory. According to these criticisms, experts are limited on rating elements and they are free of building contrary decisions. The fuzzy extended AHP improved the use of linguistic scales and provided clusters of decision spaces. The use of FAHP reduces the task load of participants to some degree, but it also increases the task load of the facilitator. The major weakness of the FAHP is based on element removal. The most used FAHP method, the extent synthesis analysis, can remove several elements in the decision hierarchy and sensitivity to the decision is lost. The proposed RPI algorithm is an extended analysis approach which reveals hidden priorities in selection problems. By using the RPI method, a rank reversal phenomenon is noted at once and the final results focus on relative priorities under sub-group clusters. The RPI process also improves the extent synthesis method by including many previously ignored elements in the problem.

The rotational approach is a post-survey treatment of rank reversal problem. Therefore, it brings an additional procedure to the conventional one. It is still an existing gap to find a pre-survey treatment for the problem. The source of rank reversal problem is the uncertainty of expert judgments and their imbalanced outcomes. From that point, AHP surveys are usually rejected from the rational choice theory. Based on the bounded rationality or rational irrationality perspectives, non-compliance can be ignored to retain expert knowledge. In such case, future work is expected to improve surveys to eliminate such drawbacks including the scale limitations.

References

- [1] G.S. Becker, The Economic Approach to Human Behavior, University of Chicago Press, 1976.
- [2] T.L. Saaty, A scaling method for priorities in hierarchical structures, J. Math. Psychol. 15 (1977) 234–281.
- [3] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338-353.
- [4] T.L. Saaty, Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process, RWS Publications, 2000.
- [5] L. Mikhailov, P. Tsvetinov, Evaluation of services using a fuzzy analytic hierarchy process, Appl. Soft Comput. 5 (2004) 23-33.
- [6] A. Awasthi, S.S. Chauhan, A hybrid approach integrating affinity diagram, AHP and fuzzy TOPSIS for sustainable city logistics planning, Appl. Math. Model. 36 (2012) 573–584.
- [7] Y.-H. Chen, T.-C. Wang, C.-Y. Wu, Multi-criteria decision making with fuzzy linguistic preference relations, Appl. Math. Model. 35 (2011) 1322–1330.
- [8] Z. Ayağ, R.G. Özdemir, A fuzzy AHP approach to evaluating machine tool alternatives, J. Intell. Manuf. 17 (2006) 179–190.
- [9] M. Dağdeviren, S. Yavuz, N. Kilınç, Weapon selection using the AHP and TOPSIS methods under fuzzy environment, Expert Syst. Appl. 36 (2009) 8143-8151.
- [10] O. Kilincci, S.A. Onal, Fuzzy AHP approach for supplier selection in a washing machine company, Expert Syst. Appl. 38 (2011) 9656–9664.
- [11] İ. Korkmaz, H. Gökçen, T. Çetinyokuş, An analytic hierarchy process and two-sided matching based decision support system for military personnel assignment, Inf. Sci. 178 (2008) 2915–2927.
- [12] C.G. Şen, G. Çınar, Evaluation and pre-allocation of operators with multiple skills: a combined fuzzy AHP and max-min approach, Expert Syst. Appl. 37 (2010) 2043–2053.
- [13] P.J.M. Van Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets Syst. 11 (1983) 199–227.
- [14] J.J. Buckley, Fuzzy hierarchical analysis, Fuzzy Sets Syst. 17 (1985) 233–247.
- [15] C.G.E. Boender, J.G. de Graan, F.A. Lootsma, Multi-criteria decision analysis with fuzzy pairwise comparisons, Fuzzy Sets Syst. 29 (1989) 133-143.
- [16] D.-Y. Chang, Applications of the extent analysis method on fuzzy AHP, Eur. J. Oper. Res. 95 (1996) 649-655.
- [17] A. Ishizaka, A. Labib, Analytic hierarchy process and expert choice: benefits and limitations, OR Insight 22 (2009) 201–220.
- [18] G.C. Roper-Lowe, J.A. Sharp, The analytic hierarchy process and its application to an information technology decision, J. Oper. Res. Soc. 41 (1990) 49–59.
- [19] R.D. Holder, Some comments on the analytic hierarchy process, J. Oper. Res. Soc. 41 (1990) 1073-1076.
- [20] T.L. Saaty, The analytic hierarchy process: planning, priority setting, resource allocation, McGraw-Hill International Book Co., 1980.
- [21] E. Bulut, O. Duru, T. Keçeci, S. Yoshida, Use of consistency index, expert prioritization and direct numerical inputs for generic fuzzy-AHP modeling: a process model for shipping asset management, Expert Syst. Appl. 39 (2012) 1911–1923.
- [22] O. Duru, E. Bulut, S. Yoshida, Regime switching fuzzy AHP model for choice-varying priorities problem and expert consistency prioritization: a cubic fuzzy-priority matrix design, Expert Syst. Appl. 39 (2012) 4954–4964.
- [23] G. Crawford, C. Williams, A note on the analysis of subjective judgment matrices, J. Math. Psychol. 29 (1985) 387-405.
- [24] J. Aguarón, J.M. Moreno-Jiménez, The geometric consistency index: approximated thresholds, Eur. J. Oper. Res. 147 (2003) 137–145.
- [25] D.G.M. Watson, Practical Ship Design, Elsevier, 2002.