2007-03-13

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CULVERT HYDRAULICS: COMPARISON OF CURRENT COMPUTER MODELS

by

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A thesis submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Master of Science

Department of Civil and Environmental Engineering

Brigham Young University

April 2007

BRIGHAM YOUNG UNIVERSITY

GRADUATE COMMITTEE APPROVAL

of a thesis submitted by

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This thesis has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

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ABSTRACT

CULVERT HYDRAULICS: COMPARISON OF CURRENT COMPUTER MODELS

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The hydraulic analysis of culverts is complicated when using hand calculations. Fortunately, several computer programs are available to assist in analyzing culvert hydraulics, some of which include HY-8, Fish X-ing, Broken-back Culvert Analysis Program (BCAP), Hydraflow Express, Culvert Master, Culvert, and Hydrologic Engineering Center River Analysis System (HEC-RAS). While all of these programs can simulate the behavior of flow through a culvert, slightly different methodologies are utilized among the programs to complete a full hydraulic analysis, resulting in different predictions for headwater depth, flow control, and outlet velocities. The purpose of this paper is to compare (1) the available hydraulic features and (2) the numerical solutions from the seven programs to manually computed values.

Four test cases were developed to test the accuracy of program results. The headwater depths and outlet velocities were compared to those obtained through calculations based on culvert hydraulic theory outlined in the Federal Highway Administration publication, Hydraulic Design Series 5.

Based on the results, Fish X-ing was unable to analyze culverts under inlet control, while Culvert incorrectly predicted inlet control headwater depths at low flow conditions. Hydraflow Express struggled to predict correct outlet control headwater depths while BCAP had difficulty analyzing straight barrel culverts acting under outlet control. Overall, HY-8, Culvert Master, and HEC-RAS produced accurate results most consistently.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Rollin H. Hotchkiss, for his patience and support in helping me understand culvert hydraulics. I would also like to thank Dr. Jim Nelson and Dr. Alan Zundel for serving on my graduate committee and providing me guidance when needed, as well as Chris Smemoe for his assistance in helping me understanding the workings of HY-8.

I also wish to thank Phil Thompson for answering the many emails requesting clarification of aspects of culvert hydraulics, especially the polynomial curves used in HY-8.

Finally, I wish to thank my husband Josh, for his love and patience during the many hours I spent working on my thesis, and for being willing to drive to campus countless times bringing me candy and support to get me through the day.

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1 Introduction and Purpose

Culverts are structures designed to pass flow underneath roadways with minimal disturbance to surrounding areas (1). Traditionally, design requirements included not exceeding policy headwater depths or outlet velocities. However, more recent design considers the ability of fish to pass to upstream areas. With these requirements, accurate analysis and design becomes quite important.

The analysis of culvert performance can be solved through a series of hand calculations based on simplifying assumptions. However, had calculations are tedious and prone to error, and as a result it became desirable to automate the calculations required to solve culvert performance. The first aids in culvert analysis came from a series of nomographs published in the 1965 version of HEC-5, Hydraulic Charts for the Selection of Highway Culverts. In the 1960's the Federal Highway Administration also began using FORTRAN and other programs to automate culvert hydraulic computations, establishing a milestone in culvert analysis (2).

Today, several computer programs exist to aid in solving culvert hydraulic problems. The purpose of this research is to study seven of those programs, listed in Table 1-1, and compare the features and capabilities of each. Test cases based on numerical hand calculations are used to compare headwater depths and outlet velocities produced by each program. No laboratory experiments are utilized. Based on the results of the research, limitations are identified to help inform users of the strengths and weakness of the various culvert hydraulic programs.

Table 1-1 Culvert Analysis Programs and Authors

Program	Author/Agency	Domain	Reference
HY-8	Federal Highway Administration	Public	(7)
Fish X-ing	US Forest Service	Public	(9)
BCAP	Nebraska Department of Roads	Public	(10)
Hydraflow Express	Intelisolve	Commercial	(11)
Culvert Master	Haestad Methods	Commercial	(12)
Culvert	Texas Department of Transportation	Public	(13)
HEC-RAS	US Army Corps of Engineers	Public	(14)

Of the programs listed in Table 1-1, HY-8 and HEC-RAS were compared in a study by Ahmed Kassem, Ahmed A. Sattar, and M. Hanif Chaudhry (3). In this study, the two programs were compared for the purpose of developing a procedure to assist in software selection. As a result of the variety of programs available to assist in culvert hydraulic analysis, this paper will focus on seven, rather than two programs. Unlike Kassem et al., this paper will include a more focused and detailed analysis of program accuracy by focusing strictly on inlet and outlet control headwater depth and outlet velocities predictions in the test cases.

2 Culvert Hydraulics Background

While culverts are simple structures, their analysis is complex because hydraulic behavior can vary dramatically. Flow through a culvert is generally characterized by gradually or rapidly varying flow, and may also include the presence of a hydraulic jump (1). To determine the flow profile through a culvert barrel, gradually varied flow calculations are completed through the use of normal, critical, and tailwater depths as boundary conditions. The depth of flow through a culvert is always approaching the boundary depths at the inlet and exit of the barrel. Energy and momentum calculations are required to determine the presence and location of a hydraulic jump (1). Flow through a culvert is controlled by either the barrel inlet or outlet, and the control may change by simply increasing or decreasing the flow rate, slope, and tailwater depth. The headwater depth at the entrances is directly affected by whether the flow control is at the inlet or the outlet of the culvert. Therefore it is important in design to determine which control produces the highest headwater depths in order to prevent problems such as flow overtopping the roadway and flooding of surrounding areas.

The classification of gradually varied flow profiles in culvert barrels is defined by the type of slope on which they exist, as well as the boundary conditions. When normal depth is greater than critical depth throughout the barrel, the slope is mild with the upstream boundary depth equal to normal, while critical or tailwater depth acts as the downstream boundary. On steep slopes, critical depth is greater than normal and is used for the upstream boundary depth; normal or tailwater depth acts as the downstream boundary (4). In the hydraulic analysis of culverts, boundary conditions govern the water depths at the inlet and outlet of the culvert. This is important in forewater and backwater calculations when computing headwater depths and outlet velocities. For a more detailed explanation of water surface profiles and boundary conditions see Appendix A.

While it is standard procedure in culvert hydraulic computations to use critical depth at the outlet, for tailwater depths less than critical the true depth at the outlet is slightly below critical; also known as brink depth (5). Critical depth actually occurs just inside the culvert outlet and from there the water surface moves through brink depth as it meets the tailwater downstream of the culvert (Figure 2-1). In this paper, all computations were made using the standard culvert hydraulic procedure where the outlet depth is equal to critical. Brink depth was not considered.

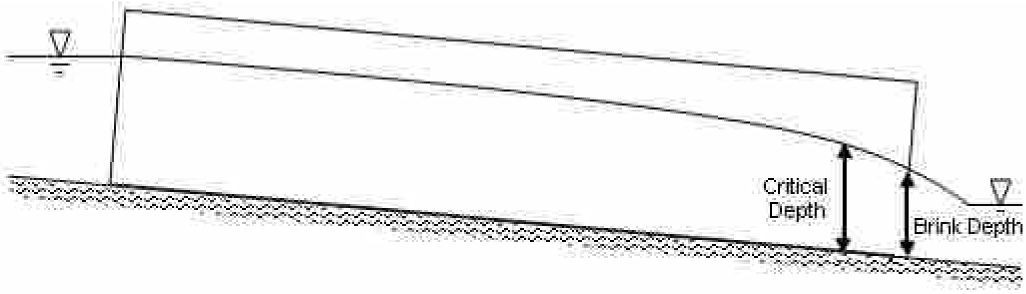

Figure 2-1 Brink Depth

3 Computer Programs

While there are several culvert hydraulic programs that exist commercially, privately, and in public domain, only seven of those programs were studied in this research: HY-8, Fish X-ing, BCAP, Hydraflow Express, Culvert Master, Culvert, and HEC-RAS (Table 1). The criteria for selecting these particular programs for research were based on functionality and availability. To obtain a useful comparison of available culvert hydraulic software it was necessary to select programs that offered a wide range of features from fish passage to broken-back culvert analysis. The programs listed in Table 1-1 were also the most readily available at the time of the study.

3.1 HY-8

The first version of HY-8 was developed for the Federal Highway Administration using a Quick Basic compiler (6). Philip Thompson of the FHWA later released other versions of HY-8 (7). Until recently, HY-8 was a DOS-based computer program with limited graphical capabilities. However, the program has recently been translated into the C++ programming language with a new graphical user interface compatible with the MS Windows operating system (8). The new Windows version of HY-8 includes superior graphics and plotting capabilities when compared to its older counterparts. The updated version also includes a new report generation feature. The primary function of HY-8 is to compute headwater depths at the entrance of culverts.

3.2 Fish X-ing

The United States Forest Service developed Fish X-ing for the purpose of assessing and designing culvert crossings suitable for fish passage. Utilizing gradually varied flow equations, Fish X-ing analyzes culvert crossings by computing water surface profiles for a range of flows (9). The program compares the hydraulic flow conditions within a culvert to the swimming abilities of fish to determine if a particular culvert is friendly to fish passage.

3.3 Broken-back Culvert Analysis Program (BCAP)

The Nebraska Department of Roads developed the Broken Back Culvert Analysis Program in 1998 (10). The primary goal behind the development of this program was to automate the analysis of culverts containing one or two break elevations. Another strength of the program is its ability to determine the presence of hydraulic jumps in culverts through the use of the momentum equation.

3.4 Hydraflow Express

Hydraflow Express, by Intelisolve, was developed for quick culvert analysis along with other hydraulic and hydrologic problems. The program is capable of calculating hydraulic profiles as well as rating tables for the following shapes: box, elliptical, circular, and arch. Also included in the program is the ability to analyze hydraulic jumps and roadway overtopping (11). The hydraulic theory utilized in the program is taken from HDS-5 (1).

3.5 Culvert Master

Culvert Master, developed by Haestad Methods, computes headwater depths at the entrance of culverts (12). The program allows for input of watershed information to obtain rainfall and runoff values that will eventually pass through the culvert barrel.

3.6 Culvert

The Texas Department of Transportation developed Culvert for use in designing highway culverts. Version 1.2, released in 2002, allows for the analysis of straight culvert barrels as well as culverts with single or double break elevations (13).

3.7 HEC-RAS

Developed by the US Army Corps of Engineers, HEC-RAS is a multi-purpose program with the capabilities of analyzing steady and unsteady flow conditions. However, HEC-RAS is a more complex program in that it was designed for river analysis and is more input intensive. As a result, a culvert is analyzed as a part of the stream network, where the upstream cross sections, velocities, and flow contractions are considered in the culvert hydraulic analysis—a feature unique only to HEC-RAS of the seven programs studied (14). In order to generate a complete culvert model, four stream cross sections are required (14).

4 Program Feature Comparison

Of the seven programs analyzed, each one has different characteristics that make it unique, ranging from the most complex program in terms of user input, HEC-RAS, to the most basic, Hydraflow Express. Table 4-1 lists and compares the hydraulic features found in each program.

Table 4-1 Program Feature and Capability Comparison

	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
Roadway Overtopping	▨			▨	▨	▨	▨
Multiple Identical Barrels	▨		▨	▨	▨	▨	▨
Inlet and Outlet Control	▨	▨	▨	▨	▨	▨	▨
Water Surface Profile Plots	▨	▨	▨	▨			▨
Full Flow Option	▨						
Hydraulic Jumps		▨	▨	▨			▨
Culvert Break Points			▨			▨	
Partially Filled Culverts		▨					▨
Adverse Slope Analysis		▨		▨	▨		
Horizontal Slope Analysis		▨	▨	▨	▨		
Fish Passage		▨					

All programs with the exception of Fish X-ing and BCAP can analyze roadway overtopping. While roadway overtopping is a major concern in the design of culverts, the primary focus of Fish X-ing is the suitability of a culvert for fish passage (9). The primary importance of BCAP is the analysis of hydraulic jumps as well as broken back culverts (10). Culvert Master and Culvert do not provide plots of the water surface profile through the culvert barrel. HY-8 is the only program with a full flow option that assumes the culvert barrel is flowing full throughout its length.

BCAP and Culvert are the only two programs that include the capability to analyze broken back culverts, while Fish X-ing and HEC-RAS are the only programs that analyze partially filled or buried culverts, utilizing a composite Manning's n value in the computations (9, 14).

While all of the programs are capable of computing inlet and outlet control headwater depths, different methods of doing so exist among the programs. The next section analyzes the difference between these methods more closely.

4.1 Inlet Control Headwater Depth

Flow through a culvert is typically controlled by one of two locations: the culvert inlet or the culvert outlet. Under inlet control, the culvert barrel is capable of passing more flow than what the inlet will allow to enter (1). The flow is supercritical under such conditions, with higher velocities and shallower depths through the culvert. Losses under inlet control do not propagate upstream and only the inlet shape and entrance type affect the computed headwater depth (1).

The headwater depth under inlet control is dependent on the whether or not the entrance is submerged. If the entrance of the culvert is not submerged, it behaves like a weir as flow enters the culvert, while a submerged culvert entrance acts as an orifice (1).

Under the sponsorship of the Federal Highway Administration, the National Bureau of Standards (NBS) developed equations defining inlet control headwater depth (1). The NBS equations were created from lab data that was collected for culvert models on a 2% slope for submerged and unsubmerged conditions (6). The data collected for inlet control was plotted with HW/D on the ordinate and $Q/AD^{0.5}$ on the abscissa. Best fit curves were identified for both the unsubmerged and submerged data sets (5) and the equations for which are represented by Equations 4-1 and 4-2 (1). Equation 4-1 represents the unsubmerged data set while Equation 4-2 is representative of submerged or orifice flow data:

$$\frac{HW_i}{D} = \frac{H_c}{D} + K\left[\frac{Q}{AD^{05}}\right]^M - 0.5S \tag{4-1}$$

$$\frac{HW_i}{D} = c\left[\frac{Q}{AD^{05}}\right]^2 + Y - 0.5S \tag{4-2}$$

where HW_i is the headwater depth for inlet control (ft), D is the barrel rise (ft), H_c is the specific head at critical depth ($d_c + V_c^2/2g$) (ft), Q is the discharge through the barrel (cfs), A is the full cross sectional area of culvert barrel (ft^2), S is the culvert barrel slope (ft/ft), d_c is critical depth (ft), V_c is the critical velocity (ft/s^2), and K, M, c, and Y are constants. Figure 4-1 shows the unsubmerged and submerged curves defined by the NBS equations.

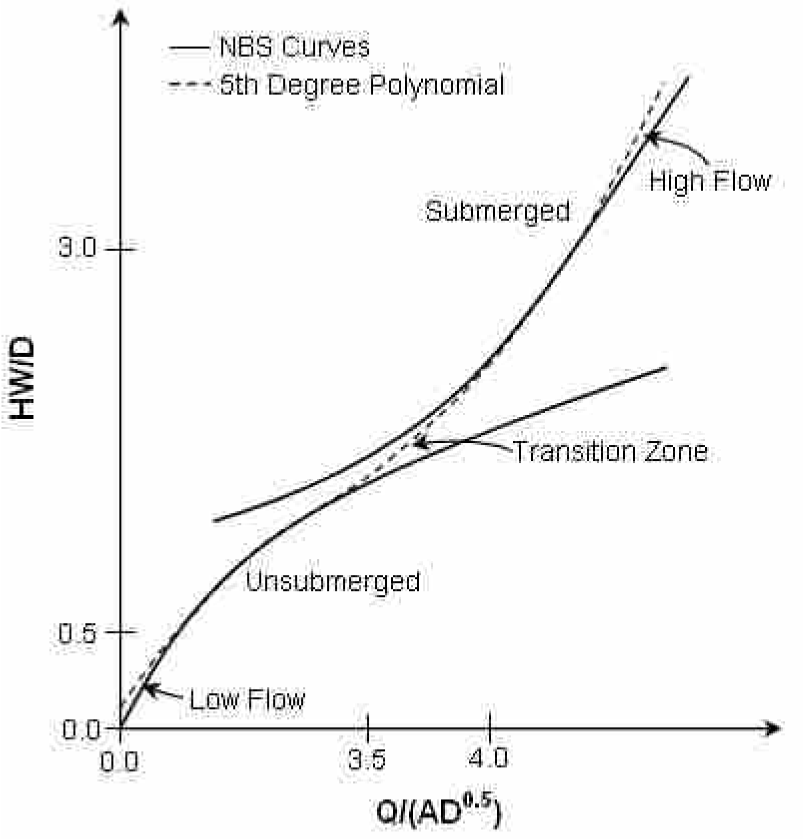

Figure 4-1 Unsubmerged, Submerged and Transition Zones

When the flow at the entrance of the culvert changes from unsubmerged to submerged flow, a transition zone develops which is not well defined. As a result, the transition zone is approximated by creating a line tangent to the submerged and unsubmerged curves. The typical range through which the transition occurs is $3.5 < Q/AD^{0.5} < 4.0$ (1). Of the seven programs studied, only three utilized these procedures for computing inlet control headwater depth outlined by the National Bureau of Standards: Culvert Master, HEC-RAS, and Hydraflow Express (Table 4-2).

An alternative to the NBS method was used to determine the headwater depths for HY-8 (7), BCAP (10), and Culvert (13). This method involves creating a best fit, fifth degree polynomial curve through all three zones of flow: unsubmerged, transition, and submerged (1). In the computational methods, the appropriate polynomial equation is used to determine the inlet control headwater depth. Polynomials must be derived for all culvert inlet types in order to be implemented into code (Appendix E). The fifth degree polynomial is expressed in Equation 4-4:

$$\frac{HW}{D} = a + b\left[\frac{Q}{AD^{0.5}}\right] + c\left[\frac{Q}{AD^{0.5}}\right]^2 + d\left[\frac{Q}{AD^{0.5}}\right]^3 + e\left[\frac{Q}{AD^{0.5}}\right]^4 + f\left[\frac{Q}{AD^{0.5}}\right]^5 - 0.5S \qquad \textbf{(4-3)}$$

where HW is the inlet control headwater depth (ft), Q is the flow through culvert barrel (cfs), A is the cross sectional area of the culvert (ft), D is the barrel rise (ft), S is the culvert slope (ft/ft), and a, b, c, d, e, and f are polynomial coefficients. Equation 4-4 is only applicable for $0.5 < HW/D < 3.0$ (7). The difference between the polynomial and NBS curves is shown for low and high flows in Figure 4-1 as the polynomial curve diverges from those defined by the NBS Equations. As a result, HY-8, BCAP, and Culvert must account for the low and high flow conditions through the use of a low flow entrance loss coefficient and high flow factor (7).

Fish X-ing has a slightly different algorithm implemented for computing inlet control headwater depths. As shown in Table 4-2, Fish X-ing computes inlet control headwater depth by adding the entrance loss and velocity head to the depth of water at the culvert inlet (9).

Table 4-2 Comparison of Inlet Control Headwater Depth Computational Methods

Inlet Control Method	**HY-8**	**Fish X-ing**	**BCAP**	**Hydraflow Express**	**Culvert Master**	**Culvert**	**HEC-RAS**
Polynomial Equations	▨		▨			▨	
NBS Equations				▨	▨		▨
$y_{HW} = (1 + k_e)\frac{v^2}{2g} + y$		▨					

4.2 Outlet Control Headwater Depth

Under outlet control, flow in the culvert barrel exceeds its capacity. Subcritical flow persists under outlet control, with greater water depths and lower velocities through the culvert (1).

When computing the outlet control headwater depth via backwater calculations, the entrance, exit, and friction losses through the barrel are added to the depth of water at the entrance (1). All of the programs utilize backwater calculations to determine the depth at the entrance of the culvert. However, a problem arises when using backwater methods for steeply sloped culverts coupled with tailwater depths less than normal. With this method, the resulting outlet control headwater depth obtained is greater than that reported for inlet control, which is incorrect. In this situation the culvert will always be inlet controlled with supercritical flow through the barrel. Outlet control will never occur.

Of the programs studied, six assume critical depth at the entrance and use the procedure described for computing outlet control headwater depths. Although HEC-RAS uses this method, it appears the program is aware that flow through the culvert is not controlled at the outlet and still reports the control as inlet. HY-8 reports 0.0 for the outlet control headwater depth on steeply sloped culverts with the tailwater depth less than normal depth. The procedure used in all programs is incorrect. To correctly represent the outlet control headwater depth for steeply sloped, low tailwater culverts, the programs should report only inlet control depths and state that the outlet control headwater depths are not applicable.

5 Performance Tests

Although most of the programs incorporate the hydraulic theory outlined in the Hydraulic Design System 5 (1), variations do exist in the way the theory was implemented into code. For this reason, four test cases were developed and analyzed using each of the programs.

Due to the importance of headwater depths and outlet velocities in culvert hydraulics, all cases were developed to test the accuracy of inlet and outlet control headwater depth and outlet velocity approximations. Case A was designed such that the culvert was inlet controlled for all flows. Case B was designed to be outlet controlled for all flows. Cases C and D were designed to test the transition from inlet to outlet control and outlet to inlet control as flow increases. All test cases involved a 5 foot diameter, 100 foot long, concrete pipe culvert with a square edge entrance and a headwall. The cases have only slight differences in order to focus strictly on headwater depth and outlet velocity predictions. A tailwater depth of 0.0 ft (perched outlet) was used in the first three cases so the tailwater would not impact the hydraulic analysis. Table 5-1 summarizes other pertinent input data for each case.

Table 5-1 Test Case Parameters

Case	Q (cfs)	Slope (%)	Tailwater Depth (ft)
A	0-300	1.0	0.0
B	0-100	0.2	0.0
C	0-150	0.3	0.0
D	0-200	0.5	4.5

Figure 5-1 depicts each test case with its corresponding slope and tailwater conditions.

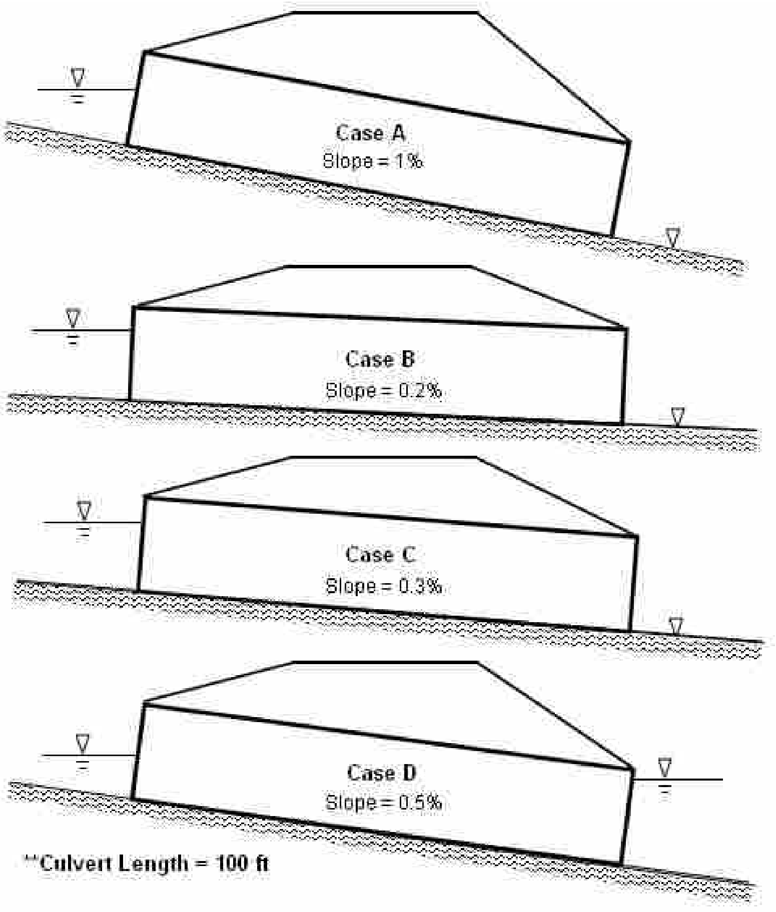

Figure 5-1 Test Cases

For each of the test cases, the inlet and outlet control headwater depths, outlet velocity, and flow control were manually computed for comparison with the values predicted by the programs. The NBS equations were used to compute the inlet control headwater depths for all cases (1). The outlet control headwater depths were computed using the direct step backwater method to determine the inlet depth and adjusted by adding the inlet loss and velocity head inside the barrel. This was done for all cases except when critical depth was greater than normal depth and tailwater was less than normal. In this case, the culvert will never be under outlet control, and therefore it is not appropriate to compute outlet control headwater depths. According to the standard procedure for backwater calculations, brink depth was not considered.

Outlet velocities were computed by determining the outlet depth (right at the exit of the culvert, as opposed to tailwater depth just outside the culvert barrel) and dividing the flow by the corresponding area. For mild slopes, the outlet depth was assumed to be critical when the tailwater depth was less than critical depth. For steep slopes, the outlet depth was determined from forewater calculations as the depth neared normal at the culvert outlet. If normal depth was reached inside the barrel, normal depth was assumed at the outlet. Tailwater depth was used as the outlet depth in cases where it exceeded the downstream boundary depth. All details regarding the hand calculations can be found in Appendix B.

5.1 Results and Discussion

The predicted inlet and outlet control headwater depths and outlet velocities from the seven computer programs were compared to manually computed values (See Appendix C). Solutions with noticeable error were identified by test case and corresponding barrel slope. Statistical analyses were not used or appropriate since all of results were strictly deterministic and without variability. Following the suggested accuracy for nomographs based on the NBS equations for inlet control found in HDS-5 (1), any error above or below 10% of the manually computed inlet control headwater depths were considered incorrect. Differences between calculated and program results for inlet control headwater depth are plotted in Figure 5-2 as a function of dimensionless discharge.

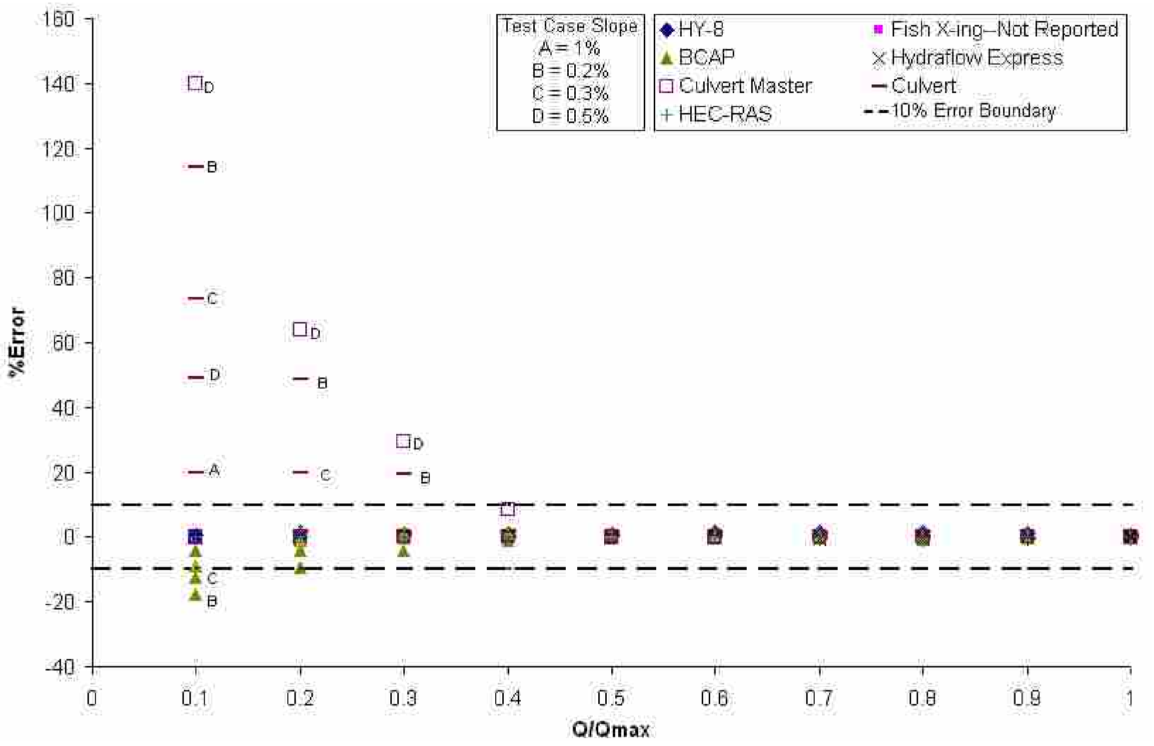

Figure 5-2 Inlet Control Headwater Depth Error

HEC-RAS and HY-8 produced correct inlet control headwater values most consistently and with the lowest average error. Differences using HY-8, although insignificant, are attributed to the fact that HY-8 uses the 5th degree polynomial approach rather than the NBS equations when computing the inlet control headwater depth. Fish

X-ing was unable to produce values for the inlet control headwater depths because for all cases the program predicted flow control at the outlet of the culvert and failed to report inlet control values.

In case D, it is unclear why Culvert Master performed poorly with errors ranging from 30-140% (0.91-2.33 ft). According to the user's manual, the program uses the same procedure followed in the manual calculations (12). However, while it appears to be a bug in the program for this condition, the exact cause of error is undetermined since the code used in Culvert Master was inaccessible for this study.

The errors found in Culvert are due to the fact that the fifth degree polynomial equation (Equation 4-4) is only accurate for HW/D ratios between 0.5 and 3.0 (13). For flows below approximately 40 cfs, the HW/D ratios were below 0.5. Instead of accounting for the low flow conditions by using a low flow entrance loss coefficient (7), it appears that Culvert sets the headwater depth equal to a lower limit of half the pipe diameter, 2.5 feet, until the depths were such that it was appropriate to use the fifth degree polynomial equation. The highest error of 114% (1.33 ft) occurred in case B and the lowest error of 20% (0.41 ft) occurred in case A.

Figure 5-3 shows the error for outlet control headwater depth predicted by each of the programs. For cases B and C, BCAP inaccurately predicted the flow control at the inlet for all flows. BCAP also overestimated the outlet control headwater depth results for case D with a maximum error of 21% (1.52 ft). These errors occur because BCAP was not intended to be used for straight culverts. In broken-back culvert operations, hydraulic control is invariably at the entrance or at the break in culvert slope. Outlet control has not been fully considered in the program and therefore results in error.

HEC-RAS, Culvert Master, Culvert, and Fish X-ing all produced relatively high average errors for case A. The high error in this case resulted from the misuse of the outlet control equation for steeply sloped culverts (critical depth greater than normal depth). Since outlet control does not exist in these cases with tailwater depth less than normal, outlet control headwater depths reported by any program are not valid and therefore were not considered in Figure 5-3. Culvert Master and HEC-RAS produced the most accurate results with HY-8 having minimal error as well.

Hydraflow Express consistently predicted outlet control headwater depths below the correct value with errors ranging from 18% (0.85 ft) in case B to 24% (approximately 1.0 ft) in case C. In part, this was due to the inappropriate outlet depths used in the standard step computations. In case C, for example, the hydraulic slope of the culvert changed from steep to mild, and therefore the boundary condition at the downstream end of the culvert changed from normal to critical as flow increased. However, for all flows, Hydraflow Express assumed outlet control and used critical depth at the outlet for all flows. The program also used a standard step procedure for outlet control headwater depth computations while the manual computations used the direct step method (10). Because the program code was unavailable, the exact cause of error in the resulting headwater depths is unclear.

Figure 5-4 shows the results of the computed outlet velocity for each test case. In case A, Hydraflow Express reported the highest error for outlet velocity (-34%, -3.0 ft below calculated value) while Fish X-ing produced the highest error for cases B (40%, 3.54 ft) and C (40%, 4.16 ft). Culvert Master produced the highest error in case D (19%, 1.82 ft).

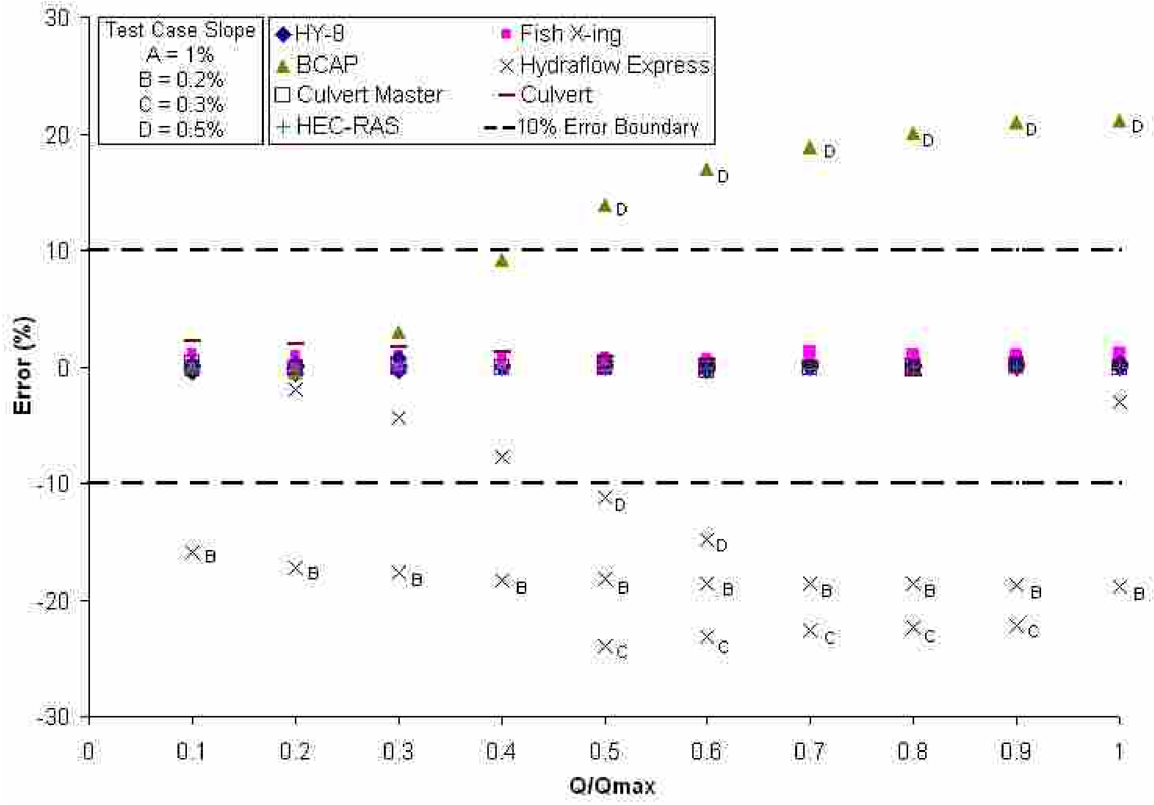

Figure 5-3 Outlet Control Headwater Depth Error

In case A, Hydraflow Express appeared to use critical rather than normal depth as the outlet depth, which resulted in outlet velocities much lower than calculated. The error in Fish X-ing was attributed to the fact that for the last flow of 300 cfs, Fish X-ing predicted a mild hydraulic slope with normal depth greater than critical, producing a higher outlet velocity than expected.

Since the hydraulic slope in case B was mild and tailwater depth was less than critical at the outlet, the downstream boundary was critical depth. However, Fish X-ing predicted outlet depths lower than critical depth, producing excessive outlet velocities for this case. The same error occurred in case C for Fish X-ing when the program produced outlet depths much lower than critical depth at higher flows. The reason for these errors is a result of the way in which Fish X-ing predicts outlet depths. For hydraulically mild slopes, the program appears to use brink depth at the outlet rather than critical depth as in standard culvert hydraulic calculations. However, the method for obtaining outlet depth in this case is unclear since the program reference manual does not clearly explain the processes implemented in Fish X-ing (9).

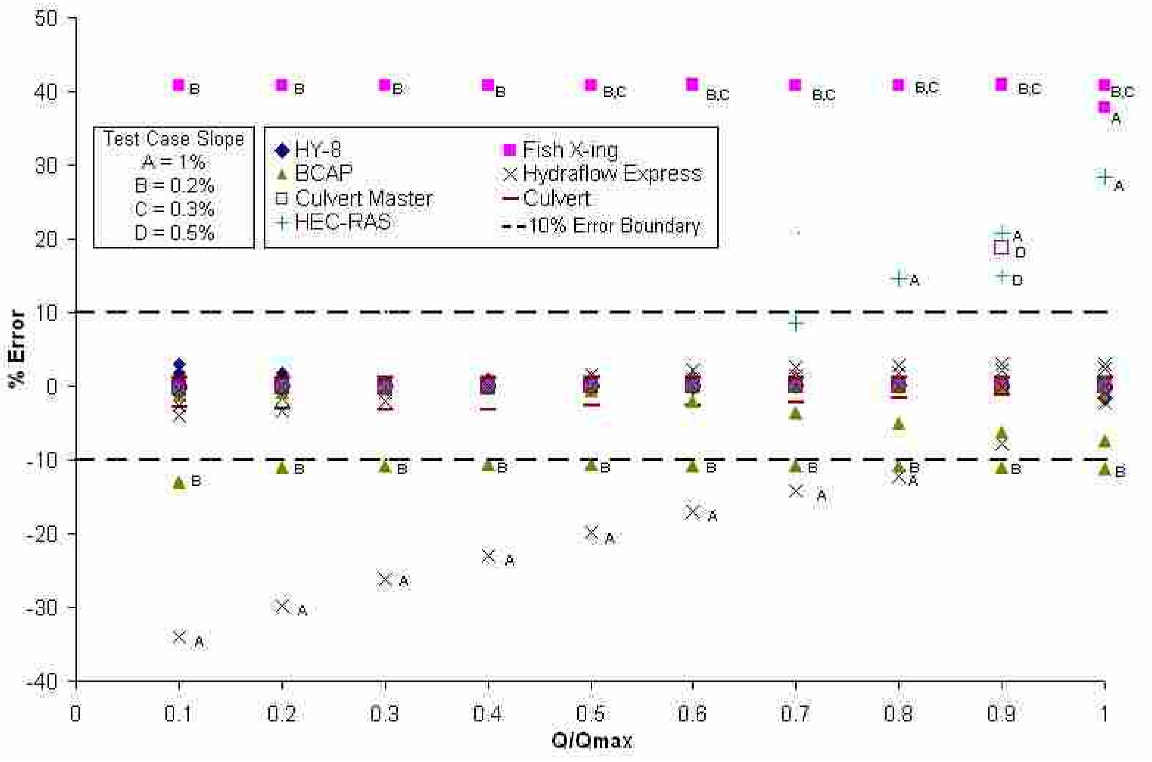

Figure 5-4 Outlet Velocity Error

In case D for the dimensionless discharge value of 0.9, HEC-RAS was unable to balance the energy equation and therefore assumed an outlet depth of critical. This assumption proved incorrect as the depth was actually normal. As a result, HEC-RAS over predicted the outlet velocity (15%, 1.45 ft). Since the code for HEC-RAS was inaccessible, it is unclear why the program produced high outlet velocity errors for higher flows in case A. It appears that Culvert Master had the same error as HEC-RAS for case D.

BCAP had a minor error in case B (11%, 1.0 ft) as a result of predicting inlet control when the actual flow control was at the outlet. Again this error is attributed to the problems with straight barrel analysis in a program designed for broken-back culverts.

5.2 Analysis of Results

Although most of the programs produce accurate results most of the time, several limitations were identified. While operating under outlet control, Hydraflow Express is unable to produce accurate headwater depths. Fish X-ing is unable to analyze culverts under inlet control. Regardless of the hydraulic slope of the culvert, all cases are assumed to operate under outlet control. BCAP is also limited in its ability to analyze straight barrel culverts operating under outlet control with high tailwater.

For low flow cases, Culvert was unable to predict accurate inlet control headwater depths. Culvert also has a problem determining the controlling headwater depths. While Culvert reports inlet and outlet control headwater depths, the controlling headwater depth is reported in a separate column. In general practice, the higher of the inlet and outlet control headwater depths is recorded as the controlling depth. In Culvert, a combination of the inlet control and outlet control depths is used, however, and the computational algorithm is not known.

HY-8, HEC-RAS, and Culvert Master predicted the most accurate results most consistently in the four test cases.

6 Summary and Conclusions

The first objective of the research was to identify and compare available features in seven culvert hydraulic programs. It was found that each program was designed to handle specific capabilities, the importance of which depends on the design constraints. Users should select the appropriate program focused for their specific needs.

The second objective was to analyze the accuracy of the results produced by each program when tested with four different hydraulic cases. While most of the programs produced accurate results for most of the cases, errors did exist. Fish X-ing was unable to analyze culverts under inlet control, while Hydraflow Express was unable to predict accurate outlet control headwater depths. Culvert could not accurately analyze culverts under low flow conditions, while BCAP was unable to properly analyze straight barrel culverts under outlet control. HEC-RAS and Culvert Master and HY-8 produced the most accurate results most consistently in the test cases.

Based on this research, Fish X-ing, HEC-RAS, BCAP, and Culvert all have unique features separating them from the other programs. Fish X-ing is the only program that analyzes culverts for fish passage, while HEC-RAS is the only program that analyzes culverts as part of a stream network. BCAP and Culvert are the only programs that will analyze broken back culverts. In terms of program accuracy, Fish X-ing, BCAP, Hydraflow Express, and Culvert predicted inaccurate results most frequently. HEC-RAS and Culvert Master and HY-8 produced the most accurate results most consistently in the test cases.

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Appendix A. Culvert Hydraulic Review

When computing water surface profiles through a culvert barrel, boundary conditions are required such that the forewater and backwater computations begin at the appropriate depth. The five classifications of gradually varied flow profiles include steep, mild, horizontal, adverse, and critical. There are three levels in each classification; however, the third level was ignored here since the upstream boundary condition is 0.0 flow depth which will never occur (1).

A.1 Steep

A slope is defined as steep when critical depth is greater than normal depth. In this situation the upstream boundary condition is critical depth. For an S1 curve, the tailwater depth is greater than both the normal and critical depth at the outlet, and therefore the tailwater depth is the downstream boundary condition. For an S2 curve, as long as the tailwater depth is below critical the downstream boundary condition will be normal depth (1).

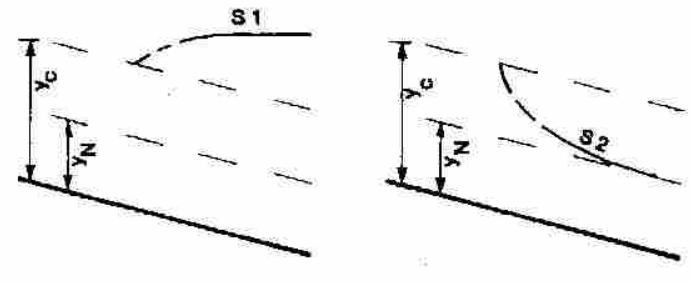

Figure A-1 Steep Slope Profiles (1)

A.2 Mild

For mild slopes, critical depth is always less than normal depth. Under these conditions, normal depth serves as the upstream boundary condition, as shown in Figure A-2. When the tailwater depth is greater than critical and normal depth, the flow profile is classified as M1 and the downstream boundary condition is the tailwater depth. When the tailwater depth is less than critical the flow profile is M2 and the downstream boundary condition is critical depth (1).

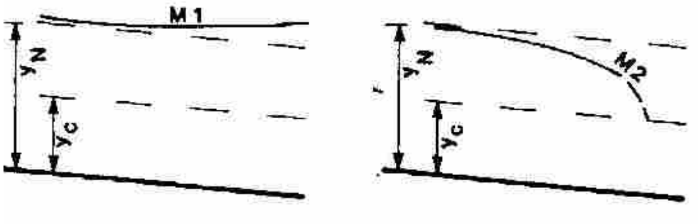

Figure A-2 Mild Slope Profiles (1)

A.3 Horizontal

Horizontal slopes are not common, although they can exist from settlement of the culvert barrel. Analysis of culverts on a 0% slope becomes difficult, for normal depth approaches infinity due to the zero slope entered in Manning's equation. The problem arises during backwater computations when approaching the upstream boundary, which in this case is infinity. Therefore, H1 curves are undefined (1).

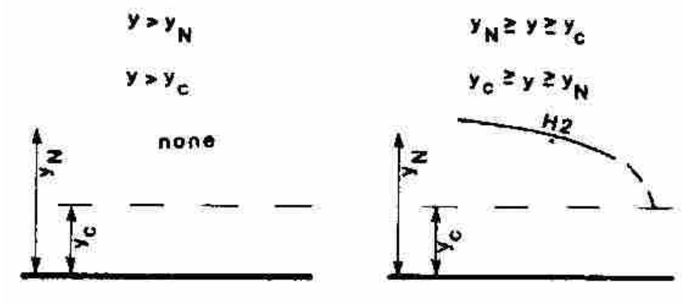

Figure A-3 Horizontal Slope Profiles (1)

A.4 Adverse

Analysis of a culvert barrel on an adverse slopes face similar challenges to those on horizontal slopes. With an adverse slope, solving Manning's equation results in an undefined value for normal depth. As with horizontal slopes, the A1 profile is undefined since the upstream boundary condition of normal depth is undefined. The A2 profile is inferred, with critical depth as the downstream boundary condition (1).

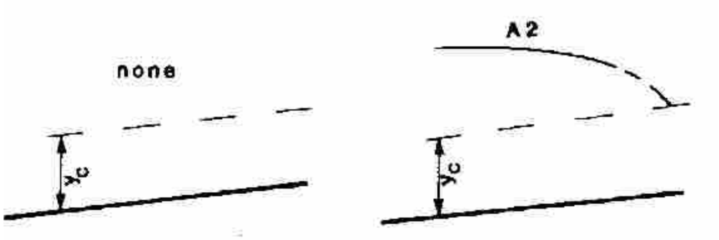

Figure A-4 Adverse Slope Profiles (1)

A.5 Critical

Critical slope profiles are rare instances when the normal depth is equal to the critical depth. A C1 profile occurs when the water depth is greater than both critical and normal depth, where the downstream boundary is the tailwater depth and the upstream boundary is equal to normal and critical depth. A C2 profile is the line equal to normal and critical depth through the length of the culvert (1).

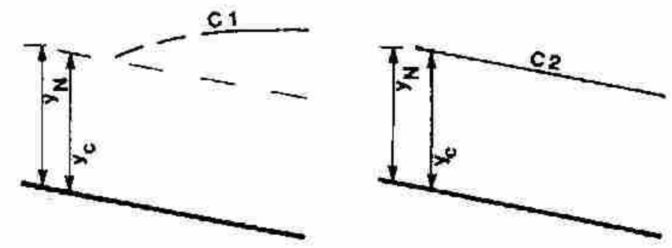

Figure A-5 Critical Slope Profiles (1)

References

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Appendix B. Hand Calculations

Four test cases were developed for the purpose of comparing the computed inlet and outlet control headwater depths and outlet velocities determined by each of the programs. Normal and critical depths were computed for all flows in each test case to determine the slope and boundary conditions defining each test case. NBS equations were used to compute inlet control headwater depths, while backwater calculations were used for computing outlet control headwater depths.

B.1 Normal and Critical Depth

Normal and critical depth values are important in determining boundary conditions in culvert hydraulics and were computed using the Goal Seek tool in Microsoft Excel. Equations B-1 through B-4 were used in the normal depth computations, while Equations B-1 and B-4 through B-6 were used in the critical depth computations (1):

$$A = \frac{1}{8}(\theta - \sin(\theta))D^2 \tag{B-1}$$

$$R = \frac{1}{4}\left(1 - \frac{\sin(\theta)}{\theta}\right)D \tag{B-2}$$

$$Q = \frac{k}{n} A R^{2/3} S^{1/2} \tag{B-3}$$

$$\theta = 2\cos^{-1}\left(\frac{\frac{D}{2} - y_n}{\frac{D}{2}}\right) \tag{B-4}$$

$$T = 2\sqrt{y_c\left(D - y_c\right)} \tag{B-5}$$

$$Q = \sqrt{g\frac{A^3}{T}} \tag{B-6}$$

where A is the area of flow (ft^2), D is the barrel rise (ft), θ is the angle that defines the top width of flow (radians), R is the hydraulic radius (ft), Q is the flow through the culvert (cfs), k is a units constant (1.49 for English), n is Manning's roughness coefficient (unitless), S is the slope of the culvert barrel (ft/ft), y_n is the normal depth of flow (ft), y_c is critical depth (ft), T is the top width of flow (ft), and g is the acceleration due to gravity (32.2 ft/s^2).

The computation of normal and critical depth values was necessary to determine the type of hydraulic slope, which in turn affected the boundary conditions when computing backwater calculations. If critical depth was greater than normal depth, the slope was steep, otherwise the slope was mild.

B.2 Inlet Control Headwater Depth

Following the procedures outlined in HDS-5 for computing inlet control headwater depth, the NBS equations (Equations 4-1 and 4-2) were used. Equation 4-1, for unsubmerged flow, is only applicable for values of $Q/(AD^{0.5}) < 3.5$ whereas Equation 4-2 is only applicable for values of $Q/(AD^{0.5}) > 4.0$ (2). The computed inlet control headwater depths for each of the four test cases are displayed in Tables B-1 through B-4. For Case D, the headwater depth value at 160 cfs occurred in the transition zone. Table B-4 contains more flow values between 140 and 180 cfs to better define the beginning and end of the transition zone ($Q/AD^{0.5} = 3.5$ and $Q/AD^{0.5} = 4.0$). A linear line was used to interpolate between the two endpoints of the transition zone. The equation defining the transition zone was used to obtain the inlet control headwater depth of 6.0 ft at 160 cfs (Figure B-1).

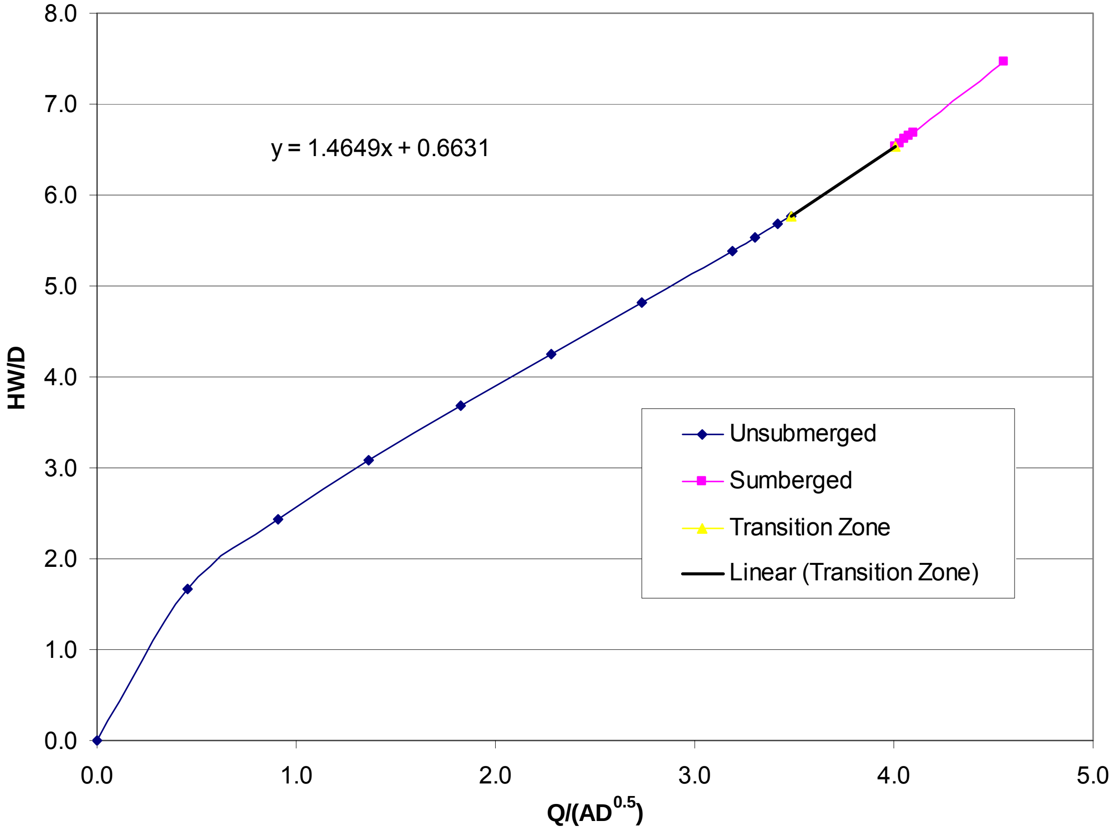


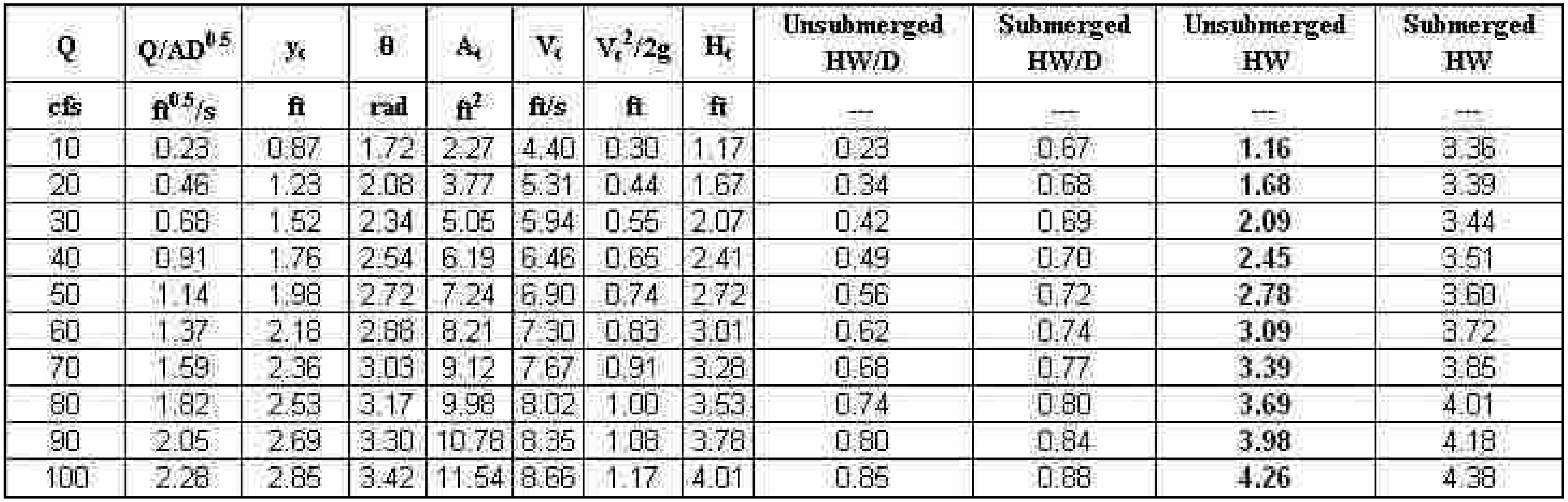
Figure B-1 Case D Linear Transition Zone

Tables B-1 through B-4 show the headwater depth values computed at each flow for both the submerged and unsubmerged conditions in all four test cases. The values for the coefficients used in the NBS equations were obtained from HDS-5 where K = 0.0098, M = 2.0, c = 0.0398, and Y = 0.67 for a concrete pipe culvert with a square edge and headwall (2).

Table B-1 Case A: Manually Computed Inlet Control Depths

Q	$Q/AD^{0.5}$	y_c	θ	A_c	V_c	$V_c^2/2g$	H_c	Unsubmerged HW/D	Submerged HW/D	Unsubmerged HW	Submerged HW
cfs	$ft^{0.5}/s$	ft	rad	ft^2	ft/s	ft	ft	---	---	---	---
30	0.68	1.52	2.34	5.05	5.94	0.55	2.07	0.41	0.68	**2.07**	3.42
60	1.37	2.18	2.88	8.21	7.30	0.83	3.01	0.61	0.74	**3.07**	3.70
90	2.05	2.69	3.30	10.78	8.35	1.08	3.78	0.79	0.83	**3.96**	4.16
120	2.73	3.13	3.65	12.93	9.28	1.34	4.47	0.96	0.96	**4.81**	4.81
150	3.42	3.51	3.97	14.73	10.18	1.61	5.12	1.13	1.13	**5.67**	5.65
180	4.10	3.84	4.28	16.19	11.12	1.92	5.76	1.31	1.33	6.56	**6.67**
210	4.78	4.13	4.56	17.33	12.12	2.28	6.41	1.50	1.58	7.50	**7.88**
240	5.47	4.36	4.82	18.16	13.22	2.71	7.07	1.70	1.85	8.51	**9.27**
270	6.15	4.54	5.05	18.72	14.42	3.23	7.77	1.92	2.17	9.59	**10.85**
300	6.83	4.67	5.24	19.08	15.72	3.84	8.51	2.15	2.52	10.77	**12.62**

Table B-2 Case B: Manually Computed Inlet Control Depths

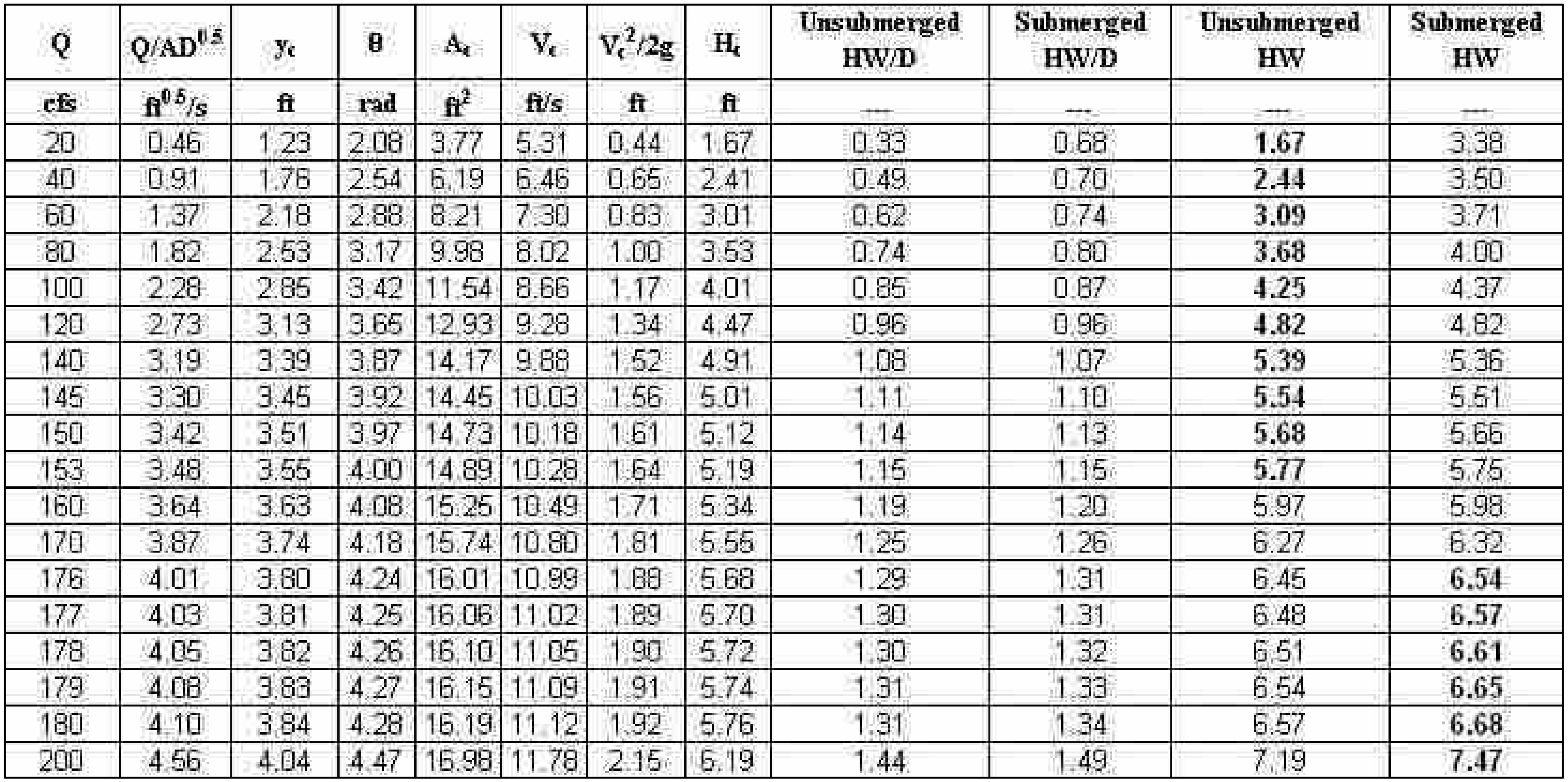


Q	$Q/AD^{0.5}$	y_c	θ	A_c	V_c	$V_c^2/2g$	H_c	Unsubmerged HW/D	Submerged HW/D	Unsubmerged HW	Submerged HW
cfs	$ft^{0.5}/s$	ft	rad	ft^2	ft/s	ft	ft	---	---	---	---
10	0.23	0.87	1.72	2.27	4.40	0.30	1.17	0.23	0.67	**1.16**	3.36
20	0.46	1.23	2.08	3.77	5.31	0.44	1.67	0.34	0.68	**1.68**	3.39
30	0.68	1.52	2.34	5.05	5.94	0.55	2.07	0.42	0.69	**2.09**	3.44
40	0.91	1.76	2.54	6.19	6.46	0.65	2.41	0.49	0.70	**2.45**	3.51
50	1.14	1.98	2.72	7.24	6.90	0.74	2.72	0.56	0.72	**2.78**	3.60
60	1.37	2.18	2.88	8.21	7.30	0.83	3.01	0.62	0.74	**3.09**	3.72
70	1.59	2.36	3.03	9.12	7.67	0.91	3.28	0.68	0.77	**3.39**	3.85
80	1.82	2.53	3.17	9.98	8.02	1.00	3.53	0.74	0.80	**3.69**	4.01
90	2.05	2.69	3.30	10.78	8.35	1.08	3.78	0.80	0.84	**3.98**	4.18
100	2.28	2.85	3.42	11.54	8.66	1.17	4.01	0.85	0.88	**4.26**	4.38

Table B-3 Case C: Manually Computed Inlet Control Depths

Q	$Q/AD^{0.5}$	y_c	θ	A_c	V_c	$V_c^2/2g$	H_c	Unsubmerged HW/D	Submerged HW/D	Unsubmerged HW	Submerged HW
cfs	$ft^{0.5}/s$	ft	rad	ft^2	ft/s	ft	ft	---	---	---	---
15	0.34	1.06	1.92	3.06	4.90	0.37	1.44	0.29	0.67	**1.44**	3.37
30	0.68	1.52	2.34	5.05	5.94	0.55	2.07	0.42	0.69	**2.08**	3.44
45	1.02	1.88	2.64	6.73	6.69	0.69	2.57	0.52	0.71	**2.61**	3.55
60	1.37	2.18	2.88	8.21	7.30	0.83	3.01	0.62	0.74	**3.09**	3.71
75	1.71	2.45	3.10	9.56	7.85	0.96	3.40	0.71	0.78	**3.54**	3.92
90	2.05	2.69	3.30	10.78	8.35	1.08	3.78	0.79	0.84	**3.97**	4.18
105	2.39	2.92	3.48	11.91	8.82	1.21	4.13	0.88	0.90	**4.40**	4.48
120	2.73	3.13	3.65	12.93	9.28	1.34	4.47	0.97	0.97	**4.83**	4.83
135	3.07	3.33	3.82	13.87	9.73	1.47	4.80	1.05	1.04	**5.25**	5.22
150	3.42	3.51	3.97	14.73	10.18	1.61	5.12	1.14	1.13	**5.69**	5.67

Table B-4 Case D: Manually Computed Inlet Control Depths



Q	$Q/AD^{0.5}$	y_c	θ	A_c	V_c	$V_c^2/2g$	H_c	Unsubmerged HW/D	Submerged HW/D	Unsubmerged HW	Submerged HW
cfs	$ft^{0.5}/s$	ft	rad	ft^2	ft/s	ft	ft	---	---	---	---
20	0.46	1.23	2.08	3.77	5.31	0.44	1.67	0.33	0.68	**1.67**	3.38
40	0.91	1.76	2.54	6.19	6.46	0.65	2.41	0.49	0.70	**2.44**	3.50
60	1.37	2.18	2.88	8.21	7.30	0.83	3.01	0.62	0.74	**3.09**	3.71
80	1.82	2.53	3.17	9.98	8.02	1.00	3.53	0.74	0.80	**3.68**	4.00
100	2.28	2.85	3.42	11.54	8.66	1.17	4.01	0.85	0.87	**4.25**	4.37
120	2.73	3.13	3.65	12.93	9.28	1.34	4.47	0.96	0.96	**4.82**	4.82
140	3.19	3.39	3.87	14.17	9.88	1.52	4.91	1.08	1.07	**5.39**	5.36
145	3.30	3.45	3.92	14.45	10.03	1.56	5.01	1.11	1.10	**5.54**	5.51
150	3.42	3.51	3.97	14.73	10.18	1.61	5.12	1.14	1.13	**5.68**	5.66
153	3.48	3.55	4.00	14.89	10.28	1.64	5.19	1.15	1.15	**5.77**	5.75
160	3.64	3.63	4.08	15.25	10.49	1.71	5.34	1.19	1.20	5.97	5.98
170	3.87	3.74	4.18	15.74	10.80	1.81	5.55	1.25	1.26	6.27	6.32
176	4.01	3.80	4.24	16.01	10.99	1.88	5.68	1.29	1.31	6.45	**6.54**
177	4.03	3.81	4.25	16.06	11.02	1.89	5.70	1.30	1.31	6.48	**6.57**
178	4.05	3.82	4.26	16.10	11.05	1.90	5.72	1.30	1.32	6.51	**6.61**
179	4.08	3.83	4.27	16.15	11.09	1.91	5.74	1.31	1.33	6.54	**6.65**
180	4.10	3.84	4.28	16.19	11.12	1.92	5.76	1.31	1.34	6.57	**6.68**
200	4.56	4.04	4.47	16.98	11.78	2.15	6.19	1.44	1.49	7.19	**7.47**

B.3 Outlet Control Headwater Depth

Outlet control headwater depths were determined using the direct step backwater method. The direct step method involves assuming a delta y value (depth of water) and computing the corresponding delta x value (distance up the culvert corresponding to the assumed water depth). The headwater depth was computed by adding the entrance loss and velocity head inside the culvert to the depth in the entrance of the culvert determined from backwater calculations. Equations B-7 through B-15 (1) were used to complete the direct step backwater computations for determining the outlet control headwater depths:

$$P = 0.5\theta D \tag{B-7}$$

$$A = \frac{1}{8}\left(\frac{\theta - \sin(\theta)}{\sin(\theta/2)}\right)D \tag{B-8}$$

where P is the wetted perimeter (ft), D is the culvert diameter (ft), and θ is the angle that defines the top of the width of flow (rad), and A is the area of flow (ft^2);

$$u = \frac{Q}{A} \tag{B-9}$$

where u is the velocity of flow in the culvert barrel (ft/s), Q is the flow through the barrel (cfs);

$$F^2 = \left(\frac{u}{\sqrt{gD}} \right)^2 \tag{B-10}$$

where F is the Froude number and g is the acceleration due to gravity (32.2 ft/s^2);

$$S_f = \frac{n^2 u^2}{k^2 R^{4/3}} \tag{B-11}$$

where n is the Manning's roughness coefficient, k is a units constant, R is the hydraulic radius (ft), and S_f is the friction slope through the barrel;

$$\Delta E = \Delta y \left(1 - F^2 \right) \tag{B-12}$$

$$\Delta x = \frac{\Delta E}{S_o - S_f} \tag{B-13}$$

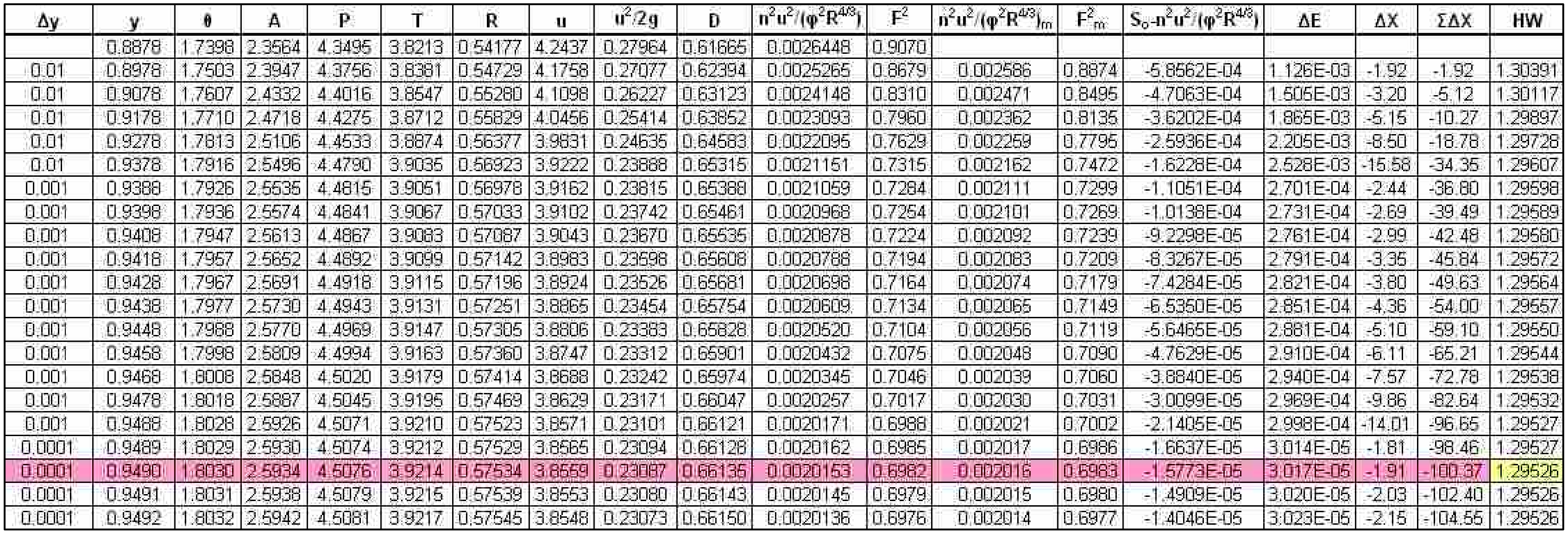
where ΔE is the change in energy through the length of the barrel (ft), Δy is the change in depth of flow (ft), Δx is the change in distance along the length of the barrel (ft), and S_o is the slope of the barrel;

$$HW = y + k_e \frac{u^2}{2g} + \frac{u^2}{2g} \tag{B-14}$$

where HW is the headwater depth just upstream of the entrance of the culvert (ft), y is the depth of flow at the entrance of the barrel (ft), and k_e is the entrance loss coefficient.

Case A was not analyzed using backwater calculations since the case was on a steep slope (critical depth greater than normal) with no tailwater and could therefore never operate under outlet control. Tables B-5 through B-14 show the backwater calculations for case B starting at critical depth downstream and continuing up the culvert until the entrance was reached (sum of delta x values = -100 ft). Once the entrance of the culvert was reached, the velocity head inside the culvert and the entrance loss were added to the corresponding water depth to obtain the headwater depth. In case C, the slope of the culvert changes from steep to mild and therefore the backwater computations were only completed for the mild cases where normal depth is less than critical (Tables B-15 through B-20). Again, the downstream starting depth was critical. Tables B-21 through B-30 show the outlet control headwater depths for case D. The computations began at the tailwater depth downstream since it was greater than critical and normal depth at all flows.

Table B-5 Case B 10 cfs: Manually Computed Outlet Control Headwater Depths

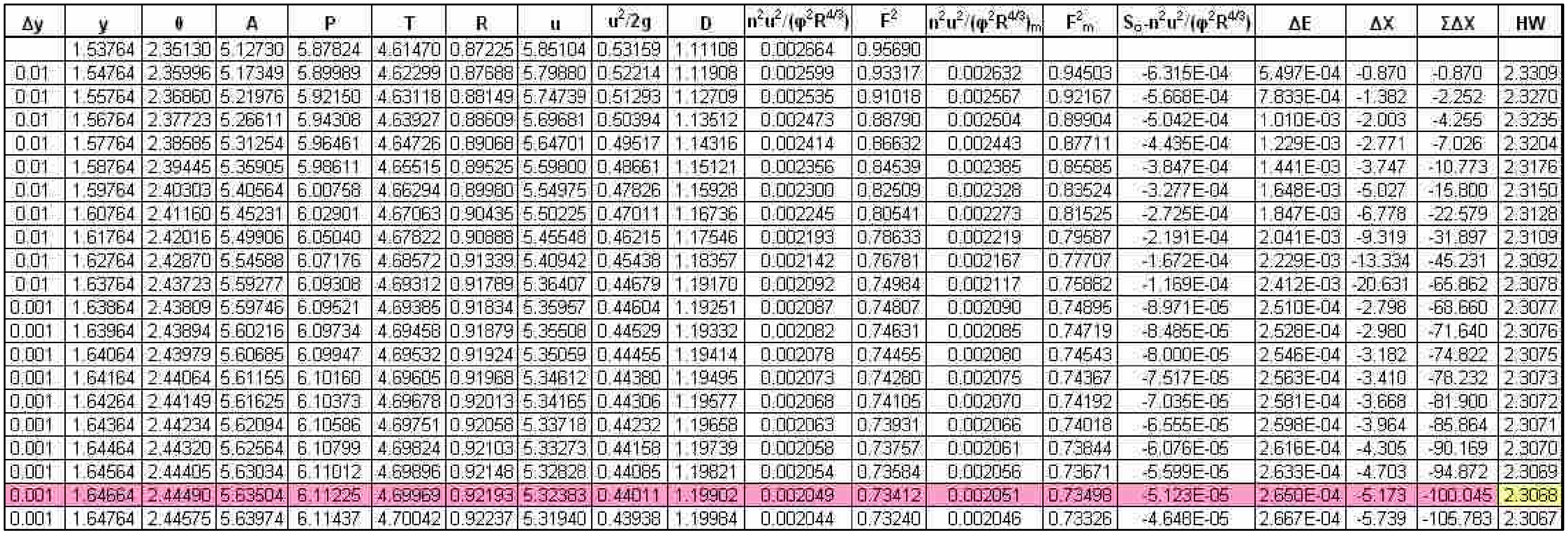


Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	0.8878	1.7398	2.3564	4.3495	3.8213	0.54177	4.2437	0.27964	0.61665	0.0026448	0.9070							
0.01	0.8978	1.7503	2.3947	4.3756	3.8381	0.54729	4.1758	0.27077	0.62394	0.0025265	0.8679	0.002586	0.8874	-5.8562E-04	1.126E-03	-1.92	-1.92	1.30391
0.01	0.9078	1.7607	2.4332	4.4016	3.8547	0.55280	4.1098	0.26227	0.63123	0.0024148	0.8310	0.002471	0.8495	-4.7063E-04	1.505E-03	-3.20	-5.12	1.30117
0.01	0.9178	1.7710	2.4718	4.4275	3.8712	0.55829	4.0456	0.25414	0.63852	0.0023093	0.7960	0.002362	0.8135	-3.6202E-04	1.865E-03	-5.15	-10.27	1.29897
0.01	0.9278	1.7813	2.5106	4.4533	3.8874	0.56377	3.9831	0.24635	0.64583	0.0022095	0.7629	0.002259	0.7795	-2.5936E-04	2.205E-03	-8.50	-18.78	1.29728
0.01	0.9378	1.7916	2.5496	4.4790	3.9035	0.56923	3.9222	0.23888	0.65315	0.0021151	0.7315	0.002162	0.7472	-1.6228E-04	2.528E-03	-15.58	-34.35	1.29607
0.001	0.9388	1.7926	2.5535	4.4815	3.9051	0.56978	3.9162	0.23815	0.65388	0.0021059	0.7284	0.002111	0.7299	-1.1051E-04	2.701E-04	-2.44	-36.80	1.29598
0.001	0.9398	1.7936	2.5574	4.4841	3.9067	0.57033	3.9102	0.23742	0.65461	0.0020968	0.7254	0.002101	0.7269	-1.0138E-04	2.731E-04	-2.69	-39.49	1.29589
0.001	0.9408	1.7947	2.5613	4.4867	3.9083	0.57087	3.9043	0.23670	0.65535	0.0020878	0.7224	0.002092	0.7239	-9.2298E-05	2.761E-04	-2.99	-42.48	1.29580
0.001	0.9418	1.7957	2.5652	4.4892	3.9099	0.57142	3.8983	0.23598	0.65608	0.0020788	0.7194	0.002083	0.7209	-8.3267E-05	2.791E-04	-3.35	-45.84	1.29572
0.001	0.9428	1.7967	2.5691	4.4918	3.9115	0.57196	3.8924	0.23526	0.65681	0.0020698	0.7164	0.002074	0.7179	-7.4284E-05	2.821E-04	-3.80	-49.63	1.29564
0.001	0.9438	1.7977	2.5730	4.4943	3.9131	0.57251	3.8865	0.23454	0.65754	0.0020609	0.7134	0.002065	0.7149	-6.5350E-05	2.851E-04	-4.36	-54.00	1.29557
0.001	0.9448	1.7988	2.5770	4.4969	3.9147	0.57305	3.8806	0.23383	0.65828	0.0020520	0.7104	0.002056	0.7119	-5.6465E-05	2.881E-04	-5.10	-59.10	1.29550
0.001	0.9458	1.7998	2.5809	4.4994	3.9163	0.57360	3.8747	0.23312	0.65901	0.0020432	0.7075	0.002048	0.7090	-4.7629E-05	2.910E-04	-6.11	-65.21	1.29544
0.001	0.9468	1.8008	2.5848	4.5020	3.9179	0.57414	3.8688	0.23242	0.65974	0.0020345	0.7046	0.002039	0.7060	-3.8840E-05	2.940E-04	-7.57	-72.78	1.29538
0.001	0.9478	1.8018	2.5887	4.5045	3.9195	0.57469	3.8629	0.23171	0.66047	0.0020257	0.7017	0.002030	0.7031	-3.0099E-05	2.969E-04	-9.86	-82.64	1.29532
0.001	0.9488	1.8028	2.5926	4.5071	3.9210	0.57523	3.8571	0.23101	0.66121	0.0020171	0.6988	0.002021	0.7002	-2.1405E-05	2.998E-04	-14.01	-96.65	1.29527
0.0001	0.9489	1.8029	2.5930	4.5074	3.9212	0.57529	3.8565	0.23094	0.66128	0.0020162	0.6985	0.002017	0.6986	-1.6637E-05	3.014E-05	-1.81	-98.46	1.29527
0.0001	0.9490	1.8030	2.5934	4.5076	3.9214	0.57534	3.8559	0.23087	0.66135	0.0020153	0.6982	0.002016	0.6983	-1.5773E-05	3.017E-05	-1.91	-100.37	1.29526
0.0001	0.9491	1.8031	2.5938	4.5079	3.9215	0.57539	3.8553	0.23080	0.66143	0.0020145	0.6979	0.002015	0.6980	-1.4909E-05	3.020E-05	-2.03	-102.40	1.29526
0.0001	0.9492	1.8032	2.5942	4.5081	3.9217	0.57545	3.8548	0.23073	0.66150	0.0020136	0.6976	0.002014	0.6977	-1.4046E-05	3.023E-05	-2.15	-104.55	1.29526

Table B-6 Case B 20 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	1.2555	2.0995	3.8624	5.2486	4.3364	0.73589	5.1781	0.41635	0.89068	0.0026177	0.9349							
0.01	1.2655	2.1087	3.9058	5.2717	4.3479	0.74091	5.1206	0.40715	0.89833	0.0025367	0.9065	0.0025772	0.9207	-5.77E-04	7.932E-04	-1.37	-1.37	1.87620
0.01	1.2755	2.1179	3.9493	5.2946	4.3591	0.74592	5.0641	0.39822	0.90599	0.0024589	0.8791	0.0024978	0.8928	-4.98E-04	1.072E-03	-2.15	-3.53	1.87281
0.01	1.2855	2.1270	3.9930	5.3175	4.3703	0.75091	5.0088	0.38956	0.91366	0.0023842	0.8528	0.0024216	0.8659	-4.22E-04	1.341E-03	-3.18	-6.71	1.86982
0.01	1.2955	2.1362	4.0367	5.3404	4.3814	0.75589	4.9545	0.38116	0.92134	0.0023123	0.8274	0.0023482	0.8401	-3.48E-04	1.599E-03	-4.59	-11.30	1.86722
0.01	1.3055	2.1453	4.0806	5.3632	4.3923	0.76086	4.9012	0.37301	0.92904	0.0022432	0.8030	0.0022777	0.8152	-2.78E-04	1.848E-03	-6.65	-17.95	1.86499
0.01	1.3155	2.1544	4.1246	5.3859	4.4031	0.76581	4.8490	0.36510	0.93674	0.0021767	0.7795	0.0022099	0.7913	-2.10E-04	2.087E-03	-9.94	-27.90	1.86313
0.01	1.3255	2.1634	4.1687	5.4086	4.4138	0.77075	4.7977	0.35742	0.94446	0.0021127	0.7569	0.0021447	0.7682	-1.45E-04	2.318E-03	-16.02	-43.92	1.86160
0.01	1.3355	2.1725	4.2129	5.4312	4.4244	0.77567	4.7474	0.34996	0.95218	0.0020511	0.7351	0.0020819	0.7460	-8.19E-05	2.540E-03	-31.02	-74.94	1.86042
0.001	1.3365	2.1734	4.2173	5.4335	4.4255	0.77617	4.7424	0.34923	0.95296	0.0020451	0.7329	0.0020481	0.7340	-4.81E-05	2.660E-04	-5.53	-80.47	1.86031
0.001	1.3375	2.1743	4.2217	5.4358	4.4265	0.77666	4.7374	0.34849	0.95373	0.0020391	0.7308	0.0020421	0.7319	-4.21E-05	2.681E-04	-6.37	-86.84	1.86022
0.001	1.3385	2.1752	4.2261	5.4380	4.4276	0.77715	4.7324	0.34776	0.95451	0.0020331	0.7287	0.0020361	0.7297	-3.61E-05	2.703E-04	-7.49	-94.34	1.86012
0.0001	1.3386	2.1753	4.2266	5.4382	4.4277	0.77720	4.7319	0.34769	0.95458	0.0020325	0.7285	0.0020328	0.7286	-3.28E-05	2.714E-05	-0.83	-95.16	1.86011
0.0001	1.3387	2.1754	4.2270	5.4385	4.4278	0.77725	4.7314	0.34762	0.95466	0.0020319	0.7283	0.0020322	0.7284	-3.22E-05	2.716E-05	-0.84	-96.01	1.86010
0.0001	1.3388	2.1755	4.2275	5.4387	4.4279	0.77730	4.7310	0.34755	0.95474	0.0020313	0.7280	0.0020316	0.7281	-3.16E-05	2.719E-05	-0.86	-96.87	1.86009
0.0001	1.3389	2.1756	4.2279	5.4389	4.4280	0.77735	4.7305	0.34747	0.95482	0.0020307	0.7278	0.0020310	0.7279	-3.10E-05	2.721E-05	-0.88	-97.75	1.86009
0.0001	1.3390	2.1757	4.2284	5.4391	4.4281	0.77740	4.7300	0.34740	0.95489	0.0020301	0.7276	0.0020304	0.7277	-3.04E-05	2.723E-05	-0.90	-98.64	1.86008
0.0001	1.3391	2.1757	4.2288	5.4394	4.4282	0.77744	4.7295	0.34733	0.95497	0.0020295	0.7274	0.0020298	0.7275	-2.98E-05	2.725E-05	-0.91	-99.56	1.86007
0.0001	1.3392	2.1758	4.2292	5.4396	4.4283	0.77749	4.7290	0.34725	0.95505	0.0020289	0.7272	0.0020292	0.7273	-2.92E-05	2.727E-05	-0.93	-100.49	1.86006
0.0001	1.3393	2.1759	4.2297	5.4398	4.4284	0.77754	4.7285	0.34718	0.95512	0.0020283	0.7270	0.0020286	0.7271	-2.86E-05	2.729E-05	-0.95	-101.45	1.86005
0.0001	1.3394	2.1760	4.2301	5.4400	4.4285	0.77759	4.7280	0.34711	0.95520	0.0020277	0.7268	0.0020280	0.7269	-2.80E-05	2.731E-05	-0.98	-102.42	1.86004

Table B-7 Case B 30 cfs: Manually Computed Outlet Control Headwater Depths

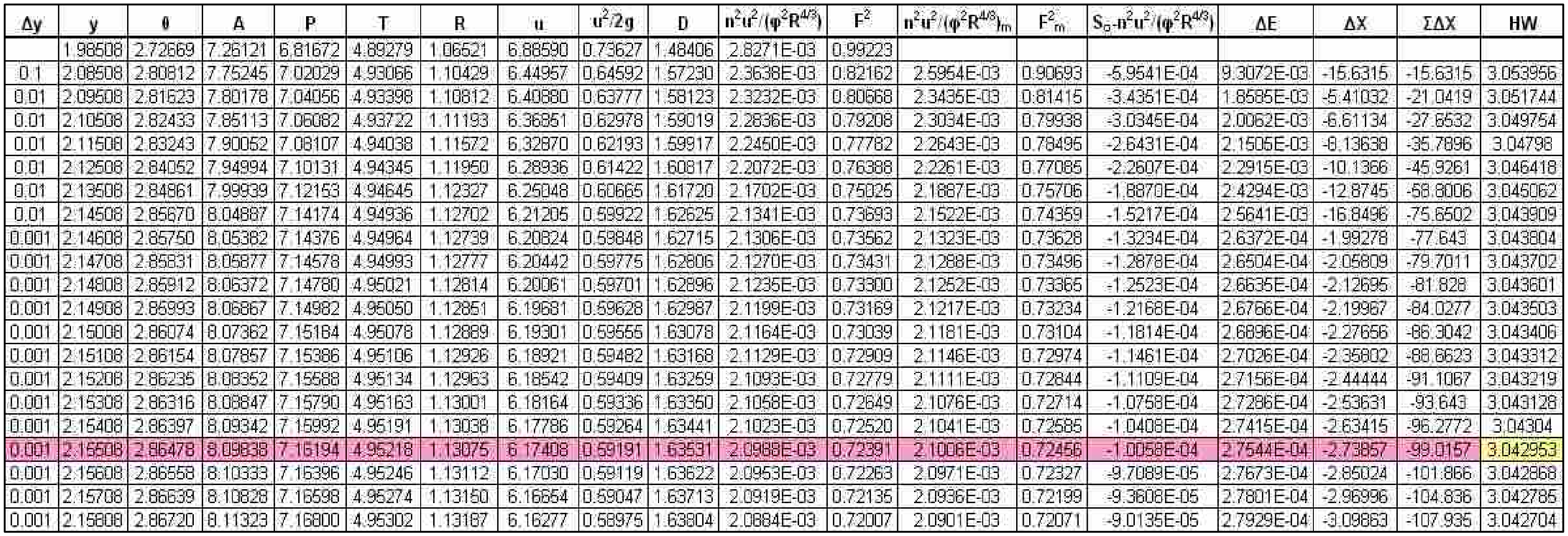


Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	1.53764	2.35130	5.12730	5.87824	4.61470	0.87225	5.85104	0.53159	1.11108	0.002664	0.95690							
0.01	1.54764	2.35996	5.17349	5.89989	4.62299	0.87688	5.79880	0.52214	1.11908	0.002599	0.93317	0.002632	0.94503	-6.315E-04	5.497E-04	-0.870	-0.870	2.3309
0.01	1.55764	2.36860	5.21976	5.92150	4.63118	0.88149	5.74739	0.51293	1.12709	0.002535	0.91018	0.002567	0.92167	-5.668E-04	7.833E-04	-1.382	-2.252	2.3270
0.01	1.56764	2.37723	5.26611	5.94308	4.63927	0.88609	5.69681	0.50394	1.13512	0.002473	0.88790	0.002504	0.89904	-5.042E-04	1.010E-03	-2.003	-4.255	2.3235
0.01	1.57764	2.38585	5.31254	5.96461	4.64726	0.89068	5.64701	0.49517	1.14316	0.002414	0.86632	0.002443	0.87711	-4.435E-04	1.229E-03	-2.771	-7.026	2.3204
0.01	1.58764	2.39445	5.35905	5.98611	4.65515	0.89525	5.59800	0.48661	1.15121	0.002356	0.84539	0.002385	0.85585	-3.847E-04	1.441E-03	-3.747	-10.773	2.3176
0.01	1.59764	2.40303	5.40564	6.00758	4.66294	0.89980	5.54975	0.47826	1.15928	0.002300	0.82509	0.002328	0.83524	-3.277E-04	1.648E-03	-5.027	-15.800	2.3150
0.01	1.60764	2.41160	5.45231	6.02901	4.67063	0.90435	5.50225	0.47011	1.16736	0.002245	0.80541	0.002273	0.81525	-2.725E-04	1.847E-03	-6.778	-22.579	2.3128
0.01	1.61764	2.42016	5.49906	6.05040	4.67822	0.90888	5.45548	0.46215	1.17546	0.002193	0.78633	0.002219	0.79587	-2.191E-04	2.041E-03	-9.319	-31.897	2.3109
0.01	1.62764	2.42870	5.54588	6.07176	4.68572	0.91339	5.40942	0.45438	1.18357	0.002142	0.76781	0.002167	0.77707	-1.672E-04	2.229E-03	-13.334	-45.231	2.3092
0.01	1.63764	2.43723	5.59277	6.09308	4.69312	0.91789	5.36407	0.44679	1.19170	0.002092	0.74984	0.002117	0.75882	-1.169E-04	2.412E-03	-20.631	-65.862	2.3078
0.001	1.63864	2.43809	5.59746	6.09521	4.69385	0.91834	5.35957	0.44604	1.19251	0.002087	0.74807	0.002090	0.74895	-8.971E-05	2.510E-04	-2.798	-68.660	2.3077
0.001	1.63964	2.43894	5.60216	6.09734	4.69458	0.91879	5.35508	0.44529	1.19332	0.002082	0.74631	0.002085	0.74719	-8.485E-05	2.528E-04	-2.980	-71.640	2.3076
0.001	1.64064	2.43979	5.60685	6.09947	4.69532	0.91924	5.35059	0.44455	1.19414	0.002078	0.74455	0.002080	0.74543	-8.000E-05	2.546E-04	-3.182	-74.822	2.3075
0.001	1.64164	2.44064	5.61155	6.10160	4.69605	0.91968	5.34612	0.44380	1.19495	0.002073	0.74280	0.002075	0.74367	-7.517E-05	2.563E-04	-3.410	-78.232	2.3073
0.001	1.64264	2.44149	5.61625	6.10373	4.69678	0.92013	5.34165	0.44306	1.19577	0.002068	0.74105	0.002070	0.74192	-7.035E-05	2.581E-04	-3.668	-81.900	2.3072
0.001	1.64364	2.44234	5.62094	6.10586	4.69751	0.92058	5.33718	0.44232	1.19658	0.002063	0.73931	0.002066	0.74018	-6.555E-05	2.598E-04	-3.964	-85.864	2.3071
0.001	1.64464	2.44320	5.62564	6.10799	4.69824	0.92103	5.33273	0.44158	1.19739	0.002058	0.73757	0.002061	0.73844	-6.076E-05	2.616E-04	-4.305	-90.169	2.3070
0.001	1.64564	2.44405	5.63034	6.11012	4.69896	0.92148	5.32828	0.44085	1.19821	0.002054	0.73584	0.002056	0.73671	-5.599E-05	2.633E-04	-4.703	-94.872	2.3069
0.001	1.64664	2.44490	5.63504	6.11225	4.69969	0.92193	5.32383	0.44011	1.19902	0.002049	0.73412	0.002051	0.73498	-5.123E-05	2.650E-04	-5.173	-100.045	2.3068
0.001	1.64764	2.44575	5.63974	6.11437	4.70042	0.92237	5.31940	0.43938	1.19984	0.002044	0.73240	0.002046	0.73326	-4.648E-05	2.667E-04	-5.739	-105.783	2.3067

Table B-8 Case B 40 cfs: Manually Computed Outlet Control Headwater Depths

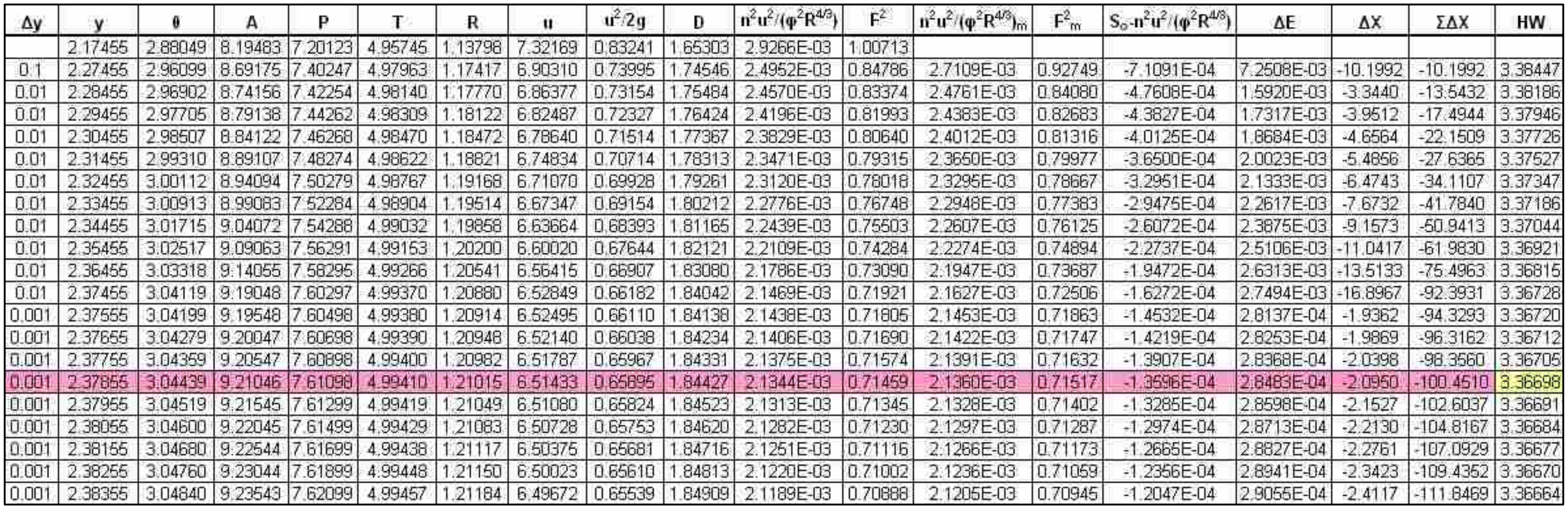
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	1.77551	2.55357	6.24640	6.38392	4.78544	0.97846	6.40369	0.63676	1.30529	0.00273817	0.97566							
0.01	1.78551	2.56192	6.29428	6.40480	4.79145	0.98274	6.35497	0.62711	1.31365	0.002681	0.95476	2.7096E-03	0.96521	-7.0959E-04	3.4793E-04	-0.49032	-0.49032	2.7262
0.01	1.79551	2.57026	6.34223	6.42566	4.79737	0.98702	6.30693	0.61766	1.32202	0.00262539	0.93442	2.6532E-03	0.94459	-6.5320E-04	5.5411E-04	-0.84831	-1.33863	2.7220
0.01	1.80551	2.57860	6.39023	6.44649	4.80320	0.99127	6.25956	0.60842	1.33041	0.0025713	0.91463	2.5983E-03	0.92453	-5.9835E-04	7.5474E-04	-1.26138	-2.60001	2.7181
0.01	1.81551	2.58692	6.43829	6.46730	4.80894	0.99551	6.21283	0.59937	1.33882	0.00251867	0.89537	2.5450E-03	0.90500	-5.4499E-04	9.5000E-04	-1.74316	-4.34317	2.7146
0.01	1.82551	2.59523	6.48641	6.48808	4.81459	0.99974	6.16674	0.59051	1.34724	0.00246746	0.87662	2.4931E-03	0.88599	-4.9307E-04	1.1401E-03	-2.31218	-6.65535	2.7113
0.01	1.83551	2.60353	6.53458	6.50884	4.82015	1.00396	6.12128	0.58183	1.35568	0.00241762	0.85836	2.4425E-03	0.86749	-4.4254E-04	1.3251E-03	-2.99426	-9.64960	2.7083
0.01	1.84551	2.61183	6.58281	6.52957	4.82562	1.00815	6.07643	0.57334	1.36414	0.00236911	0.84059	2.3934E-03	0.84948	-3.9337E-04	1.5052E-03	-3.82657	-13.47617	2.7055
0.01	1.85551	2.62011	6.63109	6.55028	4.83100	1.01234	6.03219	0.56502	1.37261	0.00232188	0.82328	2.3455E-03	0.83193	-3.4549E-04	1.6807E-03	-4.86456	-18.34073	2.7030
0.01	1.86551	2.62839	6.67943	6.57097	4.83629	1.01651	5.98854	0.55687	1.38111	0.00227589	0.80641	2.2989E-03	0.81485	-2.9889E-04	1.8515E-03	-6.19484	-24.53557	2.7008
0.01	1.87551	2.63665	6.72782	6.59164	4.84149	1.02066	5.94546	0.54889	1.38962	0.00223111	0.78999	2.2535E-03	0.79820	-2.5350E-04	2.0180E-03	-7.96055	-32.49612	2.6988
0.01	1.88551	2.64491	6.77626	6.61228	4.84661	1.02480	5.90296	0.54107	1.39814	0.00218748	0.77398	2.2093E-03	0.78199	-2.0930E-04	2.1801E-03	-10.41657	-42.91269	2.6971
0.01	1.89551	2.65316	6.82475	6.63290	4.85164	1.02892	5.86102	0.53341	1.40669	0.00214499	0.75839	2.1662E-03	0.76619	-1.6624E-04	2.3381E-03	-14.06496	-56.97765	2.6956
0.01	1.90551	2.66140	6.87329	6.65350	4.85658	1.03303	5.81963	0.52590	1.41525	0.00210359	0.74319	2.1243E-03	0.75079	-1.2429E-04	2.4921E-03	-20.05049	-77.02814	2.6944
0.001	1.90651	2.66223	6.87815	6.65556	4.85707	1.03344	5.81552	0.52516	1.41611	0.00209951	0.74169	2.1016E-03	0.74244	-1.0155E-04	2.5756E-04	-2.53626	-79.56440	2.6943
0.001	1.90751	2.66305	6.88301	6.65762	4.85755	1.03385	5.81141	0.52442	1.41697	0.00209544	0.74020	2.0975E-03	0.74094	-9.7475E-05	2.5906E-04	-2.65767	-82.22208	2.6941
0.001	1.90851	2.66387	6.88786	6.65968	4.85804	1.03426	5.80732	0.52368	1.41783	0.00209138	0.73871	2.0934E-03	0.73945	-9.3409E-05	2.6055E-04	-2.78933	-85.01141	2.6940
0.001	1.90951	2.66470	6.89272	6.66174	4.85853	1.03467	5.80322	0.52294	1.41869	0.00208733	0.73722	2.0894E-03	0.73796	-8.9354E-05	2.6204E-04	-2.93257	-87.94398	2.6939
0.001	1.91051	2.66552	6.89758	6.66380	4.85901	1.03508	5.79913	0.52220	1.41954	0.00208329	0.73574	2.0853E-03	0.73648	-8.5310E-05	2.6352E-04	-3.08901	-91.03299	2.6938
0.001	1.91151	2.66634	6.90244	6.66586	4.85950	1.03549	5.79505	0.52147	1.42040	0.00207926	0.73426	2.0813E-03	0.73500	-8.1276E-05	2.6500E-04	-3.26056	-94.29356	2.6937
0.001	1.91251	2.66717	6.90730	6.66791	4.85998	1.03590	5.79098	0.52074	1.42126	0.00207524	0.73278	2.0773E-03	0.73352	-7.7252E-05	2.6648E-04	-3.44952	-97.74307	2.6936
0.001	1.91351	2.66799	6.91216	6.66997	4.86047	1.03631	5.78690	0.52000	1.42212	0.00207123	0.73131	2.0732E-03	0.73204	-7.3239E-05	2.6796E-04	-3.65867	-101.40174	2.6935
0.001	1.91451	2.66881	6.91702	6.67203	4.86095	1.03672	5.78284	0.51927	1.42298	0.00206724	0.72984	2.0692E-03	0.73057	-6.9236E-05	2.6943E-04	-3.89143	-105.29317	2.6934
0.001	1.91551	2.66963	6.92188	6.67408	4.86143	1.03713	5.77878	0.51854	1.42384	0.00206325	0.72838	2.0652E-03	0.72911	-6.5243E-05	2.7089E-04	-4.15205	-109.44522	2.6933
0.001	1.91651	2.67046	6.92674	6.67614	4.86191	1.03754	5.77472	0.51782	1.42470	0.00205927	0.72692	2.0613E-03	0.72765	-6.1261E-05	2.7235E-04	-4.44584	-113.89105	2.6932

Table B-9 Case B 50 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	1.98508	2.72669	7.26121	6.81672	4.89279	1.06521	6.88590	0.73627	1.48406	2.8271E-03	0.99223							
0.1	2.08508	2.80812	7.75245	7.02029	4.93066	1.10429	6.44957	0.64592	1.57230	2.3638E-03	0.82162	2.5954E-03	0.90693	-5.9541E-04	9.3072E-03	-15.6315	-15.6315	3.053956
0.01	2.09508	2.81623	7.80178	7.04056	4.93398	1.10812	6.40880	0.63777	1.58123	2.3232E-03	0.80668	2.3435E-03	0.81415	-3.4351E-04	1.8585E-03	-5.41032	-21.0419	3.051744
0.01	2.10508	2.82433	7.85113	7.06082	4.93722	1.11193	6.36851	0.62978	1.59019	2.2836E-03	0.79208	2.3034E-03	0.79938	-3.0345E-04	2.0062E-03	-6.61134	-27.6532	3.049754
0.01	2.11508	2.83243	7.90052	7.08107	4.94038	1.11572	6.32870	0.62193	1.59917	2.2450E-03	0.77782	2.2643E-03	0.78495	-2.6431E-04	2.1505E-03	-8.13638	-35.7896	3.04798
0.01	2.12508	2.84052	7.94994	7.10131	4.94345	1.11950	6.28936	0.61422	1.60817	2.2072E-03	0.76388	2.2261E-03	0.77085	-2.2607E-04	2.2915E-03	-10.1366	-45.9261	3.046418
0.01	2.13508	2.84861	7.99939	7.12153	4.94645	1.12327	6.25048	0.60665	1.61720	2.1702E-03	0.75025	2.1887E-03	0.75706	-1.8870E-04	2.4294E-03	-12.8745	-58.8006	3.045062
0.01	2.14508	2.85670	8.04887	7.14174	4.94936	1.12702	6.21205	0.59922	1.62625	2.1341E-03	0.73693	2.1522E-03	0.74359	-1.5217E-04	2.5641E-03	-16.8496	-75.6502	3.043909
0.001	2.14608	2.85750	8.05382	7.14376	4.94964	1.12739	6.20824	0.59848	1.62715	2.1306E-03	0.73562	2.1323E-03	0.73628	-1.3234E-04	2.6372E-04	-1.99278	-77.643	3.043804
0.001	2.14708	2.85831	8.05877	7.14578	4.94993	1.12777	6.20442	0.59775	1.62806	2.1270E-03	0.73431	2.1288E-03	0.73496	-1.2878E-04	2.6504E-04	-2.05809	-79.7011	3.043702
0.001	2.14808	2.85912	8.06372	7.14780	4.95021	1.12814	6.20061	0.59701	1.62896	2.1235E-03	0.73300	2.1252E-03	0.73365	-1.2523E-04	2.6635E-04	-2.12695	-81.828	3.043601
0.001	2.14908	2.85993	8.06867	7.14982	4.95050	1.12851	6.19681	0.59628	1.62987	2.1199E-03	0.73169	2.1217E-03	0.73234	-1.2168E-04	2.6766E-04	-2.19967	-84.0277	3.043503
0.001	2.15008	2.86074	8.07362	7.15184	4.95078	1.12889	6.19301	0.59555	1.63078	2.1164E-03	0.73039	2.1181E-03	0.73104	-1.1814E-04	2.6896E-04	-2.27656	-86.3042	3.043406
0.001	2.15108	2.86154	8.07857	7.15386	4.95106	1.12926	6.18921	0.59482	1.63168	2.1129E-03	0.72909	2.1146E-03	0.72974	-1.1461E-04	2.7026E-04	-2.35802	-88.6623	3.043312
0.001	2.15208	2.86235	8.08352	7.15588	4.95134	1.12963	6.18542	0.59409	1.63259	2.1093E-03	0.72779	2.1111E-03	0.72844	-1.1109E-04	2.7156E-04	-2.44444	-91.1067	3.043219
0.001	2.15308	2.86316	8.08847	7.15790	4.95163	1.13001	6.18164	0.59336	1.63350	2.1058E-03	0.72649	2.1076E-03	0.72714	-1.0758E-04	2.7286E-04	-2.53631	-93.643	3.043128
0.001	2.15408	2.86397	8.09342	7.15992	4.95191	1.13038	6.17786	0.59264	1.63441	2.1023E-03	0.72520	2.1041E-03	0.72585	-1.0408E-04	2.7415E-04	-2.63415	-96.2772	3.04304
0.001	2.15508	2.86478	8.09838	7.16194	4.95218	1.13075	6.17408	0.59191	1.63531	2.0988E-03	0.72391	2.1006E-03	0.72456	-1.0058E-04	2.7544E-04	-2.73857	-99.0157	3.042953
0.001	2.15608	2.86558	8.10333	7.16396	4.95246	1.13112	6.17030	0.59119	1.63622	2.0953E-03	0.72263	2.0971E-03	0.72327	-9.7089E-05	2.7673E-04	-2.85024	-101.866	3.042868
0.001	2.15708	2.86639	8.10828	7.16598	4.95274	1.13150	6.16654	0.59047	1.63713	2.0919E-03	0.72135	2.0936E-03	0.72199	-9.3608E-05	2.7801E-04	-2.96996	-104.836	3.042785
0.001	2.15808	2.86720	8.11323	7.16800	4.95302	1.13187	6.16277	0.58975	1.63804	2.0884E-03	0.72007	2.0901E-03	0.72071	-9.0135E-05	2.7929E-04	-3.09863	-107.935	3.042704

Table B-10 Case B 60 cfs: Manually Computed Outlet Control Headwater Depths

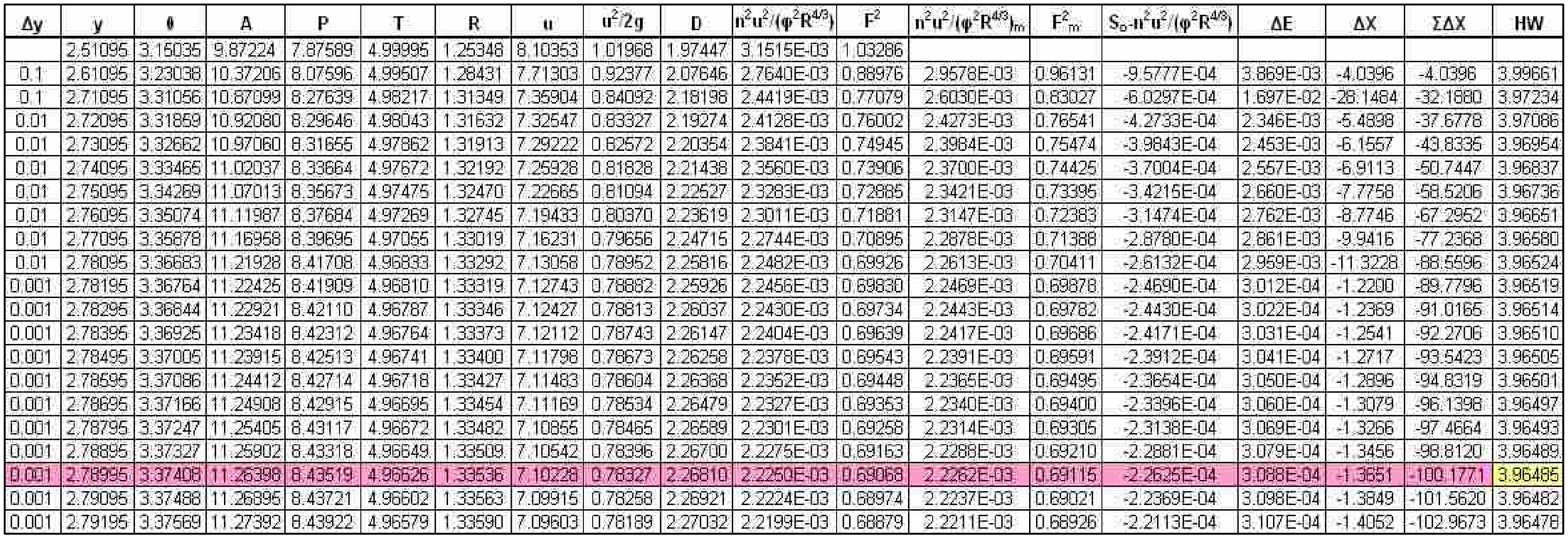


Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	2.17455	2.88049	8.19483	7.20123	4.95745	1.13798	7.32169	0.83241	1.65303	2.9266E-03	1.00713							
0.1	2.27455	2.96099	8.69175	7.40247	4.97963	1.17417	6.90310	0.73995	1.74546	2.4952E-03	0.84786	2.7109E-03	0.92749	-7.1091E-04	7.2508E-03	-10.1992	-10.1992	3.38447
0.01	2.28455	2.96902	8.74156	7.42254	4.98140	1.17770	6.86377	0.73154	1.75484	2.4570E-03	0.83374	2.4761E-03	0.84080	-4.7608E-04	1.5920E-03	-3.3440	-13.5432	3.38186
0.01	2.29455	2.97705	8.79138	7.44262	4.98309	1.18122	6.82487	0.72327	1.76424	2.4196E-03	0.81993	2.4383E-03	0.82683	-4.3827E-04	1.7317E-03	-3.9512	-17.4944	3.37946
0.01	2.30455	2.98507	8.84122	7.46268	4.98470	1.18472	6.78640	0.71514	1.77367	2.3829E-03	0.80640	2.4012E-03	0.81316	-4.0125E-04	1.8684E-03	-4.6564	-22.1509	3.37726
0.01	2.31455	2.99310	8.89107	7.48274	4.98622	1.18821	6.74834	0.70714	1.78313	2.3471E-03	0.79315	2.3650E-03	0.79977	-3.6500E-04	2.0023E-03	-5.4856	-27.6365	3.37527
0.01	2.32455	3.00112	8.94094	7.50279	4.98767	1.19168	6.71070	0.69928	1.79261	2.3120E-03	0.78018	2.3295E-03	0.78667	-3.2951E-04	2.1333E-03	-6.4743	-34.1107	3.37347
0.01	2.33455	3.00913	8.99083	7.52284	4.98904	1.19514	6.67347	0.69154	1.80212	2.2776E-03	0.76748	2.2948E-03	0.77383	-2.9475E-04	2.2617E-03	-7.6732	-41.7840	3.37186
0.01	2.34455	3.01715	9.04072	7.54288	4.99032	1.19858	6.63664	0.68393	1.81165	2.2439E-03	0.75503	2.2607E-03	0.76125	-2.6072E-04	2.3875E-03	-9.1573	-50.9413	3.37044
0.01	2.35455	3.02517	9.09063	7.56291	4.99153	1.20200	6.60020	0.67644	1.82121	2.2109E-03	0.74284	2.2274E-03	0.74894	-2.2737E-04	2.5106E-03	-11.0417	-61.9830	3.36921
0.01	2.36455	3.03318	9.14055	7.58295	4.99266	1.20541	6.56415	0.66907	1.83080	2.1786E-03	0.73090	2.1947E-03	0.73687	-1.9472E-04	2.6313E-03	-13.5133	-75.4963	3.36815
0.01	2.37455	3.04119	9.19048	7.60297	4.99370	1.20880	6.52849	0.66182	1.84042	2.1469E-03	0.71921	2.1627E-03	0.72506	-1.6272E-04	2.7494E-03	-16.8967	-92.3931	3.36728
0.001	2.37555	3.04199	9.19548	7.60498	4.99380	1.20914	6.52495	0.66110	1.84138	2.1438E-03	0.71805	2.1453E-03	0.71863	-1.4532E-04	2.8137E-04	-1.9362	-94.3293	3.36720
0.001	2.37655	3.04279	9.20047	7.60698	4.99390	1.20948	6.52140	0.66038	1.84234	2.1406E-03	0.71690	2.1422E-03	0.71747	-1.4219E-04	2.8253E-04	-1.9869	-96.3162	3.36712
0.001	2.37755	3.04359	9.20547	7.60898	4.99400	1.20982	6.51787	0.65967	1.84331	2.1375E-03	0.71574	2.1391E-03	0.71632	-1.3907E-04	2.8368E-04	-2.0398	-98.3560	3.36705
0.001	2.37855	3.04439	9.21046	7.61098	4.99410	1.21015	6.51433	0.65895	1.84427	2.1344E-03	0.71459	2.1360E-03	0.71517	-1.3596E-04	2.8483E-04	-2.0950	-100.4510	3.36698
0.001	2.37955	3.04519	9.21545	7.61299	4.99419	1.21049	6.51080	0.65824	1.84523	2.1313E-03	0.71345	2.1328E-03	0.71402	-1.3285E-04	2.8598E-04	-2.1527	-102.6037	3.36691
0.001	2.38055	3.04600	9.22045	7.61499	4.99429	1.21083	6.50728	0.65753	1.84620	2.1282E-03	0.71230	2.1297E-03	0.71287	-1.2974E-04	2.8713E-04	-2.2130	-104.8167	3.36684
0.001	2.38155	3.04680	9.22544	7.61699	4.99438	1.21117	6.50375	0.65681	1.84716	2.1251E-03	0.71116	2.1266E-03	0.71173	-1.2665E-04	2.8827E-04	-2.2761	-107.0929	3.36677
0.001	2.38255	3.04760	9.23044	7.61899	4.99448	1.21150	6.50023	0.65610	1.84813	2.1220E-03	0.71002	2.1236E-03	0.71059	-1.2356E-04	2.8941E-04	-2.3423	-109.4352	3.36670
0.001	2.38355	3.04840	9.23543	7.62099	4.99457	1.21184	6.49672	0.65539	1.84909	2.1189E-03	0.70888	2.1205E-03	0.70945	-1.2047E-04	2.9055E-04	-2.4117	-111.8469	3.36664

Table B-11 Case B 70 cfs: Manually Computed Outlet Control Headwater Depths

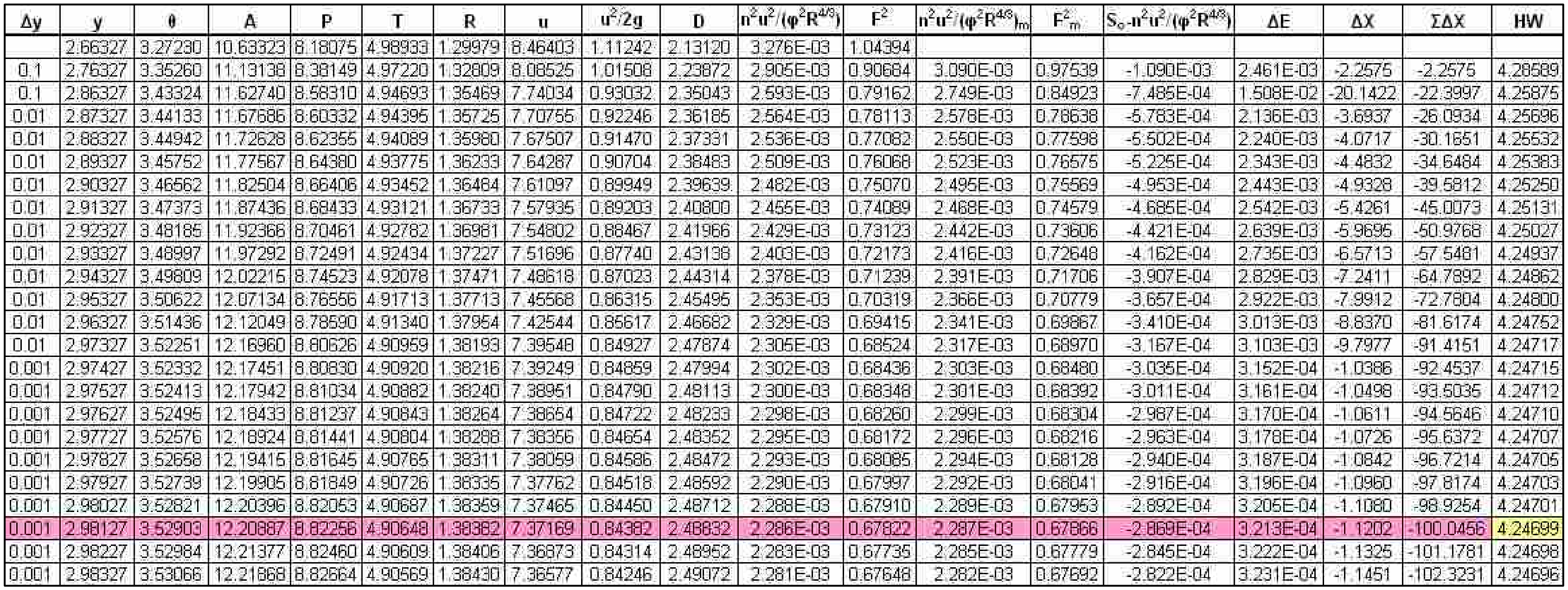
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.34878	3.02054	9.06184	7.55136	4.99084	1.20003	7.72470	0.92657	1.81569	3.0350E-03	1.02062							
0.1	2.44878	3.10061	9.56140	7.75154	4.98895	1.23348	7.32110	0.83228	1.91268	2.6280E-03	0.87027	2.8315E-03	0.94545	-8.3154E-04	5.4554E-03	-6.56062	-6.56062	3.69719
0.1	2.54878	3.18062	10.06137	7.95155	4.99905	1.26533	6.95730	0.75162	2.01266	2.2940E-03	0.74689	2.4610E-03	0.80858	-4.6103E-04	1.9142E-02	-41.51986	-48.08048	3.67621
0.01	2.55878	3.18862	10.11136	7.97155	4.99862	1.26843	6.92291	0.74420	2.02283	2.2640E-03	0.73580	2.2790E-03	0.74135	-2.7902E-04	2.5865E-03	-9.27008	-57.35056	3.67509
0.01	2.56878	3.19662	10.16134	7.99156	4.99811	1.27151	6.88886	0.73690	2.03304	2.2346E-03	0.72492	2.2493E-03	0.73036	-2.4929E-04	2.6964E-03	-10.81635	-68.16691	3.67413
0.01	2.57878	3.20463	10.21132	8.01157	4.99752	1.27457	6.85514	0.72970	2.04328	2.2057E-03	0.71425	2.2201E-03	0.71959	-2.2011E-04	2.8041E-03	-12.73999	-80.90690	3.67334
0.001	2.57978	3.20543	10.21632	8.01357	4.99745	1.27488	6.85179	0.72899	2.04430	2.2028E-03	0.71319	2.2042E-03	0.71372	-2.0422E-04	2.8628E-04	-1.40181	-82.30871	3.67327
0.001	2.58078	3.20623	10.22131	8.01557	4.99739	1.27518	6.84844	0.72828	2.04533	2.1999E-03	0.71214	2.2014E-03	0.71266	-2.0136E-04	2.8734E-04	-1.42696	-83.73567	3.67320
0.001	2.58178	3.20703	10.22631	8.01757	4.99732	1.27549	6.84509	0.72757	2.04636	2.1971E-03	0.71108	2.1985E-03	0.71161	-1.9851E-04	2.8839E-04	-1.45277	-85.18844	3.67313
0.001	2.58278	3.20783	10.23131	8.01957	4.99726	1.27579	6.84175	0.72686	2.04738	2.1942E-03	0.71003	2.1957E-03	0.71056	-1.9566E-04	2.8944E-04	-1.47929	-86.66773	3.67306
0.001	2.58378	3.20863	10.23630	8.02158	4.99719	1.27610	6.83841	0.72615	2.04841	2.1914E-03	0.70898	2.1928E-03	0.70951	-1.9282E-04	2.9049E-04	-1.50654	-88.17427	3.67300
0.001	2.58478	3.20943	10.24130	8.02358	4.99712	1.27640	6.83507	0.72544	2.04944	2.1886E-03	0.70794	2.1900E-03	0.70846	-1.8998E-04	2.9154E-04	-1.53456	-89.70883	3.67294
0.001	2.58578	3.21023	10.24630	8.02558	4.99706	1.27671	6.83174	0.72473	2.05047	2.1857E-03	0.70689	2.1872E-03	0.70741	-1.8715E-04	2.9259E-04	-1.56337	-91.27220	3.67288
0.001	2.58678	3.21103	10.25130	8.02758	4.99699	1.27701	6.82841	0.72402	2.05150	2.1829E-03	0.70585	2.1843E-03	0.70637	-1.8432E-04	2.9363E-04	-1.59300	-92.86520	3.67282
0.001	2.58778	3.21183	10.25629	8.02958	4.99692	1.27731	6.82508	0.72332	2.05252	2.1801E-03	0.70481	2.1815E-03	0.70533	-1.8150E-04	2.9467E-04	-1.62351	-94.48870	3.67276
0.001	2.58878	3.21263	10.26129	8.03158	4.99685	1.27762	6.82175	0.72261	2.05355	2.1773E-03	0.70377	2.1787E-03	0.70429	-1.7869E-04	2.9571E-04	-1.65491	-96.14362	3.67270
0.001	2.58978	3.21343	10.26629	8.03358	4.99677	1.27792	6.81843	0.72191	2.05458	2.1745E-03	0.70273	2.1759E-03	0.70325	-1.7588E-04	2.9675E-04	-1.68726	-97.83088	3.67265
0.001	2.59078	3.21423	10.27128	8.03558	4.99670	1.27822	6.81512	0.72121	2.05561	2.1717E-03	0.70170	2.1731E-03	0.70221	-1.7307E-04	2.9779E-04	-1.72060	-99.55148	3.67259
0.001	2.59178	3.21503	10.27628	8.03759	4.99663	1.27853	6.81180	0.72051	2.05664	2.1689E-03	0.70066	2.1703E-03	0.70118	-1.7027E-04	2.9882E-04	-1.75497	-101.30645	3.67254
0.001	2.59278	3.21583	10.28128	8.03959	4.99656	1.27883	6.80849	0.71981	2.05767	2.1661E-03	0.69963	2.1675E-03	0.70015	-1.6747E-04	2.9985E-04	-1.79043	-103.09688	3.67249
0.001	2.59378	3.21664	10.28627	8.04159	4.99648	1.27913	6.80519	0.71911	2.05870	2.1633E-03	0.69860	2.1647E-03	0.69912	-1.6468E-04	3.0088E-04	-1.82702	-104.92390	3.67244

Table B-12 Case B 80 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.51095	3.15035	9.87224	7.87589	4.99995	1.25348	8.10353	1.01968	1.97447	3.1515E-03	1.03286							
0.1	2.61095	3.23038	10.37206	8.07596	4.99507	1.28431	7.71303	0.92377	2.07646	2.7640E-03	0.88976	2.9578E-03	0.96131	-9.5777E-04	3.869E-03	-4.0396	-4.0396	3.99661
0.1	2.71095	3.31056	10.87099	8.27639	4.98217	1.31349	7.35904	0.84092	2.18198	2.4419E-03	0.77079	2.6030E-03	0.83027	-6.0297E-04	1.697E-02	-28.1484	-32.1880	3.97234
0.01	2.72095	3.31859	10.92080	8.29646	4.98043	1.31632	7.32547	0.83327	2.19274	2.4128E-03	0.76002	2.4273E-03	0.76541	-4.2733E-04	2.346E-03	-5.4898	-37.6778	3.97086
0.01	2.73095	3.32662	10.97060	8.31655	4.97862	1.31913	7.29222	0.82572	2.20354	2.3841E-03	0.74945	2.3984E-03	0.75474	-3.9843E-04	2.453E-03	-6.1557	-43.8335	3.96954
0.01	2.74095	3.33465	11.02037	8.33664	4.97672	1.32192	7.25928	0.81828	2.21438	2.3560E-03	0.73906	2.3700E-03	0.74425	-3.7004E-04	2.557E-03	-6.9113	-50.7447	3.96837
0.01	2.75095	3.34269	11.07013	8.35673	4.97475	1.32470	7.22665	0.81094	2.22527	2.3283E-03	0.72885	2.3421E-03	0.73395	-3.4215E-04	2.660E-03	-7.7758	-58.5206	3.96736
0.01	2.76095	3.35074	11.11987	8.37684	4.97269	1.32745	7.19433	0.80370	2.23619	2.3011E-03	0.71881	2.3147E-03	0.72383	-3.1474E-04	2.762E-03	-8.7746	-67.2952	3.96651
0.01	2.77095	3.35878	11.16958	8.39695	4.97055	1.33019	7.16231	0.79656	2.24715	2.2744E-03	0.70895	2.2878E-03	0.71388	-2.8780E-04	2.861E-03	-9.9416	-77.2368	3.96580
0.01	2.78095	3.36683	11.21928	8.41708	4.96833	1.33292	7.13058	0.78952	2.25816	2.2482E-03	0.69926	2.2613E-03	0.70411	-2.6132E-04	2.959E-03	-11.3228	-88.5596	3.96524
0.001	2.78195	3.36764	11.22425	8.41909	4.96810	1.33319	7.12743	0.78882	2.25926	2.2456E-03	0.69830	2.2469E-03	0.69878	-2.4690E-04	3.012E-04	-1.2200	-89.7796	3.96519
0.001	2.78295	3.36844	11.22921	8.42110	4.96787	1.33346	7.12427	0.78813	2.26037	2.2430E-03	0.69734	2.2443E-03	0.69782	-2.4430E-04	3.022E-04	-1.2369	-91.0165	3.96514
0.001	2.78395	3.36925	11.23418	8.42312	4.96764	1.33373	7.12112	0.78743	2.26147	2.2404E-03	0.69639	2.2417E-03	0.69686	-2.4171E-04	3.031E-04	-1.2541	-92.2706	3.96510
0.001	2.78495	3.37005	11.23915	8.42513	4.96741	1.33400	7.11798	0.78673	2.26258	2.2378E-03	0.69543	2.2391E-03	0.69591	-2.3912E-04	3.041E-04	-1.2717	-93.5423	3.96505
0.001	2.78595	3.37086	11.24412	8.42714	4.96718	1.33427	7.11483	0.78604	2.26368	2.2352E-03	0.69448	2.2365E-03	0.69495	-2.3654E-04	3.050E-04	-1.2896	-94.8319	3.96501
0.001	2.78695	3.37166	11.24908	8.42915	4.96695	1.33454	7.11169	0.78534	2.26479	2.2327E-03	0.69353	2.2340E-03	0.69400	-2.3396E-04	3.060E-04	-1.3079	-96.1398	3.96497
0.001	2.78795	3.37247	11.25405	8.43117	4.96672	1.33482	7.10855	0.78465	2.26589	2.2301E-03	0.69258	2.2314E-03	0.69305	-2.3138E-04	3.069E-04	-1.3266	-97.4664	3.96493
0.001	2.78895	3.37327	11.25902	8.43318	4.96649	1.33509	7.10542	0.78396	2.26700	2.2275E-03	0.69163	2.2288E-03	0.69210	-2.2881E-04	3.079E-04	-1.3456	-98.8120	3.96489
0.001	2.78995	3.37408	11.26398	8.43519	4.96626	1.33536	7.10228	0.78327	2.26810	2.2250E-03	0.69068	2.2262E-03	0.69115	-2.2625E-04	3.088E-04	-1.3651	-100.1771	3.96485
0.001	2.79095	3.37488	11.26895	8.43721	4.96602	1.33563	7.09915	0.78258	2.26921	2.2224E-03	0.68974	2.2237E-03	0.69021	-2.2369E-04	3.098E-04	-1.3849	-101.5620	3.96482
0.001	2.79195	3.37569	11.27392	8.43922	4.96579	1.33590	7.09603	0.78189	2.27032	2.2199E-03	0.68879	2.2211E-03	0.68926	-2.2113E-04	3.107E-04	-1.4052	-102.9673	3.96478

Table B-13 Case B 90 cfs: Manually Computed Outlet Control Headwater Depths

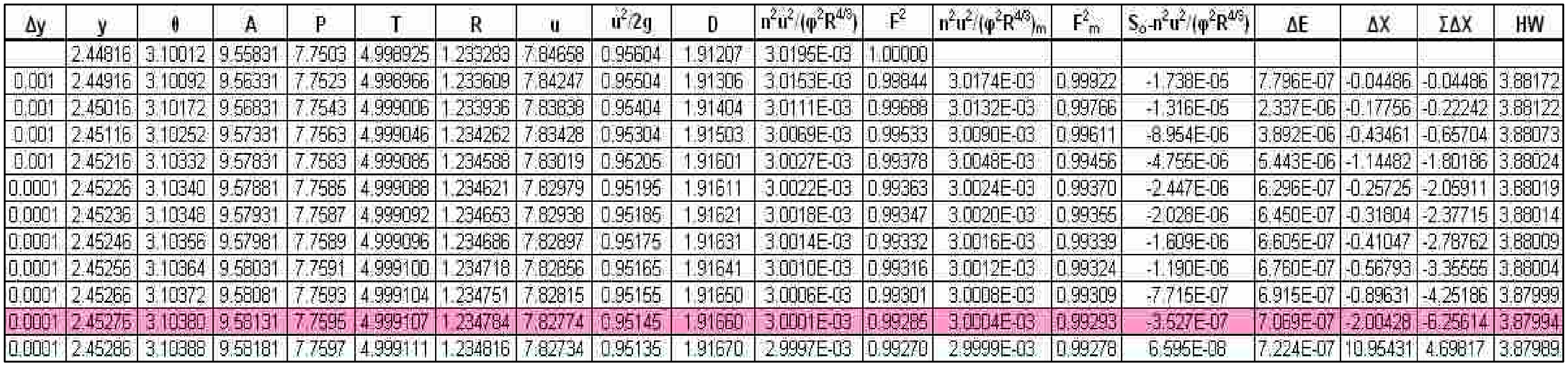


Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.66327	3.27230	10.63323	8.18075	4.98933	1.29979	8.46403	1.11242	2.13120	3.276E-03	1.04394							
0.1	2.76327	3.35260	11.13138	8.38149	4.97220	1.32809	8.08525	1.01508	2.23872	2.905E-03	0.90684	3.090E-03	0.97539	-1.090E-03	2.461E-03	-2.2575	-2.2575	4.28589
0.1	2.86327	3.43324	11.62740	8.58310	4.94693	1.35469	7.74034	0.93032	2.35043	2.593E-03	0.79162	2.749E-03	0.84923	-7.485E-04	1.508E-02	-20.1422	-22.3997	4.25875
0.01	2.87327	3.44133	11.67686	8.60332	4.94395	1.35725	7.70755	0.92246	2.36185	2.564E-03	0.78113	2.578E-03	0.78638	-5.783E-04	2.136E-03	-3.6937	-26.0934	4.25696
0.01	2.88327	3.44942	11.72628	8.62355	4.94089	1.35980	7.67507	0.91470	2.37331	2.536E-03	0.77082	2.550E-03	0.77598	-5.502E-04	2.240E-03	-4.0717	-30.1651	4.25532
0.01	2.89327	3.45752	11.77567	8.64380	4.93775	1.36233	7.64287	0.90704	2.38483	2.509E-03	0.76068	2.523E-03	0.76575	-5.225E-04	2.343E-03	-4.4832	-34.6484	4.25383
0.01	2.90327	3.46562	11.82504	8.66406	4.93452	1.36484	7.61097	0.89949	2.39639	2.482E-03	0.75070	2.495E-03	0.75569	-4.953E-04	2.443E-03	-4.9328	-39.5812	4.25250
0.01	2.91327	3.47373	11.87436	8.68433	4.93121	1.36733	7.57935	0.89203	2.40800	2.455E-03	0.74089	2.468E-03	0.74579	-4.685E-04	2.542E-03	-5.4261	-45.0073	4.25131
0.01	2.92327	3.48185	11.92366	8.70461	4.92782	1.36981	7.54802	0.88467	2.41966	2.429E-03	0.73123	2.442E-03	0.73606	-4.421E-04	2.639E-03	-5.9695	-50.9768	4.25027
0.01	2.93327	3.48997	11.97292	8.72491	4.92434	1.37227	7.51696	0.87740	2.43138	2.403E-03	0.72173	2.416E-03	0.72648	-4.162E-04	2.735E-03	-6.5713	-57.5481	4.24937
0.01	2.94327	3.49809	12.02215	8.74523	4.92078	1.37471	7.48618	0.87023	2.44314	2.378E-03	0.71239	2.391E-03	0.71706	-3.907E-04	2.829E-03	-7.2411	-64.7892	4.24862
0.01	2.95327	3.50622	12.07134	8.76556	4.91713	1.37713	7.45568	0.86315	2.45495	2.353E-03	0.70319	2.366E-03	0.70779	-3.657E-04	2.922E-03	-7.9912	-72.7804	4.24800
0.01	2.96327	3.51436	12.12049	8.78590	4.91340	1.37954	7.42544	0.85617	2.46682	2.329E-03	0.69415	2.341E-03	0.69867	-3.410E-04	3.013E-03	-8.8370	-81.6174	4.24752
0.01	2.97327	3.52251	12.16960	8.80626	4.90959	1.38193	7.39548	0.84927	2.47874	2.305E-03	0.68524	2.317E-03	0.68970	-3.167E-04	3.103E-03	-9.7977	-91.4151	4.24717
0.001	2.97427	3.52332	12.17451	8.80830	4.90920	1.38216	7.39249	0.84859	2.47994	2.302E-03	0.68436	2.303E-03	0.68480	-3.035E-04	3.152E-04	-1.0386	-92.4537	4.24715
0.001	2.97527	3.52413	12.17942	8.81034	4.90882	1.38240	7.38951	0.84790	2.48113	2.300E-03	0.68348	2.301E-03	0.68392	-3.011E-04	3.161E-04	-1.0498	-93.5035	4.24712
0.001	2.97627	3.52495	12.18433	8.81237	4.90843	1.38264	7.38654	0.84722	2.48233	2.298E-03	0.68260	2.299E-03	0.68304	-2.987E-04	3.170E-04	-1.0611	-94.5646	4.24710
0.001	2.97727	3.52576	12.18924	8.81441	4.90804	1.38288	7.38356	0.84654	2.48352	2.295E-03	0.68172	2.296E-03	0.68216	-2.963E-04	3.178E-04	-1.0726	-95.6372	4.24707
0.001	2.97827	3.52658	12.19415	8.81645	4.90765	1.38311	7.38059	0.84586	2.48472	2.293E-03	0.68085	2.294E-03	0.68128	-2.940E-04	3.187E-04	-1.0842	-96.7214	4.24705
0.001	2.97927	3.52739	12.19905	8.81849	4.90726	1.38335	7.37762	0.84518	2.48592	2.290E-03	0.67997	2.292E-03	0.68041	-2.916E-04	3.196E-04	-1.0960	-97.8174	4.24703
0.001	2.98027	3.52821	12.20396	8.82053	4.90687	1.38359	7.37465	0.84450	2.48712	2.288E-03	0.67910	2.289E-03	0.67953	-2.892E-04	3.205E-04	-1.1080	-98.9254	4.24701
0.001	2.98127	3.52903	12.20887	8.82256	4.90648	1.38382	7.37169	0.84382	2.48832	2.286E-03	0.67822	2.287E-03	0.67866	-2.869E-04	3.213E-04	-1.1202	-100.0456	4.24699
0.001	2.98227	3.52984	12.21377	8.82460	4.90609	1.38406	7.36873	0.84314	2.48952	2.283E-03	0.67735	2.285E-03	0.67779	-2.845E-04	3.222E-04	-1.1325	-101.1781	4.24698
0.001	2.98327	3.53066	12.21868	8.82664	4.90569	1.38430	7.36577	0.84246	2.49072	2.281E-03	0.67648	2.282E-03	0.67692	-2.822E-04	3.231E-04	-1.1451	-102.3231	4.24696

Table B-14 Case B 100 cfs: Manually Computed Outlet Control Headwater Depths

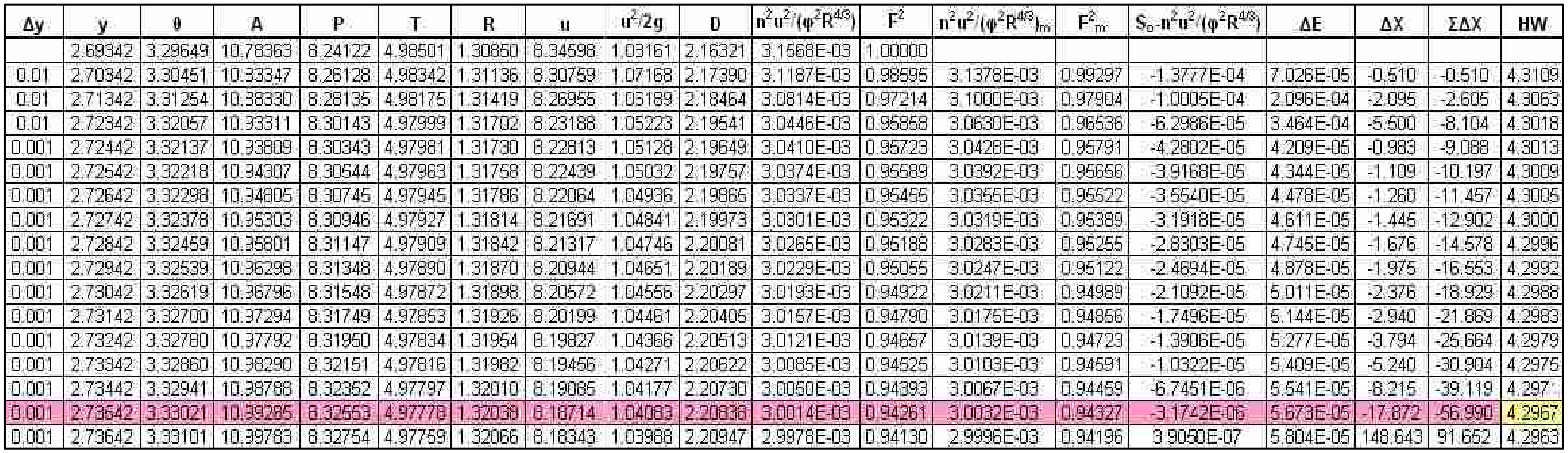
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	2.80733	3.38808	11.35025	8.47020	4.96208	1.34002	8.81038	1.20532	2.28740	3.408E-03	1.05388							
0.1	2.90733	3.46892	11.84508	8.67229	4.93319	1.36585	8.44232	1.10672	2.40110	3.051E-03	0.92184	3.229E-03	0.98786	-1.229E-03	1.214E-03	-0.9874	-0.9874	4.567411
0.1	3.00733	3.55030	12.33661	8.87574	4.89596	1.38992	8.10595	1.02029	2.51975	2.748E-03	0.80983	2.899E-03	0.86584	-8.990E-04	1.342E-02	-14.9230	-15.9104	4.537761
0.1	3.10733	3.63237	12.82399	9.08092	4.85022	1.41219	7.79788	0.94421	2.64400	2.489E-03	0.71423	2.618E-03	0.76203	-6.184E-04	2.380E-02	-38.4791	-54.3895	4.523643
0.01	3.11733	3.64062	12.87247	9.10155	4.84516	1.41432	7.76852	0.93711	2.65677	2.466E-03	0.70545	2.478E-03	0.70984	-4.775E-04	2.902E-03	-6.0765	-60.4660	4.522995
0.01	3.12733	3.64888	12.92089	9.12220	4.84002	1.41642	7.73940	0.93010	2.66959	2.442E-03	0.69681	2.454E-03	0.70113	-4.540E-04	2.989E-03	-6.5825	-67.0485	4.522478
0.01	3.13733	3.65715	12.96927	9.14287	4.83479	1.41851	7.71054	0.92317	2.68249	2.419E-03	0.68830	2.431E-03	0.69255	-4.309E-04	3.074E-03	-7.1348	-74.1833	4.522090
0.01	3.14733	3.66543	13.01759	9.16356	4.82948	1.42058	7.68191	0.91633	2.69544	2.397E-03	0.67991	2.408E-03	0.68410	-4.081E-04	3.159E-03	-7.7398	-81.9230	4.521829
0.01	3.15733	3.67371	13.06586	9.18428	4.82407	1.42263	7.65354	0.90957	2.70847	2.375E-03	0.67165	2.386E-03	0.67578	-3.857E-04	3.242E-03	-8.4054	-90.3285	4.521692
0.001	3.15833	3.67454	13.07068	9.18635	4.82353	1.42284	7.65071	0.90890	2.70978	2.372E-03	0.67083	2.373E-03	0.67124	-3.735E-04	3.288E-04	-0.8802	-91.2087	4.521686
0.001	3.15933	3.67537	13.07550	9.18843	4.82298	1.42304	7.64789	0.90823	2.71108	2.370E-03	0.67001	2.371E-03	0.67042	-3.713E-04	3.296E-04	-0.8876	-92.0963	4.521680
0.001	3.16033	3.67620	13.08033	9.19050	4.82243	1.42324	7.64507	0.90756	2.71239	2.368E-03	0.66920	2.369E-03	0.66961	-3.691E-04	3.304E-04	-0.8952	-92.9915	4.521675
0.001	3.16133	3.67703	13.08515	9.19258	4.82188	1.42345	7.64225	0.90689	2.71370	2.366E-03	0.66838	2.367E-03	0.66879	-3.669E-04	3.312E-04	-0.9027	-93.8942	4.521672
0.001	3.16233	3.67786	13.08997	9.19465	4.82134	1.42365	7.63944	0.90623	2.71501	2.364E-03	0.66757	2.365E-03	0.66798	-3.647E-04	3.320E-04	-0.9104	-94.8046	4.521670
0.001	3.16333	3.67869	13.09479	9.19672	4.82079	1.42385	7.63662	0.90556	2.71632	2.361E-03	0.66675	2.363E-03	0.66716	-3.625E-04	3.328E-04	-0.9182	-95.7228	4.521670
0.001	3.16433	3.67952	13.09961	9.19880	4.82023	1.42406	7.63381	0.90489	2.71763	2.359E-03	0.66594	2.360E-03	0.66635	-3.603E-04	3.337E-04	-0.9260	-96.6488	4.521670
0.001	3.16533	3.68035	13.10443	9.20087	4.81968	1.42426	7.63101	0.90423	2.71894	2.357E-03	0.66513	2.358E-03	0.66554	-3.581E-04	3.345E-04	-0.9339	-97.5827	4.521672
0.001	3.16633	3.68118	13.10925	9.20295	4.81913	1.42446	7.62820	0.90356	2.72025	2.355E-03	0.66432	2.356E-03	0.66473	-3.560E-04	3.353E-04	-0.9419	-98.5246	4.521675
0.001	3.16733	3.68201	13.11407	9.20502	4.81858	1.42466	7.62540	0.90290	2.72157	2.353E-03	0.66351	2.354E-03	0.66392	-3.538E-04	3.361E-04	-0.9500	-99.4745	4.521679
0.001	3.16833	3.68284	13.11889	9.20710	4.81802	1.42487	7.62260	0.90224	2.72288	2.351E-03	0.66271	2.352E-03	0.66311	-3.516E-04	3.369E-04	-0.9581	-100.4327	4.521684
0.001	3.16933	3.68367	13.12371	9.20917	4.81747	1.42507	7.61980	0.90157	2.72419	2.348E-03	0.66190	2.349E-03	0.66230	-3.494E-04	3.377E-04	-0.9664	-101.3991	4.521691

Table B-15 Case C 75 cfs: Manually Computed Outlet Control Headwater Depths



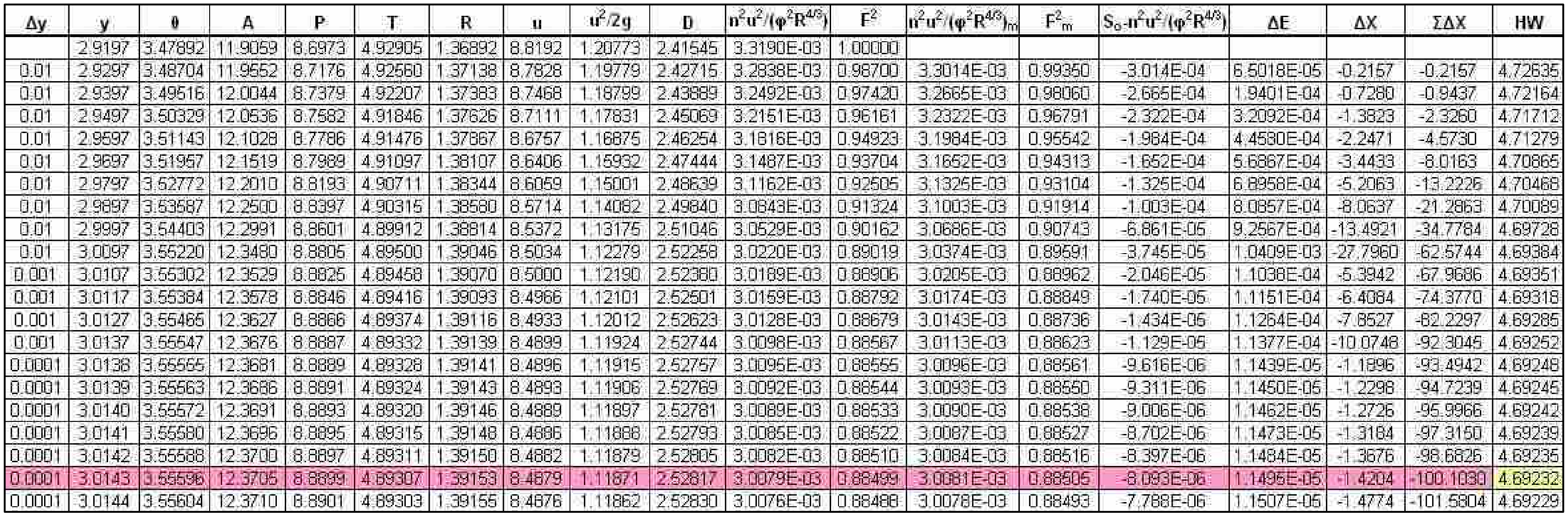
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.44816	3.10012	9.55831	7.7503	4.998925	1.233283	7.84658	0.95604	1.91207	3.0195E-03	1.00000							
0.001	2.44916	3.10092	9.56331	7.7523	4.998966	1.233609	7.84247	0.95504	1.91306	3.0153E-03	0.99844	3.0174E-03	0.99922	-1.738E-05	7.796E-07	-0.04486	-0.04486	3.88172
0.001	2.45016	3.10172	9.56831	7.7543	4.999006	1.233936	7.83838	0.95404	1.91404	3.0111E-03	0.99688	3.0132E-03	0.99766	-1.316E-05	2.337E-06	-0.17756	-0.22242	3.88122
0.001	2.45116	3.10252	9.57331	7.7563	4.999046	1.234262	7.83428	0.95304	1.91503	3.0069E-03	0.99533	3.0090E-03	0.99611	-8.954E-06	3.892E-06	-0.43461	-0.65704	3.88073
0.001	2.45216	3.10332	9.57831	7.7583	4.999085	1.234588	7.83019	0.95205	1.91601	3.0027E-03	0.99378	3.0048E-03	0.99456	-4.755E-06	5.443E-06	-1.14482	-1.80186	3.88024
0.0001	2.45226	3.10340	9.57881	7.7585	4.999088	1.234621	7.82979	0.95195	1.91611	3.0022E-03	0.99363	3.0024E-03	0.99370	-2.447E-06	6.296E-07	-0.25725	-2.05911	3.88019
0.0001	2.45236	3.10348	9.57931	7.7587	4.999092	1.234653	7.82938	0.95185	1.91621	3.0018E-03	0.99347	3.0020E-03	0.99355	-2.028E-06	6.450E-07	-0.31804	-2.37715	3.88014
0.0001	2.45246	3.10356	9.57981	7.7589	4.999096	1.234686	7.82897	0.95175	1.91631	3.0014E-03	0.99332	3.0016E-03	0.99339	-1.609E-06	6.605E-07	-0.41047	-2.78762	3.88009
0.0001	2.45256	3.10364	9.58031	7.7591	4.999100	1.234718	7.82856	0.95165	1.91641	3.0010E-03	0.99316	3.0012E-03	0.99324	-1.190E-06	6.760E-07	-0.56793	-3.35555	3.88004
0.0001	2.45266	3.10372	9.58081	7.7593	4.999104	1.234751	7.82815	0.95155	1.91650	3.0006E-03	0.99301	3.0008E-03	0.99309	-7.715E-07	6.915E-07	-0.89631	-4.25186	3.87999
0.0001	2.45276	3.10380	9.58131	7.7595	4.999107	1.234784	7.82774	0.95145	1.91660	3.0001E-03	0.99285	3.0004E-03	0.99293	-3.527E-07	7.069E-07	-2.00428	-6.25614	3.87994
0.0001	2.45286	3.10388	9.58181	7.7597	4.999111	1.234816	7.82734	0.95135	1.91670	2.9997E-03	0.99270	2.9999E-03	0.99278	6.595E-08	7.224E-07	10.95431	4.69817	3.87989

Table B-16 Case C 90 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.69342	3.29649	10.78363	8.24122	4.98501	1.30850	8.34598	1.08161	2.16321	3.1568E-03	1.00000							
0.01	2.70342	3.30451	10.83347	8.26128	4.98342	1.31136	8.30759	1.07168	2.17390	3.1187E-03	0.98595	3.1378E-03	0.99297	-1.3777E-04	7.026E-05	-0.510	-0.510	4.3109
0.01	2.71342	3.31254	10.88330	8.28135	4.98175	1.31419	8.26955	1.06189	2.18464	3.0814E-03	0.97214	3.1000E-03	0.97904	-1.0005E-04	2.096E-04	-2.095	-2.605	4.3063
0.01	2.72342	3.32057	10.93311	8.30143	4.97999	1.31702	8.23188	1.05223	2.19541	3.0446E-03	0.95858	3.0630E-03	0.96536	-6.2986E-05	3.464E-04	-5.500	-8.104	4.3018
0.001	2.72442	3.32137	10.93809	8.30343	4.97981	1.31730	8.22813	1.05128	2.19649	3.0410E-03	0.95723	3.0428E-03	0.95791	-4.2802E-05	4.209E-05	-0.983	-9.088	4.3013
0.001	2.72542	3.32218	10.94307	8.30544	4.97963	1.31758	8.22439	1.05032	2.19757	3.0374E-03	0.95589	3.0392E-03	0.95656	-3.9168E-05	4.344E-05	-1.109	-10.197	4.3009
0.001	2.72642	3.32298	10.94805	8.30745	4.97945	1.31786	8.22064	1.04936	2.19865	3.0337E-03	0.95455	3.0355E-03	0.95522	-3.5540E-05	4.478E-05	-1.260	-11.457	4.3005
0.001	2.72742	3.32378	10.95303	8.30946	4.97927	1.31814	8.21691	1.04841	2.19973	3.0301E-03	0.95322	3.0319E-03	0.95389	-3.1918E-05	4.611E-05	-1.445	-12.902	4.3000
0.001	2.72842	3.32459	10.95801	8.31147	4.97909	1.31842	8.21317	1.04746	2.20081	3.0265E-03	0.95188	3.0283E-03	0.95255	-2.8303E-05	4.745E-05	-1.676	-14.578	4.2996
0.001	2.72942	3.32539	10.96298	8.31348	4.97890	1.31870	8.20944	1.04651	2.20189	3.0229E-03	0.95055	3.0247E-03	0.95122	-2.4694E-05	4.878E-05	-1.975	-16.553	4.2992
0.001	2.73042	3.32619	10.96796	8.31548	4.97872	1.31898	8.20572	1.04556	2.20297	3.0193E-03	0.94922	3.0211E-03	0.94989	-2.1092E-05	5.011E-05	-2.376	-18.929	4.2988
0.001	2.73142	3.32700	10.97294	8.31749	4.97853	1.31926	8.20199	1.04461	2.20405	3.0157E-03	0.94790	3.0175E-03	0.94856	-1.7496E-05	5.144E-05	-2.940	-21.869	4.2983
0.001	2.73242	3.32780	10.97792	8.31950	4.97834	1.31954	8.19827	1.04366	2.20513	3.0121E-03	0.94657	3.0139E-03	0.94723	-1.3906E-05	5.277E-05	-3.794	-25.664	4.2979
0.001	2.73342	3.32860	10.98290	8.32151	4.97816	1.31982	8.19456	1.04271	2.20622	3.0085E-03	0.94525	3.0103E-03	0.94591	-1.0322E-05	5.409E-05	-5.240	-30.904	4.2975
0.001	2.73442	3.32941	10.98788	8.32352	4.97797	1.32010	8.19085	1.04177	2.20730	3.0050E-03	0.94393	3.0067E-03	0.94459	-6.7451E-06	5.541E-05	-8.215	-39.119	4.2971
0.001	2.73542	3.33021	10.99285	8.32553	4.97778	1.32038	8.18714	1.04083	2.20838	3.0014E-03	0.94261	3.0032E-03	0.94327	-3.1742E-06	5.673E-05	-17.872	-56.990	4.2967
0.001	2.73642	3.33101	10.99783	8.32754	4.97759	1.32066	8.18343	1.03988	2.20947	2.9978E-03	0.94130	2.9996E-03	0.94196	3.9050E-07	5.804E-05	148.643	91.652	4.2963

Table B-17 Case C 105 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.9197	3.47892	11.9059	8.6973	4.92905	1.36892	8.8192	1.20773	2.41545	3.3190E-03	1.00000							
0.01	2.9297	3.48704	11.9552	8.7176	4.92560	1.37138	8.7828	1.19779	2.42715	3.2838E-03	0.98700	3.3014E-03	0.99350	-3.014E-04	6.5018E-05	-0.2157	-0.2157	4.72635
0.01	2.9397	3.49516	12.0044	8.7379	4.92207	1.37383	8.7468	1.18799	2.43889	3.2492E-03	0.97420	3.2665E-03	0.98060	-2.665E-04	1.9401E-04	-0.7280	-0.9437	4.72164
0.01	2.9497	3.50329	12.0536	8.7582	4.91846	1.37626	8.7111	1.17831	2.45069	3.2151E-03	0.96161	3.2322E-03	0.96791	-2.322E-04	3.2092E-04	-1.3823	-2.3260	4.71712
0.01	2.9597	3.51143	12.1028	8.7786	4.91476	1.37867	8.6757	1.16875	2.46254	3.1816E-03	0.94923	3.1984E-03	0.95542	-1.984E-04	4.4580E-04	-2.2471	-4.5730	4.71279
0.01	2.9697	3.51957	12.1519	8.7989	4.91097	1.38107	8.6406	1.15932	2.47444	3.1487E-03	0.93704	3.1652E-03	0.94313	-1.652E-04	5.6867E-04	-3.4433	-8.0163	4.70865
0.01	2.9797	3.52772	12.2010	8.8193	4.90711	1.38344	8.6059	1.15001	2.48639	3.1162E-03	0.92505	3.1325E-03	0.93104	-1.325E-04	6.8958E-04	-5.2063	-13.2226	4.70468
0.01	2.9897	3.53587	12.2500	8.8397	4.90315	1.38580	8.5714	1.14082	2.49840	3.0843E-03	0.91324	3.1003E-03	0.91914	-1.003E-04	8.0857E-04	-8.0637	-21.2863	4.70089
0.01	2.9997	3.54403	12.2991	8.8601	4.89912	1.38814	8.5372	1.13175	2.51046	3.0529E-03	0.90162	3.0686E-03	0.90743	-6.861E-05	9.2567E-04	-13.4921	-34.7784	4.69728
0.01	3.0097	3.55220	12.3480	8.8805	4.89500	1.39046	8.5034	1.12279	2.52258	3.0220E-03	0.89019	3.0374E-03	0.89591	-3.745E-05	1.0409E-03	-27.7960	-62.5744	4.69384
0.001	3.0107	3.55302	12.3529	8.8825	4.89458	1.39070	8.5000	1.12190	2.52380	3.0189E-03	0.88906	3.0205E-03	0.88962	-2.046E-05	1.1038E-04	-5.3942	-67.9686	4.69351
0.001	3.0117	3.55384	12.3578	8.8846	4.89416	1.39093	8.4966	1.12101	2.52501	3.0159E-03	0.88792	3.0174E-03	0.88849	-1.740E-06	1.1151E-04	-6.4084	-74.3770	4.69318
0.001	3.0127	3.55465	12.3627	8.8866	4.89374	1.39116	8.4933	1.12012	2.52623	3.0128E-03	0.88679	3.0143E-03	0.88736	-1.434E-06	1.1264E-04	-7.8527	-82.2297	4.69285
0.001	3.0137	3.55547	12.3676	8.8887	4.89332	1.39139	8.4899	1.11924	2.52744	3.0098E-03	0.88567	3.0113E-03	0.88623	-1.129E-05	1.1377E-04	-10.0748	-92.3045	4.69252
0.0001	3.0138	3.55555	12.3681	8.8889	4.89328	1.39141	8.4896	1.11915	2.52757	3.0095E-03	0.88555	3.0096E-03	0.88561	-9.616E-06	1.1439E-05	-1.1896	-93.4942	4.69248
0.0001	3.0139	3.55563	12.3686	8.8891	4.89324	1.39143	8.4893	1.11906	2.52769	3.0092E-03	0.88544	3.0093E-03	0.88550	-9.311E-06	1.1450E-05	-1.2298	-94.7239	4.69245
0.0001	3.0140	3.55572	12.3691	8.8893	4.89320	1.39146	8.4889	1.11897	2.52781	3.0089E-03	0.88533	3.0090E-03	0.88538	-9.006E-06	1.1462E-05	-1.2726	-95.9966	4.69242
0.0001	3.0141	3.55580	12.3696	8.8895	4.89315	1.39148	8.4886	1.11888	2.52793	3.0085E-03	0.88522	3.0087E-03	0.88527	-8.702E-06	1.1473E-05	-1.3184	-97.3150	4.69239
0.0001	3.0142	3.55588	12.3700	8.8897	4.89311	1.39150	8.4882	1.11879	2.52805	3.0082E-03	0.88510	3.0084E-03	0.88516	-8.397E-06	1.1484E-05	-1.3676	-98.6826	4.69235
0.0001	3.0143	3.55596	12.3705	8.8899	4.89307	1.39153	8.4879	1.11871	2.52817	3.0079E-03	0.88499	3.0081E-03	0.88505	-8.093E-06	1.1495E-05	-1.4204	-100.1030	4.69232
0.0001	3.0144	3.55604	12.3710	8.8901	4.89303	1.39155	8.4876	1.11862	2.52830	3.0076E-03	0.88488	3.0078E-03	0.88493	-7.788E-06	1.1507E-05	-1.4774	-101.5804	4.69229

Table B-18 Case C 120 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	3.13009	3.65116	12.93426	9.12791	4.83859	1.41700	9.27768	1.33657	2.67315	3.5079E-03	1.00000							
0.1	3.23009	3.73431	13.41537	9.33576	4.78203	1.43699	8.94496	1.24243	2.80537	3.2004E-03	0.88575	3.3541E-03	0.94288	-3.5415E-04	5.712E-03	-16.130	-16.130	5.09374
0.01	3.24009	3.74268	13.46316	9.35669	4.77588	1.43888	8.91321	1.23362	2.81899	3.1722E-03	0.87522	3.1863E-03	0.88049	-1.8631E-04	1.195E-03	-6.415	-22.545	5.09053
0.01	3.25009	3.75106	13.51089	9.37764	4.76964	1.44076	8.88173	1.22492	2.83269	3.1444E-03	0.86485	3.1583E-03	0.87004	-1.5827E-04	1.300E-03	-8.212	-30.756	5.08748
0.01	3.26009	3.75945	13.55855	9.39862	4.76330	1.44261	8.85050	1.21633	2.84646	3.1169E-03	0.85462	3.1306E-03	0.85974	-1.3064E-04	1.403E-03	-10.737	-41.493	5.08458
0.01	3.27009	3.76785	13.60615	9.41963	4.75687	1.44445	8.81954	1.20783	2.86032	3.0899E-03	0.84454	3.1034E-03	0.84958	-1.0342E-04	1.504E-03	-14.544	-56.037	5.08184
0.01	3.28009	3.77627	13.65369	9.44067	4.75035	1.44626	8.78883	1.19943	2.87425	3.0633E-03	0.83461	3.0766E-03	0.83958	-7.6605E-05	1.604E-03	-20.942	-76.979	5.07925
0.001	3.28109	3.77711	13.65844	9.44277	4.74969	1.44644	8.78578	1.19860	2.87565	3.0607E-03	0.83362	3.0620E-03	0.83411	-6.1977E-05	1.659E-04	-2.677	-79.656	5.07899
0.001	3.28209	3.77795	13.66319	9.44488	4.74903	1.44662	8.78272	1.19777	2.87705	3.0580E-03	0.83264	3.0593E-03	0.83313	-5.9339E-05	1.669E-04	-2.812	-82.468	5.07874
0.001	3.28309	3.77879	13.66794	9.44698	4.74837	1.44680	8.77967	1.19694	2.87845	3.0554E-03	0.83165	3.0567E-03	0.83215	-5.6705E-05	1.679E-04	-2.960	-85.428	5.07850
0.001	3.28409	3.77964	13.67269	9.44909	4.74771	1.44698	8.77662	1.19610	2.87985	3.0528E-03	0.83067	3.0541E-03	0.83116	-5.4075E-05	1.688E-04	-3.122	-88.550	5.07825
0.001	3.28509	3.78048	13.67743	9.45119	4.74705	1.44716	8.77358	1.19527	2.88125	3.0501E-03	0.82969	3.0514E-03	0.83018	-5.1449E-05	1.698E-04	-3.301	-91.851	5.07800
0.001	3.28609	3.78132	13.68218	9.45330	4.74639	1.44734	8.77053	1.19444	2.88265	3.0475E-03	0.82871	3.0488E-03	0.82920	-4.8827E-05	1.708E-04	-3.498	-95.349	5.07776
0.0001	3.28619	3.78140	13.68265	9.45351	4.74632	1.44736	8.77023	1.19436	2.88279	3.0473E-03	0.82862	3.0474E-03	0.82866	-4.7386E-05	1.713E-05	-0.362	-95.711	5.07774
0.0001	3.28629	3.78149	13.68313	9.45372	4.74626	1.44738	8.76992	1.19428	2.88293	3.0470E-03	0.82852	3.0471E-03	0.82857	-4.7124E-05	1.714E-05	-0.364	-96.074	5.07771
0.0001	3.28639	3.78157	13.68360	9.45393	4.74619	1.44740	8.76962	1.19420	2.88307	3.0467E-03	0.82842	3.0469E-03	0.82847	-4.6862E-05	1.715E-05	-0.366	-96.440	5.07769
0.0001	3.28649	3.78166	13.68408	9.45414	4.74613	1.44742	8.76932	1.19411	2.88321	3.0465E-03	0.82832	3.0466E-03	0.82837	-4.6600E-05	1.716E-05	-0.368	-96.809	5.07766
0.0001	3.28659	3.78174	13.68455	9.45435	4.74606	1.44743	8.76901	1.19403	2.88335	3.0462E-03	0.82822	3.0463E-03	0.82827	-4.6339E-05	1.717E-05	-0.371	-97.179	5.07764
0.0001	3.28669	3.78183	13.68503	9.45457	4.74599	1.44745	8.76871	1.19395	2.88349	3.0459E-03	0.82813	3.0461E-03	0.82818	-4.6077E-05	1.718E-05	-0.373	-97.552	5.07761
0.0001	3.28679	3.78191	13.68550	9.45478	4.74593	1.44747	8.76840	1.19386	2.88363	3.0457E-03	0.82803	3.0458E-03	0.82808	-4.5816E-05	1.719E-05	-0.375	-97.927	5.07759
0.0001	3.28689	3.78199	13.68598	9.45499	4.74586	1.44749	8.76810	1.19378	2.88377	3.0454E-03	0.82793	3.0456E-03	0.82798	-4.5554E-05	1.720E-05	-0.378	-98.305	5.07757
0.0001	3.28699	3.78208	13.68645	9.45520	4.74579	1.44751	8.76779	1.19370	2.88391	3.0452E-03	0.82783	3.0453E-03	0.82788	-4.5292E-05	1.721E-05	-0.380	-98.685	5.07754
0.0001	3.28709	3.78216	13.68693	9.45541	4.74573	1.44752	8.76749	1.19362	2.88405	3.0449E-03	0.82774	3.0450E-03	0.82778	-4.5031E-05	1.722E-05	-0.382	-99.068	5.07752
0.0001	3.28719	3.78225	13.68740	9.45562	4.74566	1.44754	8.76719	1.19353	2.88419	3.0446E-03	0.82764	3.0448E-03	0.82769	-4.4770E-05	1.723E-05	-0.385	-99.452	5.07749
0.0001	3.28729	3.78233	13.68788	9.45583	4.74560	1.44756	8.76688	1.19345	2.88433	3.0444E-03	0.82754	3.0445E-03	0.82759	-4.4508E-05	1.724E-05	-0.387	-99.840	5.07747
0.0001	3.28739	3.78242	13.68835	9.45604	4.74553	1.44758	8.76658	1.19337	2.88447	3.0441E-03	0.82744	3.0442E-03	0.82749	-4.4247E-05	1.725E-05	-0.390	-100.230	5.07745

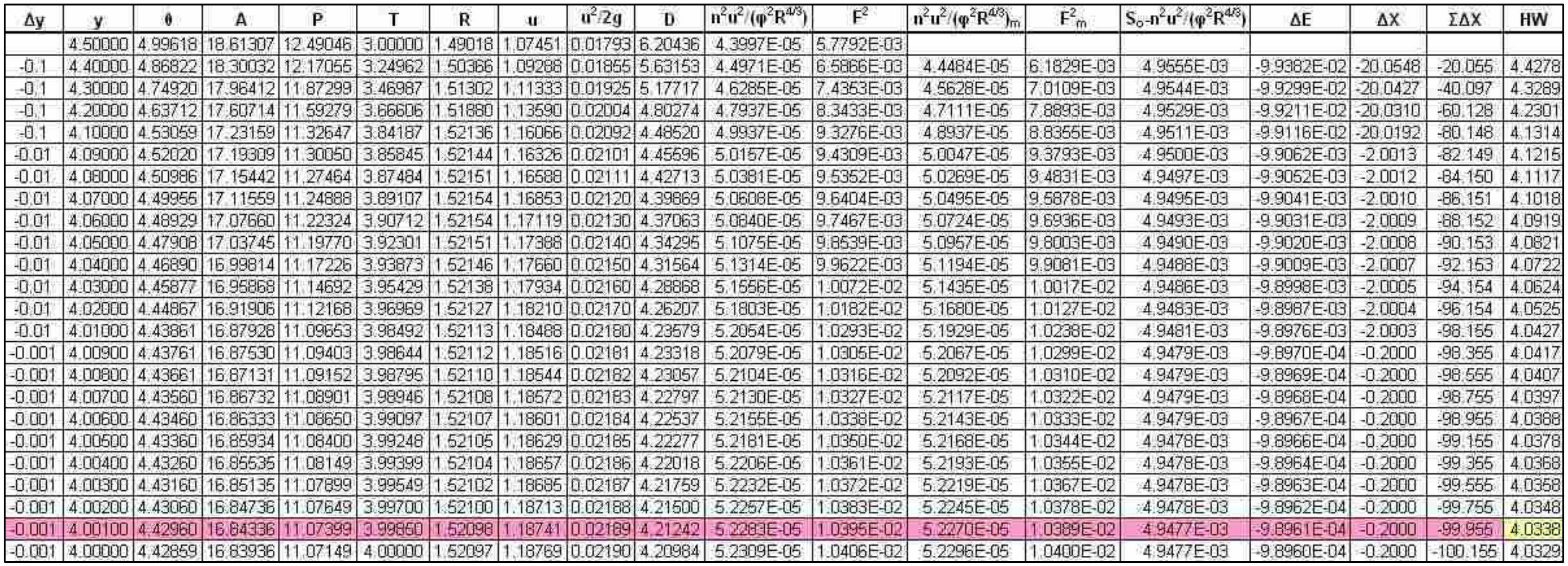
Table B-19 Case C 135 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	3.326695	3.815637	13.87433	9.539091	4.718718	1.454471	9.730196	1.470135377	2.940276243	0.00372643	0.9999981							
0.1	3.426695	3.901071	14.34254	9.752676	4.643807	1.470626	9.412557	1.375717916	3.088530994	0.00343612	0.8908558	0.003581277	0.94542698	-0.000581277	0.005457	-9.38848	-9.38848	5.490272
0.1	3.526695	3.987988	14.80276	9.96997	4.558902	1.484735	9.119919	1.29150498	3.247001757	0.00318498	0.7955062	0.003310552	0.843181003	-0.000310552	0.015682	-50.4969	-59.8854	5.463953
0.01	3.536695	3.996771	14.84831	9.991926	4.549841	1.48603	9.091946	1.283594335	3.263478233	0.00316179	0.7866419	0.003173386	0.791074032	-0.000173386	0.002089	-12.0498	-71.9352	5.462087
0.001	3.537695	3.99765	14.85286	9.994125	4.548929	1.486159	9.089161	1.282808134	3.265132576	0.00315949	0.7857617	0.003160642	0.786201815	-0.000160642	0.000214	-1.3309	-73.2661	5.461907
0.001	3.538695	3.998529	14.8574	9.996323	4.548016	1.486287	9.086378	1.282022812	3.266788146	0.00315719	0.7848827	0.003158343	0.785322241	-0.000158343	0.000215	-1.35578	-74.6219	5.461729
0.001	3.539695	3.999409	14.86195	9.998522	4.547102	1.486415	9.083598	1.281238368	3.268444943	0.0031549	0.7840049	0.003156048	0.784443801	-0.000156048	0.000216	-1.38135	-76.0032	5.461553
0.001	3.540695	4.000289	14.8665	10.00072	4.546187	1.486543	9.08082	1.280454801	3.270102971	0.00315261	0.7831281	0.003153755	0.783566494	-0.000153755	0.000216	-1.40765	-77.4108	5.461377
0.001	3.541695	4.001169	14.87104	10.00292	4.545271	1.48667	9.078044	1.279672111	3.271762231	0.00315032	0.7822525	0.003151466	0.782690317	-0.000151466	0.000217	-1.43471	-78.8456	5.461203
0.001	3.542695	4.002049	14.87559	10.00512	4.544353	1.486797	9.075271	1.278890296	3.273422725	0.00314804	0.7813780	0.003149181	0.78181527	-0.000149181	0.000218	-1.46256	-80.3081	5.461031
0.001	3.543695	4.002929	14.88013	10.00732	4.543435	1.486925	9.072499	1.278109355	3.275084456	0.00314576	0.7805047	0.003146898	0.780941348	-0.000146898	0.000219	-1.49123	-81.7993	5.460859
0.001	3.544695	4.003809	14.88468	10.00952	4.542516	1.487051	9.06973	1.277329286	3.276747426	0.00314348	0.7796324	0.003144619	0.780068552	-0.000144619	0.00022	-1.52077	-83.3201	5.460689
0.001	3.545695	4.00469	14.88922	10.01173	4.541595	1.487178	9.066963	1.27655009	3.278411636	0.00314121	0.7787613	0.003142343	0.779196878	-0.000142343	0.000221	-1.55121	-84.8713	5.46052
0.001	3.546695	4.005571	14.89376	10.01393	4.540674	1.487305	9.064199	1.275771763	3.28007709	0.00313893	0.7778913	0.00314007	0.778326325	-0.00014007	0.000222	-1.58259	-86.4539	5.460353
0.001	3.547695	4.006452	14.8983	10.01613	4.539751	1.487431	9.061437	1.274994306	3.281743789	0.00313667	0.7770225	0.003137801	0.777456892	-0.000137801	0.000223	-1.61496	-88.0689	5.460187
0.001	3.548695	4.007333	14.90284	10.01833	4.538827	1.487557	9.058677	1.274217718	3.283411735	0.0031344	0.7761547	0.003135534	0.776588575	-0.000135534	0.000223	-1.64837	-89.7172	5.460022
0.001	3.549695	4.008215	14.90738	10.02054	4.537903	1.487683	9.055919	1.273441997	3.28508093	0.00313214	0.7752881	0.003133272	0.775721374	-0.000133272	0.000224	-1.68287	-91.4001	5.459858
0.001	3.550695	4.009096	14.91191	10.02274	4.536977	1.487808	9.053163	1.272667142	3.286751378	0.00312988	0.7744225	0.003131012	0.774855286	-0.000131012	0.000225	-1.71851	-93.1186	5.459696
0.001	3.551695	4.009978	14.91645	10.02494	4.53605	1.487934	9.05041	1.271893151	3.288423079	0.00312763	0.7735581	0.003128755	0.773990309	-0.000128755	0.000226	-1.75534	-94.874	5.459535
0.001	3.552695	4.01086	14.92099	10.02715	4.535122	1.488059	9.047659	1.271120025	3.290096037	0.00312538	0.7726948	0.003126502	0.773126442	-0.000126502	0.000227	-1.79344	-96.6674	5.459375
0.001	3.553695	4.011742	14.92552	10.02935	4.534193	1.488184	9.04491	1.270347761	3.291770252	0.00312313	0.7718326	0.003124252	0.772263682	-0.000124252	0.000228	-1.83286	-98.5003	5.459217
0.001	3.554695	4.012624	14.93006	10.03156	4.533263	1.488308	9.042163	1.269576359	3.293445729	0.00312088	0.7709715	0.003122005	0.771402028	-0.000122005	0.000229	-1.87368	-100.374	5.45906

Table B-20 Case C 150 cfs: Manually Computed Outlet Control Headwater Depths

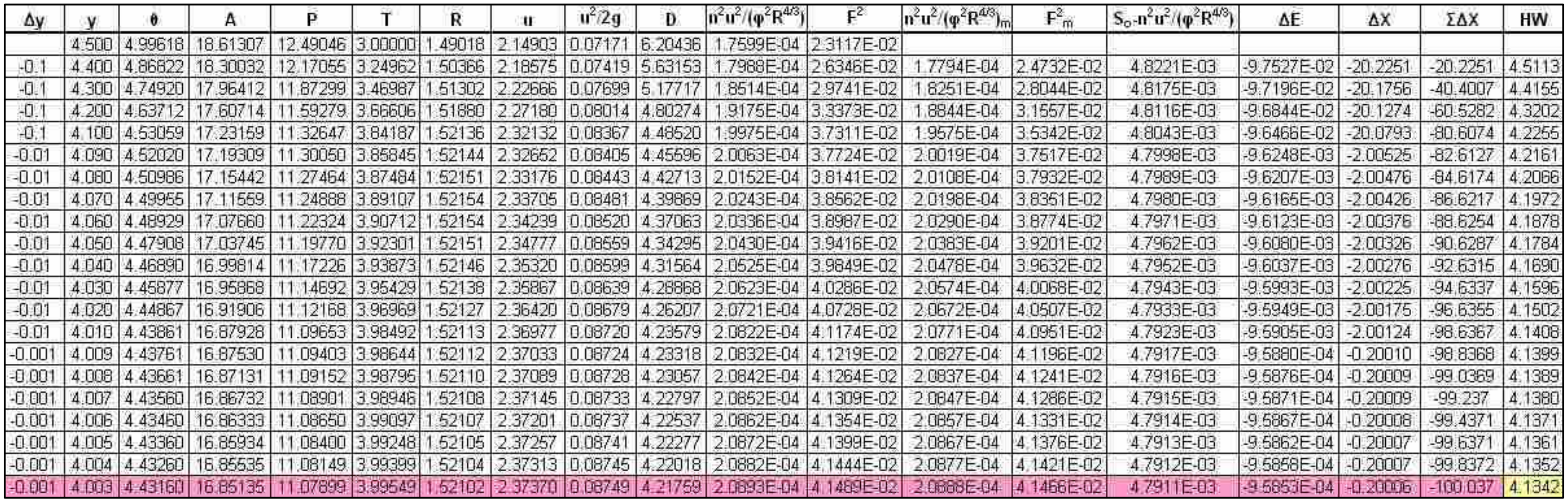
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	3.51062	3.97390	14.7294	9.9348	4.57325	1.48261	10.18374	1.61038	3.22076	3.9790E-03	1.00000							
0.1	3.61062	4.06226	15.1821	10.1557	4.47952	1.49494	9.88007	1.51577	3.38922	3.7041E-03	0.89447	3.8415E-03	0.94723	-8.4151E-04	5.277E-03	-6.2705	-6.270	5.88428
0.1	3.71062	4.15260	15.6249	10.3815	4.37466	1.50507	9.60007	1.43108	3.57168	3.4657E-03	0.80135	3.5849E-03	0.84791	-5.8490E-04	1.521E-02	-26.0035	-32.274	5.85723
0.01	3.72062	4.16175	15.6686	10.4044	4.36353	1.50596	9.57330	1.42311	3.59081	3.4437E-03	0.79264	3.4547E-03	0.79699	-4.5473E-04	2.030E-03	-4.4643	-36.738	5.85528
0.01	3.73062	4.17093	15.7122	10.4273	4.35228	1.50682	9.54675	1.41522	3.61010	3.4220E-03	0.78403	3.4329E-03	0.78834	-4.3288E-04	2.117E-03	-4.8897	-41.628	5.85345
0.01	3.74062	4.18013	15.7556	10.4503	4.34091	1.50767	9.52041	1.40743	3.62957	3.4006E-03	0.77553	3.4113E-03	0.77978	-4.1133E-04	2.202E-03	-5.3537	-46.982	5.85176
0.01	3.75062	4.18936	15.7990	10.4734	4.32941	1.50849	9.49429	1.39971	3.64922	3.3796E-03	0.76713	3.3901E-03	0.77133	-3.9010E-04	2.287E-03	-5.8618	-52.843	5.85019
0.01	3.76062	4.19861	15.8422	10.4965	4.31780	1.50928	9.46837	1.39208	3.66905	3.3588E-03	0.75882	3.3692E-03	0.76298	-3.6917E-04	2.370E-03	-6.4205	-59.264	5.84874
0.01	3.77062	4.20789	15.8853	10.5197	4.30606	1.51005	9.44267	1.38454	3.68907	3.3383E-03	0.75062	3.3485E-03	0.75472	-3.4853E-04	2.453E-03	-7.0375	-66.302	5.84742
0.01	3.78062	4.21719	15.9283	10.5430	4.29419	1.51080	9.41718	1.37707	3.70928	3.3181E-03	0.74250	3.3282E-03	0.74656	-3.2819E-04	2.534E-03	-7.7224	-74.024	5.84622
0.01	3.79062	4.22652	15.9712	10.5663	4.28220	1.51152	9.39190	1.36968	3.72968	3.2982E-03	0.73448	3.3081E-03	0.73849	-3.0815E-04	2.615E-03	-8.4866	-82.510	5.84515
0.01	3.80062	4.23587	16.0140	10.5897	4.27008	1.51222	9.36682	1.36238	3.75028	3.2786E-03	0.72655	3.2884E-03	0.73051	-2.8839E-04	2.695E-03	-9.3446	-91.855	5.84419
0.001	3.80162	4.23681	16.0182	10.5920	4.26886	1.51229	9.36432	1.36165	3.75235	3.2766E-03	0.72576	3.2776E-03	0.72615	-2.7761E-04	2.738E-04	-0.9864	-92.842	5.84410
0.001	3.80262	4.23775	16.0225	10.5944	4.26764	1.51236	9.36183	1.36093	3.75442	3.2747E-03	0.72497	3.2757E-03	0.72537	-2.7566E-04	2.746E-04	-0.9963	-93.838	5.84401
0.001	3.80362	4.23869	16.0268	10.5967	4.26642	1.51243	9.35933	1.36020	3.75650	3.2728E-03	0.72419	3.2737E-03	0.72458	-2.7372E-04	2.754E-04	-1.0062	-94.844	5.84392
0.001	3.80462	4.23962	16.0310	10.5991	4.26519	1.51250	9.35684	1.35948	3.75857	3.2708E-03	0.72340	3.2718E-03	0.72379	-2.7178E-04	2.762E-04	-1.0163	-95.860	5.84384
0.001	3.80562	4.24056	16.0353	10.6014	4.26397	1.51257	9.35435	1.35876	3.76065	3.2689E-03	0.72262	3.2698E-03	0.72301	-2.6985E-04	2.770E-04	-1.0266	-96.887	5.84375
0.001	3.80662	4.24150	16.0396	10.6037	4.26274	1.51263	9.35187	1.35803	3.76273	3.2669E-03	0.72183	3.2679E-03	0.72223	-2.6791E-04	2.778E-04	-1.0368	-97.924	5.84367
0.001	3.80762	4.24244	16.0438	10.6061	4.26152	1.51270	9.34938	1.35731	3.76482	3.2650E-03	0.72105	3.2660E-03	0.72144	-2.6598E-04	2.786E-04	-1.0473	-98.971	5.84359
0.001	3.80862	4.24338	16.0481	10.6084	4.26029	1.51277	9.34690	1.35659	3.76690	3.2631E-03	0.72027	3.2641E-03	0.72066	-2.6405E-04	2.793E-04	-1.0579	-100.029	5.84351
0.001	3.80962	4.24432	16.0524	10.6108	4.25906	1.51283	9.34442	1.35587	3.76899	3.2612E-03	0.71949	3.2621E-03	0.71988	-2.6213E-04	2.801E-04	-1.0686	-101.097	5.84343

Table B-21 Case D 20 cfs: Manually Computed Outlet Control Headwater Depths



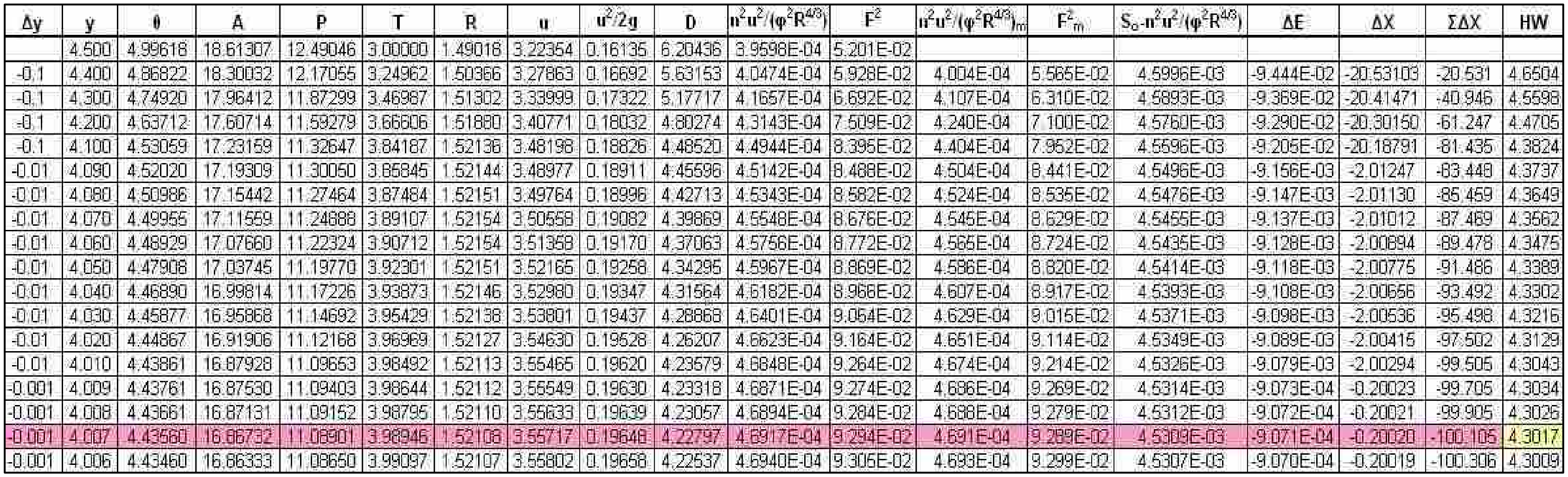
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	4.50000	4.99618	18.61307	12.49046	3.00000	1.49018	1.07451	0.01793	6.20436	4.3997E-05	5.7792E-03							
-0.1	4.40000	4.86822	18.30032	12.17055	3.24962	1.50366	1.09288	0.01855	5.63153	4.4971E-05	6.5866E-03	4.4484E-05	6.1829E-03	4.9555E-03	-9.9382E-02	-20.0548	-20.055	4.4278
-0.1	4.30000	4.74920	17.96412	11.87299	3.46987	1.51302	1.11333	0.01925	5.17717	4.6285E-05	7.4353E-03	4.5628E-05	7.0109E-03	4.9544E-03	-9.9299E-02	-20.0427	-40.097	4.3289
-0.1	4.20000	4.63712	17.60714	11.59279	3.66606	1.51880	1.13590	0.02004	4.80274	4.7937E-05	8.3433E-03	4.7111E-05	7.8893E-03	4.9529E-03	-9.9211E-02	-20.0310	-60.128	4.2301
-0.1	4.10000	4.53059	17.23159	11.32647	3.84187	1.52136	1.16066	0.02092	4.48520	4.9937E-05	9.3276E-03	4.8937E-05	8.8355E-03	4.9511E-03	-9.9116E-02	-20.0192	-80.148	4.1314
-0.01	4.09000	4.52020	17.19309	11.30050	3.85845	1.52144	1.16326	0.02101	4.45596	5.0157E-05	9.4309E-03	5.0047E-05	9.3793E-03	4.9500E-03	-9.9062E-03	-2.0013	-82.149	4.1215
-0.01	4.08000	4.50986	17.15442	11.27464	3.87484	1.52151	1.16588	0.02111	4.42713	5.0381E-05	9.5352E-03	5.0269E-05	9.4831E-03	4.9497E-03	-9.9052E-03	-2.0012	-84.150	4.1117
-0.01	4.07000	4.49955	17.11559	11.24888	3.89107	1.52154	1.16853	0.02120	4.39869	5.0608E-05	9.6404E-03	5.0495E-05	9.5878E-03	4.9495E-03	-9.9041E-03	-2.0010	-86.151	4.1018
-0.01	4.06000	4.48929	17.07660	11.22324	3.90712	1.52154	1.17119	0.02130	4.37063	5.0840E-05	9.7467E-03	5.0724E-05	9.6936E-03	4.9493E-03	-9.9031E-03	-2.0009	-88.152	4.0919
-0.01	4.05000	4.47908	17.03745	11.19770	3.92301	1.52151	1.17388	0.02140	4.34295	5.1075E-05	9.8539E-03	5.0957E-05	9.8003E-03	4.9490E-03	-9.9020E-03	-2.0008	-90.153	4.0821
-0.01	4.04000	4.46890	16.99814	11.17226	3.93873	1.52146	1.17660	0.02150	4.31564	5.1314E-05	9.9622E-03	5.1194E-05	9.9081E-03	4.9488E-03	-9.9009E-03	-2.0007	-92.153	4.0722
-0.01	4.03000	4.45877	16.95868	11.14692	3.95429	1.52138	1.17934	0.02160	4.28868	5.1556E-05	1.0072E-02	5.1435E-05	1.0017E-02	4.9486E-03	-9.8998E-03	-2.0005	-94.154	4.0624
-0.01	4.02000	4.44867	16.91906	11.12168	3.96969	1.52127	1.18210	0.02170	4.26207	5.1803E-05	1.0182E-02	5.1680E-05	1.0127E-02	4.9483E-03	-9.8987E-03	-2.0004	-96.154	4.0525
-0.01	4.01000	4.43861	16.87928	11.09653	3.98492	1.52113	1.18488	0.02180	4.23579	5.2054E-05	1.0293E-02	5.1929E-05	1.0238E-02	4.9481E-03	-9.8976E-03	-2.0003	-98.155	4.0427
-0.001	4.00900	4.43761	16.87530	11.09403	3.98644	1.52112	1.18516	0.02181	4.23318	5.2079E-05	1.0305E-02	5.2067E-05	1.0299E-02	4.9479E-03	-9.8970E-04	-0.2000	-98.355	4.0417
-0.001	4.00800	4.43661	16.87131	11.09152	3.98795	1.52110	1.18544	0.02182	4.23057	5.2104E-05	1.0316E-02	5.2092E-05	1.0310E-02	4.9479E-03	-9.8969E-04	-0.2000	-98.555	4.0407
-0.001	4.00700	4.43560	16.86732	11.08901	3.98946	1.52108	1.18572	0.02183	4.22797	5.2130E-05	1.0327E-02	5.2117E-05	1.0322E-02	4.9479E-03	-9.8968E-04	-0.2000	-98.755	4.0397
-0.001	4.00600	4.43460	16.86333	11.08650	3.99097	1.52107	1.18601	0.02184	4.22537	5.2155E-05	1.0338E-02	5.2143E-05	1.0333E-02	4.9479E-03	-9.8967E-04	-0.2000	-98.955	4.0388
-0.001	4.00500	4.43360	16.85934	11.08400	3.99248	1.52105	1.18629	0.02185	4.22277	5.2181E-05	1.0350E-02	5.2168E-05	1.0344E-02	4.9478E-03	-9.8966E-04	-0.2000	-99.155	4.0378
-0.001	4.00400	4.43260	16.85535	11.08149	3.99399	1.52104	1.18657	0.02186	4.22018	5.2206E-05	1.0361E-02	5.2193E-05	1.0355E-02	4.9478E-03	-9.8964E-04	-0.2000	-99.355	4.0368
-0.001	4.00300	4.43160	16.85135	11.07899	3.99549	1.52102	1.18685	0.02187	4.21759	5.2232E-05	1.0372E-02	5.2219E-05	1.0367E-02	4.9478E-03	-9.8963E-04	-0.2000	-99.555	4.0358
-0.001	4.00200	4.43060	16.84736	11.07649	3.99700	1.52100	1.18713	0.02188	4.21500	5.2257E-05	1.0383E-02	5.2245E-05	1.0378E-02	4.9478E-03	-9.8962E-04	-0.2000	-99.755	4.0348
-0.001	4.00100	4.42960	16.84336	11.07399	3.99850	1.52098	1.18741	0.02189	4.21242	5.2283E-05	1.0395E-02	5.2270E-05	1.0389E-02	4.9477E-03	-9.8961E-04	-0.2000	-99.955	4.0338
-0.001	4.00000	4.42859	16.83936	11.07149	4.00000	1.52097	1.18769	0.02190	4.20984	5.2309E-05	1.0406E-02	5.2296E-05	1.0400E-02	4.9477E-03	-9.8960E-04	-0.2000	-100.155	4.0329

Table B-22 Case D 40 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	4.500	4.99618	18.61307	12.49046	3.00000	1.49018	2.14903	0.07171	6.20436	1.7599E-04	2.3117E-02							
-0.1	4.400	4.86822	18.30032	12.17055	3.24962	1.50366	2.18575	0.07419	5.63153	1.7988E-04	2.6346E-02	1.7794E-04	2.4732E-02	4.8221E-03	-9.7527E-02	-20.2251	-20.2251	4.5113
-0.1	4.300	4.74920	17.96412	11.87299	3.46987	1.51302	2.22666	0.07699	5.17717	1.8514E-04	2.9741E-02	1.8251E-04	2.8044E-02	4.8175E-03	-9.7196E-02	-20.1756	-40.4007	4.4155
-0.1	4.200	4.63712	17.60714	11.59279	3.66606	1.51880	2.27180	0.08014	4.80274	1.9175E-04	3.3373E-02	1.8844E-04	3.1557E-02	4.8116E-03	-9.6844E-02	-20.1274	-60.5282	4.3202
-0.1	4.100	4.53059	17.23159	11.32647	3.84187	1.52136	2.32132	0.08367	4.48520	1.9975E-04	3.7311E-02	1.9575E-04	3.5342E-02	4.8043E-03	-9.6466E-02	-20.0793	-80.6074	4.2255
-0.01	4.090	4.52020	17.19309	11.30050	3.85845	1.52144	2.32652	0.08405	4.45596	2.0063E-04	3.7724E-02	2.0019E-04	3.7517E-02	4.7998E-03	-9.6248E-03	-2.00525	-82.6127	4.2161
-0.01	4.080	4.50986	17.15442	11.27464	3.87484	1.52151	2.33176	0.08443	4.42713	2.0152E-04	3.8141E-02	2.0108E-04	3.7932E-02	4.7989E-03	-9.6207E-03	-2.00476	-84.6174	4.2066
-0.01	4.070	4.49955	17.11559	11.24888	3.89107	1.52154	2.33705	0.08481	4.39869	2.0243E-04	3.8562E-02	2.0198E-04	3.8351E-02	4.7980E-03	-9.6165E-03	-2.00426	-86.6217	4.1972
-0.01	4.060	4.48929	17.07660	11.22324	3.90712	1.52154	2.34239	0.08520	4.37063	2.0336E-04	3.8987E-02	2.0290E-04	3.8774E-02	4.7971E-03	-9.6123E-03	-2.00376	-88.6254	4.1878
-0.01	4.050	4.47908	17.03745	11.19770	3.92301	1.52151	2.34777	0.08559	4.34295	2.0430E-04	3.9416E-02	2.0383E-04	3.9201E-02	4.7962E-03	-9.6080E-03	-2.00326	-90.6287	4.1784
-0.01	4.040	4.46890	16.99814	11.17226	3.93873	1.52146	2.35320	0.08599	4.31564	2.0525E-04	3.9849E-02	2.0478E-04	3.9632E-02	4.7952E-03	-9.6037E-03	-2.00276	-92.6315	4.1690
-0.01	4.030	4.45877	16.95868	11.14692	3.95429	1.52138	2.35867	0.08639	4.28868	2.0623E-04	4.0286E-02	2.0574E-04	4.0068E-02	4.7943E-03	-9.5993E-03	-2.00225	-94.6337	4.1596
-0.01	4.020	4.44867	16.91906	11.12168	3.96969	1.52127	2.36420	0.08679	4.26207	2.0721E-04	4.0728E-02	2.0672E-04	4.0507E-02	4.7933E-03	-9.5949E-03	-2.00175	-96.6355	4.1502
-0.01	4.010	4.43861	16.87928	11.09653	3.98492	1.52113	2.36977	0.08720	4.23579	2.0822E-04	4.1174E-02	2.0771E-04	4.0951E-02	4.7923E-03	-9.5905E-03	-2.00124	-98.6367	4.1408
-0.001	4.009	4.43761	16.87530	11.09403	3.98644	1.52112	2.37033	0.08724	4.23318	2.0832E-04	4.1219E-02	2.0827E-04	4.1196E-02	4.7917E-03	-9.5880E-04	-0.20010	-98.8368	4.1399
-0.001	4.008	4.43661	16.87131	11.09152	3.98795	1.52110	2.37089	0.08728	4.23057	2.0842E-04	4.1264E-02	2.0837E-04	4.1241E-02	4.7916E-03	-9.5876E-04	-0.20009	-99.0369	4.1389
-0.001	4.007	4.43560	16.86732	11.08901	3.98946	1.52108	2.37145	0.08733	4.22797	2.0852E-04	4.1309E-02	2.0847E-04	4.1286E-02	4.7915E-03	-9.5871E-04	-0.20009	-99.237	4.1380
-0.001	4.006	4.43460	16.86333	11.08650	3.99097	1.52107	2.37201	0.08737	4.22537	2.0862E-04	4.1354E-02	2.0857E-04	4.1331E-02	4.7914E-03	-9.5867E-04	-0.20008	-99.4371	4.1371
-0.001	4.005	4.43360	16.85934	11.08400	3.99248	1.52105	2.37257	0.08741	4.22277	2.0872E-04	4.1399E-02	2.0867E-04	4.1376E-02	4.7913E-03	-9.5862E-04	-0.20007	-99.6371	4.1361
-0.001	4.004	4.43260	16.85535	11.08149	3.99399	1.52104	2.37313	0.08745	4.22018	2.0882E-04	4.1444E-02	2.0877E-04	4.1421E-02	4.7912E-03	-9.5858E-04	-0.20007	-99.8372	4.1352
-0.001	4.003	4.43160	16.85135	11.07899	3.99549	1.52102	2.37370	0.08749	4.21759	2.0893E-04	4.1489E-02	2.0888E-04	4.1466E-02	4.7911E-03	-9.5853E-04	-0.20006	-100.037	4.1342

Table B-23 Case D 60 cfs: Manually Computed Outlet Control Headwater Depths

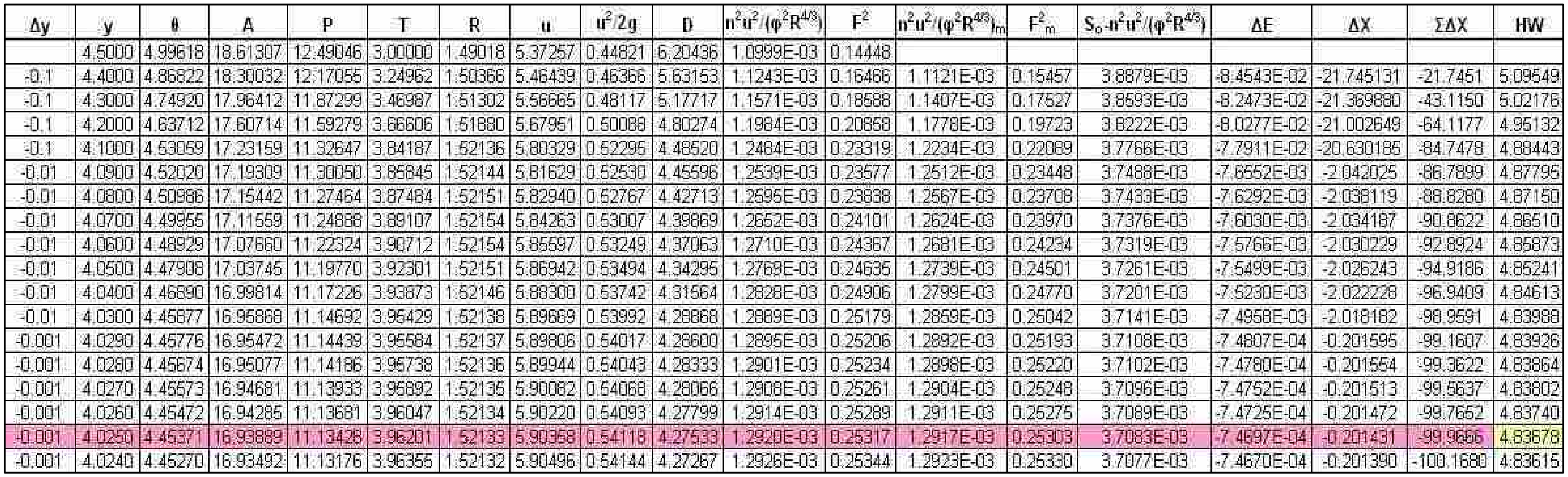


Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	4.500	4.99618	18.61307	12.49046	3.00000	1.49018	3.22354	0.16135	6.20436	3.9598E-04	5.201E-02							
-0.1	4.400	4.86822	18.30032	12.17055	3.24962	1.50366	3.27863	0.16692	5.63153	4.0474E-04	5.928E-02	4.004E-04	5.565E-02	4.5996E-03	-9.444E-02	-20.53103	-20.531	4.6504
-0.1	4.300	4.74920	17.96412	11.87299	3.46987	1.51302	3.33999	0.17322	5.17717	4.1657E-04	6.692E-02	4.107E-04	6.310E-02	4.5893E-03	-9.369E-02	-20.41471	-40.946	4.5598
-0.1	4.200	4.63712	17.60714	11.59279	3.66606	1.51880	3.40771	0.18032	4.80274	4.3143E-04	7.509E-02	4.240E-04	7.100E-02	4.5760E-03	-9.290E-02	-20.30150	-61.247	4.4705
-0.1	4.100	4.53059	17.23159	11.32647	3.84187	1.52136	3.48198	0.18826	4.48520	4.4944E-04	8.395E-02	4.404E-04	7.952E-02	4.5596E-03	-9.205E-02	-20.18791	-81.435	4.3824
-0.01	4.090	4.52020	17.19309	11.30050	3.85845	1.52144	3.48977	0.18911	4.45596	4.5142E-04	8.488E-02	4.504E-04	8.441E-02	4.5496E-03	-9.156E-03	-2.01247	-83.448	4.3737
-0.01	4.080	4.50986	17.15442	11.27464	3.87484	1.52151	3.49764	0.18996	4.42713	4.5343E-04	8.582E-02	4.524E-04	8.535E-02	4.5476E-03	-9.147E-03	-2.01130	-85.459	4.3649
-0.01	4.070	4.49955	17.11559	11.24888	3.89107	1.52154	3.50558	0.19082	4.39869	4.5548E-04	8.676E-02	4.545E-04	8.629E-02	4.5455E-03	-9.137E-03	-2.01012	-87.469	4.3562
-0.01	4.060	4.48929	17.07660	11.22324	3.90712	1.52154	3.51358	0.19170	4.37063	4.5756E-04	8.772E-02	4.565E-04	8.724E-02	4.5435E-03	-9.128E-03	-2.00894	-89.478	4.3475
-0.01	4.050	4.47908	17.03745	11.19770	3.92301	1.52151	3.52165	0.19258	4.34295	4.5967E-04	8.869E-02	4.586E-04	8.820E-02	4.5414E-03	-9.118E-03	-2.00775	-91.486	4.3389
-0.01	4.040	4.46890	16.99814	11.17226	3.93873	1.52146	3.52980	0.19347	4.31564	4.6182E-04	8.968E-02	4.607E-04	8.917E-02	4.5393E-03	-9.108E-03	-2.00656	-93.492	4.3302
-0.01	4.030	4.45877	16.95868	11.14692	3.95429	1.52138	3.53801	0.19437	4.28868	4.6401E-04	9.054E-02	4.629E-04	9.015E-02	4.5371E-03	-9.098E-03	-2.00536	-95.498	4.3216
-0.01	4.020	4.44867	16.91906	11.12168	3.96969	1.52127	3.54630	0.19528	4.26207	4.6623E-04	9.164E-02	4.651E-04	9.114E-02	4.5349E-03	-9.089E-03	-2.00415	-97.502	4.3129
-0.01	4.010	4.43861	16.87928	11.09653	3.98492	1.52113	3.55465	0.19620	4.23579	4.6848E-04	9.264E-02	4.674E-04	9.214E-02	4.5326E-03	-9.079E-03	-2.00294	-99.505	4.3043
-0.001	4.009	4.43761	16.87530	11.09403	3.98644	1.52112	3.55549	0.19630	4.23318	4.6871E-04	9.274E-02	4.686E-04	9.269E-02	4.5314E-03	-9.073E-04	-0.20023	-99.705	4.3034
-0.001	4.008	4.43661	16.87131	11.09152	3.98795	1.52110	3.55633	0.19639	4.23057	4.6894E-04	9.284E-02	4.688E-04	9.279E-02	4.5312E-03	-9.072E-04	-0.20021	-99.905	4.3026
-0.001	4.007	4.43560	16.86732	11.08901	3.98946	1.52108	3.55717	0.19648	4.22797	4.6917E-04	9.294E-02	4.691E-04	9.289E-02	4.5309E-03	-9.071E-04	-0.20020	-100.105	4.3017
-0.001	4.006	4.43460	16.86333	11.08650	3.99097	1.52107	3.55802	0.19658	4.22537	4.6940E-04	9.305E-02	4.693E-04	9.299E-02	4.5307E-03	-9.070E-04	-0.20019	-100.306	4.3009

Table B-24 Case D 80 cfs: Manually Computed Outlet Control Headwater Depths

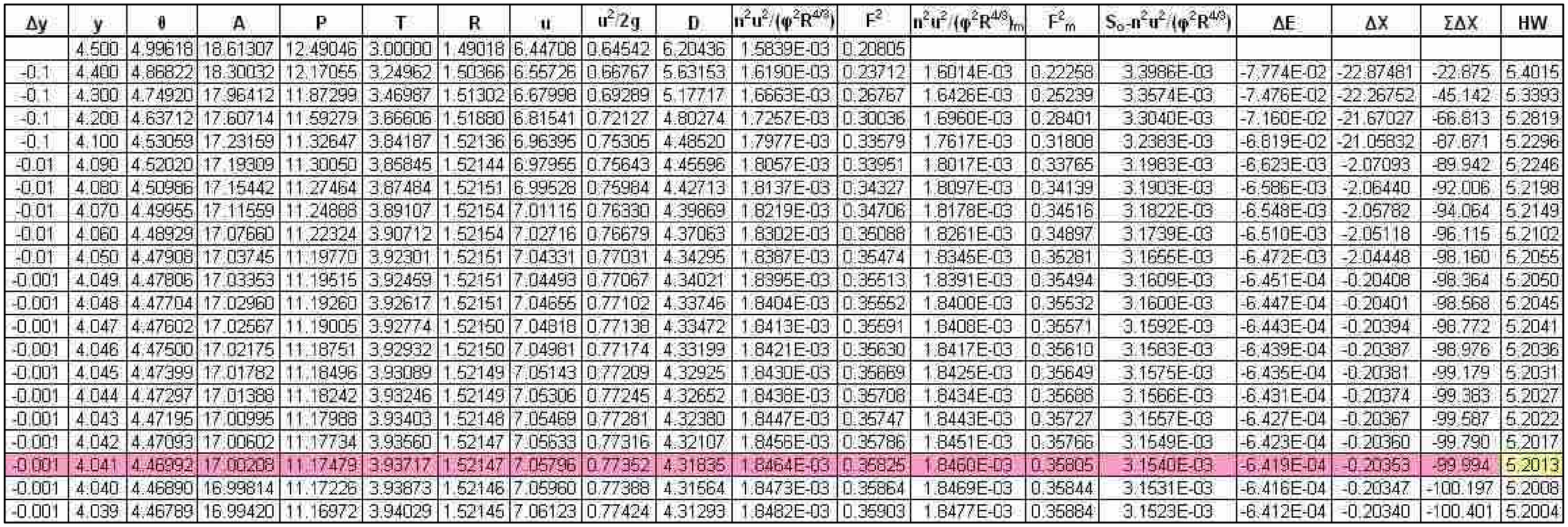
Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	4.500	4.99618	18.61307	12.49046	3.00000	1.49018	4.29805	0.28685	6.20436	7.0396E-04	0.09247							
-0.1	4.400	4.86822	18.30032	12.17055	3.24962	1.50366	4.37151	0.29674	5.63153	7.1954E-04	0.10539	7.1175E-04	0.09893	4.2883E-03	-9.011E-02	-21.01261	-21.013	4.8451
-0.1	4.300	4.74920	17.96412	11.87299	3.46967	1.51302	4.45332	0.30795	5.17717	7.4057E-04	0.11896	7.3005E-04	0.11218	4.2699E-03	-8.878E-02	-20.79241	-41.805	4.7619
-0.1	4.200	4.63712	17.60714	11.59279	3.66606	1.51880	4.54361	0.32057	4.80274	7.6699E-04	0.13349	7.5378E-04	0.12623	4.2462E-03	-8.738E-02	-20.57762	-62.383	4.6808
-0.1	4.100	4.53059	17.23159	11.32647	3.84187	1.52136	4.64264	0.33469	4.48520	7.9900E-04	0.14924	7.8299E-04	0.14137	4.2170E-03	-8.586E-02	-20.36120	-82.744	4.6020
-0.01	4.090	4.52020	17.19309	11.30050	3.85845	1.52144	4.65303	0.33619	4.45596	8.0252E-04	0.15089	8.0076E-04	0.15007	4.1992E-03	-8.499E-03	-2.02401	-84.768	4.5943
-0.01	4.080	4.50986	17.15442	11.27464	3.87484	1.52151	4.66352	0.33771	4.42713	8.0610E-04	0.15256	8.0431E-04	0.15173	4.1957E-03	-8.483E-03	-2.02177	-86.790	4.5866
-0.01	4.070	4.49955	17.11559	11.24888	3.89107	1.52154	4.67410	0.33924	4.39869	8.0973E-04	0.15425	8.0791E-04	0.15341	4.1921E-03	-8.466E-03	-2.01951	-88.809	4.5789
-0.01	4.060	4.48929	17.07660	11.22324	3.90712	1.52154	4.68477	0.34079	4.37063	8.1343E-04	0.15595	8.1158E-04	0.15510	4.1884E-03	-8.449E-03	-2.01724	-90.826	4.5712
-0.01	4.050	4.47908	17.03745	11.19770	3.92301	1.52151	4.69554	0.34236	4.34295	8.1720E-04	0.15766	8.1532E-04	0.15680	4.1847E-03	-8.432E-03	-2.01495	-92.841	4.5635
-0.01	4.040	4.46890	16.99814	11.17226	3.93873	1.52146	4.70640	0.34395	4.31564	8.2102E-04	0.15940	8.1911E-04	0.15853	4.1809E-03	-8.415E-03	-2.01266	-94.854	4.5559
-0.01	4.030	4.45877	16.95868	11.14692	3.95429	1.52138	4.71735	0.34555	4.28868	8.2490E-04	0.16114	8.2296E-04	0.16027	4.1770E-03	-8.397E-03	-2.01035	-96.864	4.5483
-0.01	4.020	4.44867	16.91906	11.12168	3.96969	1.52127	4.72840	0.34717	4.26207	8.2885E-04	0.16291	8.2688E-04	0.16203	4.1731E-03	-8.380E-03	-2.00802	-98.872	4.5408
-0.001	4.019	4.44766	16.91509	11.11916	3.97122	1.52126	4.72951	0.34733	4.25942	8.2925E-04	0.16309	8.2905E-04	0.16300	4.1709E-03	-8.370E-04	-0.20067	-99.073	4.5400
-0.001	4.018	4.44666	16.91111	11.11664	3.97275	1.52124	4.73062	0.34750	4.25678	8.2965E-04	0.16327	8.2945E-04	0.16318	4.1706E-03	-8.368E-04	-0.20065	-99.274	4.5392
-0.001	4.017	4.44565	16.90714	11.11412	3.97427	1.52123	4.73173	0.34766	4.25415	8.3005E-04	0.16344	8.2985E-04	0.16336	4.1702E-03	-8.366E-04	-0.20063	-99.474	4.5385
-0.001	4.016	4.44464	16.90317	11.11161	3.97580	1.52122	4.73284	0.34782	4.25151	8.3045E-04	0.16362	8.3025E-04	0.16353	4.1698E-03	-8.365E-04	-0.20060	-99.675	4.5377
-0.001	4.015	4.44364	16.89919	11.10909	3.97732	1.52120	4.73395	0.34799	4.24889	8.3085E-04	0.16380	8.3065E-04	0.16371	4.1694E-03	-8.363E-04	-0.20058	-99.875	4.5370
-0.001	4.014	4.44263	16.89521	11.10658	3.97885	1.52119	4.73507	0.34815	4.24626	8.3125E-04	0.16398	8.3105E-04	0.16389	4.1690E-03	-8.361E-04	-0.20056	-100.076	4.5362
-0.001	4.013	4.44163	16.89123	11.10407	3.98037	1.52118	4.73619	0.34831	4.24364	8.3165E-04	0.16416	8.3145E-04	0.16407	4.1685E-03	-8.359E-04	-0.20053	-100.277	4.5355
-0.001	4.012	4.44062	16.88725	11.10156	3.98189	1.52116	4.73730	0.34848	4.24102	8.3205E-04	0.16434	8.3185E-04	0.16425	4.1681E-03	-8.358E-04	-0.20051	-100.477	4.5347

Table B-25 Case D 100 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	4.5000	4.99618	18.61307	12.49046	3.00000	1.49018	5.37257	0.44821	6.20436	1.0999E-03	0.14448							
-0.1	4.4000	4.86822	18.30032	12.17055	3.24962	1.50366	5.46439	0.46366	5.63153	1.1243E-03	0.16466	1.1121E-03	0.15457	3.8879E-03	-8.4543E-02	-21.745131	-21.7451	5.09549
-0.1	4.3000	4.74920	17.96412	11.87299	3.46987	1.51302	5.56665	0.48117	5.17717	1.1571E-03	0.18588	1.1407E-03	0.17527	3.8593E-03	-8.2473E-02	-21.369880	-43.1150	5.02176
-0.1	4.2000	4.63712	17.60714	11.59279	3.66606	1.51880	5.67951	0.50088	4.80274	1.1984E-03	0.20858	1.1778E-03	0.19723	3.8222E-03	-8.0277E-02	-21.002649	-64.1177	4.95132
-0.1	4.1000	4.53059	17.23159	11.32647	3.84187	1.52136	5.80329	0.52295	4.48520	1.2484E-03	0.23319	1.2234E-03	0.22089	3.7766E-03	-7.7911E-02	-20.630185	-84.7478	4.88443
-0.01	4.0900	4.52020	17.19309	11.30050	3.85845	1.52144	5.81629	0.52530	4.45596	1.2539E-03	0.23577	1.2512E-03	0.23448	3.7488E-03	-7.6552E-03	-2.042025	-86.7899	4.87795
-0.01	4.0800	4.50986	17.15442	11.27464	3.87484	1.52151	5.82940	0.52767	4.42713	1.2595E-03	0.23838	1.2567E-03	0.23708	3.7433E-03	-7.6292E-03	-2.038119	-88.8280	4.87150
-0.01	4.0700	4.49955	17.11559	11.24888	3.89107	1.52154	5.84263	0.53007	4.39869	1.2652E-03	0.24101	1.2624E-03	0.23970	3.7376E-03	-7.6030E-03	-2.034187	-90.8622	4.86510
-0.01	4.0600	4.48929	17.07660	11.22324	3.90712	1.52154	5.85597	0.53249	4.37063	1.2710E-03	0.24367	1.2681E-03	0.24234	3.7319E-03	-7.5766E-03	-2.030229	-92.8924	4.85873
-0.01	4.0500	4.47908	17.03745	11.19770	3.92301	1.52151	5.86942	0.53494	4.34295	1.2768E-03	0.24635	1.2739E-03	0.24501	3.7261E-03	-7.5499E-03	-2.026243	-94.9186	4.85241
-0.01	4.0400	4.46890	16.99814	11.17226	3.93873	1.52146	5.88300	0.53742	4.31564	1.2828E-03	0.24906	1.2799E-03	0.24770	3.7201E-03	-7.5230E-03	-2.022228	-96.9409	4.84613
-0.01	4.0300	4.45877	16.95868	11.14692	3.95429	1.52138	5.89669	0.53992	4.28868	1.2889E-03	0.25179	1.2859E-03	0.25042	3.7141E-03	-7.4958E-03	-2.018182	-98.9591	4.83988
-0.001	4.0290	4.45776	16.95472	11.14439	3.95584	1.52137	5.89806	0.54017	4.28600	1.2895E-03	0.25206	1.2892E-03	0.25193	3.7108E-03	-7.4807E-04	-0.201595	-99.1607	4.83926
-0.001	4.0280	4.45674	16.95077	11.14186	3.95738	1.52136	5.89944	0.54043	4.28333	1.2901E-03	0.25234	1.2898E-03	0.25220	3.7102E-03	-7.4780E-04	-0.201554	-99.3622	4.83864
-0.001	4.0270	4.45573	16.94681	11.13933	3.95892	1.52135	5.90082	0.54068	4.28066	1.2908E-03	0.25261	1.2904E-03	0.25248	3.7096E-03	-7.4752E-04	-0.201513	-99.5637	4.83802
-0.001	4.0260	4.45472	16.94285	11.13681	3.96047	1.52134	5.90220	0.54093	4.27799	1.2914E-03	0.25289	1.2911E-03	0.25275	3.7089E-03	-7.4725E-04	-0.201472	-99.7652	4.83740
-0.001	4.0250	4.45371	16.93889	11.13428	3.96201	1.52133	5.90358	0.54118	4.27533	1.2920E-03	0.25317	1.2917E-03	0.25303	3.7083E-03	-7.4697E-04	-0.201431	-99.9666	4.83678
-0.001	4.0240	4.45270	16.93492	11.13176	3.96355	1.52132	5.90496	0.54144	4.27267	1.2926E-03	0.25344	1.2923E-03	0.25330	3.7077E-03	-7.4670E-04	-0.201390	-100.1680	4.83615

Table B-26 Case D 120 cfs: Manually Computed Outlet Control Headwater Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	4.500	4.99618	18.61307	12.49046	3.00000	1.49018	6.44708	0.64542	6.20436	1.5839E-03	0.20805							
-0.1	4.400	4.86822	18.30032	12.17055	3.24962	1.50366	6.55726	0.66767	5.63153	1.6190E-03	0.23712	1.6014E-03	0.22258	3.3986E-03	-7.774E-02	-22.87481	-22.875	5.4015
-0.1	4.300	4.74920	17.96412	11.87299	3.46987	1.51302	6.67998	0.69289	5.17717	1.6663E-03	0.26767	1.6426E-03	0.25239	3.3574E-03	-7.476E-02	-22.26752	-45.142	5.3393
-0.1	4.200	4.63712	17.60714	11.59279	3.66606	1.51880	6.81541	0.72127	4.80274	1.7257E-03	0.30036	1.6960E-03	0.28401	3.3040E-03	-7.160E-02	-21.67027	-66.813	5.2819
-0.1	4.100	4.53059	17.23159	11.32647	3.84187	1.52136	6.96395	0.75305	4.48520	1.7977E-03	0.33579	1.7617E-03	0.31808	3.2383E-03	-6.819E-02	-21.05832	-87.871	5.2296
-0.01	4.090	4.52020	17.19309	11.30050	3.85845	1.52144	6.97955	0.75643	4.45696	1.8057E-03	0.33951	1.8017E-03	0.33765	3.1983E-03	-6.623E-03	-2.07093	-89.942	5.2246
-0.01	4.080	4.50986	17.15442	11.27464	3.87484	1.52151	6.99528	0.75984	4.42713	1.8137E-03	0.34327	1.8097E-03	0.34139	3.1903E-03	-6.586E-03	-2.06440	-92.006	5.2198
-0.01	4.070	4.49955	17.11559	11.24888	3.89107	1.52154	7.01115	0.76330	4.39869	1.8219E-03	0.34706	1.8178E-03	0.34516	3.1822E-03	-6.548E-03	-2.05782	-94.064	5.2149
-0.01	4.060	4.48929	17.07660	11.22324	3.90712	1.52154	7.02716	0.76679	4.37063	1.8302E-03	0.35088	1.8261E-03	0.34897	3.1739E-03	-6.510E-03	-2.05118	-96.115	5.2102
-0.01	4.050	4.47908	17.03745	11.19770	3.92301	1.52151	7.04331	0.77031	4.34295	1.8387E-03	0.35474	1.8345E-03	0.35281	3.1655E-03	-6.472E-03	-2.04448	-98.160	5.2055
-0.001	4.049	4.47806	17.03353	11.19515	3.92459	1.52151	7.04493	0.77067	4.34021	1.8395E-03	0.35513	1.8391E-03	0.35494	3.1609E-03	-6.451E-04	-0.20408	-98.364	5.2050
-0.001	4.048	4.47704	17.02960	11.19260	3.92617	1.52151	7.04655	0.77102	4.33746	1.8404E-03	0.35552	1.8400E-03	0.35532	3.1600E-03	-6.447E-04	-0.20401	-98.568	5.2045
-0.001	4.047	4.47602	17.02567	11.19005	3.92774	1.52150	7.04818	0.77138	4.33472	1.8413E-03	0.35591	1.8408E-03	0.35571	3.1592E-03	-6.443E-04	-0.20394	-98.772	5.2041
-0.001	4.046	4.47500	17.02175	11.18751	3.92932	1.52150	7.04981	0.77174	4.33199	1.8421E-03	0.35630	1.8417E-03	0.35610	3.1583E-03	-6.439E-04	-0.20387	-98.976	5.2036
-0.001	4.045	4.47399	17.01782	11.18496	3.93089	1.52149	7.05143	0.77209	4.32925	1.8430E-03	0.35669	1.8425E-03	0.35649	3.1575E-03	-6.435E-04	-0.20381	-99.179	5.2031
-0.001	4.044	4.47297	17.01388	11.18242	3.93246	1.52149	7.05306	0.77245	4.32652	1.8438E-03	0.35708	1.8434E-03	0.35688	3.1566E-03	-6.431E-04	-0.20374	-99.383	5.2027
-0.001	4.043	4.47195	17.00995	11.17988	3.93403	1.52148	7.05469	0.77281	4.32380	1.8447E-03	0.35747	1.8443E-03	0.35727	3.1557E-03	-6.427E-04	-0.20367	-99.587	5.2022
-0.001	4.042	4.47093	17.00602	11.17734	3.93560	1.52147	7.05633	0.77316	4.32107	1.8456E-03	0.35786	1.8451E-03	0.35766	3.1549E-03	-6.423E-04	-0.20360	-99.790	5.2017
-0.001	4.041	4.46992	17.00208	11.17479	3.93717	1.52147	7.05796	0.77352	4.31835	1.8464E-03	0.35825	1.8460E-03	0.35805	3.1540E-03	-6.419E-04	-0.20353	-99.994	5.2013
-0.001	4.040	4.46890	16.99814	11.17226	3.93873	1.52146	7.05960	0.77388	4.31564	1.8473E-03	0.35864	1.8469E-03	0.35844	3.1531E-03	-6.416E-04	-0.20347	-100.197	5.2008
-0.001	4.039	4.46789	16.99420	11.16972	3.94029	1.52145	7.06123	0.77424	4.31293	1.8482E-03	0.35903	1.8477E-03	0.35884	3.1523E-03	-6.412E-04	-0.20340	-100.401	5.2004

Table B-27 Case D 140 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	4.500	4.9962	18.6131	12.4905	3.0000	1.4902	7.5216	0.8785	6.2044	2.1559E-03	0.28318							
-0.1	4.400	4.8682	18.3003	12.1705	3.2496	1.5037	7.6501	0.9088	5.6315	2.2036E-03	0.32274	2.1797E-03	0.30296	2.8203E-03	-6.970E-02	-24.7153	-24.7153	5.7632
-0.1	4.300	4.7492	17.9641	11.8730	3.4699	1.5130	7.7933	0.9431	5.1772	2.2680E-03	0.36433	2.2358E-03	0.34354	2.7642E-03	-6.565E-02	-23.7486	-48.4639	5.7147
-0.1	4.200	4.6371	17.6071	11.5928	3.6661	1.5188	7.9513	0.9817	4.8027	2.3489E-03	0.40882	2.3084E-03	0.38658	2.6916E-03	-6.134E-02	-22.7907	-71.2547	5.6726
-0.01	4.190	4.6262	17.5704	11.5656	3.6845	1.5192	7.9679	0.9858	4.7687	2.3579E-03	0.41346	2.3534E-03	0.41114	2.6466E-03	-5.889E-03	-2.2250	-73.4796	5.6688
-0.01	4.180	4.6154	17.5335	11.5385	3.7028	1.5196	7.9847	0.9900	4.7352	2.3671E-03	0.41814	2.3625E-03	0.41580	2.6375E-03	-5.842E-03	-2.2150	-75.6946	5.6650
-0.01	4.170	4.6046	17.4963	11.5116	3.7208	1.5199	8.0017	0.9942	4.7023	2.3765E-03	0.42286	2.3718E-03	0.42050	2.6282E-03	-5.795E-03	-2.2049	-77.8996	5.6613
-0.01	4.160	4.5939	17.4590	11.4848	3.7387	1.5202	8.0188	0.9985	4.6699	2.3860E-03	0.42762	2.3813E-03	0.42524	2.6187E-03	-5.748E-03	-2.1948	-80.0944	5.6577
-0.01	4.150	4.5832	17.4216	11.4581	3.7563	1.5205	8.0360	1.0028	4.6379	2.3957E-03	0.43242	2.3909E-03	0.43002	2.6091E-03	-5.700E-03	-2.1846	-82.2789	5.6541
-0.01	4.140	4.5726	17.3839	11.4315	3.7738	1.5207	8.0534	1.0071	4.6065	2.4056E-03	0.43726	2.4007E-03	0.43484	2.5993E-03	-5.652E-03	-2.1743	-84.4532	5.6507
-0.01	4.130	4.5620	17.3461	11.4051	3.7911	1.5209	8.0710	1.0115	4.5755	2.4157E-03	0.44214	2.4106E-03	0.43970	2.5894E-03	-5.603E-03	-2.1639	-86.6171	5.6473
-0.01	4.120	4.5515	17.3081	11.3788	3.8082	1.5211	8.0887	1.0159	4.5450	2.4259E-03	0.44707	2.4208E-03	0.44460	2.5792E-03	-5.554E-03	-2.1534	-88.7704	5.6439
-0.01	4.110	4.5410	17.2699	11.3526	3.8251	1.5212	8.1066	1.0204	4.5149	2.4363E-03	0.45204	2.4311E-03	0.44955	2.5689E-03	-5.504E-03	-2.1428	-90.9132	5.6407
-0.01	4.100	4.5306	17.2316	11.3265	3.8419	1.5214	8.1246	1.0250	4.4852	2.4469E-03	0.45705	2.4416E-03	0.45455	2.5584E-03	-5.455E-03	-2.1320	-93.0452	5.6375
-0.01	4.090	4.5202	17.1931	11.3005	3.8584	1.5214	8.1428	1.0296	4.4560	2.4577E-03	0.46212	2.4523E-03	0.45958	2.5477E-03	-5.404E-03	-2.1212	-95.1664	5.6344
-0.01	4.080	4.5099	17.1544	11.2746	3.8748	1.5215	8.1612	1.0342	4.4271	2.4687E-03	0.46722	2.4632E-03	0.46467	2.5368E-03	-5.353E-03	-2.1102	-97.2767	5.6313
-0.01	4.070	4.4996	17.1156	11.2489	3.8911	1.5215	8.1797	1.0389	4.3987	2.4798E-03	0.47238	2.4742E-03	0.46980	2.5258E-03	-5.302E-03	-2.0992	-99.3758	5.6284
-0.001	4.069	4.4985	17.1117	11.2463	3.8927	1.5215	8.1815	1.0394	4.3959	2.4809E-03	0.47290	2.4804E-03	0.47264	2.5196E-03	-5.274E-04	-0.2093	-99.5851	5.6281
-0.001	4.068	4.4975	17.1078	11.2437	3.8943	1.5215	8.1834	1.0399	4.3930	2.4821E-03	0.47342	2.4815E-03	0.47316	2.5185E-03	-5.268E-04	-0.2092	-99.7943	5.6278
-0.001	4.067	4.4965	17.1039	11.2412	3.8959	1.5215	8.1853	1.0403	4.3902	2.4832E-03	0.47394	2.4826E-03	0.47368	2.5174E-03	-5.263E-04	-0.2091	-100.0034	5.6275
-0.001	4.066	4.4954	17.1000	11.2386	3.8975	1.5215	8.1871	1.0408	4.3874	2.4843E-03	0.47446	2.4838E-03	0.47420	2.5162E-03	-5.258E-04	-0.2090	-100.2124	5.6272

Table B-28 Case D 160 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	4.500	4.9962	18.6131	12.4905	3.0000	1.4902	8.5961	1.1474	6.2044	2.8158E-03	0.3699							
-0.1	4.400	4.8682	18.3003	12.1705	3.2496	1.5037	8.7430	1.1870	5.6315	2.8782E-03	0.4215	2.8470E-03	0.3957	2.1530E-03	-6.043E-02	-28.06747	-28.067	6.1804
-0.1	4.300	4.7492	17.9641	11.8730	3.4699	1.5130	8.9066	1.2318	5.1772	2.9623E-03	0.4759	2.9202E-03	0.4487	2.0798E-03	-5.513E-02	-26.50745	-54.575	6.1477
-0.1	4.200	4.6371	17.6071	11.5928	3.6661	1.5188	9.0872	1.2823	4.8027	3.0680E-03	0.5340	3.0151E-03	0.5049	1.9849E-03	-4.951E-02	-24.94276	-79.518	6.1234
-0.01	4.190	4.6262	17.5704	11.5656	3.6845	1.5192	9.1062	1.2876	4.7687	3.0797E-03	0.5400	3.0739E-03	0.5370	1.9261E-03	-4.630E-03	-2.40377	-81.921	6.1214
-0.01	4.180	4.6154	17.5335	11.5386	3.7028	1.5196	9.1254	1.2931	4.7352	3.0918E-03	0.5461	3.0858E-03	0.5431	1.9142E-03	-4.569E-03	-2.38690	-84.308	6.1196
-0.01	4.170	4.6046	17.4963	11.5116	3.7208	1.5199	9.1448	1.2986	4.7023	3.1040E-03	0.5523	3.0979E-03	0.5492	1.9021E-03	-4.508E-03	-2.36984	-86.678	6.1178
-0.01	4.160	4.5939	17.4590	11.4848	3.7387	1.5202	9.1643	1.3041	4.6699	3.1164E-03	0.5585	3.1102E-03	0.5554	1.8898E-03	-4.446E-03	-2.35257	-89.031	6.1162
-0.01	4.150	4.5832	17.4216	11.4581	3.7563	1.5205	9.1840	1.3097	4.6379	3.1291E-03	0.5648	3.1228E-03	0.5617	1.8772E-03	-4.383E-03	-2.33507	-91.366	6.1146
-0.01	4.140	4.5726	17.3839	11.4315	3.7738	1.5207	9.2039	1.3154	4.6065	3.1420E-03	0.5711	3.1356E-03	0.5680	1.8644E-03	-4.320E-03	-2.31733	-93.683	6.1131
-0.01	4.130	4.5620	17.3461	11.4051	3.7911	1.5209	9.2240	1.3211	4.5755	3.1552E-03	0.5775	3.1486E-03	0.5743	1.8514E-03	-4.257E-03	-2.29933	-95.982	6.1117
-0.01	4.120	4.5515	17.3081	11.3788	3.8082	1.5211	9.2442	1.3270	4.5450	3.1685E-03	0.5839	3.1619E-03	0.5807	1.8381E-03	-4.193E-03	-2.28106	-98.264	6.1104
-0.001	4.119	4.5505	17.3043	11.3761	3.8099	1.5211	9.2463	1.3275	4.5419	3.1699E-03	0.5846	3.1692E-03	0.5842	1.8308E-03	-4.158E-04	-0.22709	-98.491	6.1103
-0.001	4.118	4.5494	17.3005	11.3735	3.8116	1.5211	9.2483	1.3281	4.5389	3.1712E-03	0.5852	3.1706E-03	0.5849	1.8294E-03	-4.151E-04	-0.22690	-98.718	6.1102
-0.001	4.117	4.5484	17.2967	11.3709	3.8133	1.5211	9.2503	1.3287	4.5359	3.1726E-03	0.5859	3.1719E-03	0.5855	1.8281E-03	-4.145E-04	-0.22672	-98.944	6.1101
-0.001	4.116	4.5473	17.2928	11.3683	3.8150	1.5212	9.2524	1.3293	4.5329	3.1740E-03	0.5865	3.1733E-03	0.5862	1.8267E-03	-4.138E-04	-0.22653	-99.171	6.1099
-0.001	4.115	4.5463	17.2890	11.3656	3.8167	1.5212	9.2544	1.3299	4.5299	3.1753E-03	0.5872	3.1746E-03	0.5868	1.8254E-03	-4.132E-04	-0.22634	-99.397	6.1098
-0.001	4.114	4.5452	17.2852	11.3630	3.8184	1.5212	9.2565	1.3305	4.5268	3.1767E-03	0.5878	3.1760E-03	0.5875	1.8240E-03	-4.125E-04	-0.22616	-99.623	6.1097
-0.001	4.113	4.5442	17.2814	11.3604	3.8201	1.5212	9.2585	1.3311	4.5238	3.1780E-03	0.5885	3.1774E-03	0.5881	1.8226E-03	-4.119E-04	-0.22597	-99.849	6.1096
-0.001	4.112	4.5431	17.2776	11.3578	3.8218	1.5212	9.2606	1.3316	4.5208	3.1794E-03	0.5891	3.1787E-03	0.5888	1.8213E-03	-4.112E-04	-0.22578	-100.075	6.1095
-0.001	4.111	4.5421	17.2738	11.3552	3.8234	1.5212	9.2626	1.3322	4.5179	3.1808E-03	0.5898	3.1801E-03	0.5894	1.8199E-03	-4.106E-04	-0.22559	-100.301	6.1094

Table B-29 Case D 180 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	4.500	4.99618	18.61307	12.49046	3.00000	1.49018	9.67062	1.45219	6.20436	3.5638E-03	0.46812							
-0.1	4.400	4.86822	18.30032	12.17055	3.24962	1.50366	9.83589	1.50225	5.63153	3.6427E-03	0.53351	3.6032E-03	0.50082	1.3968E-03	-4.992E-02	-35.7385	-35.7385	6.65337
-0.1	4.300	4.74920	17.96412	11.87299	3.46987	1.51302	10.01997	1.55900	5.17717	3.7491E-03	0.60226	3.6959E-03	0.56789	1.3041E-03	-4.321E-02	-33.1347	-68.8732	6.63850
-0.01	4.290	4.73770	17.92932	11.84426	3.49050	1.51376	10.03942	1.56506	5.13660	3.7613E-03	0.60938	3.7552E-03	0.60582	1.2448E-03	-3.942E-03	-3.1666	-72.0398	6.63759
-0.01	4.280	4.72628	17.89432	11.81569	3.51090	1.51445	10.05906	1.57119	5.09679	3.7737E-03	0.61654	3.7675E-03	0.61296	1.2325E-03	-3.870E-03	-3.1402	-75.1800	6.63679
-0.01	4.270	4.71492	17.85911	11.78729	3.53106	1.51512	10.07889	1.57739	5.05771	3.7864E-03	0.62376	3.7800E-03	0.62015	1.2200E-03	-3.799E-03	-3.1136	-78.2935	6.63609
-0.01	4.260	4.70362	17.82370	11.75905	3.55100	1.51574	10.09892	1.58367	5.01935	3.7993E-03	0.63102	3.7928E-03	0.62739	1.2072E-03	-3.726E-03	-3.0866	-81.3802	6.63550
-0.01	4.250	4.69239	17.78809	11.73097	3.57071	1.51634	10.11913	1.59001	4.98166	3.8126E-03	0.63835	3.8059E-03	0.63469	1.1941E-03	-3.653E-03	-3.0594	-84.4396	6.63502
-0.01	4.240	4.68122	17.75228	11.70304	3.59021	1.51689	10.13954	1.59643	4.94464	3.8261E-03	0.64572	3.8193E-03	0.64203	1.1807E-03	-3.580E-03	-3.0318	-87.4714	6.63465
-0.01	4.230	4.67010	17.71628	11.67526	3.60949	1.51742	10.16014	1.60293	4.90825	3.8399E-03	0.65316	3.8330E-03	0.64944	1.1670E-03	-3.506E-03	-3.0038	-90.4753	6.63439
-0.01	4.220	4.65905	17.68009	11.64763	3.62855	1.51791	10.18094	1.60950	4.87249	3.8539E-03	0.66065	3.8469E-03	0.65690	1.1531E-03	-3.431E-03	-2.9754	-93.4507	6.63424
-0.01	4.210	4.64806	17.64371	11.62014	3.64741	1.51837	10.20193	1.61614	4.83733	3.8683E-03	0.66820	3.8611E-03	0.66442	1.1389E-03	-3.356E-03	-2.9465	-96.3972	6.63421
-0.01	4.200	4.63712	17.60714	11.59279	3.66606	1.51880	10.22312	1.62286	4.80274	3.8829E-03	0.67581	3.8756E-03	0.67200	1.1244E-03	-3.280E-03	-2.9171	-99.3143	6.63429
-0.001	4.199	4.63603	17.60348	11.59007	3.66791	1.51884	10.22525	1.62354	4.79932	3.8844E-03	0.67657	3.8836E-03	0.67619	1.1164E-03	-3.238E-04	-0.2901	-99.6043	6.63431
-0.001	4.198	4.63494	17.59981	11.58734	3.66977	1.51888	10.22738	1.62421	4.79589	3.8859E-03	0.67734	3.8851E-03	0.67695	1.1149E-03	-3.230E-04	-0.2898	-99.8941	6.63432
-0.001	4.197	4.63385	17.59614	11.58462	3.67162	1.51892	10.22952	1.62489	4.79248	3.8873E-03	0.67810	3.8866E-03	0.67772	1.1134E-03	-3.223E-04	-0.2895	-100.1836	6.63434
-0.001	4.196	4.63276	17.59247	11.58189	3.67346	1.51896	10.23165	1.62557	4.78907	3.8888E-03	0.67887	3.8881E-03	0.67848	1.1119E-03	-3.215E-04	-0.2892	-100.4727	6.63436
-0.001	4.195	4.63167	17.58879	11.57917	3.67531	1.51900	10.23379	1.62625	4.78566	3.8903E-03	0.67963	3.8896E-03	0.67925	1.1104E-03	-3.207E-04	-0.2889	-100.7616	6.63437

Table B-30 Case D 200 cfs: Manually Computed Outlet Control Headwater Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	4.500	4.9962	18.6131	12.4905	3.0000	1.4902	10.7451	1.7928	6.2044	4.3997E-03	0.57792						0	
-0.1	4.400	4.8682	18.3003	12.1705	3.2496	1.5037	10.9288	1.8546	5.6315	4.4971E-03	0.65866	4.4484E-03	0.61829	5.5157E-04	-3.817E-02	-69.20442	-69.2044	7.18194
-0.01	4.390	4.8560	18.2677	12.1399	3.2729	1.5048	10.9483	1.8613	5.5816	4.5087E-03	0.66693	4.5029E-03	0.66279	4.9707E-04	-3.372E-03	-6.78395	-75.9884	7.18188
-0.01	4.380	4.8438	18.2349	12.1094	3.2958	1.5058	10.9680	1.8680	5.5327	4.5207E-03	0.67524	4.5147E-03	0.67109	4.8527E-04	-3.289E-03	-6.77798	-82.7663	7.18195
-0.01	4.370	4.8317	18.2018	12.0792	3.3185	1.5069	10.9879	1.8748	5.4850	4.5330E-03	0.68360	4.5269E-03	0.67942	4.7314E-04	-3.206E-03	-6.77560	-89.5419	7.18214
-0.001	4.369	4.8305	18.1985	12.0762	3.3207	1.5070	10.9899	1.8754	5.4802	4.5343E-03	0.68444	4.5336E-03	0.68402	4.6636E-04	-3.160E-04	-0.67754	-90.2195	7.18217
-0.001	4.368	4.8293	18.1951	12.0732	3.3230	1.5071	10.9919	1.8761	5.4755	4.5355E-03	0.68528	4.5349E-03	0.68486	4.6511E-04	-3.151E-04	-0.67756	-90.8971	7.18219
-0.001	4.367	4.8281	18.1918	12.0702	3.3252	1.5072	10.9939	1.8768	5.4708	4.5368E-03	0.68612	4.5361E-03	0.68570	4.6386E-04	-3.143E-04	-0.67758	-91.5746	7.18222
-0.001	4.366	4.8269	18.1885	12.0672	3.3275	1.5073	10.9960	1.8775	5.4661	4.5380E-03	0.68696	4.5374E-03	0.68654	4.6260E-04	-3.135E-04	-0.67761	-92.2522	7.18225
-0.001	4.365	4.8257	18.1852	12.0642	3.3297	1.5074	10.9980	1.8782	5.4615	4.5393E-03	0.68780	4.5387E-03	0.68738	4.6134E-04	-3.126E-04	-0.67764	-92.9299	7.18228
-0.001	4.364	4.8245	18.1818	12.0612	3.3320	1.5075	11.0000	1.8789	5.4568	4.5406E-03	0.68864	4.5399E-03	0.68822	4.6008E-04	-3.118E-04	-0.67767	-93.6076	7.18232
-0.001	4.363	4.8233	18.1785	12.0582	3.3342	1.5076	11.0020	1.8796	5.4521	4.5418E-03	0.68948	4.5412E-03	0.68906	4.5881E-04	-3.109E-04	-0.67771	-94.2853	7.18235
-0.001	4.362	4.8221	18.1752	12.0552	3.3364	1.5077	11.0040	1.8803	5.4475	4.5431E-03	0.69032	4.5425E-03	0.68990	4.5754E-04	-3.101E-04	-0.67776	-94.9630	7.18238
-0.001	4.361	4.8209	18.1718	12.0522	3.3387	1.5078	11.0060	1.8809	5.4428	4.5444E-03	0.69116	4.5437E-03	0.69074	4.5626E-04	-3.093E-04	-0.67780	-95.6408	7.18242
-0.001	4.360	4.8197	18.1685	12.0492	3.3409	1.5079	11.0081	1.8816	5.4382	4.5457E-03	0.69201	4.5450E-03	0.69159	4.5499E-04	-3.084E-04	-0.67785	-96.3187	7.18246
-0.001	4.359	4.8185	18.1652	12.0462	3.3431	1.5080	11.0101	1.8823	5.4336	4.5469E-03	0.69285	4.5463E-03	0.69243	4.5371E-04	-3.076E-04	-0.67791	-96.9966	7.18250
-0.001	4.358	4.8173	18.1618	12.0432	3.3453	1.5081	11.0121	1.8830	5.4290	4.5482E-03	0.69369	4.5476E-03	0.69327	4.5242E-04	-3.067E-04	-0.67797	-97.6746	7.18254
-0.001	4.357	4.8161	18.1585	12.0402	3.3476	1.5082	11.0141	1.8837	5.4244	4.5495E-03	0.69454	4.5489E-03	0.69412	4.5113E-04	-3.059E-04	-0.67803	-98.3526	7.18258
-0.001	4.356	4.8149	18.1551	12.0372	3.3498	1.5082	11.0162	1.8844	5.4198	4.5508E-03	0.69538	4.5502E-03	0.69496	4.4984E-04	-3.050E-04	-0.67810	-99.0307	7.18262
-0.001	4.355	4.8137	18.1518	12.0342	3.3520	1.5083	11.0182	1.8851	5.4152	4.5521E-03	0.69623	4.5514E-03	0.69581	4.4855E-04	-3.042E-04	-0.67817	-99.7089	7.18266
-0.001	4.354	4.8125	18.1484	12.0312	3.3542	1.5084	11.0203	1.8858	5.4106	4.5534E-03	0.69707	4.5527E-03	0.69665	4.4725E-04	-3.033E-04	-0.67825	-100.3871	7.18271

B.4 Outlet Velocity Computations

To compute the appropriate outlet velocities, critical and normal depth values were first compared to determine the hydraulic slope of the culvert. For hydraulically steep culverts, the downstream boundary condition was normal depth as long as the tailwater depth was less than normal. Tables B-31 through B-40 show the forewater computations used to determine the appropriate outlet depths for case A. Table B-41 shows the results of the outlet velocity computations for case A. Equation B-4 was used to compute the angle defining the top width of flow in the culvert, from which the area of flow was computed (Equation B-1). The velocity at the outlet was then computed by dividing the flow through the culvert by the area of flow.

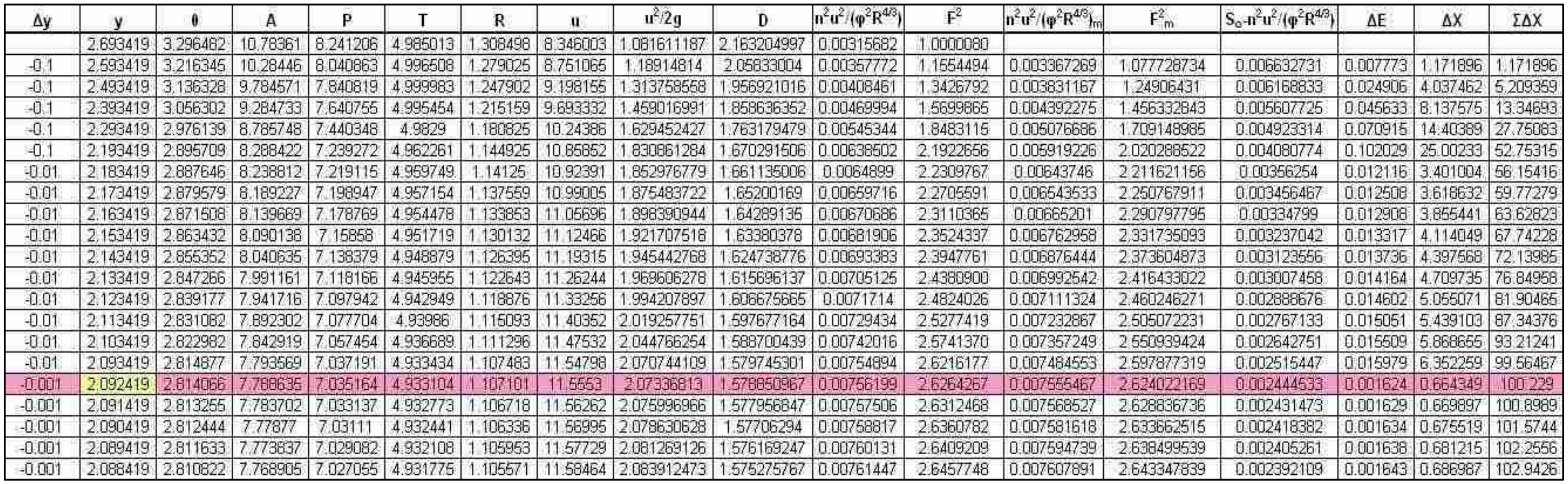
Table B-31 Case A 30 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX
	1.52025	2.33620	5.04718	5.84060	4.60004	0.86417	5.94392	0.54861	1.09720	2.7840E-03	1.0000						
-0.1	1.42025	2.24839	4.59160	5.62099	4.50961	0.81687	6.53366	0.66287	1.01818	3.6261E-03	1.3021	3.2050E-03	1.1510	6.7950E-03	1.510E-02	2.2227	2.223
-0.1	1.32025	2.15870	4.14562	5.39675	4.40826	0.76817	7.23656	0.81316	0.94042	4.8281E-03	1.7294	4.2271E-03	1.5157	5.7729E-03	5.157E-02	8.9333	11.156
-0.1	1.22025	2.06680	3.71034	5.16700	4.29522	0.71808	8.08551	1.01515	0.86383	6.5943E-03	2.3503	5.7112E-03	2.0398	4.2888E-03	1.040E-01	24.2459	35.402
-0.01	1.21025	2.05748	3.66745	5.14369	4.28324	0.71300	8.18007	1.03903	0.85623	6.8137E-03	2.4270	6.7040E-03	2.3887	3.2960E-03	1.389E-02	4.2132	39.615
-0.01	1.20025	2.04812	3.62468	5.12031	4.27113	0.70790	8.27660	1.06370	0.84865	7.0425E-03	2.5068	6.9281E-03	2.4669	3.0719E-03	1.467E-02	4.7753	44.390
-0.01	1.19025	2.03874	3.58203	5.09686	4.25890	0.70279	8.37514	1.08918	0.84107	7.2813E-03	2.5900	7.1619E-03	2.5484	2.8381E-03	1.548E-02	5.4557	49.846
-0.01	1.18025	2.02934	3.53950	5.07335	4.24653	0.69767	8.47577	1.11551	0.83350	7.5304E-03	2.6767	7.4058E-03	2.6333	2.5942E-03	1.633E-02	6.2961	56.142
-0.01	1.17025	2.01991	3.49710	5.04976	4.23403	0.69253	8.57854	1.14272	0.82595	7.7905E-03	2.7671	7.6605E-03	2.7219	2.3395E-03	1.722E-02	7.3599	63.502
-0.01	1.16025	2.01044	3.45482	5.02611	4.22141	0.68737	8.68352	1.17086	0.81841	8.0623E-03	2.8613	7.9264E-03	2.8142	2.0736E-03	1.814E-02	8.7490	72.251
-0.01	1.15025	2.00095	3.41267	5.00239	4.20864	0.68221	8.79077	1.19996	0.81087	8.3462E-03	2.9597	8.2042E-03	2.9105	1.7958E-03	1.911E-02	10.6389	82.890
-0.01	1.14025	1.99144	3.37065	4.97859	4.19575	0.67703	8.90037	1.23007	0.80335	8.6430E-03	3.0624	8.4946E-03	3.0110	1.5054E-03	2.011E-02	13.3585	96.248
-0.001	1.13925	1.99048	3.36645	4.97621	4.19445	0.67651	8.91146	1.23314	0.80260	8.6734E-03	3.0729	8.6582E-03	3.0676	1.3418E-03	2.068E-03	1.5409	97.789
-0.0001	1.13915	1.99039	3.36603	4.97597	4.19432	0.67646	8.91257	1.23344	0.80252	8.6764E-03	3.0739	8.6749E-03	3.0734	1.3251E-03	2.073E-04	0.1565	97.946
-0.0001	1.13905	1.99029	3.36561	4.97573	4.19419	0.67641	8.91368	1.23375	0.80245	8.6795E-03	3.0750	8.6780E-03	3.0745	1.3220E-03	2.074E-04	0.1569	98.103
-0.0001	1.13895	1.99020	3.36519	4.97549	4.19406	0.67635	8.91479	1.23406	0.80237	8.6825E-03	3.0760	8.6810E-03	3.0755	1.3190E-03	2.076E-04	0.1574	98.260
-0.0001	1.13885	1.99010	3.36478	4.97525	4.19393	0.67630	8.91590	1.23437	0.80230	8.6856E-03	3.0771	8.6841E-03	3.0766	1.3159E-03	2.077E-04	0.1578	98.418
-0.0001	1.13875	1.99001	3.36436	4.97501	4.19380	0.67625	8.91701	1.23468	0.80222	8.6886E-03	3.0781	8.6871E-03	3.0776	1.3129E-03	2.078E-04	0.1582	98.576
-0.0001	1.13865	1.98991	3.36394	4.97477	4.19367	0.67620	8.91812	1.23498	0.80215	8.6917E-03	3.0792	8.6902E-03	3.0787	1.3098E-03	2.079E-04	0.1587	98.735
-0.0001	1.13855	1.98981	3.36352	4.97454	4.19354	0.67615	8.91923	1.23529	0.80207	8.6947E-03	3.0803	8.6932E-03	3.0797	1.3068E-03	2.080E-04	0.1591	98.894
-0.0001	1.13845	1.98972	3.36310	4.97430	4.19341	0.67610	8.92035	1.23560	0.80200	8.6978E-03	3.0813	8.6963E-03	3.0808	1.3037E-03	2.081E-04	0.1596	99.054
-0.0001	1.13835	1.98962	3.36268	4.97406	4.19328	0.67604	8.92146	1.23591	0.80192	8.7009E-03	3.0824	8.6993E-03	3.0818	1.3007E-03	2.082E-04	0.1601	99.214
-0.0001	1.13825	1.98953	3.36226	4.97382	4.19315	0.67599	8.92257	1.23622	0.80185	8.7039E-03	3.0834	8.7024E-03	3.0829	1.2976E-03	2.083E-04	0.1605	99.374
-0.0001	1.13815	1.98943	3.36184	4.97358	4.19302	0.67594	8.92368	1.23652	0.80177	8.7070E-03	3.0845	8.7055E-03	3.0840	1.2945E-03	2.084E-04	0.1610	99.535
-0.0001	1.13805	1.98934	3.36142	4.97334	4.19289	0.67589	8.92480	1.23683	0.80169	8.7101E-03	3.0856	8.7085E-03	3.0850	1.2915E-03	2.085E-04	0.1614	99.697
-0.0001	1.13795	1.98924	3.36100	4.97311	4.19276	0.67584	8.92591	1.23714	0.80162	8.7131E-03	3.0866	8.7116E-03	3.0861	1.2884E-03	2.086E-04	0.1619	99.859
-0.0001	1.13785	1.98915	3.36058	4.97287	4.19263	0.67578	8.92702	1.23745	0.80154	8.7162E-03	3.0877	8.7147E-03	3.0871	1.2853E-03	2.087E-04	0.1624	100.021

Table B-32 Case A 60 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	S_o-$n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX
	2.17858	2.88374	8.21482	7.20936	4.95850	1.13947	7.30387	0.82836	1.65671	2.9073E-03	1.00001						
-0.1	2.07858	2.80284	7.72040	7.00710	4.92845	1.10180	7.77161	0.93786	1.56650	3.4425E-03	1.19739	3.1749E-03	1.09870	6.8251E-03	9.870E-03	1.4461	1.4461
-0.1	1.97858	2.72137	7.22941	6.80343	4.89004	1.06261	8.29943	1.06957	1.47839	4.1202E-03	1.44694	3.7814E-03	1.32217	6.2186E-03	3.222E-02	5.1807	6.6268
-0.1	1.87858	2.63919	6.74268	6.59798	4.84307	1.02193	8.89854	1.22956	1.39223	4.9896E-03	1.76632	4.5549E-03	1.60663	5.4451E-03	6.066E-02	11.1409	17.7677
-0.1	1.77858	2.55613	6.26109	6.39033	4.78730	0.97978	9.58300	1.42599	1.30785	6.1210E-03	2.18066	5.5553E-03	1.97349	4.4447E-03	9.735E-02	21.9023	39.6700
-0.1	1.67858	2.47202	5.78553	6.18005	4.72240	0.93616	10.37071	1.67006	1.22512	7.6174E-03	2.72635	6.8692E-03	2.45350	3.1308E-03	1.454E-01	46.4259	86.0959
-0.01	1.66858	2.46354	5.73834	6.15886	4.71540	0.93172	10.45599	1.69764	1.21694	7.7924E-03	2.79002	7.7049E-03	2.75818	2.2951E-03	1.758E-02	7.6606	93.7565
-0.001	1.66758	2.46269	5.73362	6.15674	4.71469	0.93128	10.46459	1.70043	1.21612	7.8102E-03	2.79649	7.8013E-03	2.79325	2.1987E-03	1.793E-03	0.8156	94.5721
-0.001	1.66658	2.46185	5.72891	6.15462	4.71398	0.93083	10.47320	1.70323	1.21530	7.8281E-03	2.80298	7.8191E-03	2.79973	2.1809E-03	1.800E-03	0.8252	95.3973
-0.001	1.66558	2.46100	5.72419	6.15249	4.71328	0.93039	10.48183	1.70604	1.21448	7.8460E-03	2.80948	7.8370E-03	2.80623	2.1630E-03	1.806E-03	0.8351	96.2324
-0.001	1.66458	2.46015	5.71948	6.15037	4.71257	0.92994	10.49046	1.70885	1.21367	7.8639E-03	2.81601	7.8549E-03	2.81275	2.1451E-03	1.813E-03	0.8451	97.0775
-0.001	1.66358	2.45930	5.71477	6.14825	4.71186	0.92950	10.49911	1.71167	1.21285	7.8819E-03	2.82256	7.8729E-03	2.81929	2.1271E-03	1.819E-03	0.8553	97.9328
-0.001	1.66258	2.45846	5.71006	6.14613	4.71115	0.92905	10.50778	1.71449	1.21203	7.9000E-03	2.82912	7.8910E-03	2.82584	2.1090E-03	1.826E-03	0.8657	98.7985
-0.001	1.66158	2.45760	5.70535	6.14400	4.71044	0.92860	10.51645	1.71733	1.21121	7.9181E-03	2.83571	7.9090E-03	2.83242	2.0910E-03	1.832E-03	0.8764	99.6749
-0.0001	1.66148	2.45752	5.70488	6.14379	4.71037	0.92856	10.51732	1.71761	1.21113	7.9199E-03	2.83637	7.9190E-03	2.83604	2.0810E-03	1.836E-04	0.0882	99.7631
-0.0001	1.66138	2.45743	5.70440	6.14358	4.71029	0.92851	10.51819	1.71789	1.21105	7.9217E-03	2.83703	7.9208E-03	2.83670	2.0792E-03	1.837E-04	0.0883	99.8514
-0.0001	1.66128	2.45735	5.70393	6.14337	4.71022	0.92847	10.51906	1.71818	1.21097	7.9235E-03	2.83769	7.9226E-03	2.83736	2.0774E-03	1.837E-04	0.0884	99.9399
-0.0001	1.66118	2.45726	5.70346	6.14316	4.71015	0.92843	10.51993	1.71846	1.21089	7.9254E-03	2.83835	7.9245E-03	2.83802	2.0755E-03	1.838E-04	0.0886	100.0284
-0.0001	1.66108	2.45718	5.70299	6.14294	4.71008	0.92838	10.52079	1.71874	1.21081	7.9272E-03	2.83901	7.9263E-03	2.83868	2.0737E-03	1.839E-04	0.0887	100.1171
-0.0001	1.66098	2.45709	5.70252	6.14273	4.71001	0.92834	10.52166	1.71903	1.21072	7.9290E-03	2.83967	7.9281E-03	2.83934	2.0719E-03	1.839E-04	0.0888	100.2059
-0.0001	1.66088	2.45701	5.70205	6.14252	4.70994	0.92829	10.52253	1.71931	1.21064	7.9308E-03	2.84033	7.9299E-03	2.84000	2.0701E-03	1.840E-04	0.0889	100.2947

Table B-33 Case A 90 cfs: Manually Computed Outlet Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$
	2.693419	3.296482	10.78361	8.241206	4.985013	1.308498	8.346003	1.081611187	2.163204997	0.00315682	1.0000080						
-0.1	2.593419	3.216345	10.28446	8.040863	4.996508	1.279025	8.751065	1.18914814	2.05833004	0.00357772	1.1554494	0.003367269	1.077728734	0.006632731	0.007773	1.171896	1.171896
-0.1	2.493419	3.136328	9.784571	7.840819	4.999983	1.247902	9.198155	1.313758558	1.956921016	0.00408461	1.3426792	0.003831167	1.24906431	0.006168833	0.024906	4.037462	5.209359
-0.1	2.393419	3.056302	9.284733	7.640755	4.995454	1.215159	9.693332	1.459016991	1.858636352	0.00469994	1.5699865	0.004392275	1.456332843	0.005607725	0.045633	8.137575	13.34693
-0.1	2.293419	2.976139	8.785748	7.440348	4.9829	1.180825	10.24386	1.629452427	1.763179479	0.00545344	1.8483115	0.005076686	1.709148985	0.004923314	0.070915	14.40389	27.75083
-0.1	2.193419	2.895709	8.288422	7.239272	4.962261	1.144925	10.85852	1.830861284	1.670291506	0.00638502	2.1922656	0.005919226	2.020288522	0.004080774	0.102029	25.00233	52.75315
-0.01	2.183419	2.887646	8.238812	7.219115	4.959749	1.14125	10.92391	1.852976779	1.661135006	0.0064899	2.2309767	0.00643746	2.211621156	0.00356254	0.012116	3.401004	56.15416
-0.01	2.173419	2.879579	8.189227	7.198947	4.957154	1.137559	10.99005	1.875483722	1.65200169	0.00659716	2.2705591	0.006543533	2.250767911	0.003456467	0.012508	3.618632	59.77279
-0.01	2.163419	2.871508	8.139669	7.178769	4.954478	1.133853	11.05696	1.898390944	1.64289135	0.00670686	2.3110365	0.00665201	2.290797795	0.00334799	0.012908	3.855441	63.62823
-0.01	2.153419	2.863432	8.090138	7.15858	4.951719	1.130132	11.12466	1.921707518	1.63380378	0.00681906	2.3524337	0.006762958	2.331735093	0.003237042	0.013317	4.114049	67.74228
-0.01	2.143419	2.855352	8.040635	7.138379	4.948879	1.126395	11.19315	1.945442768	1.624738776	0.00693383	2.3947761	0.006876444	2.373604873	0.003123556	0.013736	4.397568	72.13985
-0.01	2.133419	2.847266	7.991161	7.118166	4.945955	1.122643	11.26244	1.969606278	1.615696137	0.00705125	2.4380900	0.006992542	2.416433022	0.003007458	0.014164	4.709735	76.84958
-0.01	2.123419	2.839177	7.941716	7.097942	4.942949	1.118876	11.33256	1.994207897	1.606675665	0.0071714	2.4824026	0.007111324	2.460246271	0.002888676	0.014602	5.055071	81.90465
-0.01	2.113419	2.831082	7.892302	7.077704	4.93986	1.115093	11.40352	2.019257751	1.597677164	0.00729434	2.5277419	0.007232867	2.505072231	0.002767133	0.015051	5.439103	87.34376
-0.01	2.103419	2.822982	7.842919	7.057454	4.936689	1.111296	11.47532	2.044766254	1.588700439	0.00742016	2.5741370	0.007357249	2.550939424	0.002642751	0.015509	5.868655	93.21241
-0.01	2.093419	2.814877	7.793569	7.037191	4.933434	1.107483	11.54798	2.070744109	1.579745301	0.00754894	2.6216177	0.007484553	2.597877319	0.002515447	0.015979	6.352259	99.56467
-0.001	2.092419	2.814066	7.788635	7.035164	4.933104	1.107101	11.5553	2.07336813	1.578850967	0.00756199	2.6264267	0.007555467	2.624022169	0.002444533	0.001624	0.664349	100.229
-0.001	2.091419	2.813255	7.783702	7.033137	4.932773	1.106718	11.56262	2.075996966	1.577956847	0.00757506	2.6312468	0.007568527	2.628836736	0.002431473	0.001629	0.669897	100.8989
-0.001	2.090419	2.812444	7.77877	7.03111	4.932441	1.106336	11.56995	2.078630628	1.57706294	0.00758817	2.6360782	0.007581618	2.633662515	0.002418382	0.001634	0.675519	101.5744
-0.001	2.089419	2.811633	7.773837	7.029082	4.932108	1.105953	11.57729	2.081269126	1.576169247	0.00760131	2.6409209	0.007594739	2.638499539	0.002405261	0.001638	0.681215	102.2556
-0.001	2.088419	2.810822	7.768905	7.027055	4.931775	1.105571	11.58464	2.083912473	1.575275767	0.00761447	2.6457748	0.007607891	2.643347839	0.002392109	0.001643	0.686987	102.9426

Table B-34 Case A 120 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX
	3.1301	3.6512	12.9343	9.1279	4.8386	1.4170	9.2777	1.3366	2.6731	3.5079E-03	1.0000						
-0.1	3.0301	3.5689	12.4479	8.9223	4.8863	1.3952	9.6401	1.4430	2.5476	3.8666E-03	1.1329	3.6872E-03	1.0665	6.3128E-03	6.645E-03	1.0527	1.0527
-0.1	2.9301	3.4874	11.9573	8.7185	4.9255	1.3715	10.0357	1.5639	2.4277	4.2871E-03	1.2884	4.0768E-03	1.2107	5.9232E-03	2.107E-02	3.5565	4.6092
-0.1	2.8301	3.4064	11.4631	8.5161	4.9562	1.3461	10.4883	1.7016	2.3129	4.7826E-03	1.4715	4.5348E-03	1.3799	5.4652E-03	3.799E-02	6.9519	11.5612
-0.1	2.7301	3.3259	10.9663	8.3148	4.9788	1.3189	10.9426	1.8593	2.2026	5.3697E-03	1.6883	5.0762E-03	1.5799	4.9238E-03	5.799E-02	11.7769	23.3380
-0.1	2.6301	3.2457	10.4676	8.1143	4.9932	1.2900	11.4639	2.0407	2.0964	6.0700E-03	1.9469	5.7199E-03	1.8176	4.2801E-03	8.176E-02	19.1020	42.4400
-0.1	2.5301	3.1657	9.9679	7.9142	4.9996	1.2595	12.0386	2.2504	1.9937	6.9110E-03	2.2575	6.4905E-03	2.1022	3.5095E-03	1.102E-01	31.4062	73.8461
-0.01	2.5201	3.1577	9.9179	7.8942	4.9998	1.2564	12.0993	2.2732	1.9837	7.0042E-03	2.2919	6.9576E-03	2.2747	3.0424E-03	1.275E-02	4.1898	78.0359
-0.01	2.5101	3.1497	9.8679	7.8742	5.0000	1.2532	12.1606	2.2963	1.9736	7.0991E-03	2.3270	7.0516E-03	2.3094	2.9484E-03	1.309E-02	4.4413	82.4772
-0.01	2.5001	3.1417	9.8179	7.8542	5.0000	1.2500	12.2225	2.3197	1.9636	7.1959E-03	2.3627	7.1475E-03	2.3449	2.8525E-03	1.345E-02	4.7147	87.1919
-0.01	2.4901	3.1337	9.7679	7.8342	5.0000	1.2468	12.2851	2.3435	1.9536	7.2946E-03	2.3992	7.2452E-03	2.3810	2.7548E-03	1.381E-02	5.0130	92.2048
-0.01	2.4801	3.1257	9.7179	7.8142	4.9998	1.2436	12.3483	2.3677	1.9437	7.3952E-03	2.4364	7.3449E-03	2.4178	2.6551E-03	1.418E-02	5.3397	97.5446
-0.001	2.4791	3.1249	9.7129	7.8122	4.9998	1.2433	12.3546	2.3701	1.9427	7.4053E-03	2.4401	7.4003E-03	2.4382	2.5997E-03	1.438E-03	0.5532	98.0978
-0.001	2.4781	3.1241	9.7079	7.8102	4.9998	1.2430	12.3610	2.3726	1.9417	7.4155E-03	2.4439	7.4104E-03	2.4420	2.5896E-03	1.442E-03	0.5568	98.6546
-0.001	2.4771	3.1233	9.7029	7.8082	4.9998	1.2427	12.3674	2.3750	1.9407	7.4257E-03	2.4476	7.4206E-03	2.4458	2.5794E-03	1.446E-03	0.5605	99.2151
-0.001	2.4761	3.1225	9.6979	7.8062	4.9998	1.2423	12.3738	2.3775	1.9397	7.4360E-03	2.4514	7.4309E-03	2.4495	2.5691E-03	1.450E-03	0.5642	99.7793
-0.0001	2.4760	3.1224	9.6974	7.8060	4.9998	1.2423	12.3744	2.3777	1.9396	7.4370E-03	2.4518	7.4365E-03	2.4516	2.5635E-03	1.452E-04	0.0566	99.8360
-0.0001	2.4759	3.1223	9.6969	7.8058	4.9998	1.2423	12.3750	2.3780	1.9395	7.4380E-03	2.4522	7.4375E-03	2.4520	2.5625E-03	1.452E-04	0.0567	99.8926
-0.0001	2.4758	3.1222	9.6964	7.8056	4.9998	1.2422	12.3757	2.3782	1.9394	7.4390E-03	2.4526	7.4385E-03	2.4524	2.5615E-03	1.452E-04	0.0567	99.9493
-0.0001	2.4757	3.1221	9.6959	7.8054	4.9998	1.2422	12.3763	2.3785	1.9393	7.4401E-03	2.4529	7.4395E-03	2.4527	2.5605E-03	1.453E-04	0.0567	100.0061
-0.0001	2.4756	3.1221	9.6954	7.8052	4.9998	1.2422	12.3769	2.3787	1.9392	7.4411E-03	2.4533	7.4406E-03	2.4531	2.5594E-03	1.453E-04	0.0568	100.0628

Table B-35 Case A 150 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$
	3.5106	3.9739	14.7294	9.9348	4.5732	1.4826	10.1837	1.6104	3.2208	3.9790E-03	1.00000						
-0.1	3.4106	3.8872	14.2678	9.7181	4.6565	1.4682	10.5132	1.7163	3.0641	4.2963E-03	1.12026	4.1376E-03	1.06013	5.8624E-03	6.013E-03	1.0257	1.0257
-0.1	3.3106	3.8020	13.7984	9.5051	4.7299	1.4517	10.8708	1.8350	2.9173	4.6632E-03	1.25803	4.4797E-03	1.18914	5.5203E-03	1.891E-02	3.4263	4.4520
-0.1	3.2106	3.7180	13.3221	9.2951	4.7938	1.4332	11.2595	1.9686	2.7791	5.0886E-03	1.41671	4.8759E-03	1.33737	5.1241E-03	3.374E-02	6.5840	11.0360
-0.1	3.1106	3.6351	12.8399	9.0877	4.8486	1.4129	11.6823	2.1192	2.6482	5.5834E-03	1.60048	5.3360E-03	1.50860	4.6640E-03	5.086E-02	10.9048	21.9408
-0.1	3.0106	3.5530	12.3527	8.8825	4.8946	1.3907	12.1431	2.2897	2.5237	6.1614E-03	1.81450	5.8724E-03	1.70749	4.1276E-03	7.075E-02	17.1405	39.0813
-0.1	2.9106	3.4716	11.8613	8.6790	4.9321	1.3667	12.6462	2.4833	2.4049	6.8395E-03	2.06519	6.5004E-03	1.93985	3.4996E-03	9.398E-02	26.8559	65.9372
-0.01	2.9006	3.4636	11.8120	8.6587	4.9354	1.3642	12.6990	2.5041	2.3933	6.9136E-03	2.09258	6.8765E-03	2.07889	3.1235E-03	1.079E-02	3.4541	69.3912
-0.01	2.8906	3.4554	11.7626	8.6384	4.9386	1.3617	12.7523	2.5252	2.3818	6.9889E-03	2.12041	6.9512E-03	2.10649	3.0488E-03	1.106E-02	3.6293	73.0206
-0.01	2.8806	3.4473	11.7132	8.6182	4.9417	1.3591	12.8061	2.5465	2.3703	7.0655E-03	2.14871	7.0272E-03	2.13456	2.9728E-03	1.135E-02	3.8165	76.8370
-0.01	2.8706	3.4392	11.6638	8.5980	4.9448	1.3566	12.8603	2.5681	2.3588	7.1434E-03	2.17749	7.1044E-03	2.16310	2.8956E-03	1.163E-02	4.0168	80.8539
-0.01	2.8606	3.4311	11.6143	8.5777	4.9477	1.3540	12.9151	2.5901	2.3474	7.2226E-03	2.20674	7.1830E-03	2.19211	2.8170E-03	1.192E-02	4.2318	85.0857
-0.01	2.8506	3.4230	11.5648	8.5575	4.9506	1.3514	12.9704	2.6123	2.3360	7.3031E-03	2.23649	7.2629E-03	2.22162	2.7371E-03	1.222E-02	4.4631	89.5489
-0.01	2.8406	3.4149	11.5153	8.5373	4.9534	1.3488	13.0262	2.6348	2.3247	7.3851E-03	2.26675	7.3441E-03	2.25162	2.6559E-03	1.252E-02	4.7126	94.2615
-0.01	2.8306	3.4069	11.4657	8.5172	4.9561	1.3462	13.0825	2.6576	2.3135	7.4684E-03	2.29752	7.4267E-03	2.28213	2.5733E-03	1.282E-02	4.9825	99.2439
-0.001	2.8296	3.4061	11.4608	8.5151	4.9564	1.3459	13.0881	2.6599	2.3123	7.4768E-03	2.30062	7.4726E-03	2.29907	2.5274E-03	1.299E-03	0.5140	99.7579
-0.001	2.8286	3.4053	11.4558	8.5131	4.9566	1.3457	13.0938	2.6622	2.3112	7.4852E-03	2.30373	7.4810E-03	2.30218	2.5190E-03	1.302E-03	0.5169	100.2748
-0.001	2.8276	3.4044	11.4509	8.5111	4.9569	1.3454	13.0994	2.6645	2.3101	7.4936E-03	2.30685	7.4894E-03	2.30529	2.5106E-03	1.305E-03	0.5199	100.7948
-0.001	2.8266	3.4036	11.4459	8.5091	4.9571	1.3451	13.1051	2.6668	2.3090	7.5021E-03	2.30997	7.4979E-03	2.30841	2.5021E-03	1.308E-03	0.5229	101.3177
-0.001	2.8256	3.4028	11.4410	8.5071	4.9574	1.3449	13.1108	2.6691	2.3079	7.5105E-03	2.31310	7.5063E-03	2.31154	2.4937E-03	1.312E-03	0.5259	101.8436
-0.001	2.8246	3.4020	11.4360	8.5051	4.9577	1.3446	13.1165	2.6715	2.3067	7.5190E-03	2.31623	7.5148E-03	2.31467	2.4852E-03	1.315E-03	0.5290	102.3726

Table B-36 Case A 180 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$
	3.8423	4.2752	16.1909	10.6879	4.2182	1.5149	11.1174	1.9192	3.8384	4.6078E-03	1.00001						
-0.1	3.7423	4.1817	15.7629	10.4542	4.3390	1.5078	11.4192	2.0248	3.6328	4.8918E-03	1.11473	4.7498E-03	1.05737	5.2502E-03	5.737E-03	1.0927	1.0927
-0.1	3.6423	4.0906	15.3235	10.2266	4.4475	1.4984	11.7467	2.1426	3.4454	5.2198E-03	1.24376	5.0558E-03	1.17925	4.9442E-03	1.792E-02	3.6254	4.7181
-0.1	3.5423	4.0017	14.8738	10.0042	4.5447	1.4867	12.1018	2.2741	3.2728	5.5981E-03	1.38974	5.4090E-03	1.31675	4.5910E-03	3.168E-02	6.8994	11.6175
-0.1	3.4423	3.9145	14.4149	9.7863	4.6312	1.4730	12.4871	2.4212	3.1125	6.0347E-03	1.55580	5.8164E-03	1.47277	4.1836E-03	4.728E-02	11.3006	22.9181
-0.1	3.3423	3.8289	13.9479	9.5722	4.7077	1.4571	12.9052	2.5861	2.9628	6.5392E-03	1.74571	6.2870E-03	1.65076	3.7130E-03	6.508E-02	17.5262	40.4443
-0.1	3.2423	3.7445	13.4737	9.3613	4.7745	1.4393	13.3594	2.7713	2.8220	7.1236E-03	1.96409	6.8314E-03	1.85490	3.1686E-03	8.549E-02	26.9803	67.4246
-0.01	3.2323	3.7361	13.4259	9.3404	4.7807	1.4374	13.4069	2.7911	2.8084	7.1869E-03	1.98770	7.1552E-03	1.97589	2.8448E-03	9.759E-03	3.4305	70.8551
-0.01	3.2223	3.7278	13.3781	9.3195	4.7868	1.4355	13.4549	2.8111	2.7948	7.2513E-03	2.01165	7.2191E-03	1.99968	2.7809E-03	9.997E-03	3.5948	74.4499
-0.01	3.2123	3.7194	13.3302	9.2986	4.7928	1.4336	13.5032	2.8313	2.7813	7.3166E-03	2.03596	7.2839E-03	2.02381	2.7161E-03	1.024E-02	3.7694	78.2193
-0.01	3.2023	3.7111	13.2822	9.2777	4.7987	1.4316	13.5520	2.8518	2.7679	7.3829E-03	2.06063	7.3497E-03	2.04829	2.6503E-03	1.048E-02	3.9554	82.1747
-0.01	3.1923	3.7028	13.2342	9.2569	4.8045	1.4297	13.6011	2.8725	2.7546	7.4502E-03	2.08566	7.4165E-03	2.07314	2.5835E-03	1.073E-02	4.1538	86.3285
-0.01	3.1823	3.6944	13.1861	9.2361	4.8102	1.4277	13.6507	2.8935	2.7413	7.5185E-03	2.11106	7.4843E-03	2.09836	2.5157E-03	1.098E-02	4.3661	90.6946
-0.01	3.1723	3.6861	13.1380	9.2153	4.8158	1.4257	13.7007	2.9148	2.7281	7.5879E-03	2.13685	7.5532E-03	2.12396	2.4468E-03	1.124E-02	4.5936	95.2882
-0.001	3.1713	3.6853	13.1332	9.2132	4.8164	1.4255	13.7058	2.9169	2.7268	7.5949E-03	2.13945	7.5914E-03	2.13815	2.4086E-03	1.138E-03	0.4725	95.7607
-0.001	3.1703	3.6845	13.1283	9.2112	4.8169	1.4253	13.7108	2.9190	2.7255	7.6019E-03	2.14205	7.5984E-03	2.14075	2.4016E-03	1.141E-03	0.4750	96.2357
-0.001	3.1693	3.6836	13.1235	9.2091	4.8175	1.4251	13.7158	2.9212	2.7241	7.6089E-03	2.14466	7.6054E-03	2.14335	2.3946E-03	1.143E-03	0.4775	96.7132
-0.001	3.1683	3.6828	13.1187	9.2070	4.8180	1.4249	13.7209	2.9233	2.7228	7.6159E-03	2.14727	7.6124E-03	2.14596	2.3876E-03	1.146E-03	0.4800	97.1931
-0.001	3.1673	3.6820	13.1139	9.2049	4.8186	1.4247	13.7259	2.9255	2.7215	7.6230E-03	2.14988	7.6195E-03	2.14858	2.3805E-03	1.149E-03	0.4825	97.6756
-0.001	3.1663	3.6811	13.1091	9.2029	4.8192	1.4245	13.7309	2.9276	2.7202	7.6300E-03	2.15250	7.6265E-03	2.15119	2.3735E-03	1.151E-03	0.4850	98.1606
-0.001	3.1653	3.6803	13.1043	9.2008	4.8197	1.4243	13.7360	2.9298	2.7189	7.6371E-03	2.15513	7.6336E-03	2.15381	2.3664E-03	1.154E-03	0.4876	98.6482
-0.001	3.1643	3.6795	13.0994	9.1987	4.8203	1.4240	13.7411	2.9319	2.7176	7.6442E-03	2.15775	7.6406E-03	2.15644	2.3594E-03	1.156E-03	0.4901	99.1384
-0.001	3.1633	3.6787	13.0946	9.1966	4.8208	1.4238	13.7461	2.9341	2.7163	7.6512E-03	2.16038	7.6477E-03	2.15907	2.3523E-03	1.159E-03	0.4927	99.6311
-0.001	3.1623	3.6778	13.0898	9.1946	4.8214	1.4236	13.7512	2.9363	2.7150	7.6583E-03	2.16302	7.6548E-03	2.16170	2.3452E-03	1.162E-03	0.4954	100.1264
-0.001	3.1613	3.6770	13.0850	9.1925	4.8219	1.4234	13.7562	2.9384	2.7137	7.6654E-03	2.16566	7.6619E-03	2.16434	2.3381E-03	1.164E-03	0.4980	100.6244

Table B-37 Case A 210 cfs: Manually Computed Outlet Depths

Δy	y	ϕ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$
	4.1253	4.5571	17.3283	11.3927	3.7991	1.5210	12.1189	2.2806	4.5611	5.4460E-03	1.0000						
-0.1	4.0253	4.4540	16.9401	11.1351	3.9615	1.5213	12.3966	2.3863	4.2762	5.6968E-03	1.1161	5.5714E-03	1.0580	4.4286E-03	5.804E-03	1.3106	1.310585
-0.1	3.9253	4.3549	16.5365	10.8873	4.1078	1.5189	12.6992	2.5042	4.0257	5.9911E-03	1.2441	5.8439E-03	1.1801	4.1561E-03	1.801E-02	4.3333	5.643882
-0.1	3.8253	4.2591	16.1190	10.6477	4.2396	1.5138	13.0281	2.6356	3.8020	6.3334E-03	1.3864	6.1623E-03	1.3153	3.8377E-03	3.153E-02	8.2146	13.8585
-0.1	3.7253	4.1661	15.6890	10.4151	4.3583	1.5064	13.3851	2.7820	3.5998	6.7297E-03	1.5456	6.5316E-03	1.4660	3.4684E-03	4.660E-02	13.4359	27.29443
-0.1	3.6253	4.0754	15.2478	10.1885	4.4648	1.4966	13.7725	2.9454	3.4151	7.1871E-03	1.7249	6.9584E-03	1.6353	3.0416E-03	6.353E-02	20.8859	48.18032
-0.1	3.5253	3.9868	14.7965	9.9669	4.5601	1.4846	14.1926	3.1278	3.2447	7.7147E-03	1.9279	7.4509E-03	1.8264	2.5491E-03	8.264E-02	32.4196	80.59991
-0.01	3.5153	3.9780	14.7508	9.9450	4.5691	1.4832	14.2365	3.1472	3.2284	7.7717E-03	1.9497	7.7432E-03	1.9388	2.2568E-03	9.388E-03	4.1599	84.75979
-0.01	3.5053	3.9693	14.7051	9.9232	4.5779	1.4819	14.2808	3.1668	3.2122	7.8296E-03	1.9717	7.8006E-03	1.9607	2.1994E-03	9.607E-03	4.3682	89.12794
-0.01	3.4953	3.9605	14.6593	9.9013	4.5867	1.4805	14.3254	3.1866	3.1961	7.8883E-03	1.9941	7.8589E-03	1.9829	2.1411E-03	9.829E-03	4.5807	93.71866
-0.01	3.4853	3.9518	14.6133	9.8796	4.5953	1.4791	14.3704	3.2067	3.1801	7.9478E-03	2.0167	7.9180E-03	2.0054	2.0820E-03	1.005E-02	4.8291	98.54775
-0.001	3.4843	3.9510	14.6087	9.8774	4.5961	1.4790	14.3749	3.2087	3.1785	7.9538E-03	2.0190	7.9508E-03	2.0179	2.0492E-03	1.018E-03	0.4967	99.04446
-0.001	3.4833	3.9501	14.6042	9.8752	4.5970	1.4789	14.3795	3.2107	3.1769	7.9598E-03	2.0213	7.9568E-03	2.0201	2.0432E-03	1.020E-03	0.4993	99.54375
-0.001	3.4823	3.9492	14.5996	9.8730	4.5979	1.4787	14.3840	3.2127	3.1753	7.9658E-03	2.0236	7.9628E-03	2.0224	2.0372E-03	1.022E-03	0.5019	100.0456
-0.001	3.4813	3.9483	14.5950	9.8709	4.5987	1.4786	14.3885	3.2148	3.1737	7.9719E-03	2.0259	7.9688E-03	2.0247	2.0312E-03	1.025E-03	0.5045	100.5501

Table B-38 Case A 240 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX
	4.3571	4.8162	18.1588	12.0405	3.3473	1.5081	13.2167	2.7124	5.4249	6.5511E-03	1.0000						
-0.1	4.2571	4.7004	17.8134	11.7509	3.5567	1.5159	13.4730	2.8187	5.0084	6.7611E-03	1.1256	6.6561E-03	1.0628	3.3439E-03	6.279E-03	1.87762	1.8776
-0.1	4.1571	4.5908	17.4482	11.4770	3.7438	1.5203	13.7550	2.9379	4.6606	7.0202E-03	1.2607	6.8906E-03	1.1932	3.1094E-03	1.932E-02	6.21195	8.0896
-0.1	4.0571	4.4863	17.0653	11.2158	3.9117	1.5215	14.0636	3.0712	4.3626	7.3306E-03	1.4080	7.1754E-03	1.3344	2.8246E-03	3.344E-02	11.83709	19.9267
-0.1	3.9571	4.3860	16.6664	10.9651	4.0629	1.5200	14.4002	3.2200	4.1021	7.6964E-03	1.5699	7.5135E-03	1.4889	2.4865E-03	4.889E-02	19.66386	39.5905
-0.1	3.8571	4.2892	16.2532	10.7231	4.1992	1.5157	14.7663	3.3858	3.8706	8.1228E-03	1.7495	7.9096E-03	1.6597	2.0904E-03	6.597E-02	31.55863	71.1492
-0.01	3.8471	4.2797	16.2112	10.6993	4.2120	1.5152	14.8046	3.4034	3.8488	8.1691E-03	1.7685	8.1459E-03	1.7590	1.8541E-03	7.590E-03	4.09381	75.2430
-0.01	3.8371	4.2702	16.1690	10.6756	4.2248	1.5146	14.8432	3.4211	3.8272	8.2160E-03	1.7878	8.1925E-03	1.7782	1.8075E-03	7.782E-03	4.30530	79.5483
-0.01	3.8271	4.2608	16.1267	10.6520	4.2373	1.5140	14.8822	3.4391	3.8058	8.2636E-03	1.8073	8.2398E-03	1.7975	1.7602E-03	7.975E-03	4.53098	84.0792
-0.01	3.8171	4.2514	16.0842	10.6284	4.2498	1.5133	14.9214	3.4573	3.7847	8.3119E-03	1.8270	8.2878E-03	1.8171	1.7122E-03	8.171E-03	4.77232	88.8516
-0.01	3.8071	4.2420	16.0417	10.6049	4.2621	1.5127	14.9610	3.4757	3.7638	8.3609E-03	1.8469	8.3364E-03	1.8369	1.6636E-03	8.369E-03	5.03104	93.8826
-0.01	3.7971	4.2326	15.9990	10.5815	4.2743	1.5120	15.0009	3.4942	3.7430	8.4107E-03	1.8671	8.3858E-03	1.8570	1.6142E-03	8.570E-03	5.30910	99.1917
-0.001	3.7961	4.2317	15.9947	10.5791	4.2756	1.5119	15.0050	3.4961	3.7410	8.4157E-03	1.8691	8.4132E-03	1.8681	1.5868E-03	8.681E-04	0.54706	99.7388
-0.0001	3.7960	4.2316	15.9943	10.5789	4.2757	1.5119	15.0054	3.4963	3.7408	8.4162E-03	1.8693	8.4160E-03	1.8692	1.5840E-03	8.692E-05	0.05487	99.7936
-0.0001	3.7959	4.2315	15.9939	10.5787	4.2758	1.5119	15.0058	3.4965	3.7406	8.4167E-03	1.8695	8.4165E-03	1.8694	1.5835E-03	8.694E-05	0.05490	99.8485
-0.0001	3.7958	4.2314	15.9934	10.5784	4.2759	1.5119	15.0062	3.4967	3.7403	8.4172E-03	1.8697	8.4170E-03	1.8696	1.5830E-03	8.696E-05	0.05493	99.9035
-0.0001	3.7957	4.2313	15.9930	10.5782	4.2760	1.5119	15.0066	3.4968	3.7401	8.4177E-03	1.8699	8.4175E-03	1.8698	1.5825E-03	8.698E-05	0.05496	99.9584
-0.0001	3.7956	4.2312	15.9926	10.5780	4.2762	1.5119	15.0070	3.4970	3.7399	8.4182E-03	1.8701	8.4180E-03	1.8700	1.5820E-03	8.700E-05	0.05499	100.0134
-0.0001	3.7955	4.2311	15.9922	10.5777	4.2763	1.5119	15.0074	3.4972	3.7397	8.4187E-03	1.8703	8.4185E-03	1.8702	1.5815E-03	8.702E-05	0.05502	100.0684

Table B-39 Case A 270 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX
	4.5370	5.0464	18.7222	12.6159	2.8987	1.4840	14.4214	3.2294	6.4589	7.9692E-03	1.00000						
-0.1	4.4370	4.9144	18.4190	12.2860	3.1610	1.4992	14.6588	3.3367	5.8269	8.1230E-03	1.14526	8.0461E-03	1.07263	1.9539E-03	7.263E-03	3.71712	3.7171
-0.1	4.3370	4.7923	18.0911	11.9809	3.3914	1.5100	14.9245	3.4587	5.3344	8.3397E-03	1.29675	8.2314E-03	1.22100	1.7686E-03	2.210E-02	12.49563	16.2128
-0.1	4.2370	4.6779	17.7415	11.6947	3.5960	1.5171	15.2185	3.6963	4.9337	8.6179E-03	1.45787	8.4788E-03	1.37731	1.5212E-03	3.773E-02	24.80337	41.0161
-0.1	4.1370	4.5694	17.3726	11.4236	3.7790	1.5208	15.5417	3.7507	4.5971	8.9585E-03	1.63175	8.7882E-03	1.54481	1.2118E-03	5.448E-02	44.95862	85.9747
-0.01	4.1270	4.5589	17.3347	11.3972	3.7962	1.5210	15.5757	3.7671	4.5663	8.9962E-03	1.64996	8.9773E-03	1.64085	1.0227E-03	6.409E-03	6.26655	92.2413
-0.001	4.1260	4.5578	17.3309	11.3946	3.7979	1.5210	15.5791	3.7688	4.5632	9.0000E-03	1.65179	8.9981E-03	1.65087	1.0019E-03	6.509E-04	0.64961	92.8909
-0.001	4.1250	4.5568	17.3271	11.3919	3.7997	1.5210	15.5825	3.7704	4.5602	9.0036E-03	1.65362	9.0019E-03	1.65270	9.9814E-04	6.527E-04	0.65392	93.5448
-0.001	4.1240	4.5557	17.3233	11.3893	3.8014	1.5210	15.5859	3.7721	4.5571	9.0076E-03	1.65545	9.0057E-03	1.65453	9.9434E-04	6.545E-04	0.65826	94.2031
-0.001	4.1230	4.5547	17.3195	11.3867	3.8031	1.5210	15.5893	3.7737	4.5541	9.0114E-03	1.65729	9.0095E-03	1.65637	9.9052E-04	6.564E-04	0.66265	94.8657
-0.001	4.1220	4.5536	17.3157	11.3840	3.8048	1.5211	15.5928	3.7754	4.5510	9.0152E-03	1.65912	9.0133E-03	1.65820	9.8670E-04	6.582E-04	0.66708	95.5328
-0.001	4.1210	4.5526	17.3119	11.3814	3.8065	1.5211	15.5962	3.7770	4.5480	9.0190E-03	1.66096	9.0171E-03	1.66004	9.8287E-04	6.600E-04	0.67154	96.2043
-0.001	4.1200	4.5515	17.3081	11.3788	3.8082	1.5211	15.5996	3.7787	4.5450	9.0229E-03	1.66280	9.0210E-03	1.66188	9.7904E-04	6.619E-04	0.67605	96.8804
-0.001	4.1190	4.5505	17.3043	11.3762	3.8099	1.5211	15.6030	3.7804	4.5419	9.0267E-03	1.66464	9.0248E-03	1.66372	9.7520E-04	6.637E-04	0.68060	97.5610
-0.001	4.1180	4.5494	17.3005	11.3735	3.8116	1.5211	15.6065	3.7820	4.5389	9.0306E-03	1.66648	9.0287E-03	1.66556	9.7135E-04	6.656E-04	0.68519	98.2462
-0.001	4.1170	4.5484	17.2967	11.3709	3.8133	1.5211	15.6099	3.7837	4.5359	9.0344E-03	1.66833	9.0325E-03	1.66741	9.6750E-04	6.674E-04	0.68983	98.9360
-0.001	4.1160	4.5473	17.2929	11.3683	3.8150	1.5212	15.6134	3.7854	4.5329	9.0383E-03	1.67018	9.0364E-03	1.66925	9.6363E-04	6.693E-04	0.69451	99.6305
-0.0001	4.1159	4.5472	17.2925	11.3680	3.8151	1.5212	15.6137	3.7855	4.5326	9.0387E-03	1.67036	9.0385E-03	1.67027	9.6151E-04	6.703E-05	0.06971	99.7002
-0.0001	4.1158	4.5471	17.2921	11.3678	3.8153	1.5212	15.6141	3.7857	4.5323	9.0391E-03	1.67055	9.0389E-03	1.67045	9.6112E-04	6.705E-05	0.06976	99.7700
-0.0001	4.1157	4.5470	17.2917	11.3675	3.8155	1.5212	15.6144	3.7859	4.5320	9.0395E-03	1.67073	9.0393E-03	1.67064	9.6073E-04	6.706E-05	0.06980	99.8398
-0.0001	4.1156	4.5469	17.2914	11.3672	3.8157	1.5212	15.6147	3.7860	4.5317	9.0398E-03	1.67091	9.0397E-03	1.67082	9.6035E-04	6.708E-05	0.06985	99.9097
-0.0001	4.1155	4.5468	17.2910	11.3670	3.8158	1.5212	15.6151	3.7862	4.5314	9.0402E-03	1.67110	9.0400E-03	1.67101	9.5996E-04	6.710E-05	0.06990	99.9796
-0.0001	4.1154	4.5467	17.2906	11.3667	3.8160	1.5212	15.6154	3.7864	4.5311	9.0406E-03	1.67128	9.0404E-03	1.67119	9.5957E-04	6.712E-05	0.06995	100.0495
-0.0001	4.1153	4.5466	17.2902	11.3665	3.8162	1.5212	15.6158	3.7865	4.5308	9.0410E-03	1.67147	9.0408E-03	1.67138	9.5919E-04	6.714E-05	0.06999	100.1195

Table B-40 Case A 300 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$
	4.6694	5.2429	19.0796	13.1073	2.4850	1.4556	15.7236	3.8390	7.6780	9.7205E-03	1.00000						
-0.1	4.5694	5.0918	18.8146	12.7295	2.8054	1.4780	15.9451	3.9479	6.7065	9.7948E-03	1.17734	9.7577E-03	1.08867	2.4234E-04	8.867E-03	36.5893	36.5893
-0.01	4.5594	5.0776	18.7864	12.6940	2.8347	1.4799	15.9690	3.9598	6.6272	9.8074E-03	1.19500	9.8011E-03	1.18617	1.9892E-04	1.862E-03	9.3590	45.9482
-0.01	4.5494	5.0636	18.7579	12.6589	2.8636	1.4818	15.9933	3.9718	6.5505	9.8208E-03	1.21267	9.8141E-03	1.20383	1.8594E-04	2.038E-03	10.9625	56.9107
-0.01	4.5394	5.0497	18.7291	12.6242	2.8920	1.4836	16.0178	3.9840	6.4762	9.8351E-03	1.23035	9.8279E-03	1.22151	1.7209E-04	2.215E-03	12.8720	69.7828
-0.01	4.5294	5.0359	18.7001	12.5897	2.9200	1.4853	16.0427	3.9964	6.4042	9.8502E-03	1.24807	9.8426E-03	1.23921	1.5738E-04	2.392E-03	15.1991	84.9818
-0.001	4.5284	5.0345	18.6971	12.5863	2.9228	1.4855	16.0452	3.9977	6.3971	9.8517E-03	1.24984	9.8510E-03	1.24895	1.4904E-04	2.490E-04	1.6703	86.6522
-0.001	4.5274	5.0332	18.6942	12.5829	2.9255	1.4857	16.0477	3.9989	6.3900	9.8533E-03	1.25161	9.8525E-03	1.25073	1.4748E-04	2.507E-04	1.7001	88.3523
-0.001	4.5264	5.0318	18.6913	12.5795	2.9283	1.4859	16.0503	4.0002	6.3830	9.8549E-03	1.25339	9.8541E-03	1.25250	1.4591E-04	2.525E-04	1.7305	90.0828
-0.001	4.5254	5.0304	18.6884	12.5761	2.9311	1.4860	16.0528	4.0014	6.3759	9.8565E-03	1.25516	9.8557E-03	1.25427	1.4433E-04	2.543E-04	1.7618	91.8445
-0.001	4.5244	5.0291	18.6854	12.5727	2.9338	1.4862	16.0553	4.0027	6.3689	9.8581E-03	1.25694	9.8573E-03	1.25605	1.4274E-04	2.560E-04	1.7938	93.6383
-0.001	4.5234	5.0277	18.6825	12.5693	2.9366	1.4864	16.0578	4.0039	6.3620	9.8597E-03	1.25871	9.8589E-03	1.25782	1.4115E-04	2.578E-04	1.8266	95.4650
-0.001	4.5224	5.0263	18.6796	12.5658	2.9394	1.4865	16.0603	4.0052	6.3550	9.8613E-03	1.26049	9.8605E-03	1.25960	1.3954E-04	2.596E-04	1.8604	97.3253
-0.001	4.5214	5.0250	18.6766	12.5624	2.9421	1.4867	16.0629	4.0065	6.3481	9.8629E-03	1.26226	9.8621E-03	1.26137	1.3793E-04	2.614E-04	1.8950	99.2203
-0.0001	4.5213	5.0248	18.6763	12.5621	2.9424	1.4867	16.0631	4.0066	6.3474	9.8630E-03	1.26244	9.8630E-03	1.26235	1.3704E-04	2.623E-05	0.1914	99.4117
-0.0001	4.5212	5.0247	18.6760	12.5618	2.9427	1.4867	16.0634	4.0067	6.3467	9.8632E-03	1.26262	9.8631E-03	1.26253	1.3688E-04	2.625E-05	0.1918	99.6035
-0.0001	4.5211	5.0246	18.6757	12.5614	2.9429	1.4868	16.0636	4.0068	6.3460	9.8634E-03	1.26279	9.8633E-03	1.26271	1.3672E-04	2.627E-05	0.1922	99.7957
-0.0001	4.5210	5.0244	18.6754	12.5611	2.9432	1.4868	16.0639	4.0070	6.3453	9.8635E-03	1.26297	9.8634E-03	1.26288	1.3655E-04	2.629E-05	0.1925	99.9882
-0.0001	4.5209	5.0243	18.6751	12.5607	2.9435	1.4868	16.0641	4.0071	6.3446	9.8637E-03	1.26315	9.8636E-03	1.26306	1.3639E-04	2.631E-05	0.1929	100.1811
-0.0001	4.5208	5.0242	18.6749	12.5604	2.9437	1.4868	16.0644	4.0072	6.3439	9.8639E-03	1.26333	9.8638E-03	1.26324	1.3623E-04	2.632E-05	0.1932	100.3743
-0.0001	4.5207	5.0240	18.6746	12.5601	2.9440	1.4868	16.0646	4.0073	6.3432	9.8640E-03	1.26350	9.8639E-03	1.26342	1.3607E-04	2.634E-05	0.1936	100.5679

Table B-41 Case A Outlet Velocity Computations

Q	y_n	y_c	Tailwater	Slope	Boundary	y_{outlet}	Angle	Area	V_{outlet}
cfs	ft	ft	ft	---	---	ft	rad	ft^2	ft/s
30	1.10	1.52	0.00	Steep	Normal	1.14	1.99	3.36	**8.93**
60	1.56	2.18	0.00	Steep	Normal	1.66	2.46	5.71	**10.52**
90	1.94	2.69	0.00	Steep	Normal	2.09	2.81	7.79	**11.56**
120	2.27	3.13	0.00	Steep	Normal	2.48	3.12	9.69	**12.38**
150	2.59	3.51	0.00	Steep	Normal	2.83	3.40	11.45	**13.10**
180	2.90	3.84	0.00	Steep	Normal	3.16	3.68	13.09	**13.76**
210	3.21	4.13	0.00	Steep	Normal	3.48	3.95	14.59	**14.39**
240	3.54	4.36	0.00	Steep	Normal	3.80	4.23	15.99	**15.01**
270	3.91	4.54	0.00	Steep	Normal	4.11	4.54	17.28	**15.62**
300	4.45	4.67	0.00	Steep	Normal	4.52	5.02	18.68	**16.06**

In case B, the hydraulic slope was determined to be mild, and with a tailwater depth of 0.0 feet, the depth at the outlet was assumed to be critical. Table B-42 shows the results of the outlet velocity computations for case B.

Table B-42 Case B Outlet Velocity Computations

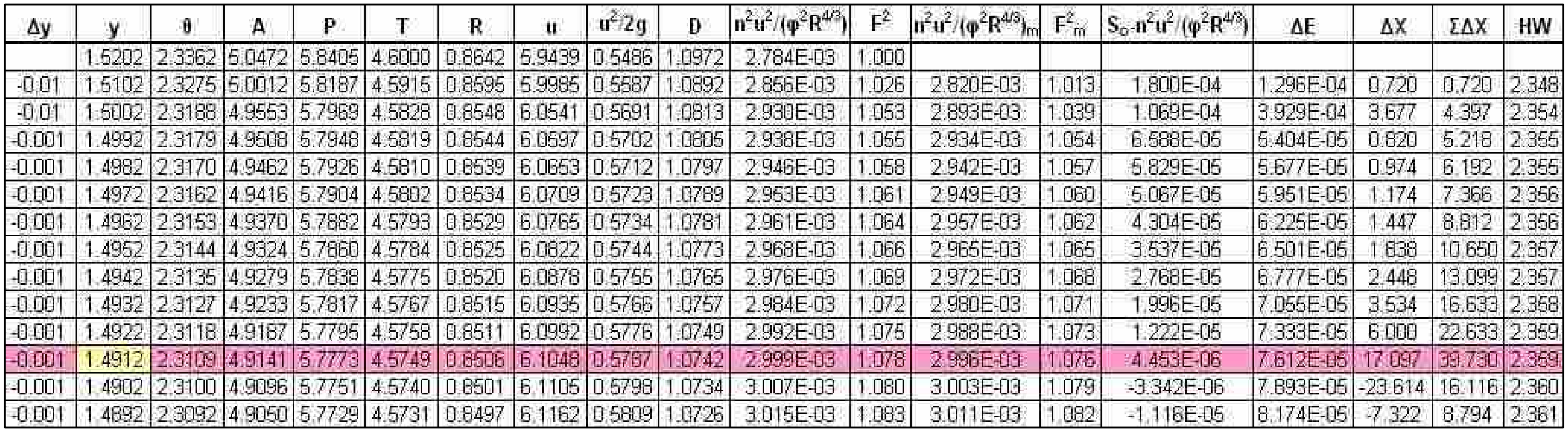
Q	y_n	y_c	Tailwater	Slope	Boundary	y_{outlet}	Angle	Area	V_{outlet}
cfs	ft	ft	ft	---	---	ft	rad	ft^2	ft/s
10	0.95	0.87	0.00	Mild	Critical	0.87	1.72	2.27	**4.40**
20	1.34	1.23	0.00	Mild	Critical	1.23	2.08	3.77	**5.31**
30	1.66	1.52	0.00	Mild	Critical	1.52	2.34	5.05	**5.94**
40	1.93	1.76	0.00	Mild	Critical	1.76	2.54	6.19	**6.46**
50	2.18	1.98	0.00	Mild	Critical	1.98	2.72	7.24	**6.90**
60	2.42	2.18	0.00	Mild	Critical	2.18	2.88	8.21	**7.30**
70	2.66	2.36	0.00	Mild	Critical	2.36	3.03	9.12	**7.67**
80	2.89	2.53	0.00	Mild	Critical	2.53	3.17	9.98	**8.02**
90	3.12	2.69	0.00	Mild	Critical	2.69	3.30	10.78	**8.35**
100	3.35	2.85	0.00	Mild	Critical	2.85	3.42	11.54	**8.66**

Forewater calculations (Tables B-43 through B-46) were required for case C when normal depth was less than critical (steep slope). Table B-47 shows the results of the outlet velocity computations for case C.

Table B-43 Case C 15 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	1.0649	1.9188	3.0585	4.7969	4.0942	0.6376	4.9044	0.3735	0.7470	2.8430E-03	0.9999							
-0.001	1.0639	1.9178	3.0544	4.7945	4.0928	0.6371	4.9110	0.3745	0.7463	2.8537E-03	1.0036	2.8484E-03	1.0018	1.5164E-04	1.790E-06	0.0118	0.0118	1.6257
-0.001	1.0629	1.9168	3.0503	4.7921	4.0914	0.6365	4.9176	0.3755	0.7455	2.8646E-03	1.0073	2.8592E-03	1.0055	1.4084E-04	5.480E-06	0.0389	0.0507	1.6262
-0.001	1.0619	1.9158	3.0462	4.7896	4.0900	0.6360	4.9242	0.3765	0.7448	2.8755E-03	1.0110	2.8700E-03	1.0092	1.2998E-04	9.188E-06	0.0707	0.1214	1.6267
-0.001	1.0609	1.9149	3.0421	4.7872	4.0886	0.6355	4.9308	0.3775	0.7441	2.8864E-03	1.0148	2.8809E-03	1.0129	1.1906E-04	1.291E-05	0.1084	0.2299	1.6272
-0.001	1.0599	1.9139	3.0380	4.7847	4.0872	0.6349	4.9374	0.3785	0.7433	2.8974E-03	1.0185	2.8919E-03	1.0167	1.0810E-04	1.666E-05	0.1541	0.3839	1.6278
-0.001	1.0589	1.9129	3.0339	4.7823	4.0858	0.6344	4.9441	0.3796	0.7426	2.9084E-03	1.0223	2.9029E-03	1.0204	9.7082E-05	2.041E-05	0.2103	0.5942	1.6283
-0.001	1.0579	1.9119	3.0299	4.7798	4.0844	0.6339	4.9507	0.3806	0.7418	2.9195E-03	1.0261	2.9140E-03	1.0242	8.6011E-05	2.419E-05	0.2813	0.8755	1.6288
-0.001	1.0569	1.9110	3.0258	4.7774	4.0829	0.6334	4.9574	0.3816	0.7411	2.9307E-03	1.0299	2.9251E-03	1.0280	7.4886E-05	2.799E-05	0.3737	1.2492	1.6294
-0.001	1.0559	1.9100	3.0217	4.7749	4.0815	0.6328	4.9641	0.3826	0.7403	2.9419E-03	1.0337	2.9363E-03	1.0318	6.3707E-05	3.180E-05	0.4991	1.7483	1.6299
-0.001	1.0549	1.9090	3.0176	4.7725	4.0801	0.6323	4.9708	0.3837	0.7396	2.9532E-03	1.0375	2.9475E-03	1.0356	5.2474E-05	3.563E-05	0.6790	2.4273	1.6305
-0.001	1.0539	1.9080	3.0135	4.7700	4.0787	0.6318	4.9775	0.3847	0.7388	2.9645E-03	1.0414	2.9588E-03	1.0395	4.1186E-05	3.948E-05	0.9585	3.3858	1.6310
-0.001	1.0529	1.9070	3.0095	4.7676	4.0773	0.6312	4.9843	0.3858	0.7381	2.9758E-03	1.0453	2.9702E-03	1.0433	2.9843E-05	4.334E-05	1.4524	4.8382	1.6316
-0.001	1.0519	1.9060	3.0054	4.7651	4.0759	0.6307	4.9911	0.3868	0.7374	2.9873E-03	1.0492	2.9816E-03	1.0472	1.8445E-05	4.723E-05	2.5605	7.3987	1.6322
-0.001	1.0509	1.9051	3.0013	4.7627	4.0744	0.6302	4.9978	0.3879	0.7366	2.9987E-03	1.0531	2.9930E-03	1.0511	6.9916E-06	5.113E-05	7.3133	14.7120	1.6327
-0.0001	1.0508	1.9050	3.0009	4.7624	4.0743	0.6301	4.9985	0.3880	0.7365	2.9999E-03	1.0535	2.9993E-03	1.0533	6.7526E-07	5.328E-06	7.8908	22.6028	1.6328
-0.0001	1.0507	1.9049	3.0005	4.7622	4.0741	0.6301	4.9992	0.3881	0.7365	3.0011E-03	1.0539	3.0005E-03	1.0537	-4.7625E-07	5.366E-06	-11.2706	11.3322	1.6329

Table B-44 Case C 30 cfs: Manually Computed Outlet Depths

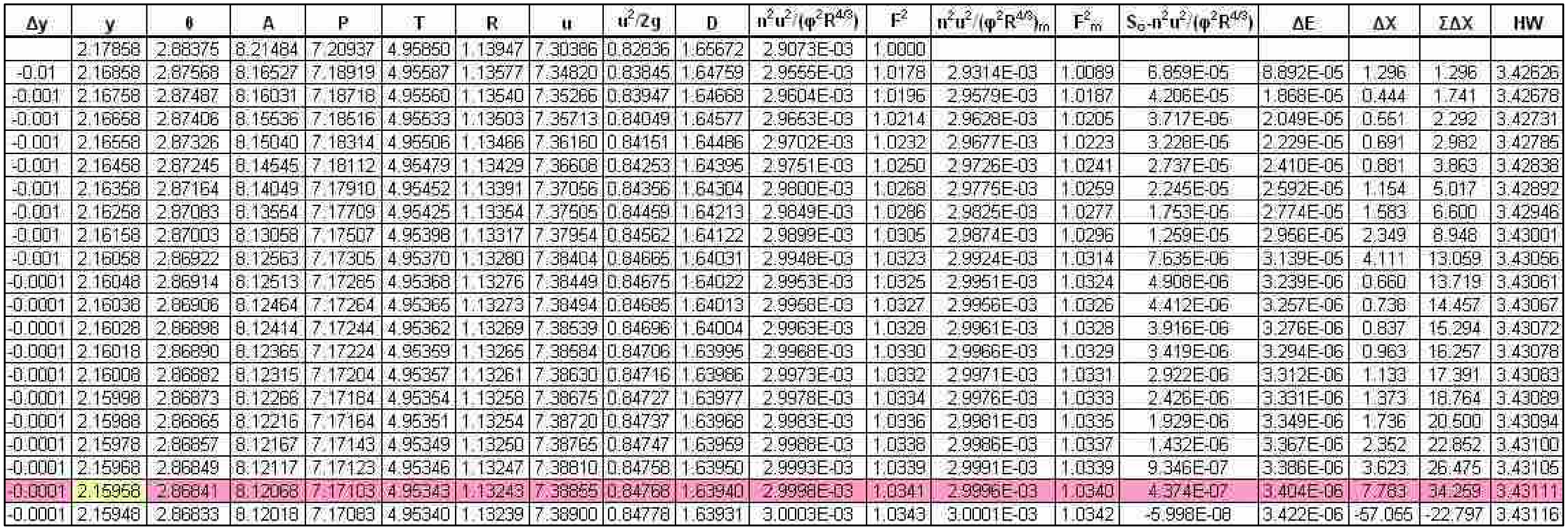


Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	1.5202	2.3362	5.0472	5.8405	4.6000	0.8642	5.9439	0.5486	1.0972	2.784E-03	1.000							
-0.01	1.5102	2.3275	5.0012	5.8187	4.5915	0.8595	5.9985	0.5587	1.0892	2.856E-03	1.026	2.820E-03	1.013	1.800E-04	1.296E-04	0.720	0.720	2.348
-0.01	1.5002	2.3188	4.9553	5.7969	4.5828	0.8548	6.0541	0.5691	1.0813	2.930E-03	1.053	2.893E-03	1.039	1.069E-04	3.929E-04	3.677	4.397	2.354
-0.001	1.4992	2.3179	4.9508	5.7948	4.5819	0.8544	6.0597	0.5702	1.0805	2.938E-03	1.055	2.934E-03	1.054	6.588E-05	5.404E-05	0.820	5.218	2.355
-0.001	1.4982	2.3170	4.9462	5.7926	4.5810	0.8539	6.0653	0.5712	1.0797	2.946E-03	1.058	2.942E-03	1.057	5.829E-05	5.677E-05	0.974	6.192	2.355
-0.001	1.4972	2.3162	4.9416	5.7904	4.5802	0.8534	6.0709	0.5723	1.0789	2.953E-03	1.061	2.949E-03	1.060	5.067E-05	5.951E-05	1.174	7.366	2.356
-0.001	1.4962	2.3153	4.9370	5.7882	4.5793	0.8529	6.0765	0.5734	1.0781	2.961E-03	1.064	2.957E-03	1.062	4.304E-05	6.225E-05	1.447	8.812	2.356
-0.001	1.4952	2.3144	4.9324	5.7860	4.5784	0.8525	6.0822	0.5744	1.0773	2.968E-03	1.066	2.965E-03	1.065	3.537E-05	6.501E-05	1.838	10.650	2.357
-0.001	1.4942	2.3135	4.9279	5.7838	4.5775	0.8520	6.0878	0.5755	1.0765	2.976E-03	1.069	2.972E-03	1.068	2.768E-05	6.777E-05	2.448	13.099	2.357
-0.001	1.4932	2.3127	4.9233	5.7817	4.5767	0.8515	6.0935	0.5766	1.0757	2.984E-03	1.072	2.980E-03	1.071	1.996E-05	7.055E-05	3.534	16.633	2.358
-0.001	1.4922	2.3118	4.9187	5.7795	4.5758	0.8511	6.0992	0.5776	1.0749	2.992E-03	1.075	2.988E-03	1.073	1.222E-05	7.333E-05	6.000	22.633	2.359
-0.001	1.4912	2.3109	4.9141	5.7773	4.5749	0.8506	6.1048	0.5787	1.0742	2.999E-03	1.078	2.996E-03	1.076	4.453E-06	7.612E-05	17.097	39.730	2.359
-0.001	1.4902	2.3100	4.9096	5.7751	4.5740	0.8501	6.1105	0.5798	1.0734	3.007E-03	1.080	3.003E-03	1.079	-3.342E-06	7.893E-05	-23.614	16.116	2.360
-0.001	1.4892	2.3092	4.9050	5.7729	4.5731	0.8497	6.1162	0.5809	1.0726	3.015E-03	1.083	3.011E-03	1.082	-1.116E-05	8.174E-05	-7.322	8.794	2.361

Table B-45 Case C 45 cfs: Manually Computed Outlet Depths

Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o\text{-}n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	$\Sigma\Delta X$	HW
	1.87542	2.63658	6.72740	6.59146	4.84145	1.02062	6.68906	0.69478	1.38954	2.8242E-03	1.0000							
-0.01	1.86542	2.62832	6.67901	6.57079	4.83624	1.01647	6.73753	0.70488	1.38103	2.8809E-03	1.0208	2.8526E-03	1.0104	1.4742E-04	1.040E-04	0.706	0.706	2.9227
-0.01	1.85542	2.62004	6.63067	6.55010	4.83095	1.01230	6.78664	0.71519	1.37254	2.9391E-03	1.0421	2.9100E-03	1.0315	8.9968E-05	3.147E-04	3.498	4.204	2.9282
-0.001	1.85442	2.61921	6.62584	6.54803	4.83042	1.01188	6.79159	0.71624	1.37169	2.9450E-03	1.0443	2.9421E-03	1.0432	5.7906E-05	4.323E-05	0.747	4.951	2.9288
-0.001	1.85342	2.61839	6.62101	6.54596	4.82988	1.01147	6.79654	0.71728	1.37084	2.9510E-03	1.0465	2.9480E-03	1.0454	5.1990E-05	4.540E-06	0.873	5.824	2.9293
-0.001	1.85242	2.61756	6.61618	6.54389	4.82935	1.01105	6.80150	0.71833	1.37000	2.9569E-03	1.0487	2.9539E-03	1.0476	4.6059E-05	4.757E-05	1.033	6.857	2.9299
-0.001	1.85142	2.61673	6.61135	6.54182	4.82881	1.01063	6.80647	0.71938	1.36915	2.9629E-03	1.0508	2.9599E-03	1.0498	4.0112E-05	4.975E-05	1.240	8.097	2.9305
-0.001	1.85042	2.61590	6.60653	6.53975	4.82827	1.01021	6.81145	0.72043	1.36830	2.9688E-03	1.0530	2.9659E-03	1.0519	3.4149E-05	5.194E-05	1.521	9.618	2.9311
-0.001	1.84942	2.61507	6.60170	6.53768	4.82773	1.00979	6.81643	0.72149	1.36745	2.9748E-03	1.0552	2.9718E-03	1.0541	2.8171E-05	5.413E-05	1.921	11.539	2.9317
-0.001	1.84842	2.61424	6.59687	6.53561	4.82719	1.00937	6.82142	0.72254	1.36661	2.9808E-03	1.0574	2.9778E-03	1.0563	2.2176E-05	5.633E-05	2.540	14.079	2.9322
-0.001	1.84742	2.61341	6.59204	6.53354	4.82665	1.00895	6.82641	0.72360	1.36576	2.9868E-03	1.0596	2.9838E-03	1.0585	1.6165E-05	5.853E-05	3.621	17.700	2.9328
-0.001	1.84642	2.61259	6.58722	6.53146	4.82611	1.00854	6.83141	0.72466	1.36491	2.9929E-03	1.0618	2.9899E-03	1.0607	1.0138E-05	6.074E-05	5.991	23.691	2.9334
-0.001	1.84542	2.61176	6.58239	6.52939	4.82557	1.00812	6.83642	0.72572	1.36406	2.9989E-03	1.0641	2.9959E-03	1.0630	4.0949E-06	6.295E-05	15.374	39.065	2.9340
-0.001	1.84442	2.61093	6.57757	6.52732	4.82503	1.00770	6.84144	0.72679	1.36322	3.0050E-03	1.0663	3.0020E-03	1.0652	-1.9645E-06	6.517E-05	-33.176	5.889	2.9346

Table B-46 Case C 60 cfs: Manually Computed Outlet Depths



Δy	y	θ	A	P	T	R	u	$u^2/2g$	D	$n^2u^2/(\varphi^2R^{4/3})$	F^2	$n^2u^2/(\varphi^2R^{4/3})_m$	F^2_m	$S_o-n^2u^2/(\varphi^2R^{4/3})$	ΔE	ΔX	ΣΔX	HW
	2.17858	2.88375	8.21484	7.20937	4.95850	1.13947	7.30386	0.82836	1.65672	2.9073E-03	1.0000							
-0.01	2.16858	2.87568	8.16527	7.18919	4.95587	1.13577	7.34820	0.83845	1.64759	2.9555E-03	1.0178	2.9314E-03	1.0089	6.859E-05	8.892E-05	1.296	1.296	3.42626
-0.001	2.16758	2.87487	8.16031	7.18718	4.95560	1.13540	7.35266	0.83947	1.64668	2.9604E-03	1.0196	2.9579E-03	1.0187	4.206E-05	1.868E-05	0.444	1.741	3.42678
-0.001	2.16658	2.87406	8.15536	7.18516	4.95533	1.13503	7.35713	0.84049	1.64577	2.9653E-03	1.0214	2.9628E-03	1.0205	3.717E-05	2.049E-05	0.551	2.292	3.42731
-0.001	2.16558	2.87326	8.15040	7.18314	4.95506	1.13466	7.36160	0.84151	1.64486	2.9702E-03	1.0232	2.9677E-03	1.0223	3.228E-05	2.229E-05	0.691	2.982	3.42785
-0.001	2.16458	2.87245	8.14545	7.18112	4.95479	1.13429	7.36608	0.84253	1.64395	2.9751E-03	1.0250	2.9726E-03	1.0241	2.737E-05	2.410E-05	0.881	3.863	3.42838
-0.001	2.16358	2.87164	8.14049	7.17910	4.95452	1.13391	7.37056	0.84356	1.64304	2.9800E-03	1.0268	2.9775E-03	1.0259	2.245E-05	2.592E-05	1.154	5.017	3.42892
-0.001	2.16258	2.87083	8.13554	7.17709	4.95425	1.13354	7.37505	0.84459	1.64213	2.9849E-03	1.0286	2.9825E-03	1.0277	1.753E-05	2.774E-05	1.583	6.600	3.42946
-0.001	2.16158	2.87003	8.13058	7.17507	4.95398	1.13317	7.37954	0.84562	1.64122	2.9899E-03	1.0305	2.9874E-03	1.0296	1.259E-05	2.956E-05	2.349	8.948	3.43001
-0.001	2.16058	2.86922	8.12563	7.17305	4.95370	1.13280	7.38404	0.84665	1.64031	2.9948E-03	1.0323	2.9924E-03	1.0314	7.635E-06	3.139E-05	4.111	13.059	3.43056
-0.0001	2.16048	2.86914	8.12513	7.17285	4.95368	1.13276	7.38449	0.84675	1.64022	2.9953E-03	1.0325	2.9951E-03	1.0324	4.908E-06	3.239E-06	0.660	13.719	3.43061
-0.0001	2.16038	2.86906	8.12464	7.17264	4.95365	1.13273	7.38494	0.84685	1.64013	2.9958E-03	1.0327	2.9956E-03	1.0326	4.412E-06	3.257E-06	0.738	14.457	3.43067
-0.0001	2.16028	2.86898	8.12414	7.17244	4.95362	1.13269	7.38539	0.84696	1.64004	2.9963E-03	1.0328	2.9961E-03	1.0328	3.916E-06	3.276E-06	0.837	15.294	3.43072
-0.0001	2.16018	2.86890	8.12365	7.17224	4.95359	1.13265	7.38584	0.84706	1.63995	2.9968E-03	1.0330	2.9966E-03	1.0329	3.419E-06	3.294E-06	0.963	16.257	3.43078
-0.0001	2.16008	2.86882	8.12315	7.17204	4.95357	1.13261	7.38630	0.84716	1.63986	2.9973E-03	1.0332	2.9971E-03	1.0331	2.922E-06	3.312E-06	1.133	17.391	3.43083
-0.0001	2.15998	2.86873	8.12266	7.17184	4.95354	1.13258	7.38675	0.84727	1.63977	2.9978E-03	1.0334	2.9976E-03	1.0333	2.426E-06	3.331E-06	1.373	18.764	3.43089
-0.0001	2.15988	2.86865	8.12216	7.17164	4.95351	1.13254	7.38720	0.84737	1.63968	2.9983E-03	1.0336	2.9981E-03	1.0335	1.929E-06	3.349E-06	1.736	20.500	3.43094
-0.0001	2.15978	2.86857	8.12167	7.17143	4.95349	1.13250	7.38765	0.84747	1.63959	2.9988E-03	1.0338	2.9986E-03	1.0337	1.432E-06	3.367E-06	2.352	22.852	3.43100
-0.0001	2.15968	2.86849	8.12117	7.17123	4.95346	1.13247	7.38810	0.84758	1.63950	2.9993E-03	1.0339	2.9991E-03	1.0339	9.346E-07	3.386E-06	3.623	26.475	3.43105
-0.0001	2.15958	2.86841	8.12068	7.17103	4.95343	1.13243	7.38855	0.84768	1.63940	2.9998E-03	1.0341	2.9996E-03	1.0340	4.374E-07	3.404E-06	7.783	34.259	3.43111
-0.0001	2.15948	2.86833	8.12018	7.17083	4.95340	1.13239	7.38900	0.84778	1.63931	3.0003E-03	1.0343	3.0001E-03	1.0342	-5.998E-08	3.422E-06	-57.065	-22.797	3.43116

Table B-47 Case C: Outlet Velocity Computations

Q	y_n	y_c	Tailwater	Slope	Boundary	y_{outlet}	Angle	Area	V_{outlet}
cfs	ft	ft	ft	---	---	ft	rad	ft^2	ft/s
15	1.05	1.06	0.00	Steep	Normal	1.05	1.90	3.00	**5.00**
30	1.49	1.52	0.00	Steep	Normal	1.49	2.31	4.91	**6.11**
45	1.85	1.88	0.00	Steep	Normal	1.85	2.61	6.58	**6.84**
60	2.16	2.18	0.00	Steep	Normal	2.16	2.87	8.12	**7.39**
75	2.45	2.45	0.00	Mild	Critical	2.45	3.10	9.56	**7.85**
90	2.74	2.69	0.00	Mild	Critical	2.69	3.30	10.78	**8.35**
105	3.02	2.92	0.00	Mild	Critical	2.92	3.48	11.91	**8.82**
120	3.30	3.13	0.00	Mild	Critical	3.13	3.65	12.93	**9.28**
135	3.61	3.33	0.00	Mild	Critical	3.33	3.82	13.87	**9.73**
150	3.96	3.51	0.00	Mild	Critical	3.51	3.97	14.73	**10.18**

Table B-48 shows the outlet depths and velocities computed for case D. Since the tailwater depth was greater than both normal and critical depths, the outlet depth was assumed to be the tailwater depth of 4.5 feet for all flows.

Table B-48 Case D: Outlet Velocity Computations

Q	y_n	y_c	Tailwater	Slope	Boundary	y_{outlet}	Angle	Area	V_{outlet}
cfs	ft	ft	ft	---	---	ft	rad	ft^2	ft/s
20	1.07	1.23	4.50	Steep	Tailwater	4.5	5.00	18.61	**1.07**
40	1.52	1.76	4.50	Steep	Tailwater	4.5	5.00	18.61	**2.15**
60	1.88	2.18	4.50	Steep	Tailwater	4.5	5.00	18.61	**3.22**
80	2.20	2.53	4.50	Steep	Tailwater	4.5	5.00	18.61	**4.30**
100	2.50	2.85	4.50	Steep	Tailwater	4.5	5.00	18.61	**5.37**
120	2.79	3.13	4.50	Steep	Tailwater	4.5	5.00	18.61	**6.45**
140	3.08	3.39	4.50	Steep	Tailwater	4.5	5.00	18.61	**7.52**
160	3.38	3.63	4.50	Steep	Tailwater	4.5	5.00	18.61	**8.60**
180	3.71	3.84	4.50	Steep	Tailwater	4.5	5.00	18.61	**9.67**
200	4.10	4.04	4.50	Mild	Tailwater	4.5	5.00	18.61	**10.75**

References

(1) French, R. H. (1985). *Open-Channel Hydraulics*. McGraw-Hill: New York.

(2) Normann, J. M., R. J. Houghtalen, and W. J. Johnson (1985). "Hydraulic Design of Highway Culverts." *Hydraulic Design Series No. 5*, 2nd Ed., Federal Highway Administration: Washington, D.C.

Appendix C. Program Testing

Four test cases were developed for the purpose of testing and comparing the computed inlet and outlet control headwater depths as well as the outlet velocity determined by each of the programs. Flow control was determined by comparing the predicted inlet and outlet control headwater depths and selecting the larger of the two as the control (1).

Tables C-1 through C-4 show the program outputs for inlet control headwater depth from all seven programs. The programs that did not specify inlet control headwater depths at certain flows are identified.

Table C-1 Case A Inlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
30	2.07	2.08	Not Reported	1.98	Not Reported	2.07	2.48	2.07
60	3.07	3.12	Not Reported	3.12	Not Reported	3.07	3.12	3.07
90	3.96	4.00	Not Reported	4.00	Not Reported	3.96	4.00	3.96
120	4.81	4.81	Not Reported	4.81	Not Reported	4.81	4.81	4.81
150	5.67	5.68	Not Reported	5.68	5.65	5.67	5.68	5.67
180	6.67	6.67	Not Reported	6.67	6.67	6.67	6.67	6.67
210	7.88	7.84	Not Reported	7.84	7.88	7.88	7.84	7.88
240	9.27	9.22	Not Reported	9.22	9.27	9.27	9.21	9.27
270	10.85	10.80	Not Reported	10.80	10.85	10.85	10.79	10.85
300	12.62	12.58	Not Reported	12.58	12.62	12.62	12.57	12.62

Table C-2 Case B Inlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
10	1.16	1.17	Not Reported	0.96	Not Reported	1.16	2.50	1.16
20	1.68	1.68	Not Reported	1.52	Not Reported	1.68	2.50	1.68
30	2.09	2.09	Not Reported	2.00	Not Reported	2.09	2.50	2.09
40	2.45	2.44	Not Reported	2.42	Not Reported	2.45	2.50	2.45
50	2.78	2.80	Not Reported	2.80	Not Reported	2.78	2.80	2.78
60	3.09	3.14	Not Reported	3.14	Not Reported	3.09	3.14	3.09
70	3.39	3.45	Not Reported	3.45	Not Reported	3.40	3.45	3.39
80	3.69	3.74	Not Reported	3.74	Not Reported	3.69	3.74	3.69
90	3.98	4.02	Not Reported	4.02	Not Reported	3.98	4.02	3.98
100	4.26	4.29	Not Reported	4.29	Not Reported	4.26	4.29	4.26

Table C-3 Case C Inlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
15	1.44	1.45	Not Reported	1.26	1.27	1.44	2.49	1.44
30	2.08	2.09	Not Reported	2.00	1.81	2.08	2.49	2.08
45	2.61	2.61	Not Reported	2.61	2.23	2.61	2.61	2.61
60	3.09	3.13	Not Reported	3.13	2.59	3.09	3.13	3.09
75	3.54	3.59	Not Reported	3.59	2.95	3.54	3.59	3.54
90	3.97	4.02	Not Reported	4.02	3.30	3.97	4.02	3.97
105	4.40	4.43	Not Reported	4.43	3.63	4.40	4.43	4.40
120	4.83	4.83	Not Reported	4.83	3.94	4.83	4.83	4.83
135	5.25	5.25	Not Reported	5.25	4.25	5.25	5.25	5.25
150	5.69	5.69	Not Reported	5.69	5.67	5.69	5.69	5.69

Table C-4 Case D Inlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
20	1.67	1.68	Not Reported	1.52	Not Reported	4.00	2.49	1.67
40	2.44	2.43	Not Reported	2.41	Not Reported	4.00	2.49	2.44
60	3.09	3.13	Not Reported	3.13	Not Reported	4.00	3.13	3.09
80	3.68	3.73	Not Reported	3.73	Not Reported	4.00	3.73	3.68
100	4.25	4.29	Not Reported	4.29	Not Reported	4.25	4.29	4.25
120	4.82	4.83	Not Reported	4.83	Not Reported	4.82	4.83	4.82
140	5.39	5.39	Not Reported	5.39	5.36	5.39	5.39	5.39
160	6.00	6.00	Not Reported	6.00	5.98	6.00	6.00	5.98
180	6.68	6.68	Not Reported	6.68	6.68	6.68	6.68	6.68
200	7.47	7.44	Not Reported	7.44	7.47	7.47	7.44	7.47

Tables C-5 through C-8 show the program output values for outlet control headwater depth for test cases A through D, respectively.

Table C-5 Case A Outlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
30	N/A	0.00	2.37	Not Reported	1.80	2.34	3.14	2.34
60	N/A	0.00	3.46	Not Reported	2.59	3.42	4.60	3.42
90	N/A	0.00	4.37	Not Reported	3.21	4.32	5.74	4.32
120	N/A	0.00	5.20	Not Reported	3.76	5.14	6.71	5.14
150	N/A	0.00	6.00	Not Reported	Not Reported	5.93	7.56	5.93
180	N/A	0.00	6.81	Not Reported	Not Reported	6.72	8.31	6.72
210	N/A	0.00	7.66	Not Reported	Not Reported	7.55	9.00	7.55
240	N/A	0.00	8.56	Not Reported	Not Reported	8.43	9.61	8.43
270	N/A	0.00	9.54	Not Reported	Not Reported	9.38	10.15	9.38
300	N/A	0.00	10.72	Not Reported	Not Reported	10.43	10.58	10.43

Table C-6 Case B Outlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
10	1.30	1.29	1.31	Not Reported	1.09	1.30	1.30	1.30
20	1.86	1.85	1.88	Not Reported	1.54	1.86	1.86	1.86
30	2.31	2.30	2.33	Not Reported	1.90	2.31	2.31	2.31
40	2.69	2.69	2.72	Not Reported	2.20	2.69	2.70	2.69
50	3.04	3.04	3.07	Not Reported	2.49	3.04	3.05	3.04
60	3.38	3.37	3.40	Not Reported	2.75	3.37	3.37	3.37
70	3.67	3.67	3.71	Not Reported	2.99	3.67	3.68	3.67
80	3.96	3.97	4.00	Not Reported	3.23	3.97	3.97	3.96
90	4.25	4.25	4.29	Not Reported	3.45	4.25	4.25	4.25
100	4.52	4.53	4.57	Not Reported	3.67	4.52	4.52	4.52

Table C-7 Case C Outlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
15	N/A	0.00	1.64	Not Reported	1.27	1.63	1.63	1.63
30	N/A	0.00	2.37	Not Reported	1.81	2.34	2.36	2.34
45	N/A	0.00	2.95	Not Reported	2.23	2.92	2.93	2.92
60	N/A	0.00	3.46	Not Reported	2.59	3.42	3.43	3.42
75	3.88	3.88	3.88	Not Reported	2.95	3.88	3.88	3.88
90	4.30	4.29	4.31	Not Reported	3.30	4.30	4.30	4.29
105	4.69	4.69	4.75	Not Reported	3.63	4.69	4.70	4.69
120	5.08	5.08	5.13	Not Reported	3.94	5.08	5.04	5.08
135	5.46	5.46	5.51	Not Reported	4.25	5.46	5.47	5.46
150	5.84	5.85	5.91	Not Reported	5.67	5.84	5.85	5.84

Table C-8 Case D Outlet Control Headwater Depths (ft)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
20	4.03	4.04	4.03	4.03	4.02	4.03	4.12	4.03
40	4.13	4.14	4.13	4.11	4.05	4.13	4.21	4.13
60	4.30	4.33	4.30	4.43	4.11	4.30	4.37	4.30
80	4.54	4.54	4.55	4.96	4.19	4.54	4.60	4.53
100	4.84	4.86	4.86	5.51	4.30	4.84	4.88	4.83
120	5.20	5.22	5.24	6.08	4.43	5.20	5.23	5.20
140	5.63	5.65	5.68	6.69	Not Reported	5.63	5.65	5.62
160	6.11	6.12	6.17	7.34	Not Reported	6.11	6.12	6.11
180	6.63	6.65	6.71	8.02	Not Reported	6.64	6.64	6.63
200	7.18	7.20	7.27	8.70	Not Reported	7.18	7.19	7.18

Tables C-9 through C-12 show the program output values for predicted flow control for test cases A through D, respectively. If the program did not specifically state the predicted control, the higher of the inlet and outlet control headwater depths was used to define the control.

Table C-9 Case A Flow Control

Q	Manually Determined Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
30	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Outlet	Inlet
60	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Outlet	Inlet
90	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Outlet	Inlet
120	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Outlet	Inlet
150	Inlet	Inlet	Outlet	Inlet	Inlet	Outlet	Outlet	Inlet
180	Inlet	Inlet	Outlet	Inlet	Inlet	Outlet	Outlet	Inlet
210	Inlet	Inlet	Outlet	Inlet	Inlet	Inlet	Outlet	Inlet
240	Inlet	Inlet	Outlet	Inlet	Inlet	Inlet	Outlet	Inlet
270	Inlet	Inlet	Outlet	Inlet	Inlet	Inlet	Inlet	Inlet
300	Inlet	Inlet	Outlet	Inlet	Inlet	Inlet	Inlet	Inlet

Table C-10 Case B Flow Control

Q	Manually Determined Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
10	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Inlet	Outlet
20	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Inlet	Outlet
30	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Inlet	Outlet
40	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
50	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
60	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
70	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
80	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
90	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
100	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet

Table C-11 Case C Flow Control

Q	Manually Determined Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
15	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Inlet	Inlet
30	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Inlet	Inlet
45	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Outlet	Inlet
60	Inlet	Inlet	Outlet	Inlet	Outlet	Outlet	Outlet	Inlet
75	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
90	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
105	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
120	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
135	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet	Outlet
150	Outlet	Outlet	Outlet	Inlet	Inlet	Outlet	Outlet	Outlet

Table C-12 Case D Flow Control

Q	Manually Determined Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
20	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet
40	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet
60	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet
80	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet
100	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet
120	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet	Outlet
140	Outlet	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet
160	Outlet	Outlet	Outlet	Outlet	Inlet	Outlet	Outlet	Outlet
180	Inlet	Inlet	Outlet	Outlet	Inlet	Inlet	Inlet	Inlet
200	Inlet	Inlet	Outlet	Outlet	Inlet	Inlet	Inlet	Inlet

Tables C-13 through C-16 show the program output values for outlet velocity for test cases A through D, respectively. The outlet velocity is expressed in ft/sec.

Table C-13 Case A Outlet Velocities (ft/s)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
30	8.93	8.90	8.94	8.87	5.90	8.94	8.68	8.91
60	10.52	10.52	10.53	10.49	7.38	10.52	10.20	10.50
90	11.56	11.54	11.58	11.62	8.53	11.57	11.19	11.54
120	12.38	12.37	12.41	12.51	9.54	12.38	11.98	12.36
150	13.10	13.07	13.13	13.30	10.49	13.09	12.75	13.08
180	13.76	13.74	13.79	13.99	11.41	13.75	13.40	13.74
210	14.39	14.39	14.44	14.61	12.34	14.38	14.06	15.64
240	15.01	15.00	15.08	15.18	13.19	15.00	14.76	17.19
270	15.62	15.58	15.71	15.68	14.40	15.60	15.44	18.86
300	16.06	15.81	22.15	15.85	15.71	16.02	15.99	20.61

Table C-14 Case B Outlet Velocities (ft/s)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
10	4.40	4.53	6.19	3.83	4.28	4.40	4.40	4.40
20	5.31	5.40	7.47	4.72	5.22	5.31	5.30	5.31
30	5.94	5.96	8.37	5.30	5.90	5.94	5.94	5.94
40	6.46	6.52	9.10	5.78	6.45	6.46	6.46	6.46
50	6.90	6.91	9.72	6.17	6.94	6.90	6.90	6.90
60	7.30	7.35	10.29	6.51	7.38	7.30	7.30	7.30
70	7.67	7.71	10.80	6.84	7.78	7.67	7.67	7.67
80	8.02	8.03	11.29	7.15	8.16	8.01	8.02	8.02
90	8.35	8.39	11.76	7.43	8.53	8.34	8.34	8.35
100	8.66	8.70	12.20	7.70	8.87	8.66	8.66	8.66

Table C-15 Case C Outlet Velocities (ft/s)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
15	5.00	5.09	5.00	4.94	4.80	4.99	4.99	5.01
30	6.11	6.11	6.11	6.06	5.90	6.09	6.09	6.07
45	6.84	6.87	6.84	6.86	6.70	6.82	6.83	6.83
60	7.39	7.40	7.39	7.39	7.38	7.37	7.37	7.36
75	7.85	7.86	11.05	7.80	7.98	7.84	7.84	7.85
90	8.35	8.39	11.76	8.18	8.53	8.34	8.34	8.35
105	8.82	8.84	12.42	8.50	9.04	8.82	8.82	8.82
120	9.28	9.32	13.07	8.82	9.54	9.28	9.28	9.28
135	9.73	9.78	13.70	9.12	10.02	9.73	9.73	9.73
150	10.18	10.19	14.34	9.44	10.49	10.18	10.18	10.18

Table C-16 Case D Outlet Velocities (ft/s)

Q	Manually Computed Value	HY-8	Fish X-ing	BCAP	Hydraflow Express	Culvert Master	Culvert	HEC-RAS
20	1.07	1.07	1.07	1.07	1.07	1.07	1.09	1.07
40	2.15	2.15	2.15	2.15	2.15	2.15	2.18	2.15
60	3.22	3.22	3.22	3.22	3.22	3.22	3.26	3.23
80	4.30	4.30	4.30	4.30	4.30	4.30	4.35	4.30
100	5.37	5.37	5.37	5.37	5.37	5.37	5.44	5.38
120	6.45	6.45	6.45	6.45	6.45	6.45	6.53	6.46
140	7.52	7.52	7.52	7.52	7.52	7.52	7.62	7.55
160	8.60	8.60	8.60	8.60	8.60	8.60	8.71	8.63
180	9.67	9.67	9.67	9.67	9.67	11.49	9.79	11.12
200	10.75	10.75	10.75	10.75	10.75	10.75	10.88	10.82

Figures C-1 through C-3 show the difference between the program results and the manually computed results for inlet control headwater depth, outlet control headwater depth, and outlet velocity, respectively.

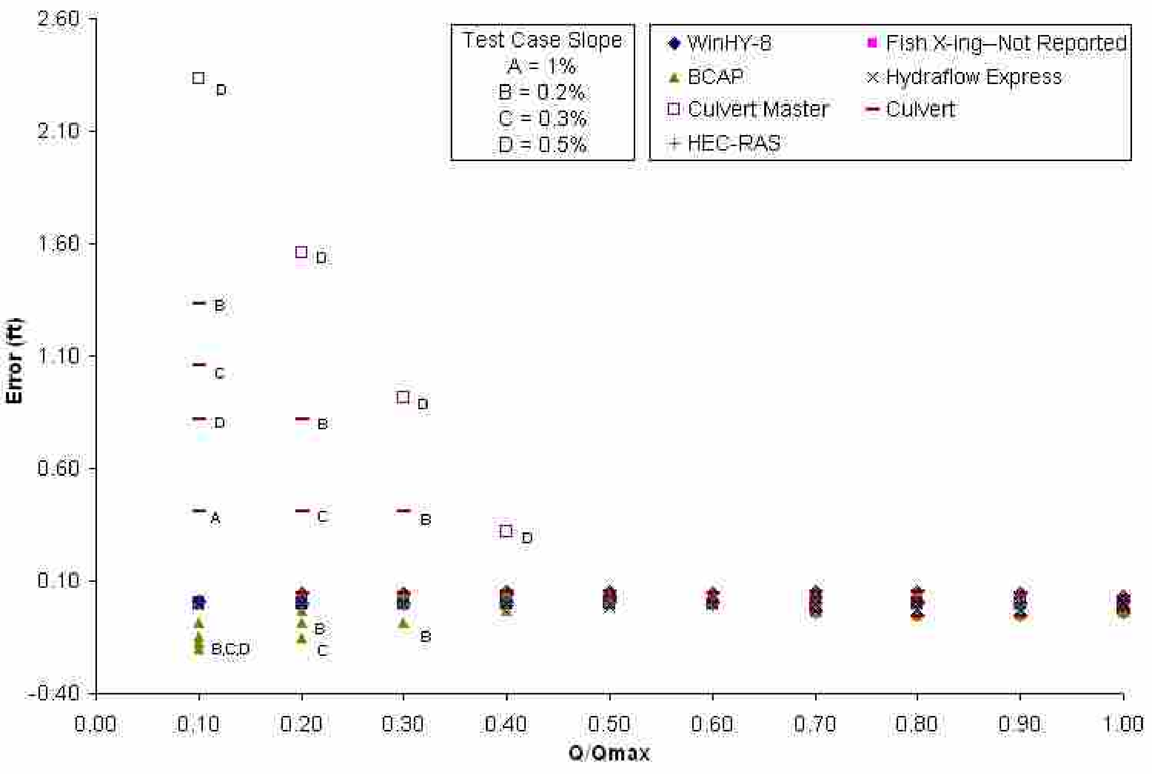


Figure C-1 Inlet Control Headwater Depth Error

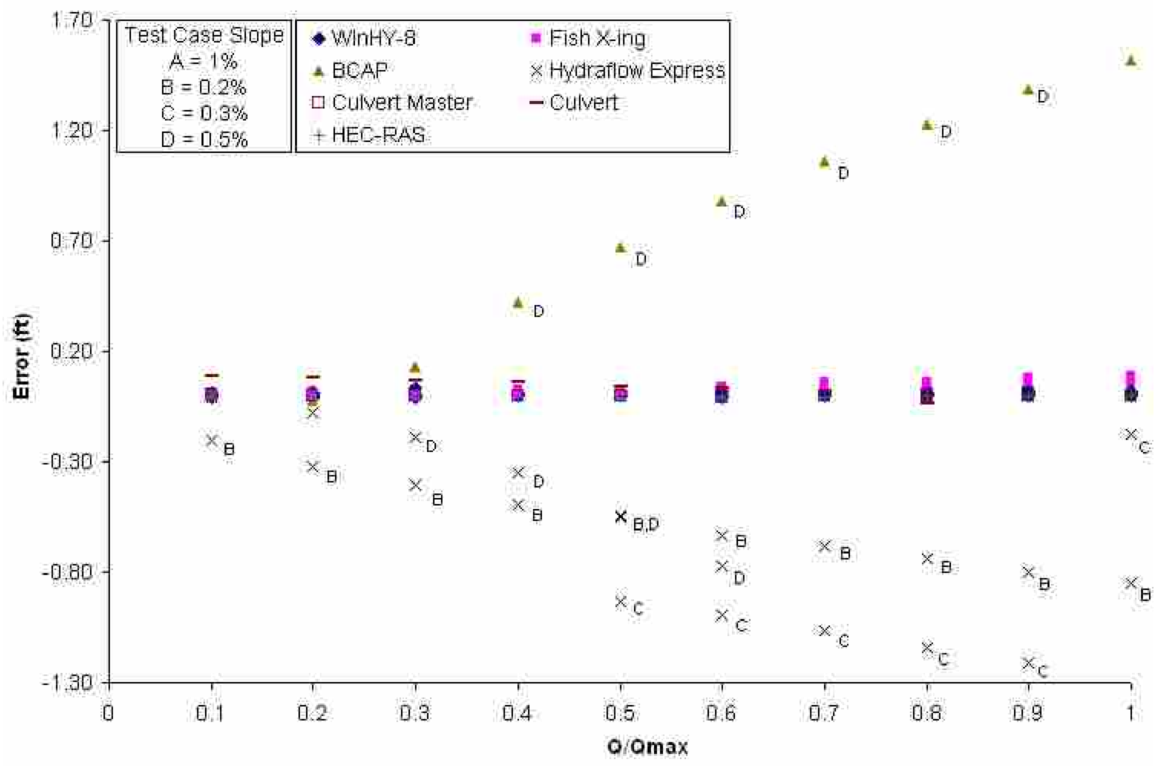


Figure C-2 Outlet Control Headwater Depth Error

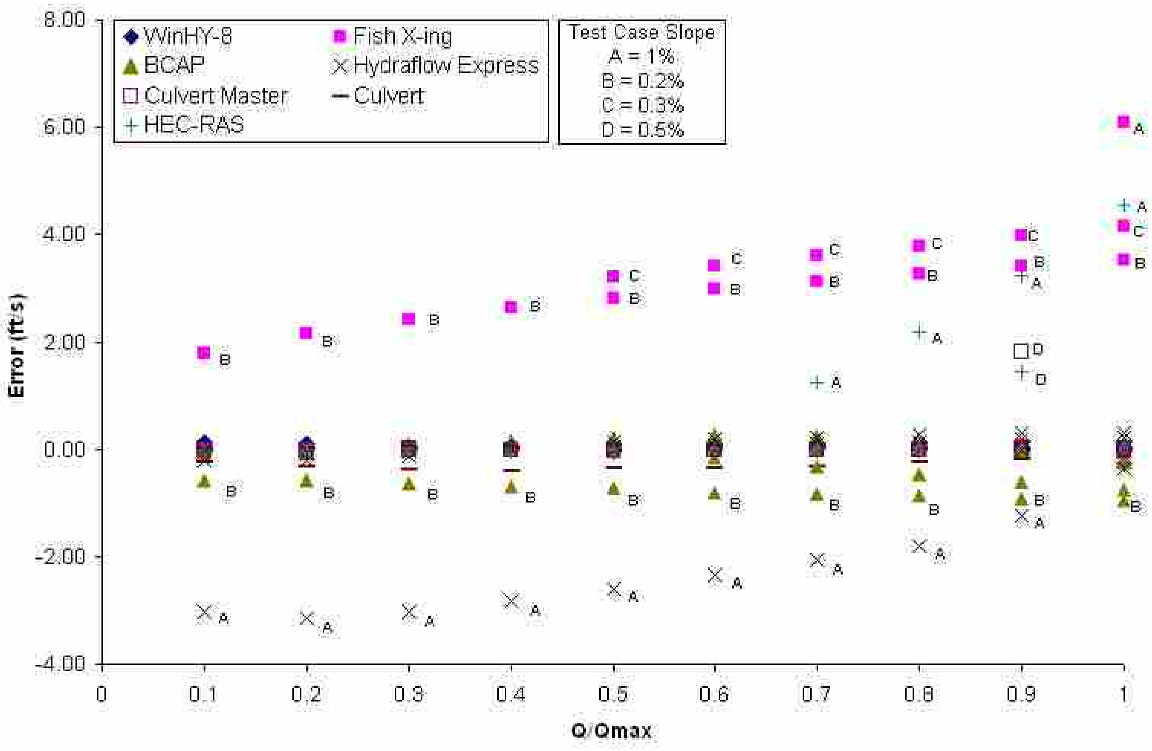


Figure C-3 Outlet Velocity Error

References

(1) Normann, J. M., R. J. Houghtalen, and W. J. Johnson (1985). “Hydraulic Design of Highway Culverts.” *Hydraulic Design Series No. 5*, 2nd Ed., Federal Highway Administration, Washington, D.C.

Appendix D. Modified Outlet Loss Coefficients

The current methodology for computing outlet loss coefficients is based on the theory described in the Federal Highway Administration's manual, HDS-5 (1). When computing outlet losses for outlet control flow, HDS-5, and therefore HY-8, uses the following equation:

$$H_o = 1.0\left[\frac{V^2}{2g} - \frac{V_d^2}{2g}\right] \qquad \textbf{(D-1)}$$

where H_o is the exit loss (ft), V is the velocity in the barrel (ft/s), V_d is the downstream channel velocity (ft/s), and g is the acceleration due to gravity (32.2 ft/sec^2). However, it is common practice to neglect the downstream velocity head, resulting in the following equation (6-2):

$$H_o = \frac{V^2}{2g} \qquad \textbf{(D-2)}$$

Clearly stated in HDS-5 is the fact that the previous two equations may overestimate exit losses and a multiplier less than 1.0 may be used (1). As a result, exit loss in HY-8 may also be overestimated. However, current research conducted at Utah State University by Blake P. Tullis and S. Collin Robinson has shown the Borda-Carnot Loss equation to better estimate exit loss (2). This equation utilizes the principles of momentum and energy, while the HDS-5 merely considerers energy conservation. The Borda-Carnot Loss is expressed by the following equation:

$$h_o = k_o \frac{(V_p - V_c)^2}{2g} \qquad \textbf{(D-3)}$$

where h_o is the exit loss (ft), V_p is the velocity in the barrel (ft/s), V_c is the downstream channel velocity (ft/s), k_o is the exit loss coefficient (1.0), and g is the acceleration due to gravity (32.2 ft/s^2)

By comparing Equations D-1 and D-3, it is clear that the numerators differ quite significantly. Equation D-2 is also rewritten as the following:

$$h_o = \frac{V_p^2}{2g}\left(1 - \frac{A_p}{A_c}\right)^2 \qquad \textbf{(D-4)}$$

where A_p is the cross sectional area of flow in the culvert (ft^2) and A_c is the cross sectional area of flow in the downstream channel (ft^2).

Through experimental process, Tullis and Robinson found that by using Equation D-3 with $k_o = 1.0$ or Equation 4 with $k_o = \left(1 - \frac{A_p}{A_c}\right)^2$, the computed exit loss is much more accurate and recommended over the HDS-5 conservative equations. To increase the accuracy of the results produced in computer models, it is recommended that through further study and experimentation the Borda-Carnot Loss method be implemented.

References

(1) Normann, J. M., Houghtalen, R. J. and Johnson, W. J. (1985). "Hydraulic design of highway culverts." *Hydraulic Design Series No. 5*, 2nd Ed., Federal Highway Administration, Washington, D.C.

(2) Robinson, S. C. and B. P. Tullis (2004). "Quantifying Culvert Exit Loss". TRB, National Research Council: Washington, D.C.

Appendix E. Auxiliary Culverts

Due to the complex nature of culvert hydraulics, improvements and new additions are continually being made in culvert type, shape, and material. Some of the most recent additions include the South Dakota Department of Transportation (DOT) research on new inlet types for concrete box culverts (1), the Kansas DOT research on various flared end sections (2, 3), new CON/SPAN culvert shapes (4), and Utah State University research on buried culverts (5). The following summarizes each of the culvert improvements and the items necessary for implementation into HY-8, BCAP, and Culvert. For all polynomial coefficients obtained, the slope correction factor in Equation 4-4 (0.5S) is incorporated into coefficient a.

E.1 South Dakota Research

The South Dakota Department of Transportation worked with the Federal Highway Administration (FHWA) to study the effects of 49 different inlet edge conditions on the performance of concrete box culverts (1). The four categories if inlet types studied include bevels and fillets, multiple barrels, span-to rise ratio, and skewed headwall. Table E-1 shows bevel and fillet inlets, Table E-2 shows span-to-rise inlets, and Table E-3 shows skewed headwall. The multiple barrel cases were not included for implementation since HY-8, BCAP, and Culvert currently includes the capabilities to analyze multiple barrel culverts.

Table E-1 Bevel and Fillet Inlet Test Cases

Inlet No.	Model ID	Wing-Wall Flare Angle	Bevel	Corner Fillets	Culvert Box Type	Culvert Slopes	Tail Water
1.1	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	0”	6’x6’	0.03, 0.007	High, Low
1.2	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	6”	6’x6’	0.03, 0.007	High, Low
1.3	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	12”	6’x6’	0.03, 0.007	High, Low
1.4	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	0”	6’x6’	0.03, 0.007	High, Low
1.5	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	6”	6’x6’	0.03, 0.007	High, Low
1.6	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	12”	6’x6’	0.03, 0.007	High, Low
1.7	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	6”	6’x12’	0.03	High, Low
1.8	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	12”	6’x12	0.03	High, Low
1.9	FC-S-0 Hybrid	0^o	4”-straight-top-bevel, 4”-radius WW bevels	0”	6’x6’	0.03	High, Low
1.10	PC-A Hybrid	0^o	8”-straight-top-bevel, no WW bevels	0”	6’x6’	0.03	High, Low

Table E-2 Span-to-Rise Ratio Inlet Test Case Parameters

Inlet No.	Model ID	Wing-Wall Flare Angle	Edge Conditions	Culvert Box Type	Culvert Slopes	Corner Fillets	Span to Rise
3.1	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	6’x6’	0.03, 0.007	0”	1:1
3.2	FC-S-30	30^o	4”-straight-top-bevel, no WW bevel	6’x6’	0.03, 0.007	0”	1:1
3.3	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	6’x6’	0.03, 0.007	0”	1:1
3.4	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	6’x12’	0.03, 0.007	0”	2:1
3.5	FC-S-30	30^o	4”-straight-top-bevel, no WW bevel	6’x12’	0.03, 0.007	0”	2:1
3.6	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	6’x12’	0.03, 0.007	0”	2:1
3.7	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	6’x18’	0.03, 0.007	0”	3:1
3.8	FC-S-30	30^o	4”-straight-top-bevel, no WW bevel	6’x18’	0.03, 0.007	0”	3:1
3.9	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	6’x18’	0.03, 0.007	0”	3:1
3.10	FC-S-0	0^o	4”-straight-top-bevel, no WW bevel	6’x24’	0.03, 0.007	0”	4:1
3.11	FC-S-30	30^o	4”-straight-top-bevel, no WW bevel	6’x24’	0.03, 0.007	0”	4:1
3.12	PC-A	0^o	8”-radius-top-bevel, 4”-radius WW bevels	6’x24’	0.03, 0.007	0”	4:1

Table E-3 Skewed Headwall Inlet Test Case Parameters

Inlet No.	Model ID	Wing-Wall Flare Angle	Edge Conditions	Culvert Box Type	Culvert Slopes	Corner Fillets	No. of Barrels	Span to Rise	Span to Rise
4.1	FC-T-0	0^{o}	4”-straight-top-bevel, no WW bevel	3’x6’x6’	0.03, 0.007	0”	3	-	0^{o}
4.2	FC-T-30	30^{o}	4”-straight-top-bevel, no WW bevel	3’x6’x6’	0.03, 0.007	0”	3	-	0^{o}
4.3	FC-T-30	30^{o}	4”-straight-top-bevel, no WW bevel	3’x6’x6’	0.03, 0.007	0”	3	-	15^{o}
4.4	FC-T-30	30^{o}	4”-straight-top-bevel, no WW bevel	3’x6’x6’	0.03, 0.007	0”	3	-	30^{o}
4.5	FC-T-30	30^{o}	4”-straight-top-bevel, no WW bevel	3’x6’x6’	0.03, 0.007	0”	3	-	45^{o}
4.6	FC-S-30	30^{o}	4”-straight-top-bevel, no WW bevel	6’x18’	0.03, 0.007	0”	-	3:1	30^{o}

In order to implement a new inlet edge condition into HY-8, Culvert, and BCAP, polynomial coefficients are required for the fifth degree polynomial that describes the performance curve (how the headwater depth changes with flow) for each inlet type. The upper and lower boundaries for which the polynomial curves accurately describe the behavior of the culvert under inlet control conditions are HW/D = 3.0 and HW/D = 0.5, respectively (6). However, when the work was completed by the South Dakota DOT for the new inlet types, the data was only collected up to HW/D ratios equaling 2.0. When the polynomial curves are plotted for values above HW/D = 2.0, the behavior of the curves is no longer reliable and are inconsistent with expectations. The polynomial coefficients obtained from the research were provided in the report by the South Dakota DOT. Figures E-1, E-2, and E-3 show the unreliable nature of the polynomial curves above HW/D = 2.0 for bevels and fillets, span-to-rise ratios, and skewed headwall inlet conditions, respectively.

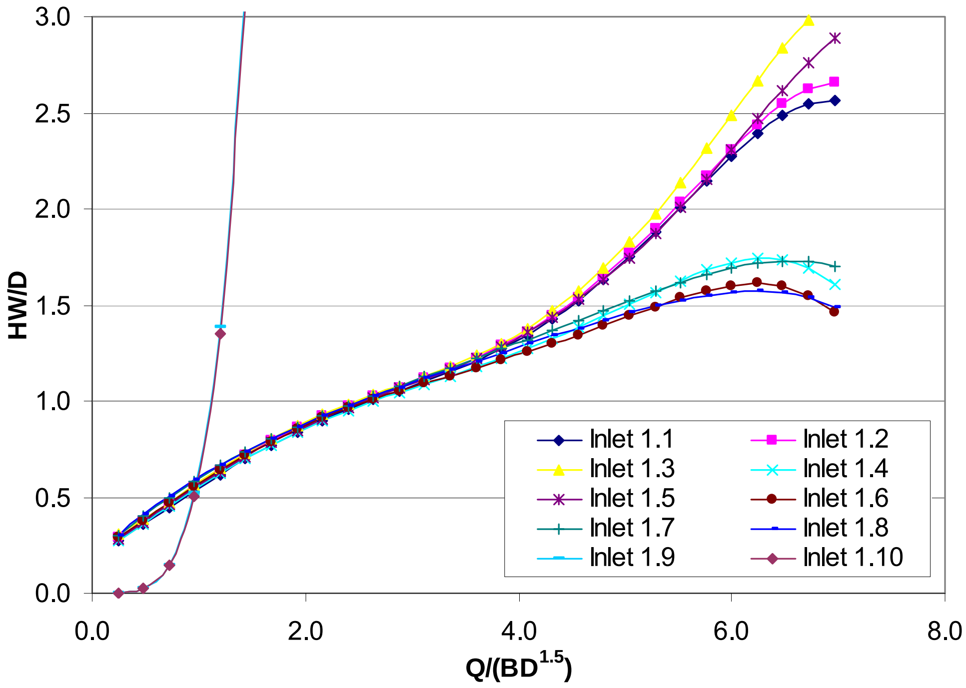


Figure E-1 Polynomial Curves for Bevel and Fillet Inlet Types

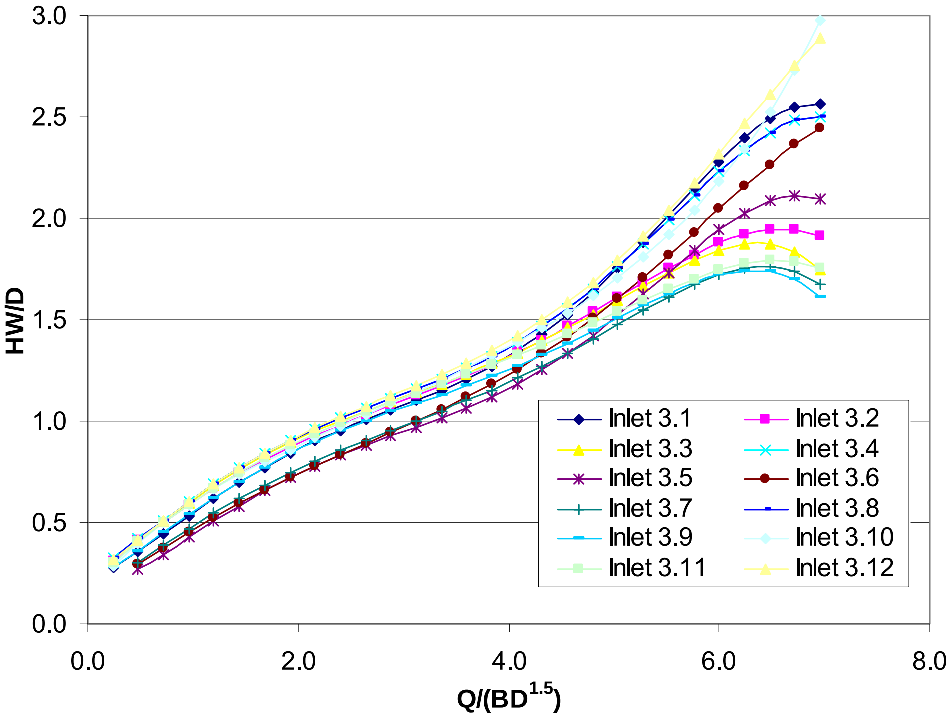


Figure E-2 Polynomial Curves for Span-to-Rise Inlet Types

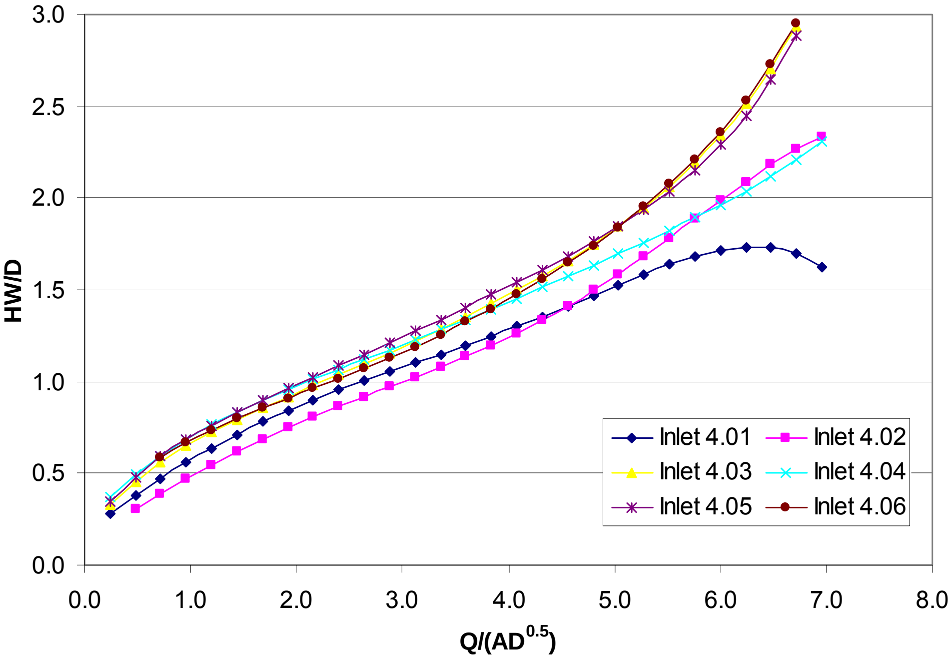


Figure E-3 Polynomial Curves for Skewed Headwall Inlet Types

To accurately implement the South Dakota research into HY-8, BCAP, or Culvert, it is recommended that new polynomial coefficients be defined such that the curves behave more appropriately for HW/D ratios between 2.0 and 3.0. This was accomplished by plotting the submerged and unsubmerged sections of each curve using the equations supplied by the National Bureau of Standards. The K, M, c, and Y coefficients used in these equations are provided in the South Dakota report (1). From there, a best-fit fifth degree polynomial curve will be applied to the unsubmerged and submerged curves. These new curves will behave more accurately for conditions above HW/D = 2.0, but since the NBS coefficients were also developed for HW/D = 2.0, the best-fit polynomial curves will not be completely accurate above HW/D = 2.0.

Another problem identified in implementing the South Dakota research into computer applications is the large number of inlet types available. It is unreasonable to add approximately 50 new inlet types to the available list of inlets in any of the programs. It is recommended that the total number of curves be reduced from 49 to 6. As previously stated, the multiple barrel inlets will be neglected. The curves from each of the three inlet conditions will be reduced to two representative curves for each, for a total of six curves. The curve reduction will be completed by locating the upper and lower 10% error boundaries for the curves and fitting one polynomial curve through the middle of that range that represents multiple curves. The maximum error of this new curve will be +/-10% (6). Figures E-4 through E-9 show the error range for each of the inlet types.

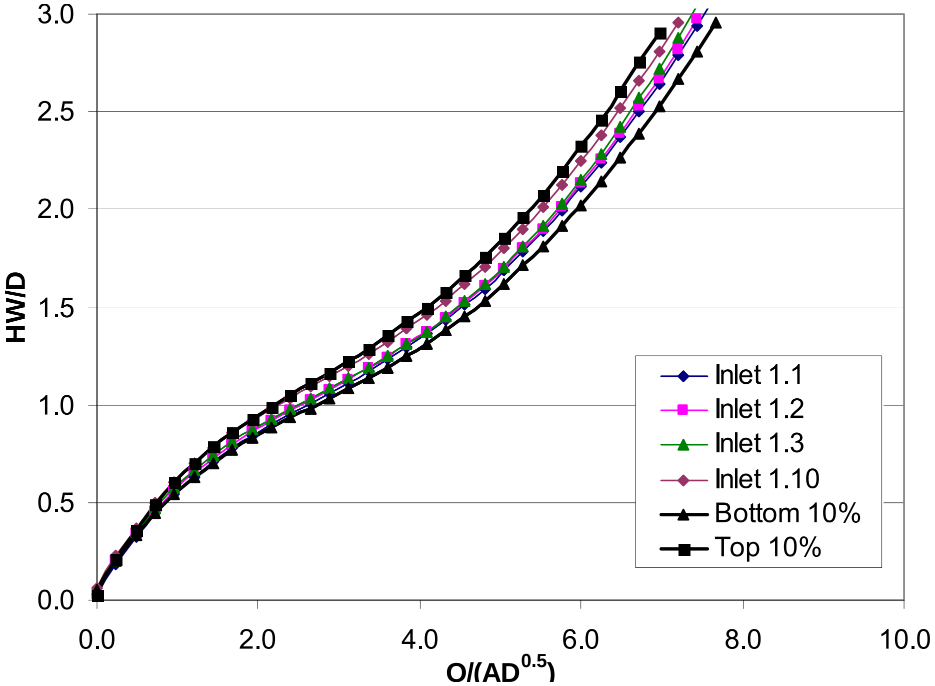


Figure E-4 10% Range for Bevel and Fillet (1) Inlet Types

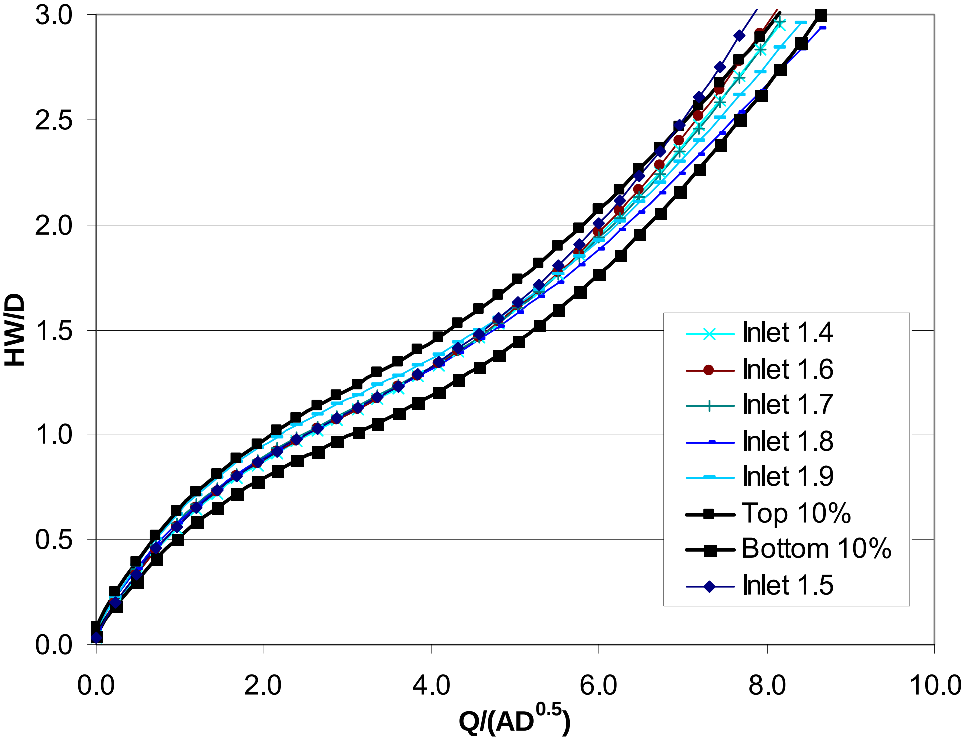


Figure E-5 10% Range for Bevel and Fillet (2) Inlet Types

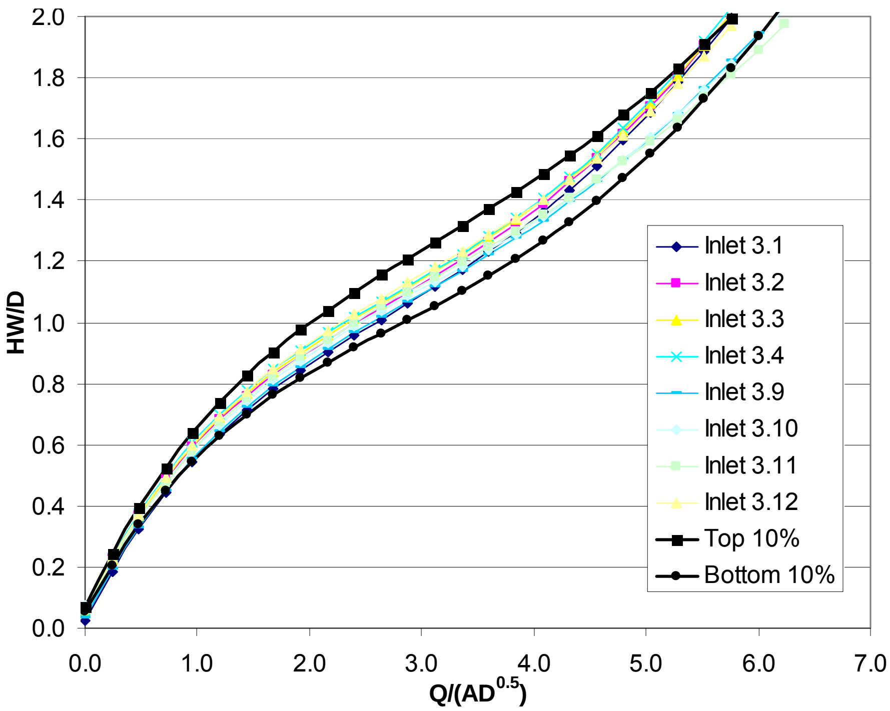


Figure E-6 10% Range for Span-to-Rise (1) Inlet Types

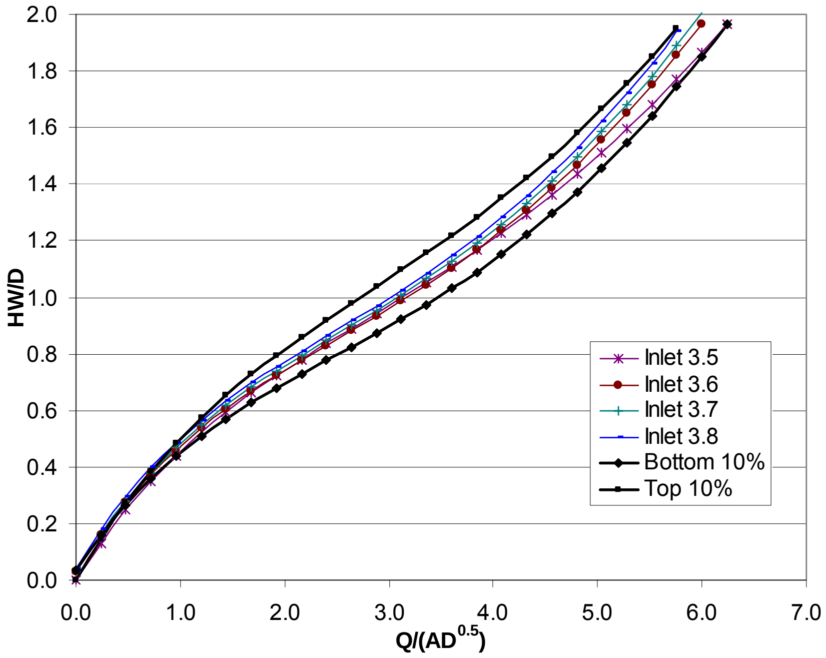


Figure E-7 10% Range for Span-to-Rise (2) Inlet Types

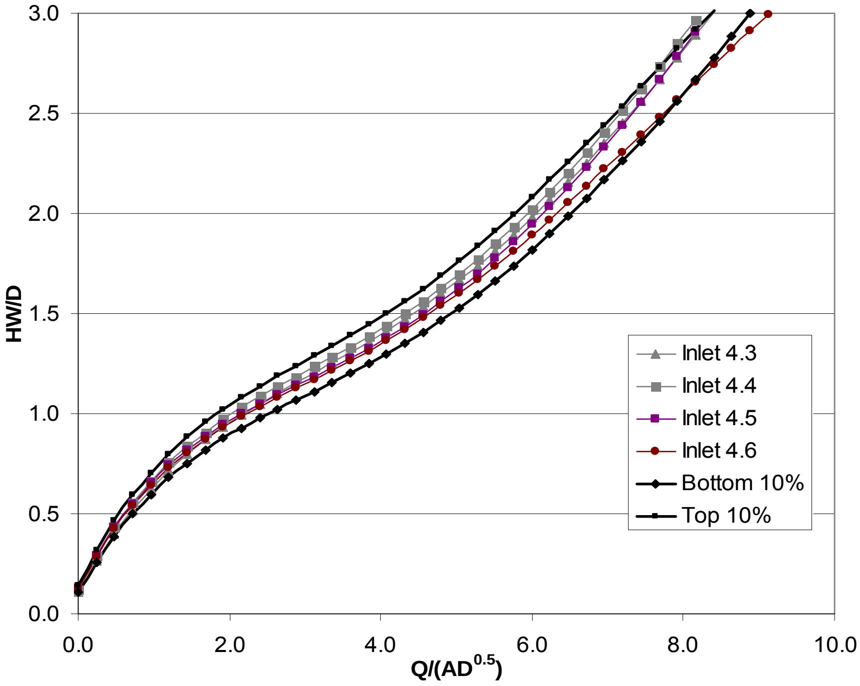


Figure E-8 10% Range for Skewed Headwall (1) Inlet Types

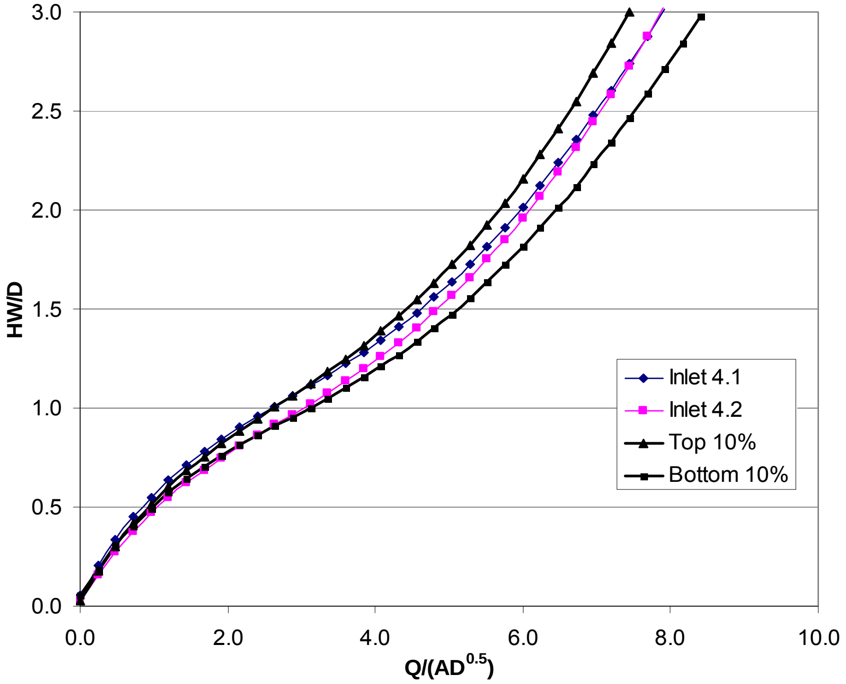


Figure E-9 10% Range for Skewed Headwall (2) Inlet Types

E.2 Kansas DOT Flared End Sections

At the University of Kansas, research was completed on the behavior of Type I, III, and IV end sections for pipe culverts through the efforts of Bruce M. McEnroe, Lance M. Johnson, and Jeffrey A. Barley (2, 3). Type I end sections are prefabricated flared end sections. Type III end sections are side tapered with a 4:1 lateral flare and no miter at the end. To analyze the performance of the culverts the submerged and unsubmerged equations developed by the National Bureau of Standards were utilized (6).

In the research conducted at University of Kansas, the regression coefficients for both unsubmerged and submerged conditions were not provided in the laboratory report; however, the dimensionless diagrams were included along with different equations other than fifth degree polynomials to represent them. These equations were used to reproduce the inlet control performance curves for each of the end sections. From the generated polynomial the fifth-degree polynomial coefficients were obtained.

In order to implement the Kansas end sections into HY-8, BCAP, and Culvert, the following equations were used (2, 3):

$$\frac{HW}{D} = \begin{cases} 1.60\left(Q^*\right)^{0.60} \\ 2.23\left(Q^*\right) + 0.023 \\ 1.289 - 1.61\left(Q^*\right) + 2.90\left(Q^*\right)^2 \end{cases} \tag{E-1}$$

$$\frac{HW}{D} = \begin{cases} 1.53\left(Q^*\right)^{0.55} \\ 2.13\left(Q^*\right) + 0.055 \\ 1.367 - 1.50\left(Q^*\right) + 2.50\left(Q^*\right)^2 \end{cases} \tag{E-2}$$

$$\frac{HW}{D} = \begin{cases} 1.65\left(Q^*\right)^{0.55} \\ 2.13\left(Q^*\right) + 0.055 \end{cases} \tag{E-3}$$

$$\frac{HW}{D} = \begin{cases} 1.53\left(Q^*\right)^{0.55} \\ 2.13\left(Q^*\right) + 0.055 \end{cases} \tag{E-4}$$

Figures E-10 through E-14 show each polynomial curve and the appropriate polynomial equation. Table E-4 shows the polynomial coefficients obtained for each test case.

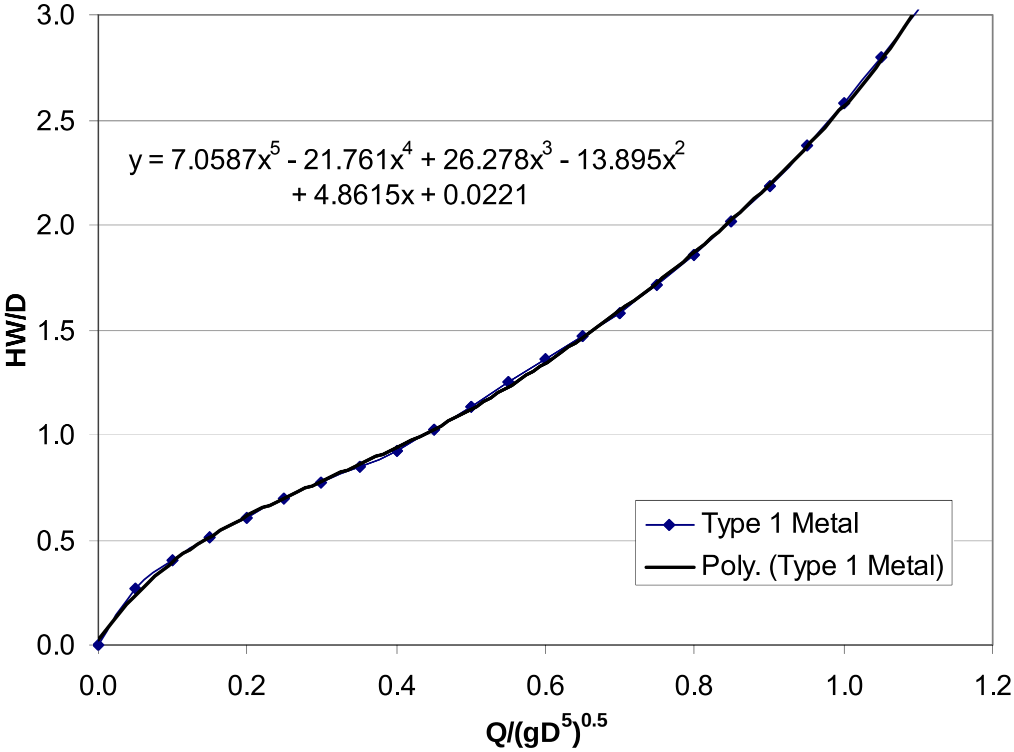


Figure E-10 Type 1-Metal Culvert

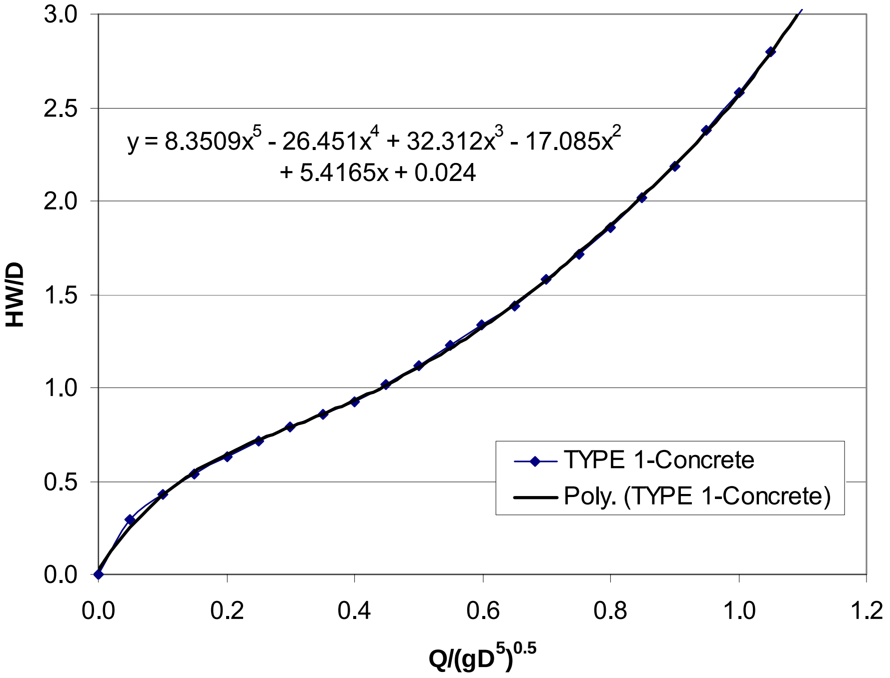


Figure E-11 Type 1 - Concrete

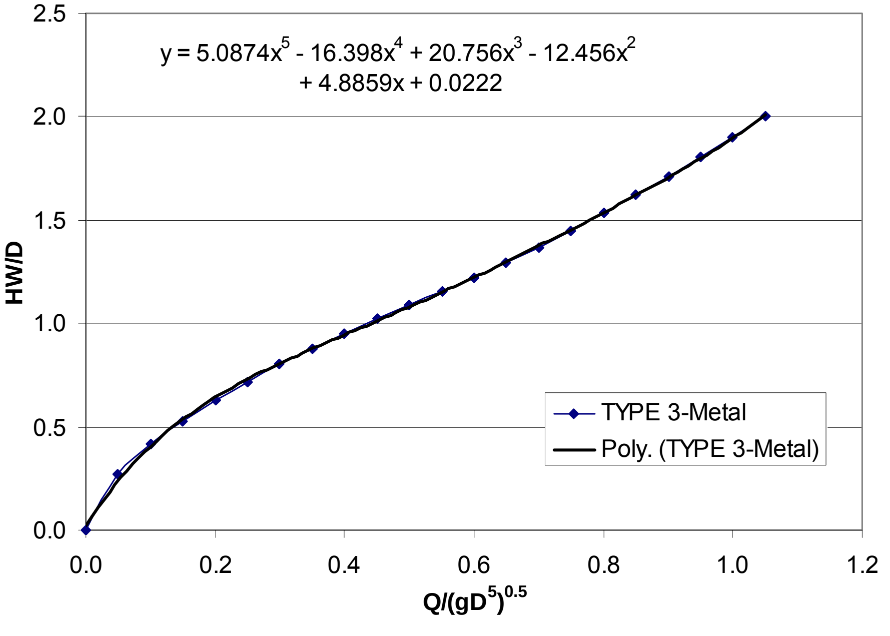


Figure E-12 Type 3 - Metal

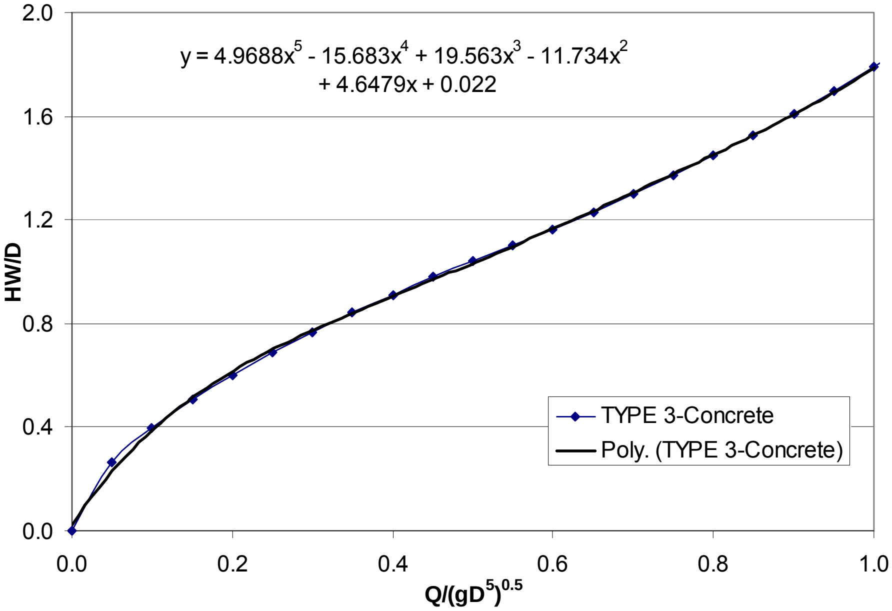


Figure E-13 Type 3 - Concrete

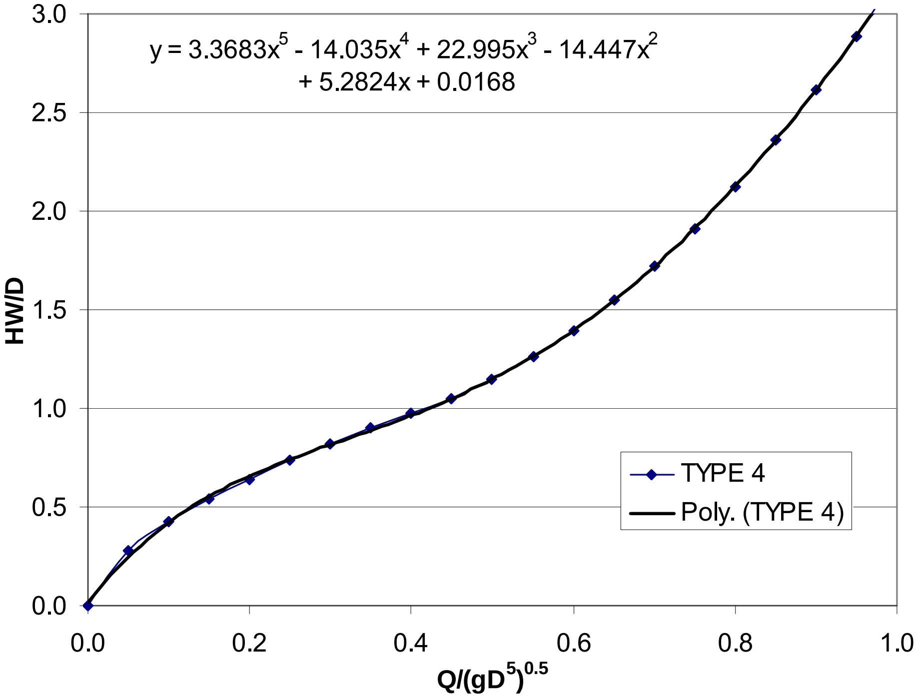


Figure E-14 Type 4

Table E-4 Polynomial Coefficients for Kansas Flared End Sections

Type and Material	a	b	c	d	e	f
Type 1: Metal	0.0221	4.8615	-13.895	26.278	-21.761	7.0587
Type 1: Concrete	0.024	5.4165	-17.085	32.312	-26.451	8.3509
Type 3: Metal	0.0222	4.8859	-12.456	20.756	-16.398	5.0874
Type 3: Concrete	0.022	4.6479	-11.734	19.563	-15.683	4.9688
Type 4	0.0168	5.2824	-14.447	22.995	-14.035	3.3683

E.3 CON/SPAN

CON/SPAN culverts are a blend of the concrete box and metal box culvert shapes. Since CON/SPAN culverts were introduced to the industry within the last 15 years, little information is available on the performance of these culvert shapes. Originally, flow through CON/SPAN culverts was analyzed using a concrete box culvert or a metal box culvert for low flows and high flows, respectively (4). Since CON/SPAN culverts generally include a headwall and/or wingwalls and metal box culverts do not include such features, it was desired to find a better approach to analyze the performance of CON/SPAN culverts.

At the University of Dayton, Ohio, research was completed on the behavior of CON/SPAN culverts through the efforts of Timothy Beach, Dr. Donald V. Chase, and Christopher Sherk. Small scale models with span to rise ratios of 2:1 and 4:1 were tested with a combination of slopes and entrance conditions of 0, 45, or 90 degree wingwalls (4). To analyze the performance of the culverts the submerged and unsubmerged equations developed by the National Bureau of Standards were utilized (6).

In the research conducted at University of Dayton, Ohio, the regression coefficients for both unsubmerged and submerged conditions were collected for the following culvert models described in Table E-5. The K, M, c, and Y coefficients determined from the lab tests are listed in Table E-6.

Table E-5 CON/SPAN Model Characteristics

Dimension	2:1 Ratio	4:1 Ratio
Span (in)	4	4
Rise (in)	2	1
Area (in^2)	7.34	3.35
Length	2.6	2.46

Table E-6 NBS Inlet Control Equation Coefficients

Shape/Wingwall Angle	K	M	c	Y
2:1 0 deg	0.475	0.667	0.043	0.543
2:1 45 deg	0.500	0.667	0.039	0.670
2:1 90 deg	0.511	0.667	0.039	0.729
4:1 0 deg	0.446	0.667	0.027	0.676
4:1 0 deg	0.455	0.667	0.035	0.595
4:1 0 deg	0.468	0.667	0.037	0.566

In order to implement the CON/SPAN culvert shapes into HY-8, BCAP, and Culvert, the K, M, c, and Y coefficients were used in conjunction with the NBS equations to develop fifth degree polynomials to represent the inlet control performance of each culvert shape. This was done by plotting the unsubmerged and submerged equations on a graph covering zones from approximately $Q/AD^{0.5} < 3.5$ and $4.0 < Q/AD^{0.5}$, respectively (6). The values of HW/D and $Q/AD^{0.5}$ for both zones of flow were plotted using only one curve. A best-fit fifth degree polynomial was then passed through the curve and the coefficients for the equation were obtained. Figures E-16 through E-20 show each polynomial curve with its respective equation. Table E-7 shows the polynomial coefficients obtained for each test case.

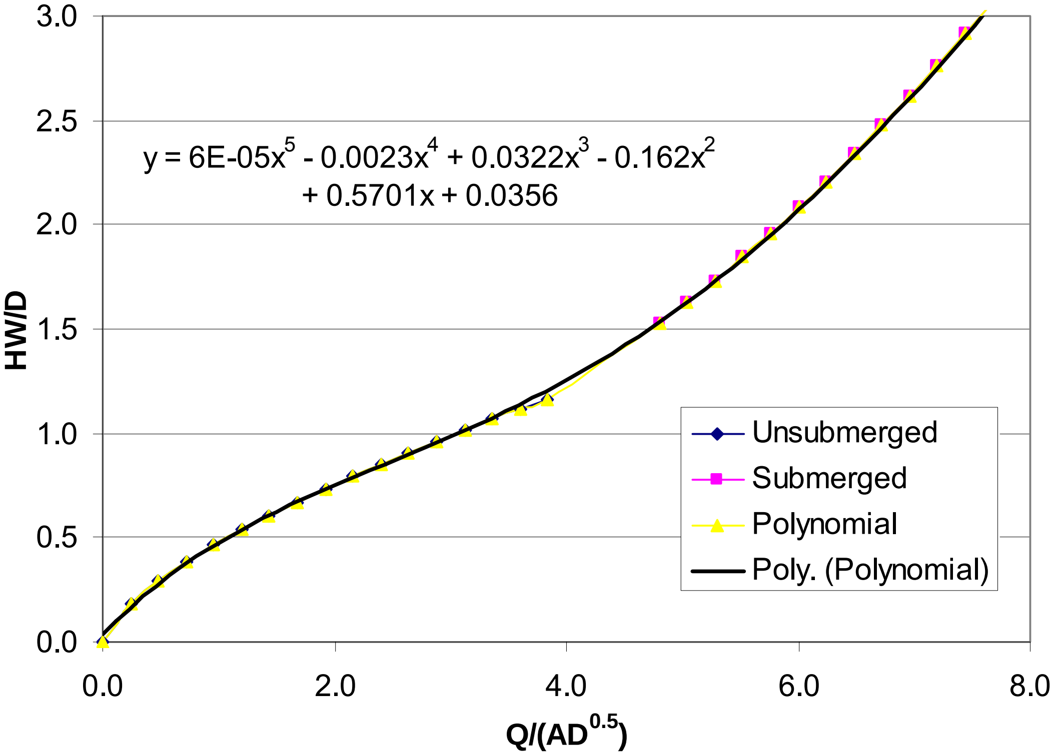


Figure E-15 2:1, 0 Degree Wingwall

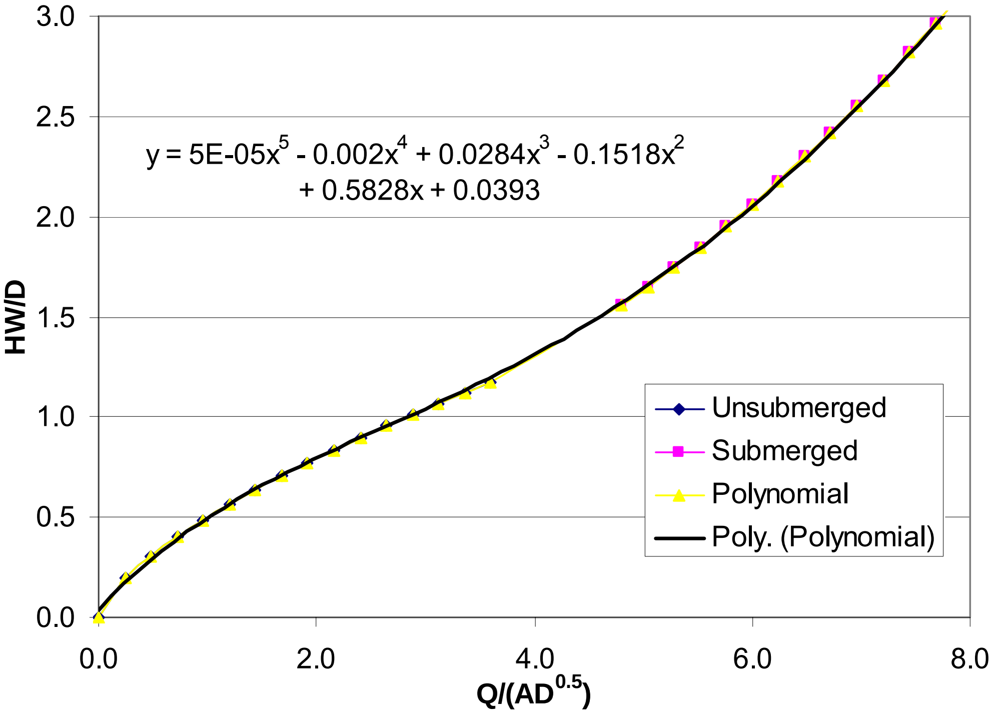


Figure E-16 2:1, 45 Degree Wingwall

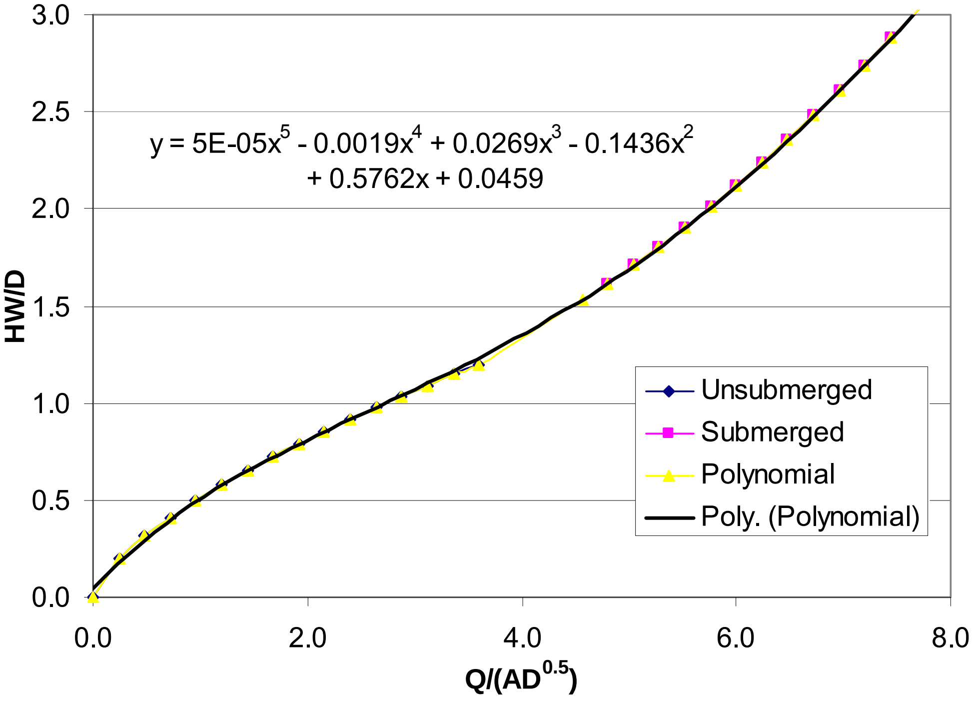


Figure E-17 2:1, 90 Degree Wingwall

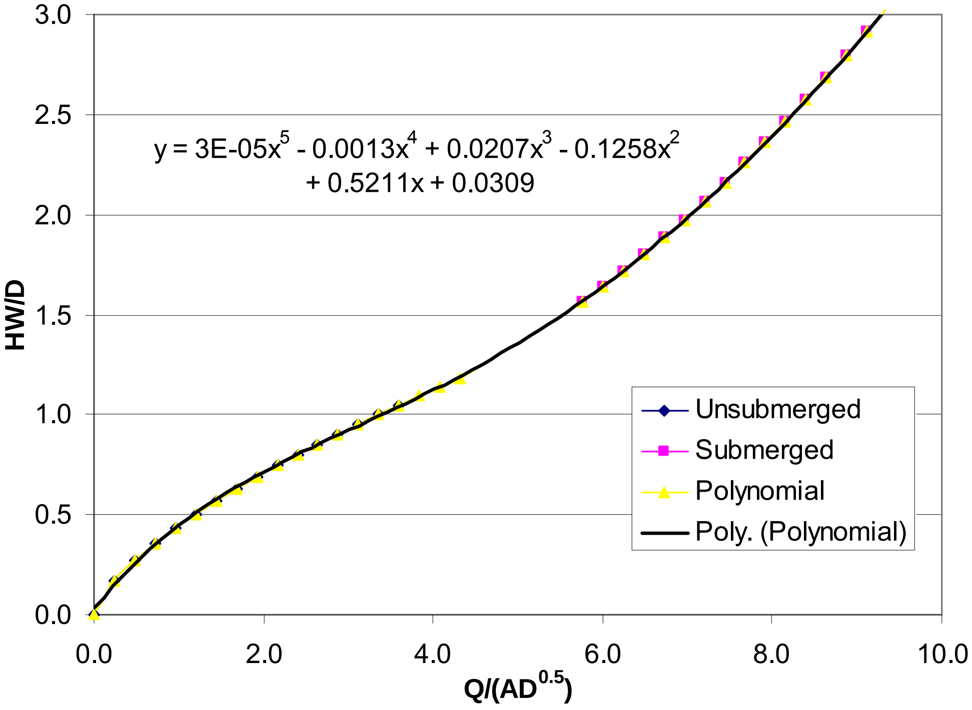


Figure E-18 4:1, 0 Degree Wingwall

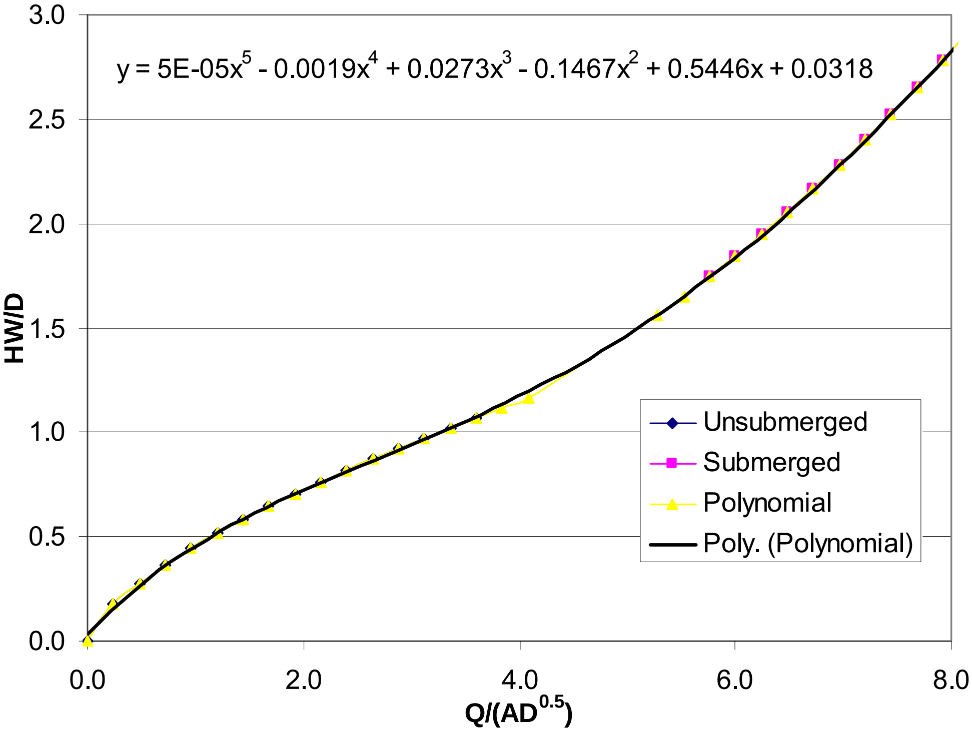


Figure E-19 4:1, 45 Degree Wingwall

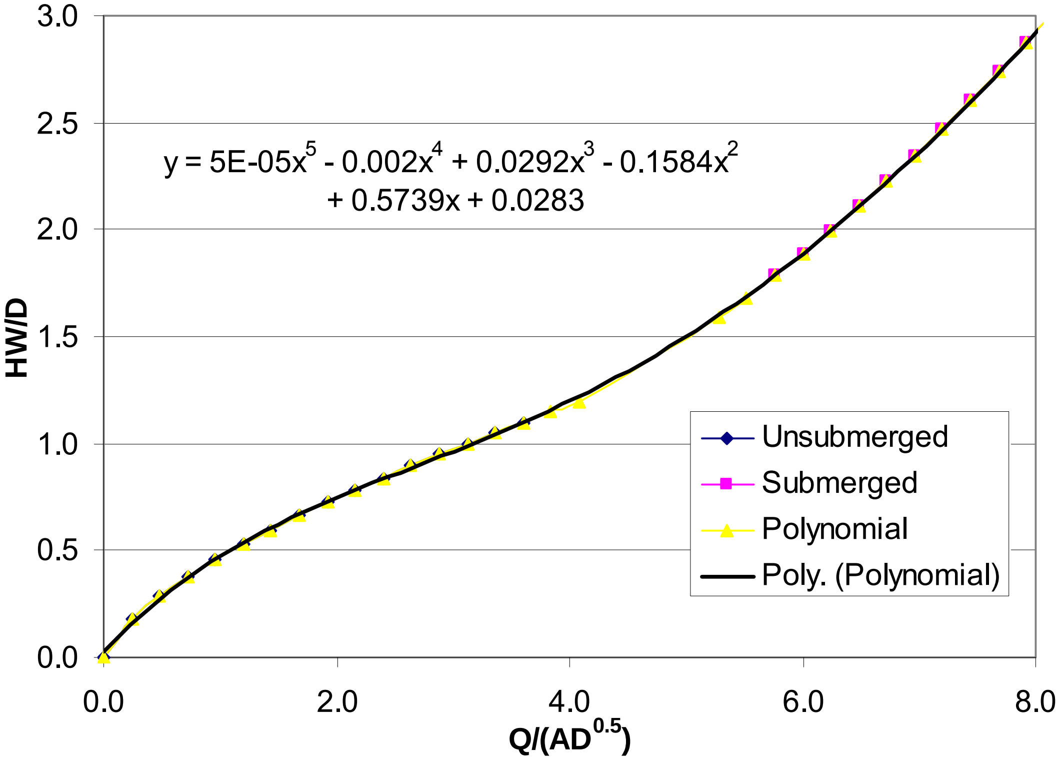


Figure E-20 4:1, 90 Degree Wingwall

Table E-7 Derived Polynomial Coefficients

Shape/Wingwall Angle	a	b	c	d	e	f
2:1 0 deg	0.0356	0.5701	-0.162	0.0322	-0.0023	0.00006
2:1 45 deg	0.0393	0.5828	-0.1518	0.0284	-0.002	0.00005
2:1 90 deg	0.0459	0.5762	-0.1436	0.0269	-0.0019	0.00005
4:1 0 deg	0.0309	0.5211	-0.1258	0.0207	-0.0013	0.00003
4:1 45 deg	0.0318	0.5446	-0.1467	0.0273	-0.0019	0.00005
4:1 90 deg	0.0283	0.5739	-0.1584	0.0292	-0.002	0.00005

E.4 Utah State University Buried Culverts

Over the years fish passage through culverts has become of increasing importance in culvert hydraulics. Traditionally, culverts were simply designed to pass flood flows, with no consideration for the ability of fish to pass through the culvert and migrate upstream for spawning. However, in recent years culvert hydraulics has been changing to include design parameters that allow for fish passage. One such design includes buried or bottomless culverts. Buried culverts are currently being studied at Utah State University under the direction of assistant professor Blake P. Tullis (5).

When a culvert barrel is partially buried in a streambed, backfill is placed inside the culvert and aligned with the naturally occurring streambed at both ends. The purpose is to create an environment that is flush with the natural surroundings and allows fish to pass through the culvert undisturbed (5). In the Utah State study, inlet loss coefficients as well as inlet control coefficients for the NBS equations were obtained for circular culverts with buried depths at 20%, 40%, and 50%, and for elliptical culverts buried at depths of 50% of the culvert rise. The inlet edge conditions tested at these buried depths included thin edge projecting (ponded and channelized), mitered to 1.5:1 fill slope, square edged inlet with vertical headwall, and 45° beveled entrance and vertical headwall (5). The K, M, c, and Y coefficients determined from the lab tests are listed in Table E-8.

Table E-8 NBS Equation Coefficients

Shape/Edge Condition	Percent Buried	K	M	c	Y
Thin Edge Projecting, (Ponded)	20	0.44	0.64	0.03	0.57
	40	0.47	0.69	0.05	0.68
	50	0.5	0.71	0.06	0.53
	50 (Ellipse)	0.53	0.67	0.07	0.46
Thin Edge Projecting, (Channelized)	20	0.42	0.62	0.03	0.62
	40	0.48	0.66	0.04	0.51
	50	0.5	0.68	0.06	0.47
	50 (Ellipse)	0.5	0.67	0.06	0.13
Mitered to 1.5:1 Fill Slope	20	0.4	0.63	0.02	0.63
	40	0.42	0.69	0.04	0.64
	50	0.44	0.68	0.05	0.48
	50 (Ellipse)	0.49	0.65	0.04	0.61
Square Edge with Vertical Headwall	20	0.4	0.63	0.02	0.67
	40	0.44	0.68	0.03	0.66
	50	0.45	0.7	0.04	0.63
	50 (Ellipse)	0.45	0.7	0.03	0.68
45^{o} Beveled Inlet with Vertical Headwall	20	0.39	0.63	0.02	0.71
	40	0.42	0.67	0.02	0.73
	50	0.44	0.69	0.03	0.66
	50 (Ellipse)	0.48	0.67	0.05	0.51

In order to implement the berried culvert shapes into HY-8, the K, M, c, and Y coefficients were used in conjunction with the NBS equations to develop fifth degree polynomials to represent the inlet control performance of each culvert shape. This was done using the same methods previously described by plotting the unsubmerged and submerged equations on a graph covering zones from approximately $Q/AD^{0.5} < 3.5$ and $4.0 < Q/AD^{0.5}$, respectively (6). The values of HW/D and $Q/AD^{0.5}$ for both zones of flow were plotted using only one curve. A best-fit fifth degree polynomial was then passed through the curve and the coefficients for the equation were obtained. Figures E-21 through E-40 show each polynomial curve with its respective equation. Table E-9 shows the polynomial coefficients obtained for each test case.

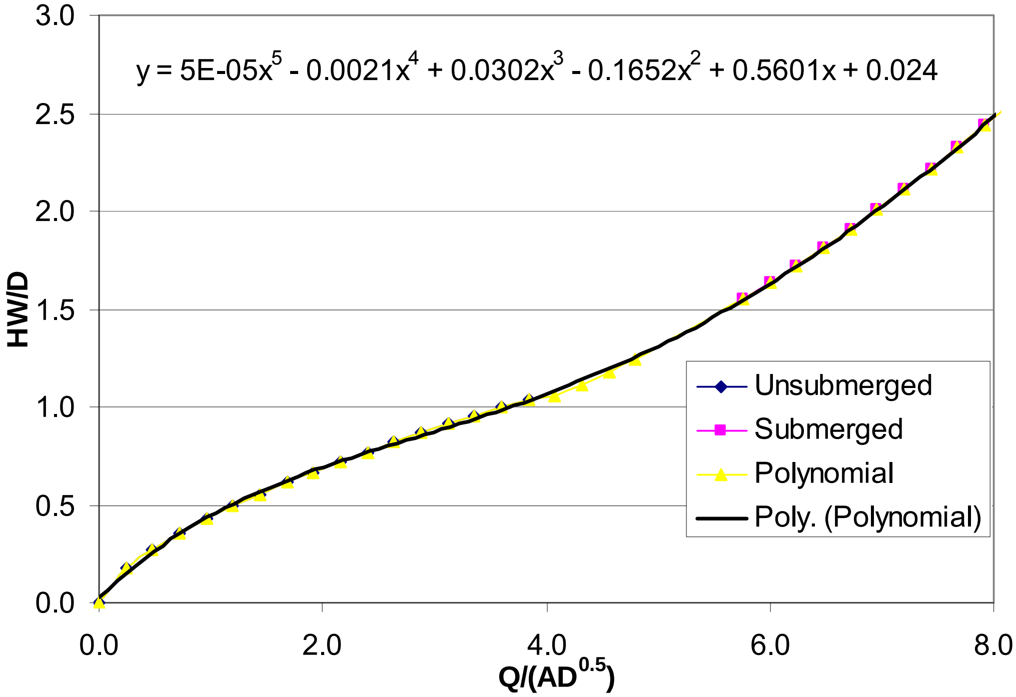


Figure E-21 Thin Edge Projecting: 20% Buried (Ponded)

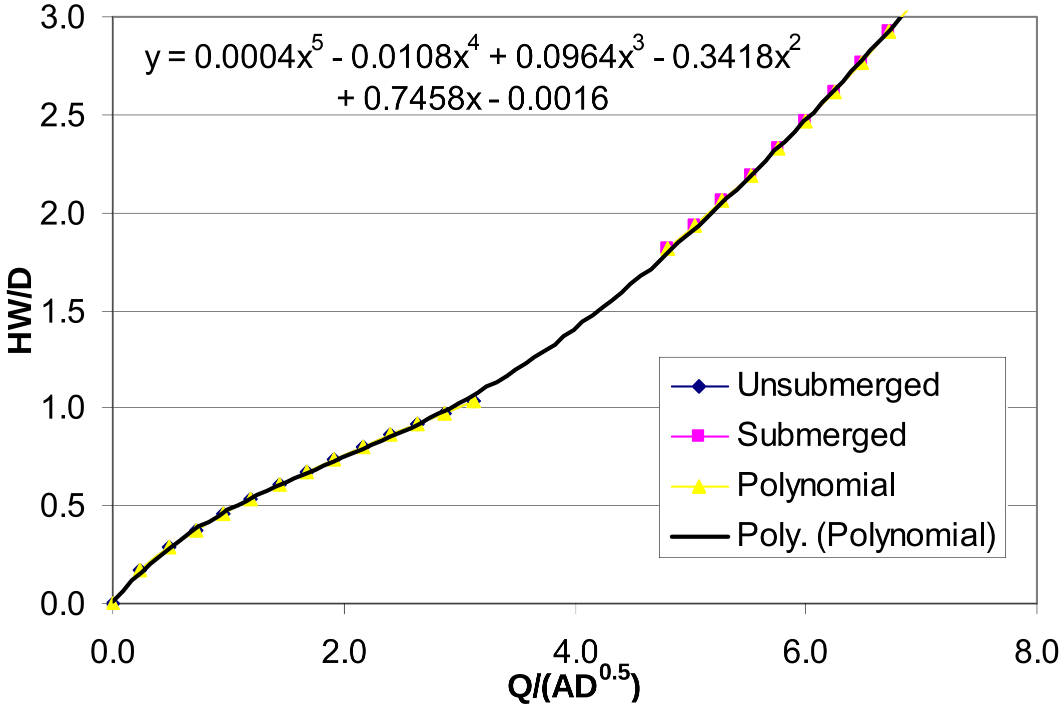


Figure E-22 Thin Edge Projecting: 40% Buried (Ponded)

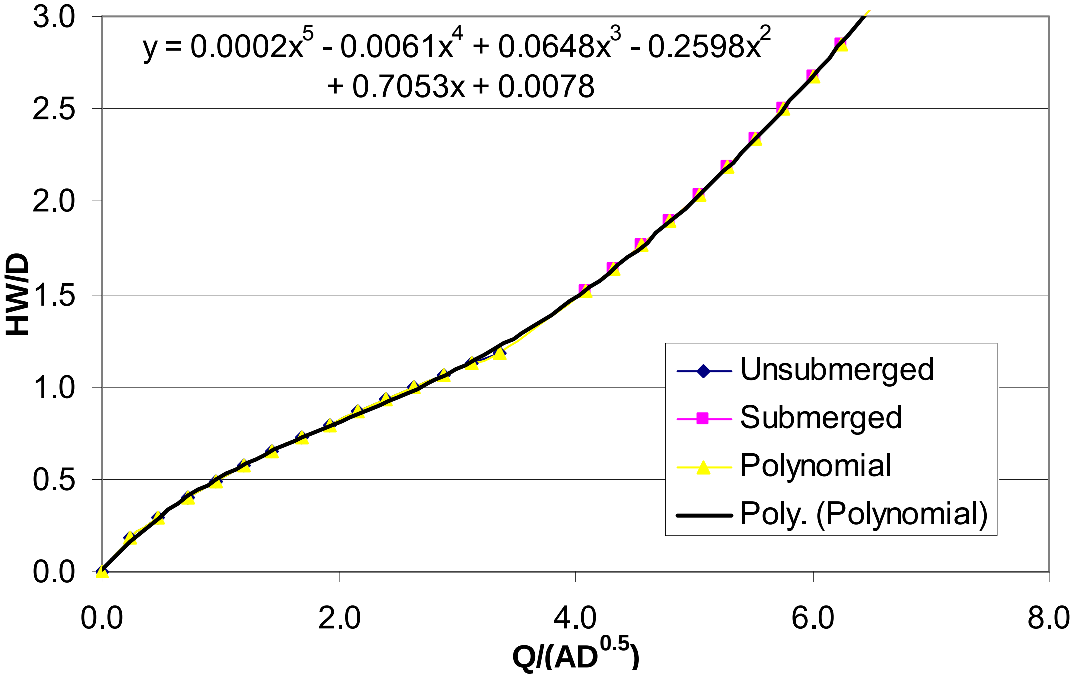


Figure E-23 Thin Edge Projecting: 50% Buried (Ponded)

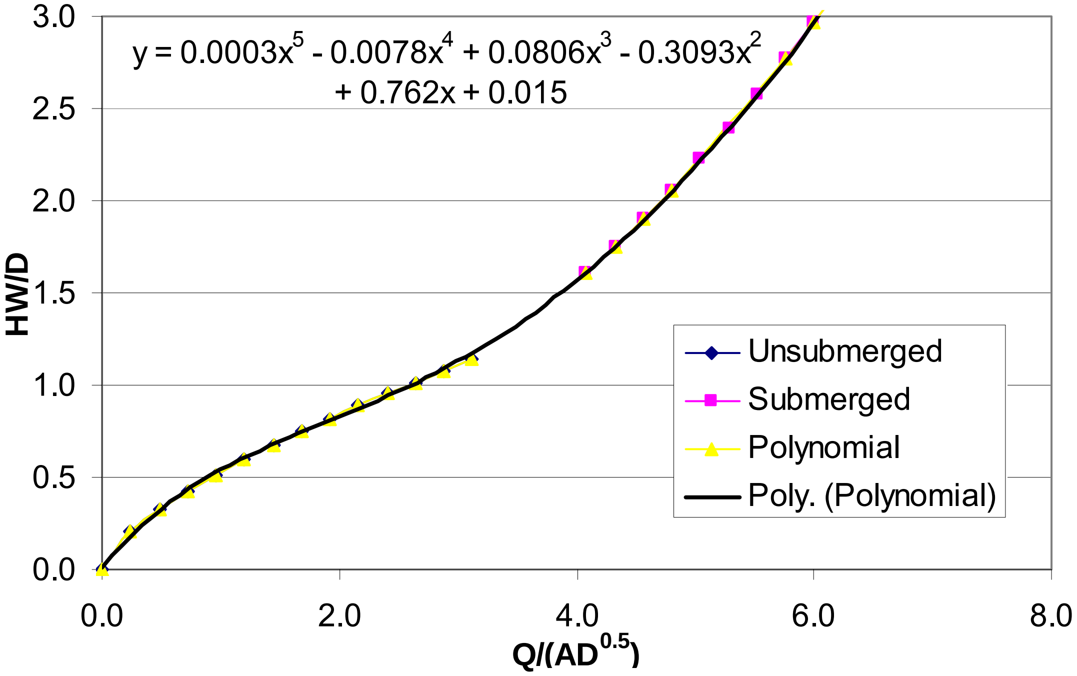


Figure E-24 Thin Edge Projecting: 50% Buried Ellipse (Ponded)

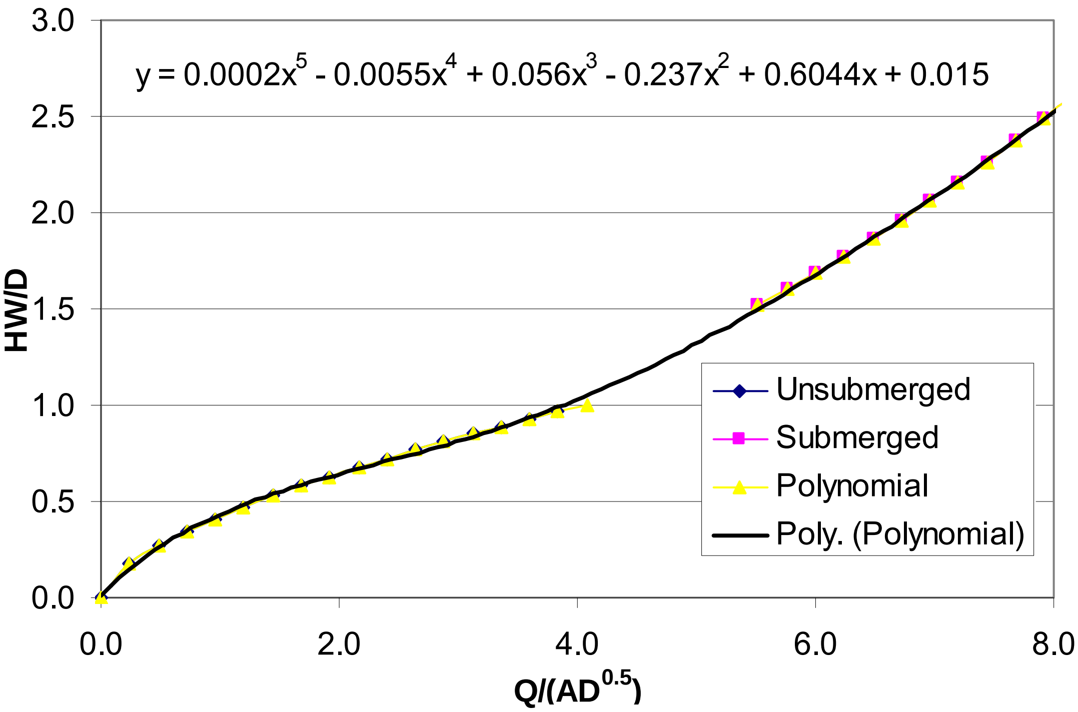


Figure E-25 Thin Edge Projecting: 20% Buried (Channelized)

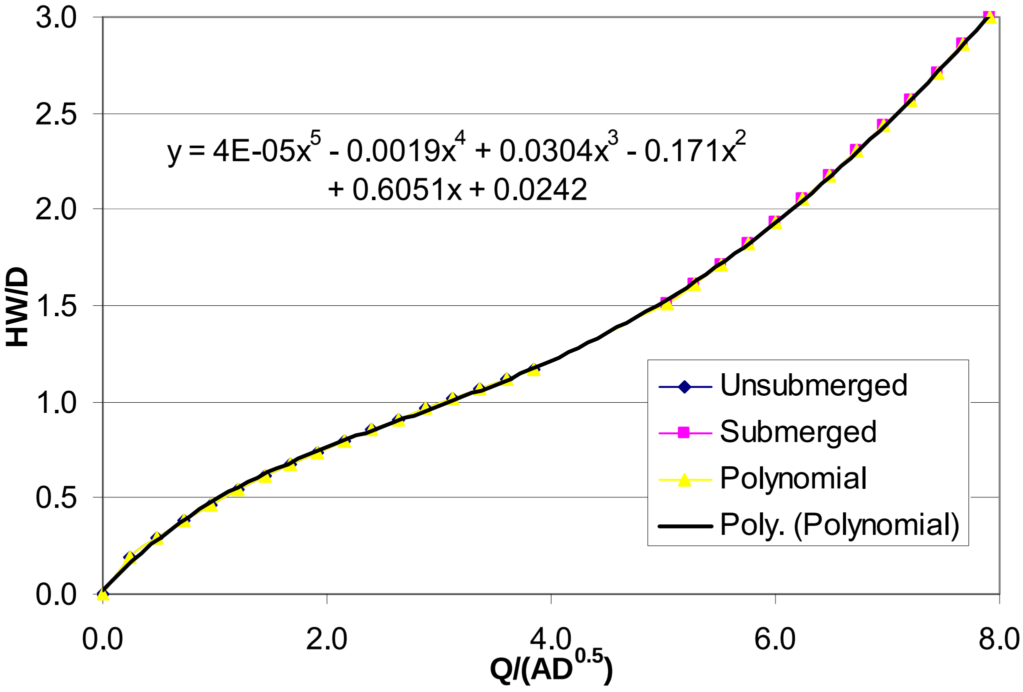


Figure E-26 Thin Edge Projecting: 40% Buried (Channelized)

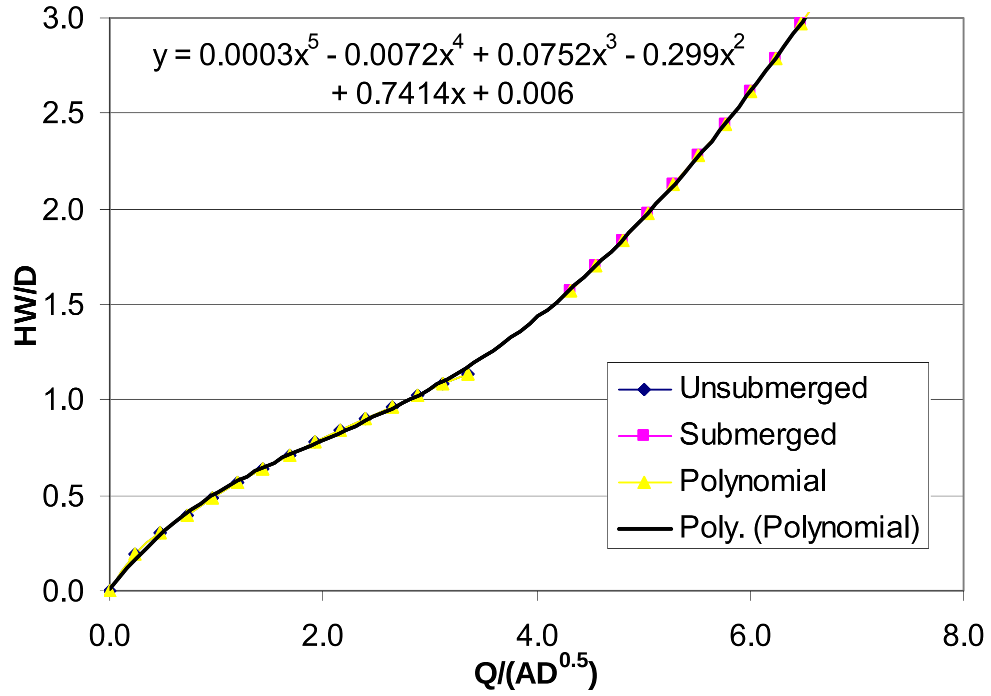


Figure E-27 Thin Edge Projecting: 50% Buried (Channelized)

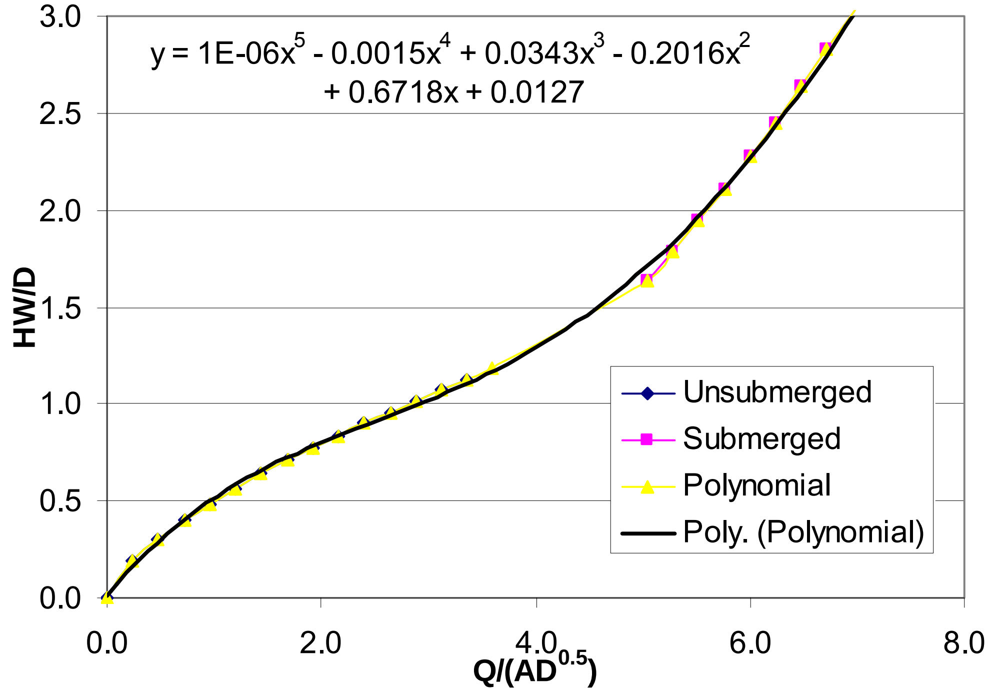


Figure E-28 Thin Edge Projecting: 50% Buried Ellipse (Channelized)

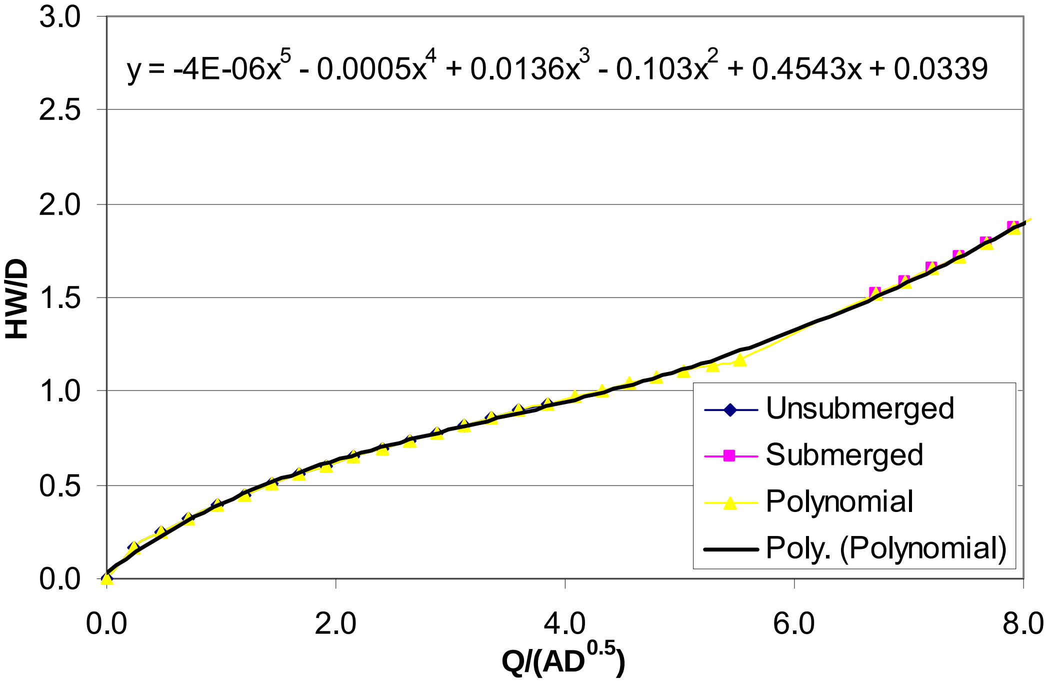


Figure E-29 Mitered to 1.5:1 Fill Slope: 20% Buried

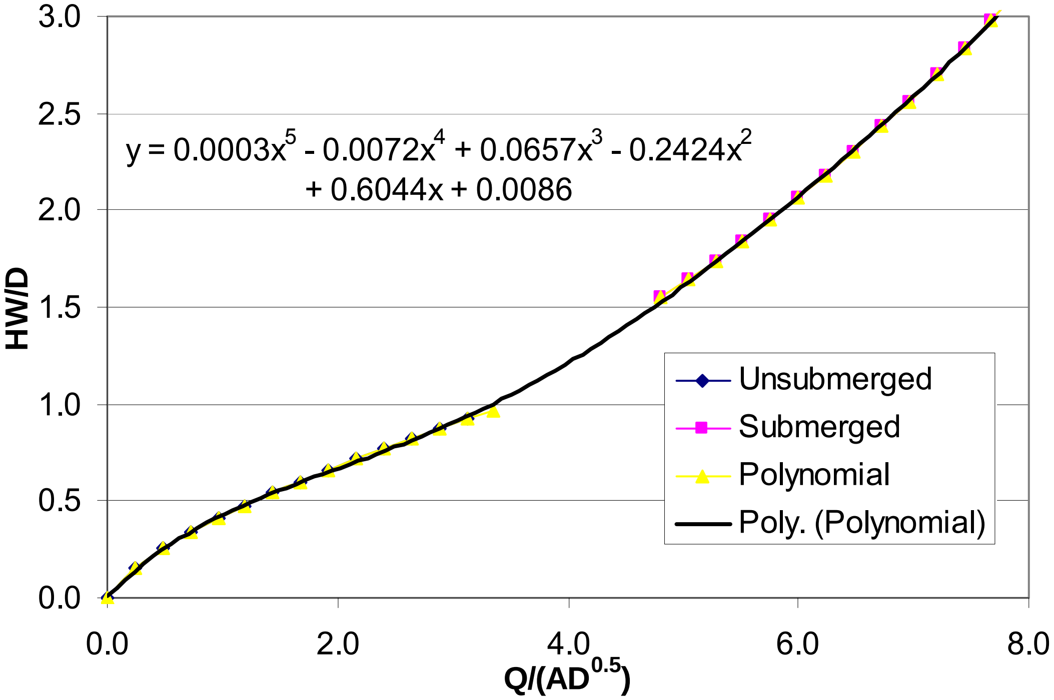


Figure E-30 Mitered to 1.5:1 Fill Slope: 40% Buried

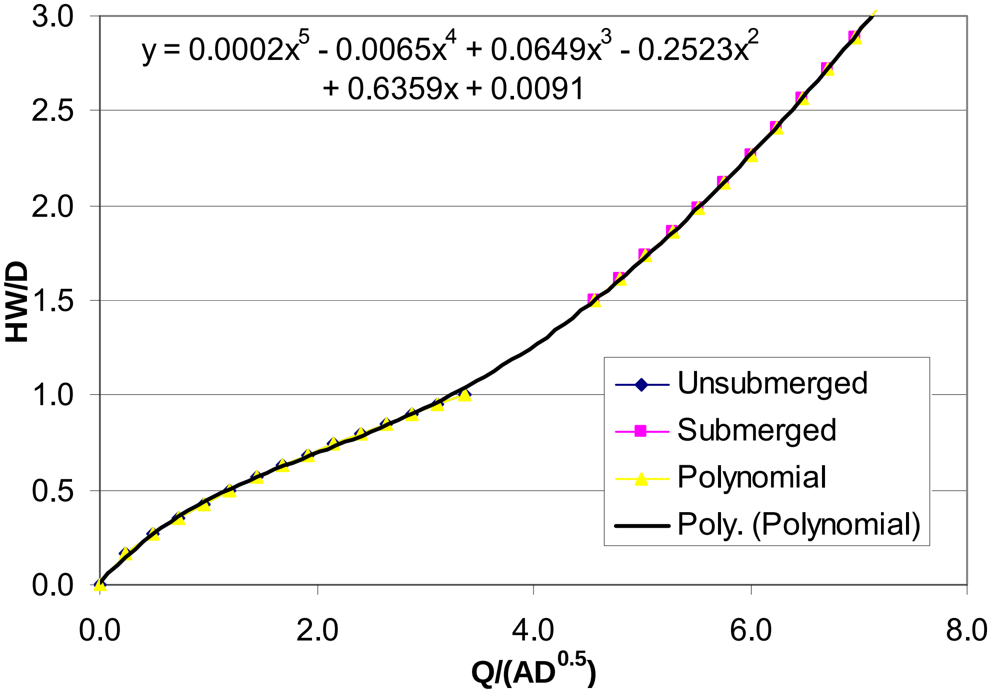


Figure E-31 Mitered to 1.5:1 Fill Slope: 50% Buried

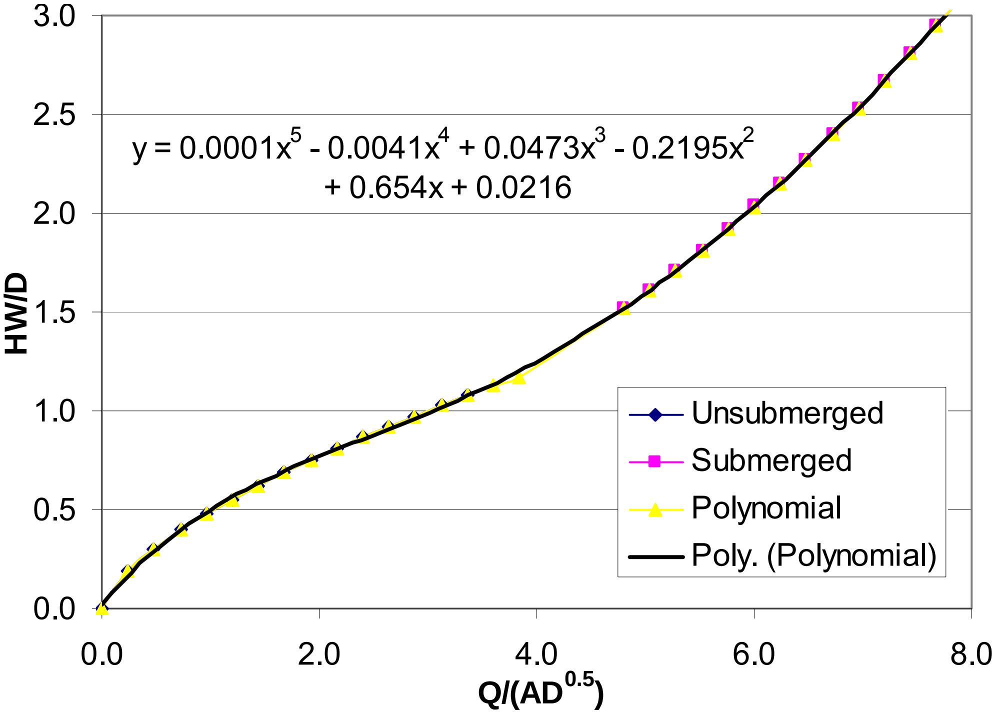


Figure E-32 Mitered to 1.5:1 Fill Slope: 50% Buried Ellipse

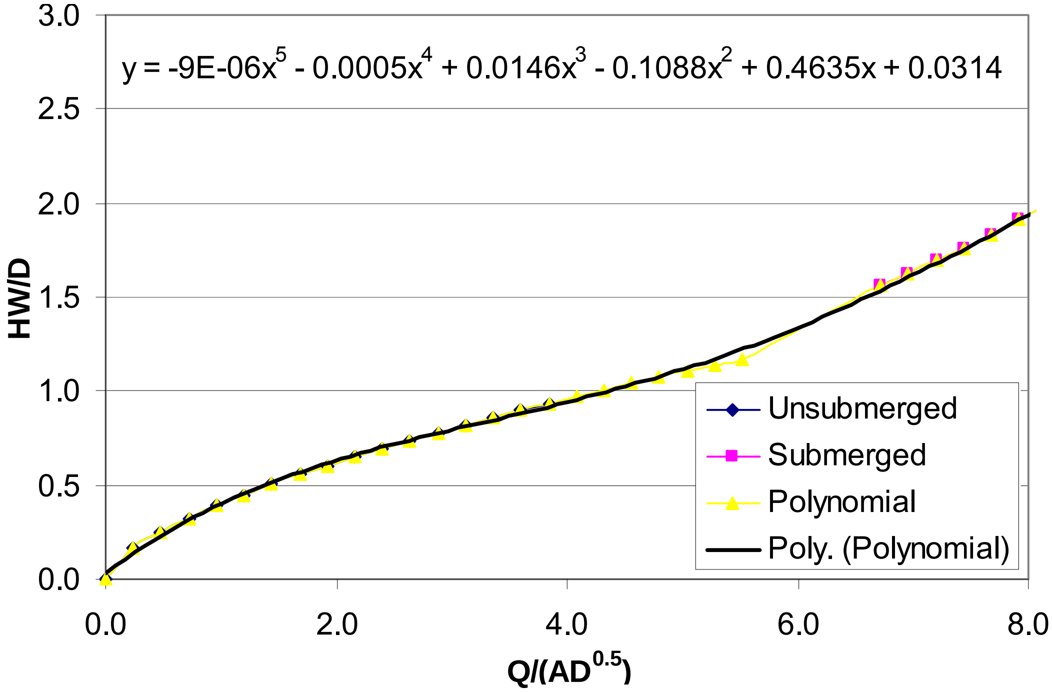


Figure E-33 Square Edge with Vertical Headwall: 20% Buried

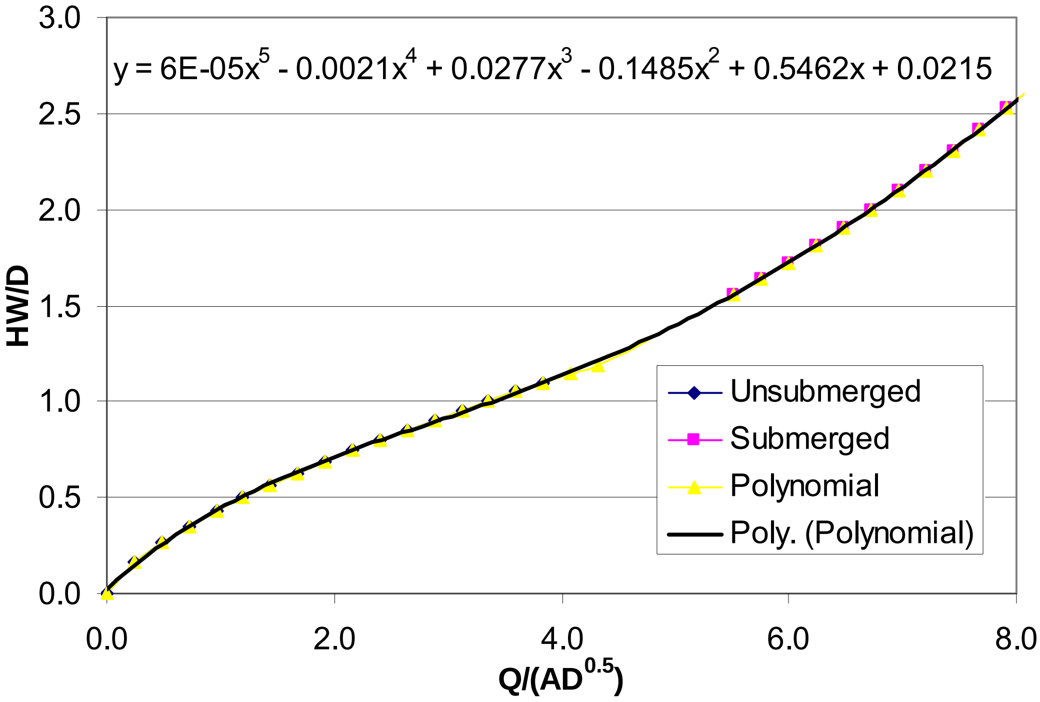


Figure E-34 Square Edge with Vertical Headwall: 40% Buried

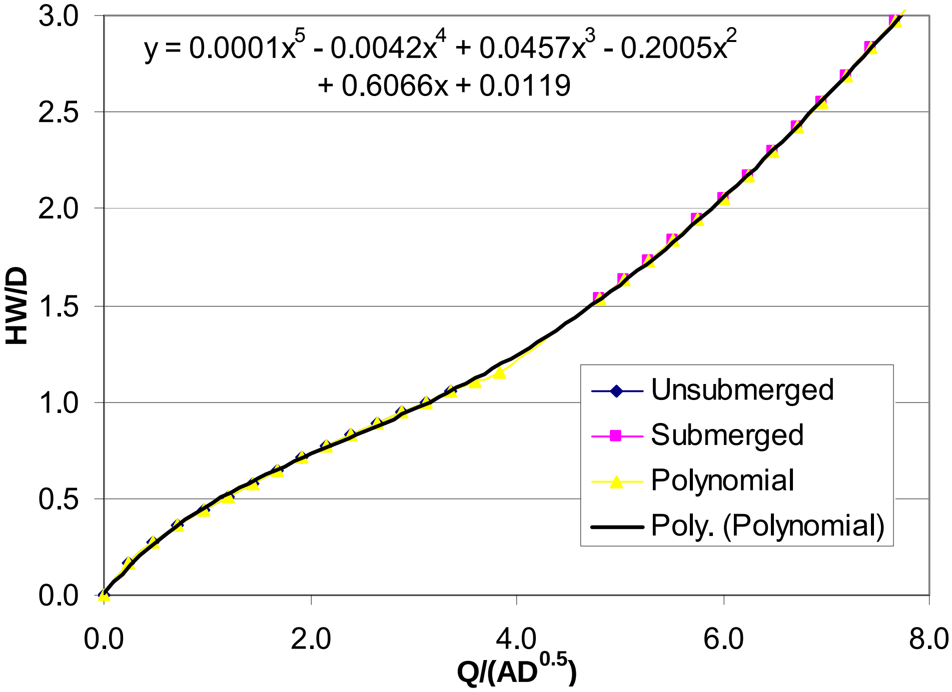


Figure E-35 Square Edge with Vertical Headwall: 50% Buried

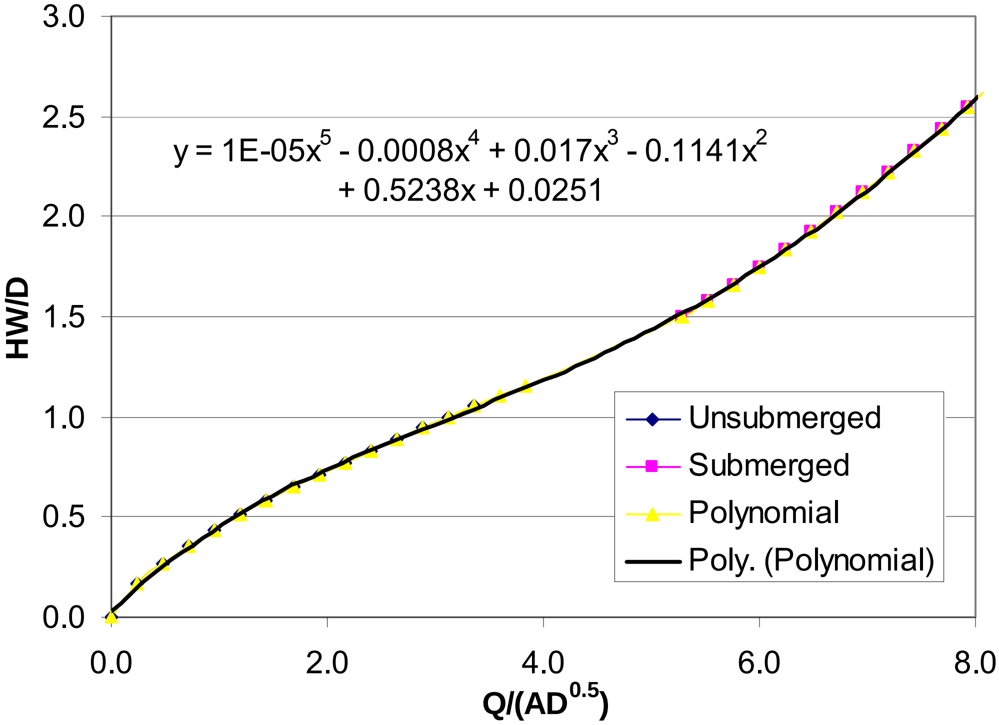


Figure E-36 Square Edge with Vertical Headwall: 50% Buried Ellipse

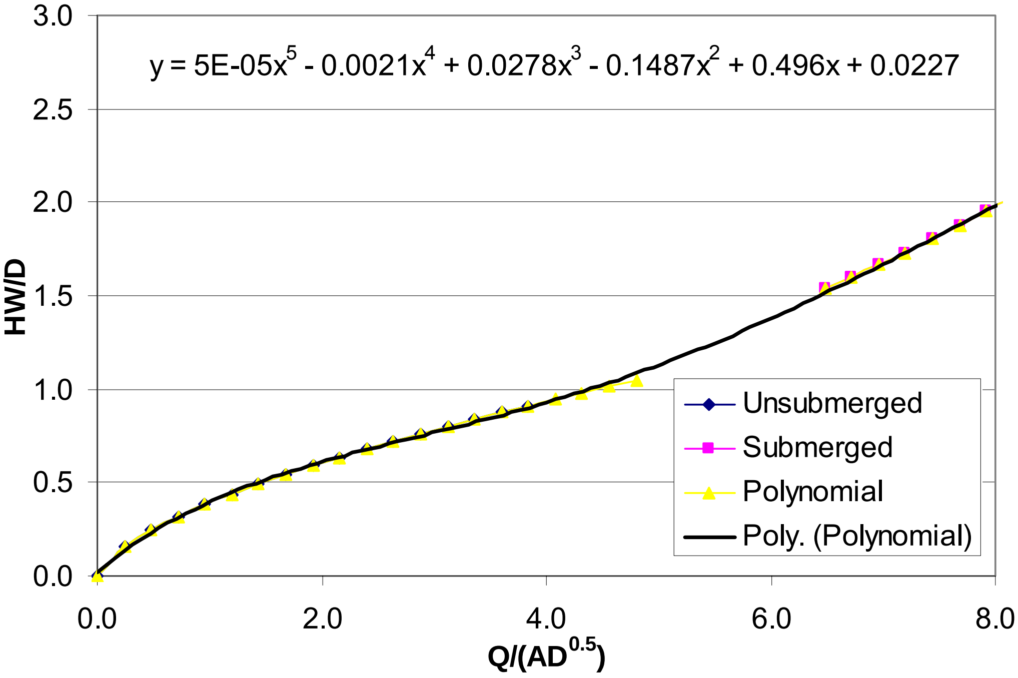


Figure E-37 45^o Bevel with Vertical Headwall: 20% Buried

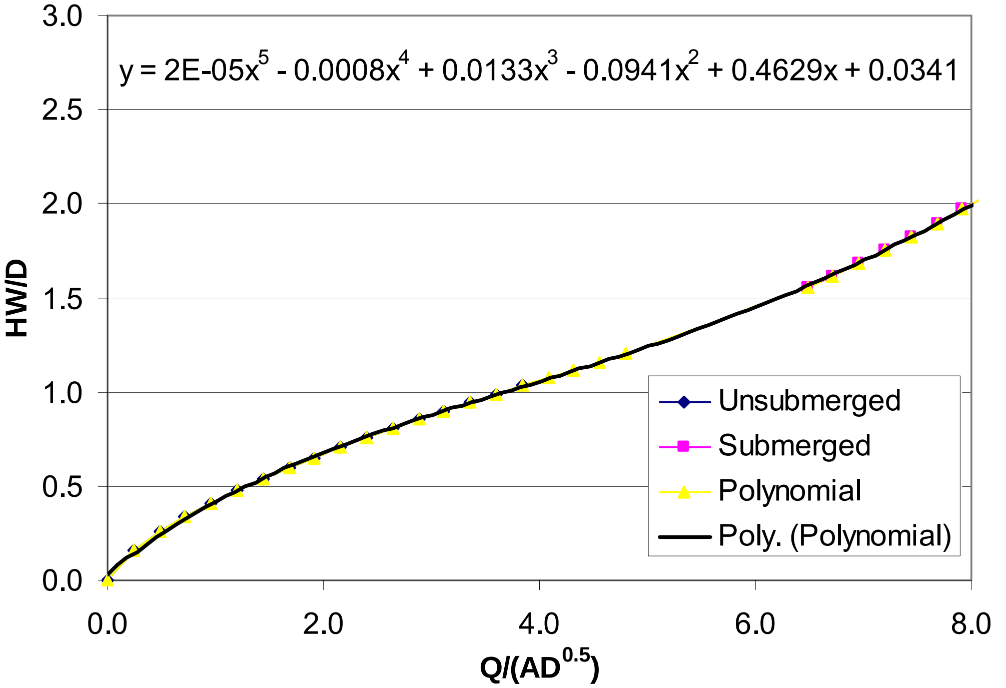


Figure E-38 45^o Bevel with Vertical Headwall: 40% Buried

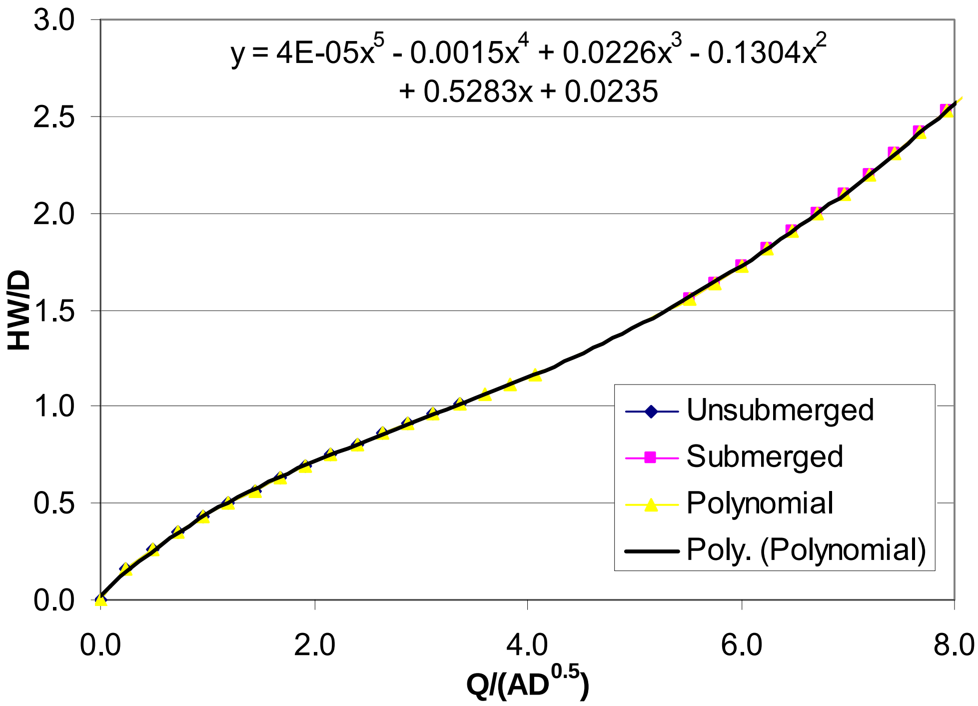


Figure E-39 45^o Bevel with Vertical Headwall: 50% Buried

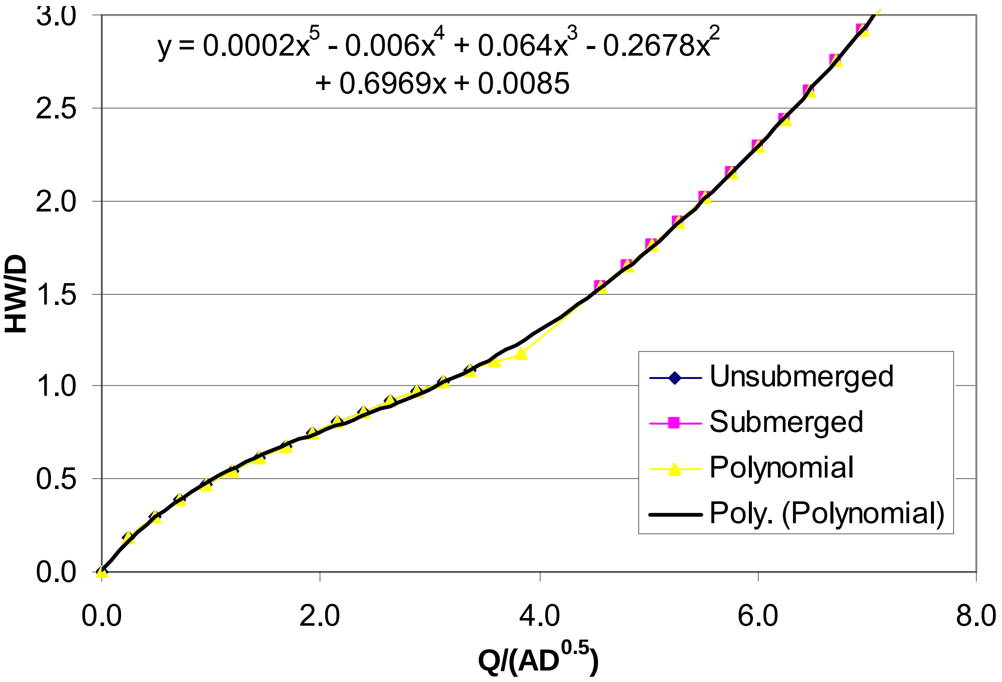


Figure E-40 45^o Bevel with Vertical Headwall: 50% Buried Ellipse

After plotting a best fit curve through all three zones of flow (submerged, unsubmerged, and transition), the coefficients of the new equation were recorded in Table E-9. These coefficients are required for programs that use the fifth degree polynomial curves for computing inlet control headwater depth.

Table E-9 Polynomial Coefficients for Each Entrance Condition

Shape/Edge Condition	Percent Buried	a	b	c	d	e	f
Thin Edge Projecting, (Ponded)	20	0.024	0.5601	-0.1652	0.0302	-0.0021	0.00005
	40	-0.0016	0.7458	-0.3418	0.0964	-0.0108	0.0004
	50	0.0078	0.7053	-0.2598	0.0648	-0.0061	0.0002
	50 (Ellipse)	0.015	0.762	-0.3093	0.0806	-0.0078	0.0003
Thin Edge Projecting, (Channelized)	20	0.015	0.6044	-0.237	0.056	-0.0055	0.0002
	40	0.0242	0.6051	-0.171	0.0304	-0.0019	0.00004
	50	0.006	0.7414	-0.299	0.0752	-0.0072	0.0003
	50 (Ellipse)	0.0127	0.6718	-0.2016	0.0343	-0.0015	0.000001
Mitered to 1.5:1 Fill Slope	20	0.0339	0.4543	-0.103	0.0136	-0.0005	-0.000004
	40	0.0086	0.6044	-0.2424	0.0657	-0.0072	0.0003
	50	0.0091	0.6359	-0.2523	0.0649	-0.0065	0.0002
	50 (Ellipse)	0.0216	0.654	-0.2195	0.0473	-0.0041	0.0001
Square Edge with Vertical Headwall	20	0.0341	0.4635	-0.1088	0.0146	-0.0005	-0.000009
	40	0.0215	0.5462	-0.1485	0.0277	-0.0021	0.00006
	50	0.0119	0.6066	-0.2005	0.0457	-0.0042	0.0001
	50 (Ellipse)	0.0251	0.5238	-0.1141	0.017	-0.0008	0.00001
45° Beveled Inlet with Vertical Headwall	20	0.0227	0.496	-0.1487	0.0278	-0.0021	0.00005
	40	0.0341	0.4629	-0.0941	0.0133	-0.0008	0.00002
	50	0.0235	0.5283	-0.1304	0.0226	-0.0015	0.00004
	50 (Ellipse)	0.0084	0.6969	-0.2678	0.064	-0.006	0.0002

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