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Secondary Preservice Mathematics Teachers' Curricular Reasoning

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ABSTRACT

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Researchers have found that teachers' decisions affect students' opportunity to learn. Prior researchers have investigated teachers' decisions while planning, implementing, or reflecting on lessons, but few researchers have studied teachers' decisions and their reasoning throughout the teaching process. It is important to study teachers' reasoning for *why* they make the decisions they do throughout the teaching process. Furthermore, because inservice and preservice teachers differ in experience and available resources that they draw on while making decisions, it is helpful to consider the resources PSTs' draw on while reasoning. *Curricular reasoning* is a framework that describes teachers' thinking processes when making decisions during the teaching process. This study investigated secondary preservice teachers' decisions and curricular reasoning throughout the teaching process. Data were collected from two groups of secondary preservice teachers in a mathematics methods course focused on student thinking and mathematics. Results revealed that the preservice teachers used all seven curricular reasoning strands, especially *drawing on mathematical meanings*, *mapping learning trajectories*, and *considering learners' perspectives*. Specifically, this study demonstrates ways in which preservice teachers reason about their decisions and the intertwined nature of their curricular reasoning. The results from this study also imply that it may be helpful to consider the resources PSTs have access to, including their instructor, and that the order of their lesson planning may allow support for the mathematical learning trajectories within individual lessons. This study also provides validation for the curricular reasoning framework described by Dingman, Teuscher, Olson, and Kasmer (in press), provides subcategories of curricular reasoning strands, and has implications for teacher education.

Keywords: curricular reasoning, secondary preservice mathematics teachers, teacher education, planning lessons, implementation of lessons, resources

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CHAPTER 1: RATIONALE

Teachers' decisions during the teaching process can affect the mathematical content that students have the opportunity to learn (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Remillard & Bryans, 2004; Stein, Smith, Henningsen, & Sliver, 2000). Figure 1 (adapted from Stein, Remillard, & Smith, 2007) displays the teaching process that teachers go through as they make decisions and transform curricula (e.g., Bernard, 2017; Callopy, 2003; Stein et al., 2000). All of these decisions teachers make impact students' opportunities to learn mathematics.

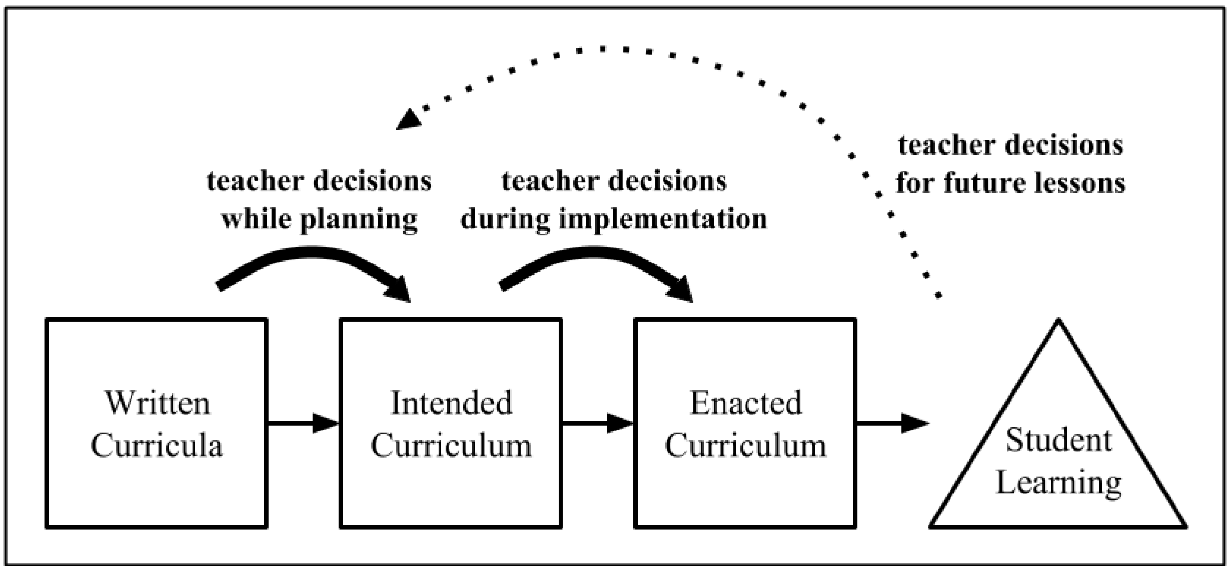

Figure 1. The teaching process that transforms curricula and affects students' opportunity to learn (adapted from Stein et al., 2007).

Research that analyzes teachers' mathematical decisions at multiple decision-points (i.e., while planning lessons, while implementing lessons, while reflecting on lessons) during the teaching process provides a more complete view of teachers' decisions than considering only one decision point within the teaching process. From a review of research on curriculum use, Remillard (2005) identified four ways teachers use written curricula as they make decisions about how to plan mathematics lessons: following or subverting, drawing on, interpreting, and participating with. *Written curricula* is defined as any "printed or electronic, often published, materials designed for use by teachers and students before, during, or after mathematics instruction" (Stein et al, 2007, p. 232). Stein, Grover, and Henningsen (1996) found that teachers often make decisions while implementing mathematics lessons that tend to lower the cognitive demand of high-level tasks. In both these studies, the researchers identified teachers' decisions when using curricula at one decision-point, either while planning or while implementing lessons. To build on this research, it is important to investigate teachers' decisions at multiple decision-points throughout the teaching process because investigating only one decision-point may not provide a complete picture of teachers' decision-making, including their reasoning for these decisions.

Often teachers' decisions are modified as they engage in the teaching process and this may leave observers wondering *why* teachers made certain decisions. For example, a teacher may decide to use a definition from a written curriculum while planning a lesson; but as she implements her lesson she modifies her decision to not include the definition from the written curriculum as she had planned. The obvious question is: why did she decide to not include the definition in her lesson? This example suggests the importance of investigating teachers' decisions throughout the teaching process, but it also suggests that investigating teachers' reasoning for their decisions is important as well to understand why teachers modify or do not modify their decisions throughout the teaching process.

Researchers have begun to investigate teachers' reasoning for their decisions as they plan and implement lessons. Roth McDuffie and colleagues identified this reasoning as teachers' *curricular reasoning* and defined it as the "thinking process that teachers engage in as they…plan, implement, and reflect on" lessons (Breyfogle, Roth McDuffie, & Wohlhuter, 2010,

p. 308; Roth McDuffie & Mather, 2009). Two research groups have identified seven thinking process that teachers reason with as they make decisions during the teaching process (Dingman, Teuscher, Olson, & Kasmer, in press; Roth McDuffie & Mather, 2009). This research provides an initial framework to describe teachers' curricular reasoning throughout the teaching process.

Although teachers engage in curricular reasoning as they make decisions, other researchers (e.g., Bush, 1986; Schoenfeld, 2015) have identified various resources that teachers draw on while making their decisions. *Resources* are entities that are either internal (e.g., knowledge, experience) or external (e.g., curriculum materials, other people) to a teacher. Some resources that teachers draw on as they make decisions are knowledge, curricula, observing other teachers, university instructors, university methods courses (e.g., Bush, 1986; Schoenfeld, 2015). While resources are available to all teachers it may be the case that preservice secondary teachers (PSTs) draw on different resources than inservice teachers. Thus, considering the resources that PSTs draw on while reasoning about their decisions throughout the teaching process is another area of research to investigate so we can better support PSTs as they entering the teaching profession.

Researchers have investigated PSTs decisions at one decision point during the teaching process, yet investigating their decisions at multiple decision points throughout the teaching process would assist mathematics teacher educators in understanding PSTs' decision-making and their reasoning for their decisions. Researchers have investigated PSTs decisions as they plan with curriculum materials (e.g., Gadanidis, Gadanidis, & Schindler, 2003; Males, Earnest, Dietiker, & Amador, 2015); however, fewer researchers have investigated PSTs' decisions during the implementation of curriculum (e.g., Nicol & Crespo, 2006) and no studies that I know of report on PSTs' decisions and reasoning throughout the teaching process. We are aware that

PSTs have access to different resources than inservice teachers, including their teaching experience (e.g., Borko & Livingston, 1989; Suh & Parker, 2010) and mathematical knowledge (e.g., Forbes & Davies, 2007); therefore, I conjecture that their curricular reasoning will also differ. Thus, understanding PSTs decision-making process provides teacher educators with more information about the decisions and ways in which PSTs reason and areas in which they may need more support.

In summary, this study about PSTs' curricular reasoning as they make decisions throughout the teaching process is valuable for multiple reasons. First, I consider multiple decision points throughout the teaching process as opposed to one decision point, which provides a clearer picture of PSTs' decision-making. Second, I identify PSTs' curricular reasoning for these decisions as well as the resources PSTs draw upon while reasoning and making decisions. The results of this study will provide teacher educators with understanding about the types of decisions PSTs make throughout the teaching process and their reasoning for these decisions. The results will also provide information about the resources that PSTs' draw upon while reasoning when making decisions. Thus, teacher educators can use the results of this study to better support PSTs' development in decision-making throughout the teaching process.

CHAPTER 2: BACKGROUND

This chapter addresses literature related to teachers' decisions during the teaching process and the need to investigate teachers' curricular reasoning for their decisions. I also outline the theoretical framework used for this study and state the research question for this study.

Literature Review

Teachers make decisions during the teaching process that affect student opportunities to learn (Huntley et al., 2000; Remillard & Bryans, 2004; Stein et al., 1996). Remillard and Bryans (2004) found that teachers' decisions to use or adapt a reform-oriented curriculum during the teaching process led to differences in the enactment of mathematics lessons, thus affecting the mathematical learning opportunities for students. Additionally, Stein et al. (1996) found that teachers' decisions during the implementation of their lessons often decreased the cognitive demand of the mathematical task, which affects students' opportunity to learn. In both studies the authors found that students' opportunities to learn mathematics were affected, yet *why* these teachers made their decisions was not investigated.

Teachers make decisions throughout the teaching process (e.g., Remillard, 2000; Sherin & Drake, 2004; Son, 2013). For instance, Sherin and Drake (2004) found that teachers made decisions *while planning* to omit or replace portions of curriculum materials based on their anticipation of student thinking. Additionally, these researchers found that the teachers also made decisions *during their lesson implementation* about the mathematical content they would include or exclude during the lesson based on their awareness of students' thinking. Therefore, when investigating teachers' decisions, it is useful to consider all possible decision-points during the teaching process.

Unfolding of Teachers' Decisions and Reasoning Throughout the Teaching Process

Teachers modify their decisions throughout the teaching process (e.g., Nicol & Crespo, 2006; Remillard, 2000; Sherin & Drake, 2004). Remillard (2000) found that one teacher decided to have students explore different representations of numbers, yet during the lesson the students brought up questions about place value. After reflecting on the lesson, this teacher decided to include place value in future lessons based on her lesson. Thus, often experiences during the implementation of a lesson lead teachers to modify their lesson when planning future lessons. Therefore, understanding teachers' reasoning for their decisions could provide an understanding of what drives teachers' decision-making and what resources teachers draw on as they reason during this decision-making.

Often teachers' intended or enacted curricula differ from the written curricula that they begin the teaching process with (e.g., Freeman & Porter, 1989; Manouchehri & Goodman, 1998; Sosniak & Stodolsky, 1993). Freeman and Porter (1989) found that none of the four teachers in their study taught lessons exactly as outlined in their textbooks and that these four teachers taught differing amounts of content from their textbooks. These teachers emphasized content closer to the textbook when they had an orientation to follow the textbook, but all of them followed the sequence of topics as presented in the textbook with few exceptions. This suggests that the extent that teachers follow a written curriculum varies. We also know that teachers' decisions about what and how to teach the content may come from the written curriculum, but often they make decisions to teach content differently than the written curricula. This begs the questions *why* do teachers follow the written curricula and *why* do teachers deviate from the written curricula?

Teachers' decisions throughout the teaching process can affect students' opportunities to learn (e.g., Brown, 2002; Stein et al., 2000). Stein et al. (2000) investigated the level of cognitive demand of mathematics tasks for students during the teaching process. Many teachers planned their mathematics tasks with a high level of cognitive demand of *doing mathematics*, but made decisions while implementing their lessons that lowered the cognitive demand of the task. For example, when a teacher saw her students struggling with the mathematical task in her lesson she showed her students a strategy to use when doing the problem and this decision caused her task which was planned and set-up in the classroom as *doing mathematics* level to become a *procedure without connections* level during the implementation of her lesson, thus lowering the cognitive demand of this task. When teachers' decisions throughout the teaching process lowers the cognitive demand of the task in the lesson, this modifies the mathematics that students have the opportunity to learn from.

Resources Teachers Draw Upon During the Teaching Process

Researchers have also found that teachers draw on various resources during the teaching process (e.g., Behm & Lloyd, 2009; Bush, 1986; Remillard & Bryans, 2004). Resources are entities that teachers draw on when reasoning rather than the thinking processes that teachers go through when making a decision. Behm and Lloyd (2009) found that PSTs drew upon curriculum materials, other PSTs, cooperating teachers, their knowledge of mathematics, and experiences in their PST education as they made decisions about the mathematics content to teach students. In contrast, Remillard and Bryans (2004) found that inservice teachers drew on their pedagogical repertoires to decide how to use curriculum materials during the teaching process. This suggests that PSTs and inservice teachers may draw upon different resources while making decisions during the teaching process. This is important because as Behm and Lloyd

(2009) conjectured based on their study on PSTs' use of curriculum materials, "differences in the availability of human and material resources" could be a contributing factor in the differing ways that PSTs made decisions compared to that of inservice teachers. These findings suggest that PSTs draw on resources while they make decisions and that it is useful to consider the resources PSTs draw on while reasoning as they make decisions because it may influence their reasoning during the teaching process.

Theoretical Framework: Curricular Reasoning

This study used the theoretical framework of curricular reasoning to frame PSTs' reasoning for their mathematical decisions during the teaching process. Roth McDuffie and colleagues (2009, 2010) identified *curricular reasoning* to describe teachers' reasoning about curricular decisions. Roth McDuffie and Mather (2009) defined curricular reasoning as "a more specific form of pedagogical reasoning…, but where curricular goals and materials remain a primary focus…throughout the reasoning process" (p. 306). Breyfogle et al. (2010) extended this definition to "the thinking processes that teachers engage in as they work with curriculum materials to plan, implement, and reflect on instruction" (p. 308). I view curricular reasoning as a combination of the two definitions: as the thinking processes that teachers engage in throughout the teaching process related to not just curriculum *materials* but also *intended and enacted curricula*. I use this definition because curricular reasoning is not only about curricular materials but also about teachers' curricular *goals*, which includes teachers' intended and enacted curricula.

The curricular reasoning framework includes seven strands found in Table 1, identified by Roth McDuffie and Mather (2009), Dingman, Teuscher, and Olson (2019), and Dingman et al. (in press). Roth McDuffie and Mather (2009) identified "analyzing curriculum materials from learners' perspectives" (p. 308) as teachers' reasoning about student thinking or difficulties students may have based on the curriculum materials. Dingman et al. (in press) split this reasoning into two strands because teachers may analyze curriculum materials for reasons other than learners' perspectives and teachers may consider learners' perspectives but not from the curriculum materials. These two strands are: (1) *analyzing curriculum materials* and (2) *considering learners' perspectives*. *Analyzing curriculum materials* is defined as when teachers reason by analyzing, comparing, or critiquing curriculum materials. It is important to note that this strand is when teachers *analyze* curriculum materials rather than *use* the curriculum. The second strand, *considering learners' perspectives,* is defined as when teachers reason about student thinking related to the mathematics of the lesson (Dingman et al., in press).

Roth McDuffie and Mather (2009) identified the curricular reasoning strand "revising plans based on work with students during instruction" (p. 312) as when teachers reason about the results of the implemented lesson during the teaching process and make modifications for future lessons. Dingman et al. (in press) expanded this to *revising based on experiences in teaching and learning* to include any experience in teaching and learning (e.g., observing someone teaching, experiencing a lesson with students) that teachers reason about as they made decisions.

Roth McDuffie and Mather (2009) identified two more curricular reasoning strands. One is "doing the task together as learners" (p. 310) – when teachers "act[ed] out what students might do when solving" a task (p. 310) in groups. This strand *doing tasks as learners* was expanded to include both when PSTs did the task as learners individually or in a group and when PSTs did the task as learners verbally as well as written, because these moments impacted PSTs' decisions just as when they did the task as a learner written down in a group.

Another strand was *mapping learning trajectories* – when teachers outlined learning trajectories that drew on students' prior knowledge, curriculum materials, and grade level expectations (Roth McDuffie & Mather, 2009). Dingman et al. (in press), modified this strand defining it to be when teachers considered the sequencing of lessons or building of mathematical content because it captured the underlying activity as described by Roth McDuffie and Mather (2009) and teachers' reasoning did not always consider students, curriculum materials, and grade level expectations when making these decisions.

Two other curricular reasoning strands that Dingman et. al (2019) and Dingman et. al (in press) identified were *positioning with regards to the mathematics* and *drawing on mathematical meanings*. *Positioning with regards to the mathematics* is defined as when a teacher's decisions is based on an explicit belief (Dingman et al., 2019). *Drawing on mathematical meanings* is defined as when teachers reason with their own mathematical meaning or the mathematical understanding they want for their students (Dingman et al., in press). To summarize these strands, Table 1 lists the seven curricular reasoning strands and their definitions for the curricular reasoning framework (Dingman et al., 2019; Dingman et al., in press; Roth McDuffie & Mather, 2009) used in this study.

Table 1

Curricular Reasoning Strands and Definitions

Strand	Definition
Analyzing Curriculum Materials	Teachers reason by analyzing, comparing, or critiquing curriculum materials.
Considering Learners' Perspectives	Teachers reason about student thinking related to the mathematics of the lesson.
Doing Tasks as Learners	Teachers reason by "acting out what students might do when solving" a task (Roth McDuffie & Mather, p. 310) either individually or as a group and either verbally or written down.
Drawing on Mathematical Meanings[1]	Teachers reason with their own mathematical meaning or the mathematical understanding they want for their students.
Mapping Learning Trajectories	Teachers reason by considering either the building of mathematical content (i.e., how a mathematical concept in their lesson connects to past and future topics) or the sequencing of lessons or units.
Positioning with Regards to the Mathematics	Teachers reason with an explicit belief to make a decision that is related to the mathematics in the lesson.
Revising Based on Experiences in Teaching and Learning	Teachers reason about any experience in teaching and learning (e.g., observing someone teaching, experiencing a lesson with students) as they make decisions.

In my study, I sought to understand the ways that PSTs reasoned, including the resources they drew on while reasoning and making their decisions. Identifying these resources is beneficial to make more sense of the ways in which PSTs' reason as they make decisions. For example, PSTs solely using a textbook to identify various student perspectives in their lesson provides a more detailed understanding of the ways in which the PSTs' reason with the strand *considering learner perspectives*.

In summary, the curricular reasoning framework includes seven strands that were identified in the research that teachers reason with as they make mathematical decisions during

[1] i.e., one's personalized mathematical understandings (Thompson, 2016; Byerley & Thompson, 2017)

the teaching process. Identifying teachers' mathematical decisions allowed me to identify the ways in which PSTs reasoned about those decisions during the teaching process. Identifying the resources PSTs' draw on during their reasoning with these strands provides insight into the ways that PSTs reason. Thus, in this study I focus on the research question: In what ways do PSTs engage in curricular reasoning as they make mathematical decisions during the teaching process?

CHAPTER 3: METHODS

In this chapter I describe the methods for my study. First I describe how I selected the participants and distinguishing characteristics among them. Next, I describe the data collection and analysis used to answer my research question.

Participants and Context

I purposefully selected PSTs enrolled at a private university in the mathematics education methods course during the fall of 2017 to participate in my study. An advantage for conducting research with the PSTs at the selected university was they were required to take five courses from the mathematics education faculty that strive to help PSTs learn to teach mathematics with a focus on student learning. The methods course is the last in the five courses sequence and the semester prior to student teaching and PSTs were introduced to unfamiliar mathematics curricula to plan and teach multiple lessons throughout the semester. Schoenfeld (2011) suggests that when people are placed in new situations their thinking and reasoning are more explicit. Therefore, the methods course was a good place to investigate the ways that PSTs make decisions and their reasoning for these decisions.

PSTs were required to collaboratively plan six secondary mathematics lessons in lesson study groups during the semester in the methods course. The PSTs planned and implemented lessons that were from a variety of different mathematical content areas. For each lesson, one PST from the group was assigned to implement their planned lesson to the other PSTs in the course (i.e., the "students" were the other PSTs in the course), while the other two PSTs in the lesson study group observed the lesson rather than participating as "students". For the rest of this thesis, I refer to the PSTs who acted as "students" as students so that they are not confused with the PSTs in the two groups who were the participants in my study. Each PST in the course implemented their planned lesson during one of the first three lesson study groups and again during one of the last three lesson study groups. As part of the course, PSTs were required to spend time during lab hours each week planning their lessons and often used additional time outside of lab hours to plan their lessons. These planning times allowed me to both hear and see PSTs' decisions and reasoning.

Furthermore, I had participated in this same course as a student two years prior, so the intricacies of the process were familiar to me and assisted me in collecting meaningful data. I knew the process of planning that PSTs went through and how soon I needed to contact participants to capture their first planning meeting related to their assigned lesson. I also shared with the participants that I knew that planning these lessons was challenging and I was not judging their decisions or reasons. This helped the participants feel at ease and express their true thoughts during the study.

For my study, I selected two groups of more vocal PSTs with one PST being a member of both groups. I selected two groups, as opposed to just one group, to compare differences and similarities between the two groups' curricular reasoning. One PST was a member of both groups so I could investigate the PST's reasoning based on the makeup of the group. Because the PSTs' lesson study groups were already assigned, I selected groups that had at least two vocal PSTs in them. Having more verbal PSTs allowed me to obtain a richer data set, because I heard the PSTs' thinking during their planning times.

To identify PSTs who were more vocal, I observed the PSTs in the course during the first couple weeks of the semester and discussed any questions I had with members of my committee. Additionally, observing the PSTs helped them become more acquainted with my quiet presence, which allowed me to gather more accurate data later because the PSTs felt comfortable around me.

The first group of PSTs prepared a lesson from the geometric transformation unit using *The University of Chicago School Mathematics Project* (*UCSMP*) *Geometry* textbook (Benson et al., 2009). This unit was the third unit during the first half of the methods course and PSTs had planned different lessons in two previous groups allowing them some experience with the lesson study process. In addition, I chose to investigate this unit because it allowed my study about the ways in which PSTs' reason about curricular decisions to be compared to the middle school teachers' curricular reasoning using the same geometric transformation unit in the NSF project (NSF #1561569).

After selecting and collecting data for the first group of PSTs, I drew from the last two lesson study groups for the semester to determine my second group. I considered the last two units because PSTs were more experienced, having taught and planned four lessons during the course and this offered a contrast to the first group. I chose the lesson that was more unfamiliar mathematics from the Gridville unit in *Mathematics: Modeling Our World* (COMAP) (Garfunkel, Godbold, Pollak, and Consortium for Mathematics and its Applications, 1998) that caused PSTs' thinking and reasoning to be more explicit (Schoenfeld, 2011).

Five PSTs participated in this study and consisted of four female students; with pseudonyms Addie, Bridget, Carrie, and Deidra; and one male student, Eric. Addie, Bridget, and Carrie were in the first group of PSTs. All the PSTs in group 1 had participated in two different lesson study groups with non-geometric topics, but this was the first time that Addie, Bridget, and Carrie had worked together. Addie and Bridget had both taught one lesson in their methods course and Carrie had not taught any lessons in the methods course. Thus, the group knew that

Carrie would teach their lesson they were planning. Their assigned lesson was on translations as a composition of reflections from *UCSMP* (Benson et al., 2009) and it was the third in the unit. Two other PST groups taught the first two lessons about reflecting points, reflecting figures and the properties of reflections.

Additionally, Bridget was retaking the methods course (had dropped the course half way through the semester), so she had experienced this same unit one year prior. From her past experience in the course; she had participated in three additional lesson study groups, had experienced the *UCSMP* (Benson et al., 2009) unit as a student, and participated in a lesson study group that planned and implemented the last lesson of the unit on isometries. On the other hand, Addie and Carrie had never seen the *UCSMP* (Benson et al., 2009) materials nor experienced this unit before their lesson study group.

Carrie, Deidra, and Eric were in the second group of PSTs. All the PSTs in group 2 had participated in four different lesson study groups on various lesson topics (e.g., rate of change, linear functions, geometric transformations, and probability). Additionally, this was the first time that Carrie, Deidra, and Eric had worked together. Carrie and Deidra had each taught a lesson during the previous lesson study groups in the second half of the course, but Eric had not. Thus, the group knew that Eric would teach their lesson in their methods course. Group 2's lesson was on minimizing the maximum distance in linear village from the *COMAP* Gridville (Garfunkel et al., 1998) unit. Group 2's lesson was the second in the unit, with another PST group teaching the first lesson – an introduction to Gridville and minimizing the total distance strategy in Linear Village (i.e., a simplified version of Gridville). Furthermore, none of the PSTs in group 2 had seen the *COMAP* materials (Garfunkel et al., 1998) nor experienced this unit before their lesson study group.

Data Collection

Data collection focused on capturing PSTs' decisions and their reasoning during the teaching process for their assigned lessons. I provide the timeline for when data were collected followed by details about how and why specific data were collected.

Figure 2 displays the timeline of PSTs participation in their lesson study group during the teaching process for a given lesson on the top row and the data that were collected during each part of the PSTs' group lesson study on the bottom row. PST groups held one lesson planning meeting with their group members (LPM 1), then as a group met with their instructor (InM), and then had three to four more group planning meetings depending on the group (e.g., LPM 2, LPM 3). I refer to the PSTs' planning across their meetings (LPM 1, InM, …, LPM 5) as the *planning process*. After all their planning meetings but prior to the teaching of their lesson in their methods course, each group emailed me their completed, joint lesson plan. Also, each PST individually completed an online lesson overview form the day before the lesson was taught so I could prepare to observe their lesson by knowing what to expect from their lesson plan and lesson overview entries. Then PSTs either taught or observed their lesson during their methods course, and I identified moments that did not align with their lesson plan because these decisions were in-the-moment and I could not capture their reasoning for these decisions during the lesson implementation; therefore, I brought up these decisions in the post-interviews. I interviewed each PST in the group individually about the development and implementation of the lesson. One reason for interviewing PSTs individually was to capture each PST's reasoning without one PST dominating or another PST being too timid to express his/her true thoughts and reasoning about the teaching process.

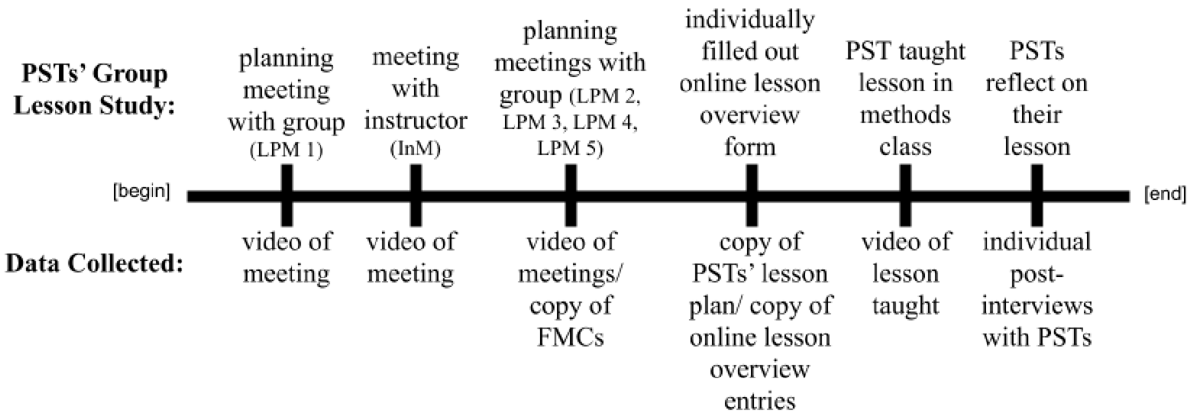

Figure 2. Timeline of PSTs group planning meetings and data collected.

In the following sections, I describe the specific data I collected from both PST groups about the teaching process for their lessons. Interviews and observations of planning meetings were video recorded to capture PSTs' decisions, their reasoning, and any resources that PSTs drew on while reasoning.

Written Documents and Video Data

Drafts of fundamental mathematics concept (FMC). Each group of PSTs was required to collaborate and write an FMC (i.e., a detailed description of the main mathematical content to be taught in a lesson) for their lesson as part of the course. These were submitted to their instructor who provided feedback for the PSTs to use as they revised their FMC. PSTs submitted updated versions of their FMC until their instructor determined that their FMC was satisfactory. I collected the FMC drafts to determine modifications the PSTs made to their FMC through the planning process and to better understand their discussions, decisions, and reasoning related to their FMC that surfaced during their planning meetings. This was in part, because sometimes PSTs would refer to only part of their FMC and the various drafts of their FMCs helped me understand the specifics they discussed.

Videos of all PST planning meetings (e.g., LPM 1, InM, LPM 2). I videoed PSTs planning meetings to identify their mathematical decisions during the planning process, their curricular reasoning, and the resources that they drew on during their curricular reasoning. From the InM I also identified PSTs' decisions and reasoning and in subsequent PST planning meetings (e.g., LPM 2, LPM 3) I determined if PSTs made any mathematical decisions or drew on reasoning from their InM. I explicitly asked PSTs to do as much work collaboratively as possible, to capture their thinking and decision-making as opposed to one of them planning by themselves and the others following.

Video of PSTs group's taught lesson. The PST's teaching video allowed me to identify instances of the teaching PST making in-the-moment mathematical decisions. I then followed-up on these decisions with each PST in the group during their post-interviews, which helped me better understand the PSTs' curricular reasoning related to the implementation of their lesson.

Videos of post-interviews with individual PSTs. Within two weekdays of when the lesson was taught, I conducted a post-interview (Appendix A) with each PST individually in both groups. These interviews allowed me to ask PSTs about specific mathematical decisions during the teaching process that were unclear without further explanation of the PST's reasoning, including asking PSTs about specific in-the-moment mathematical decisions (for the PST who taught) or would have made (for the PSTs who observed) while teaching. These interviews additionally provided insight into future mathematical decisions they would make and their reasoning for these decisions if they were to teach the lesson again. Thus, I gained more information about their reasons for mathematical decisions during the entire teaching process.

Identifying In-The-Moment Mathematical Decisions During Implementation. To identify PSTs' reasoning for in-the-moment mathematical decisions during their lesson implementation, I needed to know decisions the PSTs had already made for their lesson including anticipated student thinking and how they planned to respond to student thinking. This allowed me to identify in-the-moment mathematical decisions the teaching PST made that were not aligned with the lesson plan. These decisions were then asked about during the individual post-interviews.

PSTs' Lesson Plan. I drew upon two documents to help me know the mathematical decisions PSTs' made prior to teaching their lesson: (1) PSTs' written group lesson plan and (2) PSTs' individual online lesson overview entries. These two documents provided me with enough information about the PSTs' intended lesson both as a group and as individuals. Each group of PSTs created one joint written lesson plan, which was required as part of the course syllabus. My purpose for collecting the lesson plan was to document the group's *intended curriculum*. Each PST also individually completed an online lesson overview (Appendix B) form that documented individual PST's thinking and reasoning about the lesson. The reason for collecting these data was it allowed me to get a better grasp of each PST's plan to teach as it resided in their brain and identify whether the PSTs in the group had similar mental outlines for their lesson.

These two documents also allowed me to identify the teaching PST's in-the-moment mathematical decisions during implementation. I compared the group's lesson plan to the individual online lesson overviews prior to the lesson implementation to be aware of inconsistencies among the individual PSTs in terms of their views of the lesson plan, which helped me identify key instances during their lesson implementation that were in-the-moment decisions to follow-up on in the post-interview.

Flagging In-The-Moment Mathematical Decisions. To have an effective post-interview, I flagged in-the-moment mathematical decisions during the PST's implemented lesson. Table 2 lists the types of instances that indicated possible in-the-moment decisions that affected the mathematics of the lesson or mathematical learning opportunities for students. Using these categories required that I had a clear understanding of the PSTs' lesson plan – both as a group and individually. After a lesson was over, there were several flagged instances. I selected three to four instances that seemed to be critical decisions that affected the progression of the mathematics that students had the opportunity to learn or the PST's decisions responding to unanticipated student thinking to use in my post-interviews. For PSTs who taught, I asked them about those decisions that were flagged; and for each PST who observed, I asked what decisions they would have made in the same scenario. Following-up on these in-the-moment mathematical decisions allowed me to better understand PSTs' curricular reasoning for in-the-moment decisions they made during their lesson implementation.

Table 2

Flagging Categories of PSTs' In-The-Moment Mathematical Decisions During Lesson Implementation

Flagging Categories	Description
Deviation from Lesson Trajectory or Goals	PST deviates from their intended lesson plan, lesson goals, or FMC.
Time	PST gives excess or limited time on part of the lesson that impacts students learning.
Glitch, Jump, or Incorrect Mathematics	PST makes a jump in the flow of logic to reach the lesson goal or FMC OR demonstrates incorrect mathematics that relates to the lesson goal or FMC.
Revoicing with Major Assumptions	When revoicing a student comment, PST adds more from their own understanding than what the student said.
Teacher Surprised	PST is surprised at student responses or actions and causes question if the PST anticipated this student thinking.

Note: Table adapted from categories created from National Science Foundation project (NSF # 1561569).

Summary. In summary, both video and written data were collected from the two groups of PSTs throughout the teaching process. Videos of PSTs' planning meetings were collected to document their mathematical decisions and their reasons for those decisions. Before observing the PSTs' lesson implementation, I used the PSTs' written lesson plan and their online lesson overviews to make sense of the PSTs' intention of the lesson to identify instances of the PST's in-the-moment mathematical decisions during their lesson implementation. I flagged in-the-moment mathematical decisions that did not align with the lesson plan or the online lesson overviews, to follow-up on with individual PSTs during the post-interviews. I also followed up on any unclear reasoning for PSTs' mathematical decisions during planning, and any reasoning for PSTs' future mathematical decisions if they re-taught the lesson.

Data Analysis

The data analysis for this study used the curricular reasoning framework (Roth McDuffie & Mather, 2009; Dingman et al., in press). First, I define and explain my unit of analysis, a Big Decision. Second, I explain the labeling of the Big Decisions. Third, I explain the analysis of the data for trends and findings.

It is important to note that I only analyzed the videos from planning meetings – including the instructor meeting – and from the PSTs' individual post-interviews because these videos contained information related to their decisions and their reasoning. Additionally, before dividing my video data into decisions, I separated the parts of the post-interview videos about planning and about PST's reflection on implementation – which included both the implementation of their lesson as occurred in their methods course and decisions about future implementation of the lesson. Separating the data allowed me to compare differences in PSTs' curricular reasoning during different parts of the teaching process.

Decision as Unit of Analysis

My unit of analysis was a *Big Decision*[2]. I am using the term Big Decision as opposed to decision, because Big Decisions were overarching decisions that PSTs discussed and reasoned about in multiple lesson planning meetings. For example, group 1 PSTs made a Big Decision to include the definition of transformation in their lesson. Over the course of multiple lesson planning meetings, the PSTs discussed and reasoned about not defining transformation in their lesson, defining transformation in their lesson, connecting the transformation definition to the definition of translation, and finally deciding to define transformation at *the beginning* of their lesson. These discussions were connected and I was not able to separate the PSTs' reasoning surrounding this Big Decision because the PSTs seemed to change their decision as they discussed the lesson. Another Big Decision group 1 PSTs made was determining an accurate definition for a transformation. This Big Decision was separate from their Big Decision to include the definition of transformation in their lesson because these were two different decisions that PSTs had when making these decisions and it was distinguishable when the PSTs were discussing one Big Decision as opposed to the other.

I defined a *Big Decision* as the mathematical decision(s) or reflection(s) during the teaching process focused around one interconnected idea that affected the mathematical progression of the lesson. I included "reflection" in this definition because sometimes after their lesson implementation, PSTs reasoned about the result of a decision without forming a new decision. For example, PSTs reasoned about whether students reached a mathematical lesson goal after their lesson implementation, which revealed ways in which PSTs were reasoning

[2] I use "Big Decision" with a capitol B and D so that the reader is sure that this is my unit of analysis rather than just any decision.

related to prior decisions of a lesson goals. PSTs' conversation about a Big Decision was often spread over multiple planning meetings and sometimes followed-up on during post-interviews; therefore, all conversation related to a single mathematical decision or reflection, regardless of when it was discussed, was grouped into one Big Decision. Often these Big Decisions began with a thought, proposition, or question. Each Big Decision allowed me to view PSTs' discussion around one interconnected idea as a comprehensive whole, instead of scattered throughout different videos.

Table 3 is an example of a Big Decision from group 1 that was gathered from two planning meetings (LPM2, LPM 4) and the three post-interviews (INT A, INT B, and INT C). PSTs discussed the Big Decision of connecting the properties that are preserved and not preserved in reflections with properties that are preserved in translations. This Big Decision included the smaller decisions of (a) *whether* to connect the properties preserved and not preserved in reflections with those of translations and (b) *how* to do so in their lesson.

Table 3

Transcription of a Big Decision from Group 1's Planning

Video	Speaker	Transcription
LPM 2	*Addie:*	*[14:35]* What I'm thinking is maybe do we want to connect that…translations preserve distance and angle measure the same as reflections and then in addition orientation?
	Bridget:	Oh, gotcha.
	Addie:	…do you want to…make that connection–[3]
	Carrie:	Yeah.
	Addie:	… that it has two of the same properties as reflection and then adds another one.
	Addie:	*[20:90]* [reading part of their FMC] 'Translations preserve distance between points and angle measure like reflections, but also preserves orientation.'

[3] A dash means the speaker's sentence was interrupted either by another person and they continued their sentence afterwards or they just cut their sentence short and never finished it.

Addie: *[31:03]* And we could have them…explore the properties that it preserves or something…

Bridget: Yeah that's true.

Bridget: Oh, yeah we can…[say]: okay what properties were preserved in your very first reflection? They'[d]…[say]: *'...this one is preserved and this one is preserved.'* What about from your first to your last image?

Addie: Oh, that would be good.

Bridget: From your first image to your second image, …what properties are preserved? From your first to third image, comparing those, what properties are preserved?

Carrie: Mm-hmm.

Addie: Yeah I'd like that.

Bridget: Alright, and then we could say: from your first to second, what transformation is that? *'...That's a reflection.'* What properties are preserved in it? And then we can say from your first to third what transformation is this? *'It's a translation.'* What properties are preserved? And that can be like our very last thing so that they get that in translation …[orientation] is [preserved] and in reflection it's not.

Bridget: *[34:16]* And once they know for sure it's a translation we would…[say]: …This translation preserves what properties? …This reflection preserves what?

Carrie: …I like that a lot.

LPM4 *Addie:* *[9:39]* One of the goals that I wrote, and maybe you don't like this, but…I said: 'students will distinguish that a translation preserves distance, angle measure, and orientation by comparing translations to reflections.' Because they understand reflections, so…we'll base it off of that and then say…: …What's different? What's the same?

Carrie: And that's what we're doing: …we're doing translations from the reflection. Yeah, I think I like that a lot.

Addie: Yeah, so I was thinking about doing that.

Bridget: Yeah that sounds good.

Addie: It…went through a lot of phases, but I think that's what I want to impress.

INT A *Interviewer:* [So, as opposed to doing] activity 2 [from UCSMP], you decided…to just hav[e] students state the properties closer to the beginning of your lesson as opposed to finding them.

Addie: Oh, yeah. Finding them yeah.

Interviewer: And I was just wondering why; what was [your] reasoning for that decision?

Addie: Hmmm.

Interviewer: Why d[id] you decide to change it?

	Addie:	Well I think some [of it] was that in the previous lesson they talked about reflections and…talked about what was preserved so we just built off of that. I think that was one reason: …that they should already know and we wanted them to see that…this…related to reflection…, but it was different since we were doing two of them. So, …it was the same…because it is a reflection, but…different…because we did two. Umm, and then the other one might have been the time: cause we only had 30 minutes to teach and this would have taken a lot longer, and there was…a better way to build on that [than activity 2's way].
INT B	*Interviewer:*	And then [i]n activity 2, …it has them actually finding [the properties]…on the shapes. Why did you decide…to [just] go over the properties…at beginning of your lesson instead?
	Bridget:	So, …the two lessons previous that…had gone over were…reflecting points and then reflecting polygons, and…the lesson right before us was…on the definition of reflection and its properties. …So, we already had an entire lesson on it, and we only have 30 minutes for our [lesson], …We didn't want to spend very much time on something they should have already learned, …so we just reviewed it real fast [to] make sure that they do know it …[and then] move[d] on to…our actual lesson.
INT C	*Interviewer:*	Why did you decide to just…talk about the properties as opposed actually having...[students] identify them and drawing [them] on their figure?
	Carrie:	So, …most of those were talked about in previous lessons on reflections so since…translations have…all the same properties as reflections, just adding orientation on to it, we just…assumed they already knew that; so, it's more of…a review. …We could have had them like look more into it, but…that wasn't the main focus of our lesson, so we just…gave it to them instead

I split my video data into Big Decisions through two main steps: separating into chunks and grouping these chunks together. First, I separated each video (e.g., LPM 1, InM) into chunks of mathematical decisions that were not part of the same Big Decision. This allowed me to break apart the video data so that each chunk was at most part of one Big Decision. Second, I grouped chunks from a single video that were about the same Big Decision. Figure 3 displays a timeline with sets of chunks from a single video that were grouped together on the *Big Decisions – Segmented* row. For example, instance 14 on the *Big Decisions – Segmented* row contained all the chunks from LPM2 that included the PSTs discussing the definition of translation in their lesson plan. After grouping chunks in single videos, I grouped the sets of chunks across videos (e.g., LPM1, InM). For example, instances 14 through 19 on the *Big Decisions – Segmented* row contains the sets of chunks from LPM2 (instance 14), LPM3 (instance 15), LPM5 (instance 16), Addie's post-interview (instance 17), Bridget's post-interview (instance 18), and Carrie's post-interview (instance 19), which all contained conversation around defining translation in their lesson. Then I grouped these chunks into one Big Decision (see boxed section in Figure 3) instance on the second *Big Decisions* row, which contained an entire Big Decision from all of the PSTs' planning process or reflection on implementation.

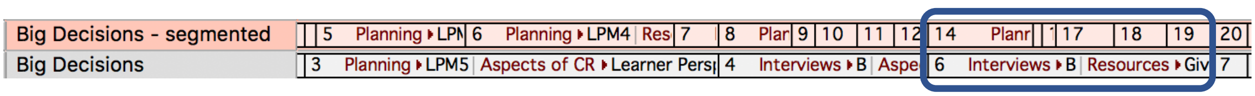

Figure 3. Example of how data were grouped and segmented.

To make sure that my chunking and grouping was reliable, I separated videos into chunks and checked my grouping of chunks multiple times. For each video, I separated it into chunks once and then a second time a day or two later after I had forgotten the first chunking, with a focus on erroring too small of chunks. Then, I went back and did a consensus of these two times separating videos into chunks with a focus on combining chunks that were part of the same Big Decision. While grouping chunks together, I double checked each set of chunks: first within each single video by listening to it as a whole and second between videos to make sure sets of chunks were about the same Big Decision. If I found a chunk that did not fit in the Big Decision, I located the correct Big Decision and added it and double checked it after placing it in a set of chunks both in a single video or among videos. Separating into chunks multiple times and checking my grouping of chunks allowed me to have greater reliability dividing my video data into Big Decisions.

Labeling for Curricular Reasoning Strands, Subcategories, and Resources

Labeling for curricular reasoning strands. After separating all videos into Big Decisions, I labeled each Big Decision with all possible curricular reasoning strands based on the definitions in Table 1. An example of labeling curricular reasoning strands within a Big Decision is provided in chapter 4. Additionally, it is important to note that I used open-coding: meaning that I used the seven curricular reasoning strands from prior research (Dingman et al., in press; Roth McDuffie & Mather, 2009).

Figure 4 is part of a timeline with Big Decisions and two curricular reasoning strands to illustrate the coding process. The segmented rows (e.g., *Big Decisions – segmented, Doing Task as Learners – segmented*) contain the sets of chunks from each video (e.g., LPM 2, INT C) that were the same Big Decision. The non-segmented rows (e.g., *Big Decisions, Doing Task as Learners*) – contain the complete Big Decision instances. I labeled both instances on the segmented and non-segmented rows with the specific curricular reasoning strands. I used the non-segmented rows for three reasons: the segmented rows (1) had instances that were clean breaks to code for the curricular reasoning strands and these smaller video instances allowed for more consistent labeling of the curricular reasoning strands, (2) made referencing the original videos easier while analyzing data, and (3) made it possible to identify the parts of each Big Decision that came from different planning meetings or post-interviews for further analysis of trends in the data.

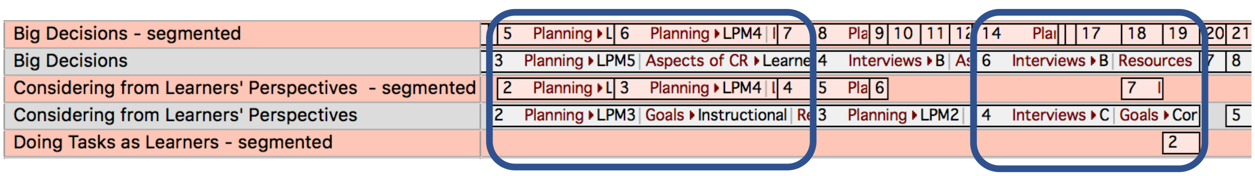

Figure 4. Example of how data were labeled.

To label the Big Decisions for curricular reasoning strands I watched each *Big Decision – segment* (first row) to determine the curricular reasoning strand that was reasoned with during each segment. When a reasoning was identified, I added the individual *Big Decision – segment* instance to the appropriate curricular reasoning strand “segmented” lines (e.g., *Considering from Learner’s Perspectives – segmented*, *Doing Tasks as Learners – segmented*). Then, for each Big Decision that had a curricular reasoning strand I added the strand to the particular curricular reasoning strand line (e.g., *Considering Learners’ Perspective*, *Doing Task as Learners*) that I used for my analysis.

Big Decisions were labeled with curricular reasoning strands according to PSTs’ reasoning. In Figure 4, Big Decision number 6 on the second row (*Big Decisions – segmented* instances 14-19 on the first row) is composed of six different videos segments with one segment labeled as *considering learners’ perspectives*, and another segment labeled as *doing the task as learners*. The entire Big Decision number 6 was labeled as *considering learners’ perspective* and *doing the task as learners* because PSTs reasoned with both curricular reasoning strands during Big Decision number 6. Thus, labeling of curricular reasoning strands was not exclusive, meaning Big Decisions were labeled for all curricular reasoning strands that were present. In contrast, in Figure 4, Big Decision number 3 on the second row is composed of four video segments (*Big Decisions – segmented* instances 4-7), all of which were labeled as *considering learners’ perspective*; thus, the Big Decision was only labeled once with this curricular reasoning strand to identify that the PSTs reasoned with this strand during this Big Decision. Each *Big Decisions – segmented* instance could also be labeled for more than one curricular reasoning strand.

When I labeled a *Big Decisions – segmented* instances from the InM videos, I only labeled verbalizations that stemmed from the PSTs. In other words, I only labeled the *Big Decisions – segmented* instances for curricular reasoning strands if it was something from, used by, or done by the PSTs themselves, because my study was about PSTs' curricular reasoning rather than their instructor's. For example, if their instructor reasoned with a curricular reasoning strand, I did not label it unless the PSTs reasoned with this strand.

Labeling for subcategories and resources. I identified subcategories within each curricular reasoning strand and labeled them in a similar way as I did for the seven curricular reasoning strands. I also identified any resources that PSTs' drew on during their curricular reasoning by labeling these resources both on a new row in my timeline and on the appropriate curricular reasoning strand rows or subcategories of curricular reasoning rows. This allowed me to analyze the different resources PSTs drew on while they reasoned with specific curricular reasoning strands or subcategories.

Reliability. To make sure my labeling of Big Decisions for curricular reasoning strands, subcategories, and resources that PSTs drew on was reliable, I went through each label at least twice. In other words, after labeling all Big Decisions for curricular reasoning strands, I went through each video and checked my labeling of curricular reasoning strands to see if I missed any or that I could identify all the ones that I had labeled. I did the same after labeling subcategories of curricular reasoning strands and for resources that PSTs drew on while reasoning with curricular reasoning strands or subcategories.

Analyze Data for Trends and Findings

Following the labeling of all Big Decisions, I quantified the curricular reasoning strands for the Big Decisions during the teaching process into percentages identifying trends and patterns in the data and comparing between the two PST groups. For each strand, subcategory and resource; I determined the percent of the Big Decisions that were made using a reasoning while planning or while reflecting on implementation [4]. It is important to note that the percentages reported do not always add up to 100% because Big Decisions were labeled with multiple curricular reasoning strands.

Similarly, I used the labeling of Big Decisions – *segmented* to analyze trends and patterns in the data during individual planning meetings (i.e., LPM1, InM, …, LPM 5). This allowed me to determine whether specific curricular reasoning strands, subcategories, or resources were used by PSTs during individual planning meetings and increased or decreased across the planning process. I then used these percentages to analyze PSTs' curricular reasoning for each strand, subcategory, or resource across the planning process.

Summary

In summary, the methods for this study focused on collecting and analyzing video data from two groups of PSTs. These PSTs were in a methods course that focused on student learning with mathematical content that was unfamiliar to them. I collected data to identify each PSTs group's Big Decisions and reasoning during the teaching process. I separated the video data based on PSTs' conversations about individual interconnected ideas and their reasoning related to those ideas to create individual Big Decisions. I labeled these Big Decisions for curricular reasoning strands, subcategories of these strands, and resources that PSTs drew on during their curricular reasoning, which allowed me to quantify PSTs' curricular reasoning and analyze the trends and patterns of PSTs' curricular reasoning during the teaching process.

[4] I use the term *reflecting on implementation* rather than 'implementation' because I could not know the PSTs thoughts and reasons behind their decisions during their lessons without asking them from their post-interviews where they *reflected* on the implementation of their lesson.

CHAPTER 4: RESULTS

This chapter presents the results to answer my research question: In what ways do PSTs engage in curricular reasoning as they make mathematical decisions during the teaching process? First, I present overall results of the curricular reasoning strands that PSTs reasoned with based on how often they occurred and when they occurred. Next, I provide an illustration of PSTs' curricular reasoning for one Big Decision during the teaching process. Drawing upon this illustration, I describe different ways that PSTs reasoned about their Big Decisions, which include descriptions of (1) significant subcategories of curricular reasoning strands, (2) the ways that PSTs reasoned while making three types of decisions, and (3) the ways that PSTs intertwined their reasoning with various strands during the teaching process. Additionally, I identify three significant resources that PSTs drew on during their curricular reasoning which provides insight into the ways PSTs reasoned.

PSTs' Overall Curricular Reasoning

PSTs reasoned with all seven curricular reasoning strands. Table 4 displays the curricular reasoning strands with the percentages of the Big Decisions separated by when the Big Decisions occurred, during the planning process or while reflecting on implementation, and by the two PST groups. Although PSTs reasoned with all strands, they reasoned the most with the strands *drawing on mathematical meanings*, *mapping learning trajectories*, and *considering learners' perspectives*. Group 1 PSTs made 115 Big Decisions during the teaching process, 85 while planning and 30 while reflection on implementation. Group 2 PSTs made 87 Big Decisions during the teaching process, 58 while planning and 29 while reflecting on implementation. I note that PSTs' Big Decisions while reflecting on implementation happened during their three individual post-interviews.

Table 4

Curricular Reasoning Strands Activated by PSTs

	Planning		Reflecting on Implementation	
Curricular reasoning strand	Group 1 (n=85)	Group 2 (n=58)	Group 1 (n=30)	Group 2 (n=29)
Drawing on Mathematical Meanings	67%	66%	47%	52%
Mapping Learning Trajectories	48%	52%	30%	62%
Considering Learners' Perspectives	41%	52%	40%	76%
Positioning with Regards to the Mathematics	38%	33%	43%	48%
Analyzing Curriculum Materials	36%	32%	37%	24%
Doing Tasks as Learners	34%	34%	3%	7%
Revising Based on Experiences in Teaching and Learning	4%	0%	37%	48%

Illustration of Ways PSTs Use Curricular Reasoning

Table 5 is an illustration of group 1's conversation during their planning process for one Big Decision of their inclusion of the definition of transformation in their lesson. As a reminder group 1's lesson was the third lesson in a geometric transformation unit from *UCSMP* (Benson et al., 2009) that introduced translations as a composite of two reflections over parallel lines. Group 1 began this Big Decision by asking the question: should we define transformation in our lesson? At first the PSTs concluded that they did not need to define transformation but later changed and choose to define it at the beginning of their lesson. PSTs' curricular reasoning is bolded with the strands labeled in the right column along with any subcategories in parenthesis for specific curricular reasoning. Dashed lines indicate the different video chunks that composed this Big Decision.

Table 5

Illustration of Curricular Reasoning Strands and Subcategories

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
1	LPM 1 [*chunk 1*]	Carrie:	Do we **need to**…**define transformation**, too**?**	Drawing on mathematical meanings (for students)
2		Bridget:	**I am guessing not**–[5]	
3		Carrie:	Okay.	
4		Bridget:	**because; well…, they don't in here** [*referring to the section in the book for their lesson*]**. They just use that word trans[formation]…right at the beginning.**	Analyzing curriculum materials
5		Carrie:	That's true.	
6		Bridget:	They just say transformation; and **I'm guessing since [section] 4.2 [in the textbook] is reflecting polygons. I'm guessing they would have it already in there.**	Analyzing curriculum materials; Mapping learning trajectories (unit connections)
7		Bridget:	**Where do they define [transformation]?**	Analyzing curriculum materials
8	LPM 1 [*chunk 2*]	Bridget:	Well, **it doesn't look like they actually define a transformation**…or did I not see it?	
9		Bridget:	Do they have like an index?	
10		Addie:	Uhmmm–	
11		Bridget:	…I don't see the word– oh, yeah–	
12		Addie:	**There's a transformation theorem.**	
13		Bridget:	Yeah. **It says "figure transformation theorem".**	
14		Addie:	[*reading from textbook*] "If a figure is determined by certain points, then its transformation image is the corresponding figure determined by the trans-" [*voice trails off*] **I don't even understand this.**	Analyzing curriculum materials
15		Carrie:	[*chuckles softly*] Sounds kind of like a definition.	
16		Bridget:	**Oh, they have vocabulary at the top [of each section], but their** [*referring*	Analyzing curriculum materials

[5] A dash means the speaker's sentence was interrupted either by another person and they continued their sentence afterwards or they just cut their sentence short and never finished it.

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
			to the prior lesson] **vocabulary doesn't include transformation.**	
17		Addie:	I bet not. …I think–	
18		Bridget:	I guess we can just ask about–	
19		Addie:	–ask about that, yeah.	
20		Bridget:	We can just ask [our instructor].	
21		Addie:	Yes; and if it ends up that we need to expound on that, then–	
22		Bridget:	–we can just add it.	
23		Addie:	–then…I'd say we deal with that when we come to it. You know what I'm saying?	
24		Bridget:	Yeah. **Because…the fact about…how transformation keeps all those things** [*referring to the characteristics of a transformation*]**– …that's important.** …I don't think we'd be hitting at that though.	Drawing on mathematical meanings (of PSTs)
25	InM	Bridget:	Also, I was wondering: does [section] 4.2 define transformation?	
26		Instructor:	Uh, I think [section] 4.1 does.	
27		Bridget:	Oh, they do? I mean, **cause we were looking at [the unit in the book] and …it's a vocabulary word in *our* section, …a *new* vocabulary word. And we were like, uh–**	Analyzing curriculum materials
28		Addie:	Well, **it's just good to reiterate [the word 'transformation'], because we're introducing translation which…is really close to transformation.**	Considering learners' perspectives (incorrect or confusion)
29		Instructor:	Right, and–	
30		Addie:	**Like, *I* get them confused.**	
31		Instructor:	Yeah, a lot of people will interchange…them: [use] transformation instead of translation. And, it's quite close.	
32		Addie:	Yeah, and so I'm sure it's just a way to emphasize that.	
33	LPM2	Carrie:	And we don't need to define transformation, right?	
34		Addie:	[*shaking head*] Nuh-uh.	
35		Bridget:	Not transformation, but *translation*.	

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
36	LPM4 [*chunk 1*]	Addie:	So, we want to introduce transformation, right?	
37		Carrie:	Yeah, we need to–	
38	LPM4 [*chunk 2*]	Carrie:	Well, [the instructor] said like it was supposed to come out in the first lesson.	
39		Addie:	Yeah, which is funny. I mean...– actually…I don't know. [transformation] was never defined. **But, I do feel like today [in class] it was like "you should just define [transformation]."**	Revising based on experiences in teaching and learning
40		Carrie:	Yeah.	
41		Addie:	Which then like– [*voice trails off*]	
42		Carrie:	Well, **we need to start off defining it**, I think.	
43		Addie:	Yeah… Cause **I feel like you can just say it, right? Like "a transformation is this", and then you can [say,] "and so reflection is a type of transformation. Okay let's review the properties that are preserved in reflections."**	Mapping learning trajectories (lesson play-by-play)
44		Carrie:	I like that.	
45		Addie:	You know?	
46		Carrie:	Yeah.	
47		Addie:	…I feel like that's just…a simple way of…–	
48		Carrie:	helping them see.	
49		Addie:	**I don't feel like we have to work to get them to…draw out the definition–**	
50		Carrie:	Oh, definitely not, cause [the PST who taught the prior lesson] was supposed to–	Positioning with regards to the mathematics
51		Addie:	**especially because we did like two activities [in prior lessons].**	
52		Carrie:	Yeah, I think it's…like you said, "tometo, tomato," but…[our instructor] said that it was supposed to have come out in the first [lesson] and even the second [lesson], so–	
53		Addie:	So, then I [thought]–	

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
54		Carrie:	**I feel like…let's just say it** [*referring to the definition of transformation*].	
55		Addie:	**let's just put it in; let's just do it.**	
56		Carrie:	Yeah.	
57	Addie's Post-Interview	Interviewer:	You reviewed the word transformation at the very beginning of your lesson. What was your reasoning for that?	
58		Addie:	So, a lot of our FMC was **based on the…fact that they** [*meaning the students*] **knew what a transformation was**. We never defined it…we [were] just [assuming], '[students] know what this is'. **…it's all based on the fact that they know that if we use this word they understand what we're talking about.** So…, 'It's a…transformation', and **…if we…say 'a translation is a transformation' and they don't even know what a transformation is, …how can we teach what a translation is?** …and so we just…wanted to put it out there. Just [say] 'this is what [a transformation] is' **because it was never talked about [in prior lessons]…**and especially cause **translation and transformation are so close.** …**we kept confusing them all the time. You probably noticed in the videos, we were always like 'wait, [Bridget], what are you talking about?' and she's like 'oh did I say transformation? I meant translation.'** You know?	Drawing on mathematical meanings (for students; of PSTs); Mapping learning trajectories (lesson connections) Revising based on experiences in teaching and learning Considering learners' perspectives (incorrect or confusion)
59		Interviewer:	Yes.	
60		Addie:	…**every time I'd be like "Okay, wait. Is she meaning [transformation] or is she's not meaning it?" So, …I think [defining transformation in the lesson] was to help preserve a distinction and then to build so**	Drawing on mathematical meanings (for students, of PSTs); Mapping learning trajectories (lesson connections)

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
			that we could build up to what a translation is.	
61	Bridget's Post-Interview [*chunk 1*]	Interviewer:	What was your reasoning for…[defining] the word transformation at the beginning of your lesson?	———
62		Bridget:	Yeah, so, we talked about that too; and…I didn't necessarily know if we needed [to define transformation] or not, …**cause the other two lessons had it in their FMC[s], but they didn't define it at all.** … So, I ended up asking them that after they taught. I [said,] **"you guys have the word 'transformation' in your FMC, but you didn't define it at all. …is that important? …the students technically still don't know what a transformation is, but you say 'a reflection is a transformation."** So, I [said,] "is that important at all?" So anyways, I guess I feel like **since we use the word transformation in our FMC, the students needed to *know* what a transformation is and know that a reflection *is a transformation*.** But [our instructor] [said,] "the previous lessons should have defined [transformation] for you guys, but they didn't so **just at the beginning of your lesson, just define it real fast." And so, we did, just to make sure– Just because it's in our FMC, and so, we wanted to make sure that the students understood that part.**	——— Revising based on experiences in teaching and learning ——— Drawing on mathematical meanings (of PSTs); Mapping learning trajectories (lesson connections) ——— Mapping learning trajectories (lesson connections); Drawing on mathematical meanings (for students) ——— Mapping learning trajectories (lesson play-by-play, lesson connections); Drawing on mathematical meanings (for students) ———
63	Bridget's Post-Interview [*chunk 2*]	Interviewer:	Did you bring this up in your conversation with [your instructor] and *ask* about it, or did [your instructor] just say 'hey and by the way' [and told it to you]?	

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
64		Bridget:	So…, you know after we teach our lessons we have like that debriefing stage? [*referring to when the class discusses the lesson just taught*] So, **in that debriefing stage, I rose my hand and asked the…person who taught [the lesson] and [said,] "you have the word transformation there. You didn't go over at once. Why is that? …What's going on?" And so, then, …the student gave their response, and then [our instructor] … responded and said "Yes I do think you guys should have gone over the definition of transformation–"** Oh yeah, cause…I also said…, "…**we…*use* that definition in *our* lesson because it's in *our* FMC.** And so [our instructor] [said], "Yeah, they should have defined it for you guys in the very first lesson, but they didn't go over it. So, you…can just write it on top of the board at the beginning [of your lesson]."	______ Revising based on experiences in teaching and learning ______ Mapping learning trajectories (lesson connections) ______
65	Carrie's Post-Interview	Interviewer:	…your decision to define transformation at the beginning of your lesson. What was your reasoning for that?	
66		Carrie:	…the previous lesson before this– Cause **each one [of the prior lessons in this unit], especially the one before us …use the word transformation in their FMC: …"a reflection is a transformation" [and] "a reflection of figure is a transformation". So, they kept using that. But then the question was brought up [from the students during the debriefing], "we don't know what a**	______ Revising based on experiences in teaching and learning

Line No.	Videos	Speaker	Transcription	Curricular Reasoning Strands (and Subcategories)
			transformation is, because it hasn't been told. So, we're just assuming– …that's in the definition, but we haven't talked about it." So, it was mainly just…**clarifying "this is a transformation" so that– …especially because we obviously used [the word transformation] in our definition [of translation] and in our FMC. So, we just wanted to make sure that students knew th[e definition of transformation] and that it was clarified.**	______ Drawing on mathematical meanings (of PSTs, for students)

As can be seen in Table 5, the group 1 PSTs discussed this Big Decision across multiple planning meetings and I revisited this Big Decision during the post-interviews to gather more information about PSTs' reasoning for their change to include the definition of transformation in their lesson. As PSTs discussed this Big Decision, they reasoned with six curricular reasoning strands. The PSTs also drew on a variety of resources while reasoning: their textbook, debriefings meetings from other PSTs' lessons in the same unit, comments from their instructor, and their understanding of the mathematics in their lesson. I remind the reader that although this Big Decision has multiple curricular reasoning stands labeled multiple times in Table 5, this Big Decision was only labeled once for any one curricular reasoning strand, subcategory, or resource that PSTs drew on. In the following sections I use this illustration to provide more detail about PSTs' curricular reasoning, subcategories of curricular reasoning, and resources that PSTs drew on during their curricular reasoning.

I explain two instances of curricular reasoning and one non-example to illustrate how Big Decisions were labeled. In Table 5 lines 1-2, PSTs expressed that they did not "need to…define transformation" and the definition of transformation was not something that students needed to have from *their* lesson. In the rest of LPM1 chunk 1, the PSTs reasoned that the definition of transformation was probably covered in other lessons. Thus, lines 1-2 were labeled as *drawing on mathematical meanings (for students)* because PSTs "expressed the mathematical understanding that students…did not need to know" (see Table 5). In Table 5 line 6, PSTs expressed that "since [section] 4.2 is reflecting polygons…they would have [the definition of transformation] in there" illustrating the PSTs connecting transformations, which is referred to as a vocabulary word in their lesson, to reflecting polygons in the prior lesson and as they expressed in line 4, transformation is also part of their lesson. Thus, this was labeled as *mapping learning trajectories (unit connections)* because PSTs expressed how a "mathematical concept in their lesson connects to past and future topics" (see Table 1). In Table 5 lines 36-37, Carrie expressed the "need to" introduce transformation but her reasoning was unclear because she never finished her sentence. Thus, line 37 was not labeled for curricular reasoning because it was not clear why they changed their decision to introduce transformations and I would have had to infer their reasoning. For instance, Carrie could have said "We need to – follow our instructor's suggestions to review it at the beginning of our lesson" or "We need to – cover it because students don't know what it is" and these would be very different reasoning strands. Therefore, labeling of curricular reasoning strands was only done when PSTs provided enough reasoning to make a valid judgement.

Curricular Reasoning Subcategories

I now focus on three curricular reasoning strands that the PSTs reasoned with the most when making decisions and subcategories that emerged from these data. There may be other subcategories within other curricular reasoning strands, but my data set was too small to identify other subcategories. I included examples of and possible subcategories for the other four curricular reasoning strands in Appendix C.

Drawing on Mathematical Meanings

Table 6 displays the two subcategories for *drawing on mathematical meanings*: (1) mathematical meanings *of PSTs* and (2) mathematical meanings *for students*. The difference between these subcategories is the purpose: whether the PSTs expressed their own mathematical meanings while reasoning during a Big Decision or whether the PSTs expressed the mathematical meanings they wanted for their students to gain from their lesson. PSTs reasoned with both subcategories in Table 5 lines 58, 60, 62, and 66. However, in Table 5 line 24 PSTs specifically reasoned with the subcategory *of PSTs* to express the importance of the definition of translation for the mathematical content in their lesson, and in lines 1-2 PSTs reasoned with the subcategory *for students* to express that the definition of transformation was not something that students needed to know during their lesson.

Table 6

Subcategories for the Strand Drawing on Mathematical Meanings

Subcategory	Definition
Of PSTs	PSTs express their own mathematical meaning (whether productive or not) of the mathematics related to their lesson.
For students	PSTs express the mathematical meaning that students either need to know or do not need to know.

Overall, both PST groups reasoned with the subcategory *of PSTs* most while planning and while reflecting on implementation. However, it is interesting to note that group 1 PSTs reasoned with the subcategory *for students* about twice as much as group 2 PSTs while planning and while reflecting on implementation.

Mapping Learning Trajectories

Table 7 displays the four subcategories for *mapping learning trajectories*: (1) *lesson* level *connections*, (2) a *lesson* level *play-by-play*, (3) *unit* level *connections*, and (4) a *unit* level *play-by-play*. In Table 5, PSTs reasoned with the subcategories *lesson connections,* and *lesson play-by-play.* In lines 58, 60, 62, and 64, PSTs reasoned with the subcategory *lesson connections* as they discussed how their definition of translation connected to the definition of transformation that they planned to use in their lesson. PSTs reasoned with the subcategory *lesson play-by-play* while discussing how to structure defining transformation in their lesson both in line 43 when Addie gives an ordering to three major points in their lesson (i.e., definition of transformation, a reflection is a type of transformation, review properties preserved in reflections) and in line 62 when Bridget states the order of when they defined transformation at the beginning of their lesson. Although PSTs reasoned little with *unit connections* in the Big Decision in Table 5, they reasoned with this subcategory in other Big Decisions. A clear example of reasoning with *unit connections* was when group 1 PSTs expressed that their lesson built on the previous lessons – reflecting points and images, but their lesson reflected an image twice. Although not included in Table 5, an example of the subcategory *unit play-by-play* was when group 2 PSTs stated the order of the lessons in the unit to make sense of how their lesson fit into the broader unit.

Table 7

Subcategories for the Strand Mapping Learning Trajectories

Subcategory	Definition
Lesson connections	PSTs explain connections between two different mathematical concepts in the same lesson.
Lesson play-by-play	PSTs state an ordering for concept(s) in their lesson or the sequencing of the lesson.
Unit connections	PSTs explain connections between two different mathematical concepts in the same unit.
Unit play-by-play	PSTs state an ordering for lesson(s) in the unit or the sequencing of the unit.

Overall, PSTs in both groups reasoned with *unit connections* more than *lesson connections*. Both PST groups reasoned with the subcategories *lesson play-by-play* and *unit connections* most while planning and reflecting on implementation. While planning, PST groups reasoned with *lesson play-by-play* about twice as much than *lesson connections* and reasoned with *unit connections* at least twice as much than *unit play-by-play*. While reflecting on implementation, PST groups reasoned with the subcategories *lesson play-by-play* and *unit connections* the same, and group 1 PSTs did not reason with the subcategory *lesson connections* while group 2 did reason with it some.

Considering Learners' Perspectives

Table 8 displays three subcategories PSTs reasoned with for *considering learners' perspectives*: (1) *correct* student understanding, (2) *incorrect* student understanding *or confusion*, and (3) *ways* students might solve the task. In Table 5 lines 28 and 58, PSTs reasoned with the subcategory *incorrect or confusion*. One PST brought up an anticipated student confusion between the words translation and transformation as PSTs discussed defining transformation in their lesson. Although PSTs did not reason with the other subcategories of *considering learners' perspectives* during the Big Decision in Table 5, they did reason with these subcategories in other Big Decisions. An example of the subcategory *correct* was when group 2 PSTs discussed the correct answer for their task to include it in their lesson plan so the PST teaching could identify whether students correctly solved the task. An example of the subcategory *ways* was when group 1 PSTs reasoned about different ways students might solve a harder task to help them decide whether or not to use the harder version of the task in their lesson.

Table 8

Subcategories for the Strand Considering Learners' Perspectives

Subcategory	Definition
Correct	PSTs express students' correct understanding/thinking for a part of the task or part of the lesson (i.e., the correct answers as intended by the PSTs).
Incorrect or confusion	PSTs express students' incorrect understanding for a part of the task or lesson OR where students might (did) struggle with the task or lesson.
Ways	PSTs express in some detail how students might or did go about solving part of the task or doing something during the lesson (rather than just the answers to parts of the task that students might come up with).

Overall, PSTs in both groups reasoned with all subcategories while planning and reflecting on implementation. While planning, both PST groups reasoned with the subcategory *incorrect or confusion* the most. While reflecting on implementation, group 1 PSTs reasoned with the subcategory *ways* the most and group 2 PSTs reasoned with the subcategories *correct* and *incorrect or confusion* the most.

Curricular Reasoning for Decision Types

PSTs generally made four decisions types during the teaching process: (1) making sense of the mathematics in their lesson for themselves, (2) determining the mathematics to include in their lesson, (3) determining the presentation and sequencing of the mathematics for their lesson (i.e., column 1 in the *Teacher Work Sample* lesson plan format in Appendix D), and (4) determining anticipated student thinking or teacher responses for such thinking (i.e., column 2 or

3 in the *Teacher Work Sample* lesson plan format in Appendix D). This section describes these types and the PSTs' reasoning for their decisions associated with each type.

First, PSTs made decisions focused on making sense of the mathematics of their lesson for themselves. An example of this was when group 2 PSTs completed the task of finding the minimax in their head to make sense of the mathematics and thus determine the different formulas – both correct and incorrect – for finding the minimax. When making decisions of this type PSTs most often reasoned with *drawing on mathematical meanings – of PSTs* and *doing tasks as learners*, though they also reasoned some with *mapping learning trajectories* and *analyzing curriculum materials*. It is important to note that during their first two planning meetings (i.e., LPM1, InM), PSTs reasoned with *drawing on mathematical meanings – of PSTs* often to make sense of the mathematics in their lesson for themselves. This seemed beneficial because during the remaining planning meetings PSTs tended to have a greater focus on the mathematics as they plan their lessons, which was partially evidenced by the verbalized conceptual learning goals that PSTs had for their students.

Second, PSTs made decisions about what mathematics to include in their lesson. An example of this is in Table 5 when group 1 PSTs decided to not include (e.g., lines 1-4) and then to include (e.g., lines 36-40) the definition of transformation in their lesson. When making decisions of this type PSTs reasoned with *drawing on mathematical meanings* – both *of PSTs* and *for students*, *doing tasks as learners*, *analyzing curriculum materials*, and *revising based on experiences in teaching and learning*, along with sometimes *positioning with regards to the mathematics* and *considering learners' perspectives*. Although PSTs reasoned about the mathematics, students, curriculum, or a combination of the three to determine what mathematics to include in their lesson, it is interesting that both PST groups tended to create their written mathematical goals for students' mathematical learning *after* developing the majority of their lesson plan.

Third, PSTs made decisions about how to present and sequence the mathematics for their lesson. An example of this is in Table 5, lines 42-56 when group 1 PSTs decided to state the definition of transformation for their students at the beginning of their lesson. When making decisions of this type PSTs reasoned with *all* curricular reasoning strands and subcategories; especially, *drawing on mathematical meanings – of PSTs* and *for students*, *mapping learning trajectories*, *considering learners' perspectives – incorrect or confusion*, *positioning with regards to the mathematics*, and *revising based on experiences in teaching and learning*.

Fourth, PSTs made decisions about anticipated student thinking or teacher responses for such thinking. An example of this was during a post-interview when a PST in group 1 expressed that she noticed students estimating the distance between the edge of the paper and the two reflection lines to prove that a composition of two reflections over parallel lines was a translation. During her post-interview the PST said she would modify her task by having her students use a figure that was not parallel to the edge of the paper because this would help her students focus on the mathematical relationships for the proof rather than a visual estimation of the proof. When making decisions of this type, PSTs most often reasoned with *considering learners' perspectives – correct* and *ways*, *doing tasks as learners*, and *revising based one experiences in teaching and learning*; and also, reasoned some with *considering learners' perspectives – incorrect and confusion*, *drawing on mathematical meanings*, and *positioning with regards to the mathematics*.

Curricular Reasoning Strands Are Intertwined

Not only did PSTs reason with all seven curricular reasoning strands, but they often reasoned with multiple strands during individual Big Decisions. The Big Decision in Table 5 illustrates PSTs' reasoning with multiple curricular reasoning strands as they made the decision of whether to define transformation in their lesson. For instance, PSTs reasoned with *analyzing curriculum materials* in Table 5 lines 4, 6-8, 12-14, 16, and 27; however, intermixed among the first three chunks of this Big Idea (lines 1-32) are four instances when PSTs reasoned with other curricular reasoning strands. In lines 1-2, PSTs reason with *drawing on mathematical meanings* to determine the mathematics that was important for students in their lesson and then in line 4 they began reasoning with *analyzing curriculum materials* to justify their decision to not include transformations in their lesson. In lines 6-8, PSTs reason with *mapping learning trajectories* and *analyzing curriculum materials* to determine the correct sequencing of the mathematics in their lesson's unit. In response to the PSTs' struggles in lines 12-14 to determine whether the definition of transformation was part of their lesson, one PST agreed that it was important to find out if the definition of transformation needed to be included in their lesson based on her reasoning with *drawing on mathematical meanings* that this definition was important for their lesson (line 24). In line 27, Bridget reasoned with *analyzing curriculum materials* to explain the oddity that transformation was listed as a new definition in their lesson but was introduced in a prior lesson in that unit, to which Addie reasoned with *considering learners' perspectives* to provide a reason for this oddity. Thus, PSTs' intertwined reasoning seemed to allow PSTs to reason in complex ways as they made decisions for their lessons. The Big Decision in Table 5 is similar to other Big Decisions where PSTs reasoned with three or more strands while planning (group 1: 51%, group 2: 40%) and while reflecting on implementation (group 1: 37%, group 2: 62%) suggesting that these PSTs reason with multiple strands often as they make their decisions.

PSTs' curricular reasoning is complex and reflects the nature of teaching. In fact, PSTs rarely reasoned with less than two strands when determining a Big Decision. PSTs reasoned with at least two strands for more than two-thirds of all planning Big Decisions (group 1: 78%, group 2: 68%) and at least two-thirds of all reflecting on implementation Big Decisions (group 1: 60%, group 2: 86%). This provides additional evidence of the intertwining nature of PSTs' curricular reasoning and that their reasons for their decisions are often multifaceted.

Significant Resources That PSTs' Drew on During Curricular Reasoning

During their curricular reasoning, PSTs drew on multiple external resources and this provides insight into the ways PSTs reasoned. I share the three most significant external resources that PSTs drew on while reasoning, which were connected to the three curricular reasoning strands most used by PSTs (i.e., *drawing upon mathematical meanings*, *mapping learning trajectories, considering learners' perspectives*).

The first resource that PSTs' drew on while reasoning was their instructor. The PSTs' instructor met with PSTs during their InM to discuss the mathematics of their lesson, was available to answer PSTs' questions about their lesson, and gave insight in the methods course during class discussions and lesson debriefings. PSTs drew on comments or suggestions from their instructor during different Big Decisions, which included their instructor's facilitation of learning through having PSTs doing the tasks as learners during their InM. Sometimes the PSTs referred to their instructor explicitly and other times I inferred that the content from their discussion came from their instructor because they did not mention the content before their instructor brought it up in the InM. In Table 5 lines 25-27, 62 and 66, group 1 PSTs drew on their instructor's comments and suggestions that students needed their group to define transformation at the beginning of their lesson. In this Big Decision, the instructor's suggestion for group 1 PSTs to define transformation at the beginning of their lesson affected Bridget's reasoning with *drawing on mathematical meanings – for students* because her desire that "the students need to know what a transformation is" (Table 5, line 62) for her lesson was fulfilled by her instructor's suggestion to "just define it at the beginning of your lesson". About half of all Big Decisions while planning, PSTs drew on comments their instructor had said during their InM or at other times while planning (group 1: 55%, group 2: 52%) and this was connected to PSTs' reasoning with *drawing on mathematical meanings* the most as well as with *considering learners' perspectives* (See Appendix E for additional figures and tables). Furthermore, Figure 5 displays the percentage of Big Decisions for each curricular reasoning strand and for both groups (G1: black, G2: gray) that PSTs activated across the planning process, and these percentages increase from either LPM1 to InM or from InM to LPM2, suggesting that the PSTs' instructor was connected to their curricular reasoning.

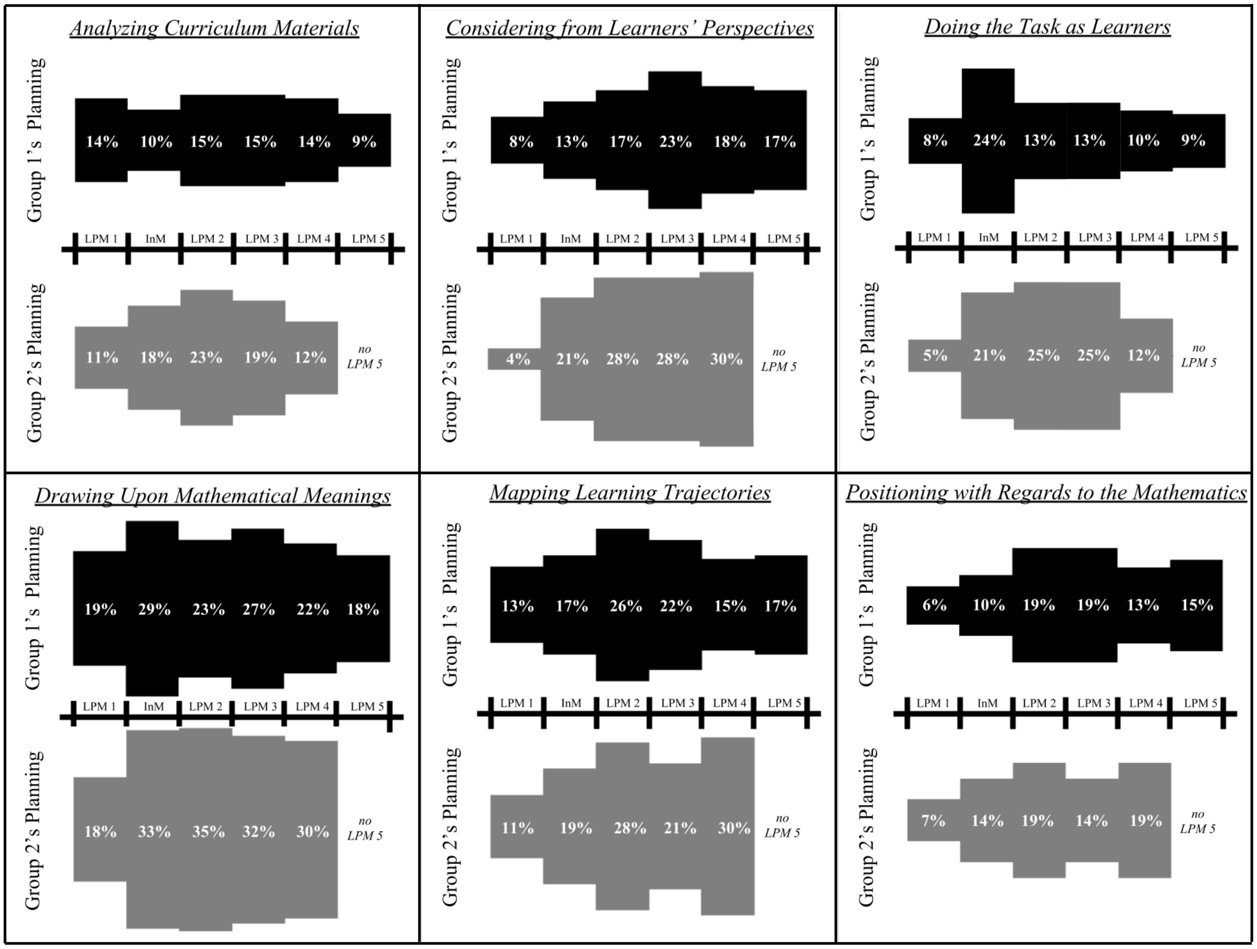

Figure 5. Timelines of curricular reasoning strands activated by PSTs during their planning process (g1: n=78, g2: n=57).

The second resource that the PSTs' drew on while reasoning was their opportunity to go through the teaching process (i.e., planning, implementing, reflecting) and receive feedback from their peers and instructor during lesson debriefings. In their methods course, PSTs were required to implement their planned lesson to their peers as "students". Following their lesson implementation, the class held a lesson debriefing where PSTs answered questions from their peers and instructor. Thus, PSTs reflected on their lesson and were provided feedback about their lesson. In Table 5 lines 58, 60, 62, 64, and 66, provides an illustration of group 1 PSTs drawing on their experience from feedback provided during the debriefing from another PST group's lesson to inform their decision to define transformation at the beginning of their lesson. The PSTs' reasoning for more than half of all reflecting on implementation Big Decisions was related to the implementation of their lesson (group 1: 54%, group 2: 86%) and was connected to their reasoning with *considering learners' perspectives* and *revising based on experiences in teaching and learning*. Furthermore, group 2 PSTs also drew on their class discussions and lesson debriefings for half of all Big Decisions while reflection on implementation (group 2: 50%), especially while reasoning with *mapping learning trajectories* and *considering learners' perspectives* (See Appendix E for additional figures and tables).

The third resource that PSTs drew on while reasoning was the design of the lesson study group schedule: that each PST group taught one lesson in a full unit with other PST groups teaching the other lessons in the same unit. The methods course was structured so that the PSTs planning their lesson were accountable to state facts the students should "know" just before their lessons if prior lessons did not cover them. As an example, in Table 5 lines 58, 62, and 66 group 1 PSTs were concerned that the definition of transformation was not brought up because if students did not understand the definition of a transformation, then this would affect whether students could understand the definition of translation that they planned in their lesson. This structural set-up of PST groups teaching one lesson in a unit seemed to guide PSTs to make many *unit* level *connections* as they reasoned with the *mapping learning trajectories* strand.

Summary

These results present the ways that PSTs reason with curricular reasoning strands when making mathematical decisions during the teaching process along with three significant external resources that PSTs drew on while reasoning. PSTs reasoned with all seven strands and I identified significant subcategories within three curricular reasoning strands. PSTs often reasoned with multiple strands within each Big Decision. The external resources of *PSTs' instructor*, *PSTs' opportunities for lesson implementation and feedback*, and *the design of the lesson study group schedule* are resources that PSTs' drew on while reasoning with *considering learners' perspectives*, *drawing on mathematical meanings*, and *mapping learning trajectories* strands.

CHAPTER 5: DISCUSSION

In this chapter I discuss four findings from my results that provide insight into PSTs' curricular reasoning in connection with other research. First, I explain the usefulness of the curricular reasoning framework. Second, I highlight PSTs' ability to make sense of the mathematics in their lesson for themselves. Third, I highlight PSTs' ability to focus on learners. Fourth, I discuss PSTs' reasoning with learning trajectories.

Curricular Reasoning Framework

Prior researchers have outlined the curricular reasoning framework, including defining curricular reasoning and seven curricular reasoning strands (Breyfogle et al., 2010; Dingman et al., 2019; Dingman et al., in press; Roth McDuffie and Mather, 2009), and this study supports the usefulness of this framework. This study supports the usefulness of these seven strands to describe PSTs' reasoning, as described by Dingman et al. (in press). The curricular reasoning framework provides a means to identify teachers' reasoning based on various entities; including student thinking, mathematics, learning trajectories, and their own beliefs.

Not only does this study justify the usefulness of the curricular reasoning framework, but this study contributes three additional details related to the curricular reasoning framework during the teaching process. First, PSTs in my study reasoned with curricular reasoning strands in *intertwined ways* by often reasoning with multiple strands as they made decisions, which allowed the PSTs to reason in more complex ways. This also reflects the complex nature of teachers' work as they make decisions during the teaching process and that it is helpful if PSTs can reason in ways that involve mathematics, curriculum, and students. For instance, PSTs' reasoning with *analyzing curriculum materials* for their Big Decision in Table 5 was supported as PSTs also reasoned with *mapping learning trajectories* and *drawing on mathematical* *meanings*. PSTs reasoned with *mapping learning trajectories* in line 6 to identify mathematical connections in the textbook they were analyzing. Furthermore, PSTs' reasoning with *drawing on mathematical meanings* in line 1 led them to reason with *analyzing curriculum* materials to determine whether they needed to include the definition of transformation in their lesson. PSTs reasoned with *drawing on mathematical meanings* in line 24 as they expressed that the definition of transformation was important to their lesson and this provided motivation to determine if they needed to define it in their lesson.

Second, as discussed by Remillard (2005), curriculum is an important part of the teaching process. The presence of coherent curriculum seemed to allow PSTs to reason with *drawing on mathematical meanings*, *mapping learning trajectories*, *positioning with regards to the mathematics*, *analyzing curriculum materials*, and *doing tasks as learners*. PSTs in my study were given a variety of unfamiliar curriculum units from which to begin their planning process that seemed to necessitate PSTs reasoning with *drawing on mathematical meanings* to make sense of the mathematics for themselves and determine the mathematics that was important for students. Similarly, these curriculum units also seemed to push PSTs to reason with *mapping learning trajectories* – specifically, with *unit connections* because their lesson content relied on content that was taught or not taught in prior lessons. Remillard (2000) proposed that curricula materials ought to support teachers' reasoning and learning, which may have been the case in my study. Perhaps if PSTs were given more familiar curriculum units, they may have reasoned differently. Thus, it may not only be helpful that PSTs are given curricula materials from which to reason but it may be important to consider *what* curricula materials they are given to reason about.

Third, it is helpful to identify the resources that PSTs draw on while reasoning because it provides understanding to the ways in which they reason. PSTs in my study drew on the resources of their instructor, lesson implementation, and lesson debriefings and may provide possible explanations for why PSTs reasoned with *considering learners' perspectives* throughout the teaching process. Similarly, the PSTs drawing on the resource of their instructor, especially during their InM, may help explain why PSTs in my study reasoned with *drawing on mathematical meanings* focused on making sense of the mathematics for themselves earlier on in their planning.

Making Sense of the Mathematics

Researchers have found that PSTs can develop their mathematical understanding in different ways. Researchers have found that PSTs developed their mathematical understanding through working with and teaching from non-self-created curriculum materials (e.g., Donna & Hick, 2017; Nicol & Crespo, 2006), reciprocal learning experiences facilitated by instructors (Suh & Parker, 2010), analyzing non-self-created curriculum materials (Ebby, 2000), working in lesson study groups (Suh & Parker, 2010), and teaching the same content multiple times (e.g., Borko & Livingston, 1989; Forbes & Davis, 2007).

My study confirmed that PSTs made sense of the mathematics as they worked with or analyzed non-self-created curriculum materials and through working in lesson study groups, but PSTs also made sense of the mathematics as they reasoned with the strands *drawing on mathematical meanings*, *analyzing curriculum materials* that were self-created, and *doing tasks as learners*. PSTs made sense of the mathematics in their lesson as they reasoned with the *doing tasks as learners* strand both as individuals and together in groups. Not only did PSTs in my study make sense of the mathematics in their lesson through working with non-self-created curriculum materials, but they also did so as they *analyzed* the curriculum materials. They analyzed not only their textbook chapter, but also *self-created* materials and online resources. Additionally, they made sense of the mathematics in their lesson as they reasoned with *drawing on mathematical meanings – of PSTs* through interacting with others. As PSTs worked in lesson study groups they learned not only from working on their lesson together but from each other while *doing tasks as learners*. Additionally, PSTs made sense of the mathematics while interacting with other PSTs in their methods course, both others' comments in their methods course and other PSTs' lessons. The PSTs in my study also made sense of the mathematics as they interacted with their instructor; who not only fostered reciprocal learning experiences of teaching multiple lessons in their methods class; but especially during their InM as they were *doing tasks as learners*, being asked questions, and listening to their instructor explain her own mathematical understanding. PSTs also made sense of the mathematics from their lesson implementation, including student thinking that arose in their lessons that allowed them to reason with *drawing upon mathematical meanings – for students*. My study confirms findings that other researchers have found about developing PSTs' mathematical knowledge and my study also reveals that PSTs made sense of the mathematics as they analyzed self-created materials, did tasks as learners, and had mathematical interactions with their instructor.

Researchers have found that PSTs draw upon their mathematical understanding when teaching (e.g., Donna & Hick, 2017; Gadanidis et al., 2003). Some researchers have found that PSTs' strengthened mathematical content knowledge led to posing of better questions that revealed student thinking (van den Kieboom, Magiera, & Moyer, 2010). Other researchers found that limited mathematical content knowledge caused student teachers to be less effective (Borko et al., 1992; Son, 2013).

Not only did PSTs in my study did draw upon their mathematical understanding during the teaching process, but the PSTs in my study also reasoned with *drawing on mathematical meanings* with a focus on making sense of the mathematics in their lesson for themselves *before* planning their lesson. In the methods course where my study was conducted, PSTs were required to write a FMC (i.e., the mathematics they wanted their students to learn from their lesson) and discuss the mathematics of their lesson with their instructor early in their planning process. These things seemed to support PSTs in this study to focus on making sense of the mathematics in their lesson for themselves at the beginning of the teaching process, which seemed to assist the PSTs to reason with the mathematics later in the teaching process as they planned the sequencing or presentation of the mathematics in their lesson as well as other student thinking and responses to that student thinking (i.e., column 3 in the *Teacher Work Sample* lesson plan format in Appendix D). Thus, it seems that PSTs taking time to make sense of the mathematics for themselves at the beginning of their planning process and discussing the mathematics with their instructor may have supported their reasoning with the strand *drawing on mathematical meaning* throughout the remainder of the teaching process.

Focus on Learners

Prior researchers have debated whether PSTs can reason about student thinking. Some researchers suggested that PSTs cannot consider student thinking until they develop their identity as a teacher and acquire classroom management skills (Fuller, 1969; Fuller and Brown, 1975; Kagan, 1992) or at least not until they first focus on themselves as teachers (Darling-Hammond & Snyder, 2000; Freese, 2006; Loughran, 2006; Mellado, 1998). Contrastingly, Shapiro (1991) found that PSTs developed their own identity as teachers *as* they interacted with students. Furthermore, Levin, Hammer, and Coffey (2009) deduced the danger of having PSTs first focus on themselves before focusing on students because it encouraged PSTs to develop habits and routines of teaching that do not include a focus on student learning and the very reason that PSTs may not focus on student thinking is because of the structure of teacher education programs. Levin et al. (2009) further suggested that PSTs will not simply remove their *inattentional blindness* (Simons & Chabris, 1999) to student thinking and begin to focus on and incorporate students' thinking into their teaching on their own, especially without support, after they leave teacher education programs (see also Grossman, Hammerness, & McDonald, 2009).

In my study, I found that PSTs *can* focus on student thinking and learning. While the PSTs in my study were in a methods course that encouraged them to focus on student thinking as they planned and implemented lessons, this may have allowed them to develop a focus on learners as suggested by Levin et al. (2009). Specifically, these PSTs had a focus on students throughout the teaching process as they reasoned with the strands *drawing on mathematical meanings*, *considering learners' perspectives*, *positioning with regards to the mathematics*, and *analyzing curriculum materials*. Specifically, PSTs reasoned with *drawing on mathematical meanings – for students* related to the mathematics students needed to know and *considering learners' perspectives* related to how students might view the mathematics in their lessons. PSTs also reasoned about student thinking to make decisions that affected the sequencing and presentation of the mathematics in their lesson, as opposed to only planning how to respond to student thinking. Additionally, Bridget, who had taken half of the methods course before, seemed to be a driving force for group 1's focus on student thinking; and this may support the implication of Levin et al.'s (2009) that PSTs who are given the supports to reason with student thinking will develop the habit of reasoning about students.

The question then becomes: why did PSTs in this study reason about student thinking and learning when other research has suggested that they are not capable of doing so? Some answers can be found in the significant external resources that PSTs drew on in this study. First, the PSTs' instructor was a resource that PSTs' drew on while reasoning with *considering learners' perspectives* and *drawing on mathematical meanings – for students*, especially during their InM. Second, the PSTs' opportunity to go through the teaching process – namely PSTs' opportunities for lesson implementation, and reflection – and receive feedback from those as "students" and their instructor – allowed PSTs' to reason with *considering learners' perspectives*. Additionally, as PSTs planned lessons in groups, PSTs brought different perspectives into their planning while working in groups; thus, one PST who was reasoning in ways that focused on learners could possibly influence another PST's development to reason with a focus on learners.

Connections in Leaning Trajectories

Prior researchers have found that novice teachers are less effective using lesson goals to guide their lesson structure and PSTs are less experienced with the sequencing of lessons. Some researchers found that PSTs' goals focused on classroom management (Gadanidis et al., 2003). Other researchers found that PSTs were not as detailed in their plans nor as selective in which parts to prioritize as inservice teachers (Borko & Livingston, 1989) and were less experienced with sequencing lessons than inservice teachers (Suh & Parker, 2010).

The PSTs in my study had mathematically focused goals; however, their sequencing of their lessons is similar to prior research. The PSTs in my study developed written mathematical goals that focused on student mathematical learning and did not have goals focused on classroom management. Yet, similar to prior research, the PSTs in my study focused less on the mathematics when sequencing content within individual lessons. Evidence for this is that the

PSTs in my study reasoned less with *mapping learning trajectories – lesson connections* than with– *unit connections*. It seems that the design of the lesson study group schedule, with each PST group only teaching one lesson in a unit, may help to explain PSTs' formation of *unit connections* when reasoning with the strand *mapping learning trajectories*. Specifically, this may be because PSTs' understanding that some of the mathematical content in their lesson builds on certain mathematical content from prior lessons in the same unit; thus, their students' learning of their lesson is influenced by whether or not students learned the content from prior lessons. If students did not learn the prior lessons' content well enough, then PSTs reviewed mathematical concepts from prior lessons that were essential to their lesson before teaching their lesson.

The fact that the PSTs reasoned less with *mapping learning trajectories – lesson connections* may be related to how they created and used their lesson goals. The PSTs in this study were similar to another less experienced inservice teacher who did not "use their goals to guide their lesson development" as a more experienced inservice teacher had done (Gadanidis et al., 2003, p. 89). This begs the question: what might allow PSTs to reason more with *mapping learning trajectories – lesson connections*? One potential connection to PSTs' reasoning with *mapping learning trajectories – lesson connections* may be PSTs' written mathematical goals for students. This study found that PSTs tended to create and solidify their written mathematical goals for students' learning *after* planning the majority of their lesson. Thus, these goals did not provide a starting point for PSTs to connect the pieces of their lesson in a way that allowed students to best meet the written mathematical goals. This is a concern because a decreased focus on connections within a lesson learning trajectory could lead to student confusion. For example, group 1's lesson trajectory contained a few areas of possible student confusion such as defining a composition "of two reflections over parallel lines" (Benson et al., 2009, p. 204) as a translation and then asking students to prove if it was a translation. Similarly, Hiebert, Morris, Berk, and Jansen (2007) suggest a teaching framework that puts the planning of lesson content goals for students at the beginning of planning a lesson *after* understanding the subject matter knowledge for the lesson.

Summary

These findings reveal areas that PSTs have the capacity to reason about as well as resources PSTs drew on while reasoning and ways that PSTs reasoned. I found that PSTs can make sense of the mathematics in the lesson they are to teach, can reason with a focus on learners, and can reason about unit level connections in learning trajectories. While reasoning with the strands *drawing on mathematical meaning*, *considering from learners' perspectives*, and *mapping learning trajectories* PSTs drew on the resources of their instructor, their opportunity to experience the entire teaching process and receive feedback, and the design of the lesson study group schedule. PSTs reasoned with many curricular reasoning strands to make sense of the mathematics and to focus on learners. This study found PSTs reasoned little with the strand *mapping learning trajectories – lesson connections*, and though it is unclear exactly why, there may be a connection between this and PSTs' use of written mathematical goals in their lesson planning.

CHAPTER 6: CONCLUSION

Teachers' mathematical decisions during the teaching process affect student learning, and understanding teachers' reasons for their decisions provides insight into their mathematical decisions. Prior researchers have studied teachers' decisions at single decision-points during the teaching process and identified that teachers often modify their decisions at the various decision-points throughout the teaching process. In this study, I used the curricular reasoning framework as shown in Table 1 to understand PSTs' reasons for their mathematical decisions throughout the teaching process. I collected data from two PST groups during the planning, implementation, and reflection of their lessons. My results indicate that PSTs in my study reasoned with the seven curricular reasoning strands when making mathematical decisions. They often reasoned with multiple curricular reasoning strands when making these decisions around a Big Decision. This adds empirical data to support the curricular reasoning framework as well as identified ways in which PSTs reason during the teaching process. Specifically, that PSTs have the capacity to reason about the mathematics, learners, and learning trajectories when given unfamiliar curricula.

Contributions

This study has two contributions to the mathematics education field related to research on teachers' mathematical decisions and their curricular reasoning during the teaching process. First, this study adds greater detail to the curricular reasoning framework. It describes PSTs' reasons for their mathematical decision during the teaching process and identifies subcategories for three curricular reasoning strands.

Second, this study contributes to the mathematics education research on teachers' decisions by examining PSTs' curricular reasoning for mathematical decisions during the teaching process prior to student teaching. By identifying the PSTs' decisions and reasoning, I found that they reasoned with the seven curricular reasoning strands at different points during the teaching process and often did so in intertwined ways, which demonstrates that the PSTs in my study had the capacity to reason with all curricular reasoning strands prior to enter student teaching. I found that PSTs in my study reasoned with multiple strands for various decision types, which shows that PSTs in my study reasoned with multiple strands to make decisions. Additionally, PSTs in my study reasoned with *drawing on mathematical meanings*, *mapping learning trajectories*, and *considering learners' perspectives* during their planning process more than the other four curricular reasoning strands; and this emphasizes that while making decisions, PSTs in my study had the capacity to reason with these strands that involved mathematics, curriculum, and students.

Implications

This study has three implications for educational practice and teacher education. First, PSTs could benefit from developing various curricular reasoning strands and have experiences reasoning with multiple curricular reasoning strands, because based on this study, PSTs' reasoning with strands were not isolated instances, but rather their decisions were based on multiple reasoning strands in intertwined ways. Based on the context of the undergraduate program from which the PSTs in my study participated, I believe that the PSTs in my study may have developed various curricular reasoning strands through their five mathematics education courses in their program and were given opportunities to reason with multiple curricular reasoning strands. The PSTs' ability to reason with multiple curricular reasoning strands seemed to assist them in reasoning with *drawing on mathematical meanings*, *mapping learning trajectories*, and *considering learners' perspectives*. Thus, it may be beneficial for teacher educators, especially those who are working with PSTs in programs that do not have multiple mathematics education courses, to provide PSTs with opportunities through tasks or activities to reason with multiple curricular reasoning strands. For example, PSTs complete activities in their coursework that require them to analyze curriculum materials; however, are the activities requiring PSTs to reason with multiple curricular reasoning strands? As PSTs are given opportunities to reason with both *analyzing curriculum materials* and *considering learners' perspectives* they may begin to consider how students might view the different tasks or definitions in the curriculum materials, including areas of incorrect answers or confusion. Similarly, as PSTs are given opportunities to reason with *analyzing curriculum materials* and *mapping learning trajectories*, PSTs may consider the connections between mathematical concepts inside one lesson, between lessons in a unit, or even between two units in a year. Thus, PSTs could gain a broader perspective of the curriculum materials and how they are related to students' learning. Thus, it seems beneficial for teacher educators to provide PSTs with activities that allow them opportunities to reason with multiple curricular reasoning strands because this may help PSTs to reason with these curricular reasoning strands in their future practice as teachers as they begin to develop habits of reasoning in intertwined ways.

Second, it may be the case that the PSTs' experiences in their methods course enhanced the ways that they reasoned. The PSTs tended to draw on different resources during the teaching process: comments from their instructor during their InM and at other times, comments from their peers who served as students during class debriefings, their lesson implementation, and the need to be aware of mathematical connections between lessons. Additionally, the structure of planning lessons in different groups of PSTs allowed them to learn ways of reasoning from each other, such as a PST who focused on student thinking could help other PSTs in their group to reason about student thinking and learning during their planning. Thus, an implication for this study is that it seems helpful to both identify resources that PSTs draw on while reasoning and future research could benefit from considering the characteristics of methods courses that support PSTs' curricular reasoning. These resources that PSTs draw on and characteristics of methods courses could reveal potential ways to support PSTs' reasoning.

Third, based on the four decision types and data on PSTs' curricular reasoning at the beginning of their planning process, teacher educators may want to explicitly teach PST to plan lessons in a suggested order and help them understand why an order is important. The results in this study indicate that PSTs made sense of the mathematics for themselves prior to lesson planning, especially during their LPM1 and InM when discussing the mathematics with their instructor and doing tasks as learners with their instructor. It seems that it may be the case that first focusing on making sense of the mathematics for themselves may have contributed to them discussing their lesson with a focus on the mathematics throughout the remainder of the teaching process. Therefore, it seems beneficial that near the beginning of the planning process PSTs receive direct support from their instructor to gain a deeper understanding the crucial mathematics in the lesson for themselves prior to planning their lesson. Additionally, though PSTs in my study did not create their mathematical goals for students' learning early in their planning and did not use the goals to guide the development of their lesson, it may assist PSTs to write their mathematical goals for students' learning and actions (i.e., what mathematics students need to know from the lesson) *before* planning how the lesson will unfold. Doing so may help PSTs focus their lessons on the most important mathematical concepts and lay the foundation for creating a mathematical learning trajectories within their lesson. The lack of clear mathematical goals driving the lesson may have affected the PSTs in my study to not have many lesson connections when reasoning with *mapping learning trajectories*. Lastly, it may be beneficial for PSTs to plan a lesson trajectory (i.e., the sequence) of the main sub-points for the lesson in a way that connects the mathematical concepts and student learning goals together so students could best achieve the written mathematical goals for the lesson in a more effective manner. After PSTs do these three steps they could then hopefully plan the remainder of their lesson with more focus and mathematical connections – namely the presentation of the mathematics to students during the lesson, including what the students will do and what the teacher will do.

Limitations and Directions for Future Research

One of the limitations of this study is that I collected data from two PST groups from one methods course at one university that differs from other universities. These data represent a small scope of PSTs' curricular reasoning as well as from a single methods course. Additionally, the PSTs in my study have had a different undergraduate experience than those at other universities, including five mathematics education courses. Despite this limitation, this study provided initial results on PSTs' decisions and their curricular reasoning for those decisions. Therefore, this research could be extended in three ways: (1) by studying PSTs' decisions and curricular reasoning in other methods courses, (2) by studying PSTs' decisions and curricular reasoning from other universities, and (3) by studying student teacher and novice teachers' decisions and curricular reasoning. By studying PSTs' decisions and curricular reasoning in other methods courses and at other universities, I would expect to find some similarities depending on the activities PSTs complete within their methods courses. However, I would expect to see differences in PSTs' decisions and curricular reasoning that may yield insight into the support structures PSTs need to make decisions and reason with different curricular reasoning strands. For example, in my study PSTs rarely reasoned with *lesson connections* within the *mapping* *learning trajectories* strand, which I hypothesize was because PST groups planned and taught one lesson within a unit rather than planning and teaching a complete unit or even consecutive lessons. By studying student teacher and novice teachers' decisions and curricular reasoning, I would hypothesize that in some cases the teachers' curricular reasoning may be different based on many resources: curricula, department, student thinking. Researchers have also documented that methods courses can influence PSTs' teaching (e.g., Amador & Weiland, 2015; Weiland, Hudson, & Amador, 2014), but it does not guarantee that PSTs will continue using what they learned (e.g., Lasley, 1980; Drake & Sherin, 2006).

Another area of potential research is to investigate the effects of the principles expressed by Hiebert et al. (2007) about teaching in ways that "support student learning" (p. 48). PSTs in my study were given opportunities to make sense of the mathematics in their lessons, were asked to debrief whether they reached their lesson's FMC and why they reached it, and for half of the class were required to re-write their lesson plan if they were to teach the lesson again. The results from this study suggest that PSTs reasoned about students during the teaching process. Thus, future research could identify if and how the principles by Hiebert et al. (2007) might allow PSTs to foster habits of reasoning about students.

Conclusion

This study investigated PSTs' curricular reasoning as they made mathematical decisions during the teaching process. It illustrates the power of investigating both PSTs' reasoning and the resources on which they draw as they reason. Describing the ways that PSTs' reason about mathematical decisions throughout the teaching process provides the field with more clarity on PSTs' reasoning about their mathematical decisions. This study also identified curricular reasoning strands that may need more attention in teacher education programs. Lastly, this study provides a perspective into PSTs' capacities to make mathematical decisions and their reasoning for those decisions. It is important that future research continue to investigate the resources PSTs are provided in order to optimize their reasoning in ways that best support student learning.

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APPENDIX A

Interview Protocols for Post-Interviews

This post-interview protocol has been adapted from a larger National Science Foundation project (NSF # 1561569).

- Follow-Up on unclear PSTs' decisions while planning:
  - Tell me about any decisions you made while planning the lesson. [*Follow up with resources used or not used as found during initial data analysis.*]

    Why did you decide not to use _________ in your lesson plan?

    Why did you decide to change _______ from your lesson plan?

    Why did you decide to add ________ to your lesson plan?

- Interview about PSTs' decision while reflecting on implementation:
  - What do you feel went well in your lesson? Why do you think it went well?

    What do you feel didn't go well in your lesson? Why do you think it didn't go well?
  - What surprised you about the implementation of the lesson?

    What surprised you about your students' learning of the content?
  - Did you achieve your FMC? How do you know?

    Which of the mathematical goals did you feel your students achieved? How do you know?
  - Do you feel your task promoted student learning in the way you had hoped? How so?

    [*If necessary*] Why did you choose to modify or change the task during the lesson as you did?

[*If necessary*] Why did you choose to change the sequence of the lesson?

[*If necessary*] Why did you choose to address or not address specific misconceptions or student thinking? [state what instances]

- If you were to teach this lesson again, what changes would you make to the lesson? Why?

  What do you plan to teach tomorrow based on how this lesson went today? Why?
- How do you see this lesson fitting in the broader unit? Or state standards?
- Compare these curriculum materials with ones you have seen or experienced before:

  In what ways did they better address the mathematics content?

  In what ways did they not address the mathematics content?
- Follow up with decisions flagged during lesson:
  - [*for PST who taught*] Why did you make ______ decision? Would you change it if you were to do it again?

    Were you surprised at ______ student thinking?
  - [*for PSTs who observed*] From this circumstance in the lesson ______, what would you have done if you were the teacher?

    Were you surprised at ______ student thinking?

APPENDIX B

Lesson Overview Form (Online)

These questions about the PSTs' lesson overview were adapted from a National Science Foundation project (NSF # 1561569) and contained the following questions.

BACKGROUND:

- What is your name?
- What unit, lesson, and grade level is this lesson for?
- What standards will this lesson address?
- List all the resources (e.g., websites, colleagues, textbooks) you used to prepare your lesson?

MATHEMATICS CONTENT:

- What mathematical content are you planning for?
- What are the big mathematical idea(s) for this lesson?
- What are your mathematical goals for students for this lesson?
- On a scale of 1 to 5 how confident are you in your understanding of the mathematics for this lesson? (1 = Not confident at all; 5 = Very confident)
- What content did you teach in the prior two lessons and how do you plan to build on that understanding in your current lesson?
- What content will you teach in the next two lessons and how do you plan to connect what student learn in this lesson to future lessons?

STUDENT THINKING:

- On a scale of 1 to 5 how confident are you in your understanding of student thinking about the mathematics you will be teaching in this lesson? (1 = Not confident at all; 5 = Very confident)
- What strategies do you think students will use to solve the task(s)?
- What misconceptions do you think students will have during your lesson?

LESSON SEQUENCE:

- How do you envision your lesson playing out during your class? (e.g., What will you do first, how long will students work, what will you discuss)
- What are the most important parts of your lesson that you will highlight for students?

APPENDIX C

Curricular Reasoning Strand Examples and Possible Subcategories

The following sections list examples and possible subcategories for the curricular reasoning strands of *positioning with regards to the mathematics*, *analyzing curriculum materials*, *doing tasks as learners*, and *revising based on experiences in teaching and learning*.

Positioning with Regards to the Mathematics:

Possible subcategories:

(1) with beliefs about the mathematics

(2) with beliefs about students

(3) with beliefs about teaching.

Examples:

- An example of beliefs about teaching: in Table 5 lines 49-56 two PSTs reasoned based on their belief that *students did not need to discover a definition if they have had some experience with it* to inform a decision to just tell students the definition of transformation.
- As an example of beliefs about students: while reflecting on implementation, a PST in group 1 shared her belief that *students get off task and learn less when teachers write on the board too long*, which led her to the decision to not write the definition on the board if teaching the lesson again.

Analyzing Curriculum Materials:

Possible subcategories:

(1) based on the mathematics

(2) based on the design for students' learning

Examples:

- An example of based on the mathematics: in Table 5, PSTs reasoned with *analyzing curriculum materials* as they sought to determine whether and where it contained the definition of transformation.
- An example of based on the design for student learning: group 1 PSTs reasoned with *analyzing curriculum materials* and chose to modify the shapes in their task to be irregular shapes so it would be easier for students to see differences in orientation when doing "two reflections over parallel lines" (Benson et al., 2009, p. 204).

<u>Doing Tasks as Learners:</u>

Possible subcategories:

(1) to make sense of the mathematics for themselves

(2) to explain the mathematics to another (usually a group member)

(3) to determine the presentation and sequencing of the mathematics for their lesson

Examples:

- An example of making sense of the mathematics for themselves: when group 2 PSTs did the task of finding the minimax with $x_1, \ldots x_n$in their head to make sense of the mathematics of finding the distance between x_1 and x_n so that they could determine the different formulas – both correct and incorrect – for finding the minimax.
- An example of explaining the mathematics to another: when a PST in group 2 did a problem of finding the minimax on the board to explain to a fellow group member why only the extreme houses matter when finding the minimax as they discussed the mathematical concept to include in their lesson.

- An example of determining the presentation and sequencing of the mathematics for their lesson: when group 1 PSTs recalled their experiences of doing a composition "of two reflections over parallel lines" (Benson et al., 2009, p. 204) on patty paper during their InM to help them decide what to include in their task.

Revising Based on Experiences in Teaching and Learning:

Possible subcategories:

(1) to modify the sequencing of the lesson

(2) to modify the mathematical content of the lesson

Examples:

- An example of modifying the sequencing of the lesson: a PST in group 2 reasoned that based on her peers' feedback during the debriefing of their lesson, she would add a class discussion about a definition that was presented during their lesson rather than rushing over it.
- An example of modifying the mathematical content of their lesson: a PST in group 2 who thought that midrange was important for their lesson reasoned during her post-interview that she would remove the term "midrange" from their lesson because it was only used in a definition and she did not think it was needed for their lesson.

APPENDIX D

Teacher Work Sample Lesson Plan Format

This appendix contains the Teacher Work Sample format from which PSTs were required to make their lesson plans during their methods course.

Department of Mathematics Education

Teacher Work Sample Lesson Plan

Column 1	Column 2	Column 3
Lesson Plan Sequence (What tasks will you pose? What clarifying and probing questions might you ask? What information and notation might you provide?)	**Anticipated Student Thinking and Students' Role in Discourse** (What do you anticipate student thinking and communication to look like and sound like?)	**Formative Assessment/ Responses to Student Thinking** (What will you *look for* and *listen for* as indicators and evidence of understanding?)

APPENDIX E

Data for Significant External Resources that PSTs Drew on While Reasoning

This appendix contains data for significant external resources that PSTs drew on from chapter 4. The first section describes the influence of PSTs' InM. The last two sections contain the tables and graphs about the resources of the PSTs' instructor and the PSTs' opportunities for lesson implementation and feedback during their methods course.

<u>PSTs' drawing on resources during their curricular reasoning:</u>

Table 9 displays the percentage of three resources that PSTs drew on while reasoning with curricular reasoning strands while planning or reflecting on implementation, with percentages out of the total number of Ideas that were labeled as resources for each group.

Table 9

Percentage of Big Decisions for Significant External Resources that PSTs while Planning and Reflecting on Implementation

	Planning		Reflecting on Implementation	
External Influence	Group 1 (n=76)	Group 2 (n=48)	Group 1 (n=26)	Group 2 (n=28)
Instructor	55%	52%	12%	18%
Methods course discussions/other PST lessons	16%	2%	12%	50%
PSTs' taught lesson	-	-	54%	86%

Figure 6, 7, and 8 display the curricular reasoning strands that PSTs reasoned where they drew on their Instructor, their lesson taught, or their methods class discussions – which include lesson debriefings, and other PST lessons.

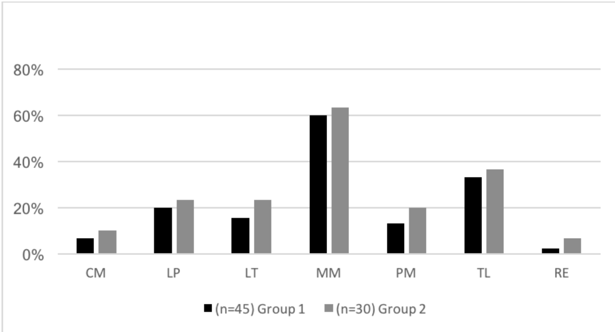

Figure 6. Curricular reasoning strands PSTs reasoned with while drawing upon comments or suggestions from their *Instructor* both while planning and while reflecting on implementation.

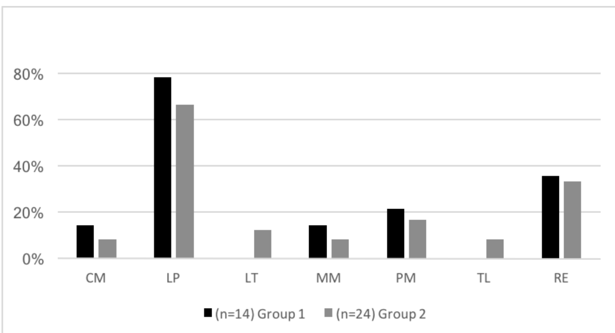

Figure 7. Curricular reasoning strands PSTs reasoned with while drawing upon *PSTs' lesson taught* both while planning and while reflecting on implementation.

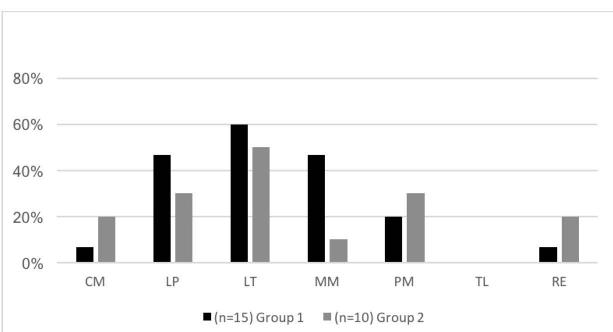

Figure 8. Curricular reasoning strands PSTs reasoned with while drawing upon *methods course discussions/other PST lessons* both while planning and while reflecting on implementation.

Additional data related to the resources of PSTs' instructor and of lesson taught:

Table 10 displays the different ways that the PSTs' drew on the influence of their instructor, with percentages out of the total number of Ideas that were labeled as *instructor* for each group. These data indicate that PSTs relied most on their instructor's facilitation of their learning of the mathematical content in their lesson, especially through questions and tasks; on their instructor's expression of mathematical meaning; and on their instructor's recommendations about things to consider in their lesson plan.

Table 10

Percentage of Ways for which PSTs Drew on Their Instructor as a Resource while Planning

Instructor Categories	Group 1 (n=42)	Group 2 (n=25)
Facilitation of PSTs' learning	55%	56%
Instructor's mathematical meaning	43%	64%
Knowledge about prior PSTs/students	5%	24%
Recommendation	48%	36%
Other	24%	8%

Table 11 displays the ways that PSTs drew on the influence of *PSTs' taught lesson* affected PSTs' curricular reasoning, with percentages out of the total number of Ideas that were labeled as *PSTs' taught lesson* for each group. This shows that as PSTs drew on the influence of their taught lesson, they focused some on what the teacher did, but PSTs focused more on student thinking and actions.

Table 11

Percentage of Ways for which PSTs Drew on Their Taught Lesson as a Resource while Reflecting on Implementation

PSTs' Taught Lesson Categories	Group 1 (n=14)	Group 2 (n=24)
Overall class experience	14%	38%
Student thinking/actions	79%	42%
Teacher did	29%	33%