



2010-04-29

Multiobjective Optimization Method for Identifying Modular Product Platforms and Modules that Account for Changing Needs over Time

Patrick K. Lewis

Brigham Young University - Provo

Follow this and additional works at: <https://scholarsarchive.byu.edu/etd>



Part of the [Mechanical Engineering Commons](#)

BYU ScholarsArchive Citation

Lewis, Patrick K., "Multiobjective Optimization Method for Identifying Modular Product Platforms and Modules that Account for Changing Needs over Time" (2010). *All Theses and Dissertations*. 2117.

<https://scholarsarchive.byu.edu/etd/2117>

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in All Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact scholarsarchive@byu.edu, ellen_amatangelo@byu.edu.

Multiobjective Optimization Method for Identifying Modular
Product Platforms and Modules That Account for
Changing Needs Over Time

Patrick K. Lewis

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of
Master of Science

Christopher A. Mattson, Chair
Spencer P. Magleby
Brian D. Jensen

Department of Mechanical Engineering
Brigham Young University
August 2010

Copyright © 2010 Patrick K. Lewis
All Rights Reserved

ABSTRACT

Multiobjective Optimization Method for Identifying Modular Product Platforms and Modules That Account for Changing Needs Over Time

Patrick K. Lewis

Department of Mechanical Engineering

Master of Science

Natural and predictable changes in consumer needs often require the development of new products. Providing solutions that anticipate, account for, and allow for these changes over time is a significant challenge to manufacturers and design engineers. Products that adapt to these changes through the addition of modules reduce production costs through product commonality and provide a set of products that cater to customization and adaptation. In this thesis, a multiobjective optimization design method using s-Pareto frontiers – sets of non-dominated designs from disparate design models – is developed and used to identify a set of optimal adaptive product designs that satisfy changing consumer needs. The novel intent of the method is to design a product that adapts to changing consumer needs by moving from one location on the s-Pareto frontier to another through the addition of a module and/or reconfiguration. The six-step method is described as follows: (A) Characterize the multiobjective design space. (B) Identify the *anticipated regions of interest* within the search space based on predicted future needs. (C) Identify the platform design variables that minimize the performance losses due to commonality across the anticipated regions of interest. (D) Assemble the s-Pareto frontier within each region of interest. (E) Determine the values of all design variables for the optimal product design in each region of interest by multiobjective optimization. (F) Identify the module design variables, and identify the platform and module designs by constrained module design. An example of the design of a simple unmanned air vehicle is used to demonstrate application of the method for a *single* Pareto frontier case. The design of a manual irrigation pump is used to demonstrate application of the method for a s-Pareto frontier case. In addition, these examples show the ability of the method to design a product that adapts to changing consumer needs by traversing the s-Pareto frontier.

Keywords: multiobjective optimization, transient pareto design, modular design, future needs

ACKNOWLEDGMENTS

This work would not have been possible without the support of my loving wife Dixie. She is both the light of my life and the often silent force that keeps me going. Likewise, without the continual support and encouragement of Dr. Christopher A. Mattson, this work would not have been possible. I appreciate the many hours he has dedicated to teaching and mentoring me throughout this project. I would also like to express my appreciation to Dr. Spencer P. Magleby and Dr. Brian D. Jensen, my other two committee members for their support and assistance throughout this research effort.

I would like to recognize the Fulton College of Engineering and Technology and the National Science Foundation Career Grant CMMI-0954580 for funding this research.

I am grateful to both my and my wifes family for their support in working toward this degree. They have given me hope on a number of occasions when I was not sure I could accomplish what I had set out to do. Finally, I am grateful to a kind Heavenly Father who has answered many prayers, enlightened my understanding, and given me ideas of how to proceed in order to be successful.

TABLE OF CONTENTS

LIST OF TABLES	vi
LIST OF FIGURES	viii
NOMENCLATURE	x
Chapter 1 Introduction	1
1.1 Objective	1
1.2 Approach, Premise, and Assumptions	2
1.3 Outline	4
Chapter 2 Literature Survey	7
2.1 Multiobjective Optimization	7
2.2 Product Modularity and Adaptability	11
2.3 Research Needs	12
Chapter 3 Phase 1 Method Development	15
3.1 Identification of Platforms and Modules that Account for Changing Needs	15
3.2 Step A: Characterize the Multiobjective Design Space	17
3.3 Step B: Define Anticipated Regions of Interest	17
3.4 Step C: Select Platform Variables	18
3.5 Step D: Select the Optimal Design Within Each Region of Interest	18
3.6 Step E: Develop Modules That Move From One Region of Interest to Another	21
Chapter 4 Phase 1 Example: Unmanned Air Vehicle (UAV) Design	25
Chapter 5 Phase 2 Method Development	35
5.1 Accounting for Changing Needs With Multiple Design Models	35
5.2 Step A: Characterize the Multiobjective Design Space	36
5.3 Step B: Define Anticipated Regions of Interest	36
5.4 Step C: Select Platform Variables	38
5.5 Step D: Assemble the s-Pareto Frontier Within Each Region of Interest	39
5.6 Step E: Select the Optimal Design Within Each Region of Interest	39
5.7 Step F: Develop Modules That Move From One Region of Interest to Another	42
Chapter 6 Phase 2 Example: Manual Irrigation Pump Design	45
Chapter 7 Conclusions	61
REFERENCES	65

LIST OF TABLES

4.1	UAV Constant Parameters	29
4.2	UAV Non-Constant Parameters	29
4.3	UAV Designs Selected Within Regions of Interest	30
4.4	UAV Platform and Module Designs	32
6.1	Pump Regions of Interest	50
6.2	Pump Discrete Design Variable Limits and Step Sizes	54
6.3	Pump Cylinder Discrete Unit Prices per Length	54
6.4	Pump Fixed Parameters	55
6.5	Pump Designs Selected Within Regions of Interest	56
6.6	Pump Platform Design	58
6.7	Pump Module Designs	58

LIST OF FIGURES

1.1	Product Family Beachhead Approach	2
1.2	Changing Market Segments	2
1.3	Generic Pareto/s-Pareto Representation	3
1.4	Graphical Intent of the Design Method	4
2.1	Generic Feasible Design Space With a Pareto Frontier	8
2.2	Generic Feasible Design Space With a s-Pareto Frontier	9
2.3	Generic 3-Dimensional Feasible Design Space and Resulting Pareto Frontier	11
2.4	Modular Architecture Types	12
3.1	Phase 1: Graphical Intent of the Design Method	15
3.2	Phase 1: Flow Chart of the Five-Step Design Method	16
3.3	Phase 1: Anticipated Regions of Interest	17
3.4	Phase 1: Shifted Pareto Frontier	19
3.5	Phase 1: Selecting the Optimal Design Within Regions of Interest	20
4.1	Concept UAV Schematic	26
4.2	Mission Profiles Schematic	26
4.3	UAV Graphical Results	33
5.1	Phase 2: Graphical Intent of the Design Method	36
5.2	Phase 2: Flow Chart of the Six-Step Design Method	37
5.3	Phase 2: Generic Feasible Design Space With Three Design Models	38
5.4	Phase 2: Anticipated Regions of Interest With Three Design Models	38
5.5	Phase 2: Shifted Pareto Frontier With Three Design Models	39
5.6	Phase 2: Assembled s-Pareto Frontier Within Regions of Interest	40
5.7	Phase 2: Selecting the s-Pareto-Optimal Design Within Regions of Interest	41
6.1	Benchmark Pump Designs	49
6.2	Pump Graphical Results and Method Validation	58
6.3	Pump CAD Renderings	60

NOMENCLATURE

δ	Matrix dictating the desired performance progression that each module provides
D_a	Set containing all design variable values of x_a and x_p
D_m	Set containing all design variable values of x_m and x_p
g	Vector of inequality constraints
h	Vector of equality constraints
J	Aggregate objective function
μ	Vector of design objectives
n_d	Number of designs comprising the adaptive design set
P	Objective space performance of the base and target designs used to develop modules
$\bar{P}^{(i)}$	Objective space performance of a design when used with the i -th module
$\Delta P^{(i)}$	Change in objective space performance from the base design to $\bar{P}^{(i)}$
p	Vector of design parameters
\hat{p}	Vector of module design parameters
x	Vector of design variables
x_a	Vector of non-platform adjustable design variables
x_m	Vector of non-platform design variables that characterize the design of modules
x_p	Vector of platform design variables

Subscripts, superscripts, and other indicators

$[]^{(i)}$	indicates current design/module
$[]^{(k)}$	indicates current design model
$n[]$	indicates the number of []
$[]_l$	indicates the lower limit of []
$[]_u$	indicates the upper limit of []
$[]^*$	indicates the optimal value of []

CHAPTER 1. INTRODUCTION

The *implementation environment* or *needed performance* of a product, commonly referred to as *consumer needs*, tend to naturally change over time [1]. When these changes result in performance needs that cannot be satisfied by a *single* product model (i.e. analytical engineering model predicting the performance of a product), additional models are developed. Products that adapt to these changes through the addition of modules reduce production costs and cater to customization and adaptation [2, 3]. In situations where product purchase costs are high and these changes occur rapidly, there is a need for products that are capable of adaptation and expandability at the consumer level [4]. However, in order to develop this type of product, a certain amount of confidence in what the future needs of that product will be is required, and there are no established methods for identifying these needs. Therefore, providing solutions that anticipate, account for, and allow for substantial change in consumer needs is a significant challenge to manufacturers and design engineers.

1.1 Objective

The objective of this thesis is to present and illustrate application of a method that includes the effects of future needs and allows for a *single* device to optimally adjust to these new and changing needs. Product families are often used to address the challenge of satisfying a variety of needs through product performance diversity, while still maintaining product commonality as seen by manufacturers [5, 6]. Two platforms for building product families are identified within the literature: Scale-based and Module-based product platforms [6, 7]. The strength of product family approaches is in their ability to provide a range of products that satisfy the *current* variation in consumer needs across *multiple market segments* [8]. One approach presented in the literature by Meyer [9] for accomplishing this is the product family beachhead approach illustrated in Figure 1.1.

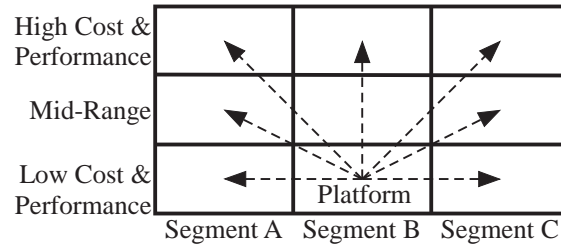


Figure 1.1: Representation of the product family beachhead approach as presented by Meyer [9].

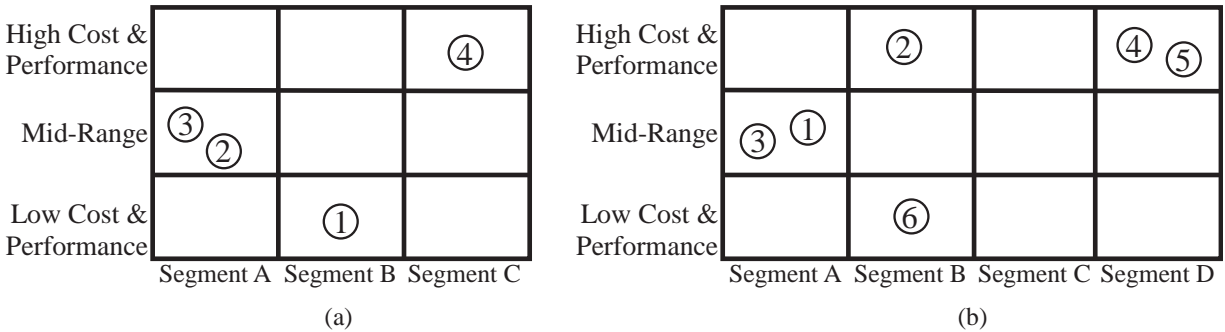


Figure 1.2: Illustration of the changes in market segment composition over time due to changing needs of consumers (groups of consumers are indicated by the numbers 1-6). The consumers and market segments of (a) represent the current market segment compositions. The consumers and market segments of (b) represent a future market segment composition.

From Figure 1.1 it is seen that product family approaches use current consumer needs of the various market segments to identify a product platform that is used to derive a variety of products to satisfy the needs of the various market segments [8,9]. However, as is illustrated through Figure 1.2, the exclusion of the effects of future needs of the various market segments, represented by the movement of consumers (groups of consumers are indicated by the numbers 1-6) from one market segment to another or the emergence of new market segments, is an important limitation of current design methods. Restated, the objective of this thesis is to overcome this limitation by developing a method that includes the effects of future needs and allows for a *single* device to optimally adjust to these new and changing needs.

1.2 Approach, Premise, and Assumptions

In this thesis, a multiobjective optimization design method using sets of non-dominated designs from disparate design models is developed and used to identify a set of optimal adaptive product

designs that satisfy changing consumer needs over time. Assuming these changes over time are known, the use of a multiobjective optimization method provides two key benefits: (1) The ability to leverage a set of non-dominated designs from multiple performance/design models to enhance the selection of the platform variables (values remain constant for all product family members) and module variables (values characterize the modules) – x_p , and x_m respectively. (2) The ability to balance the competing nature of present consumer needs against future needs. Figure 1.3 illustrates the concept of non-dominated designs (bold line) within feasible design spaces (shaded regions) based upon two design objectives (e.g. Cost, Output, etc.) represented by the horizontal and vertical axis. Figure 1.3(a) provides a generic representation of the non-dominated designs for a *single* design model, assuming that both Objective 1 and Objective 2 are to be minimized. Figure 1.3(b) provides a generic representation of the non-dominated designs in the presence of *multiple* design models.

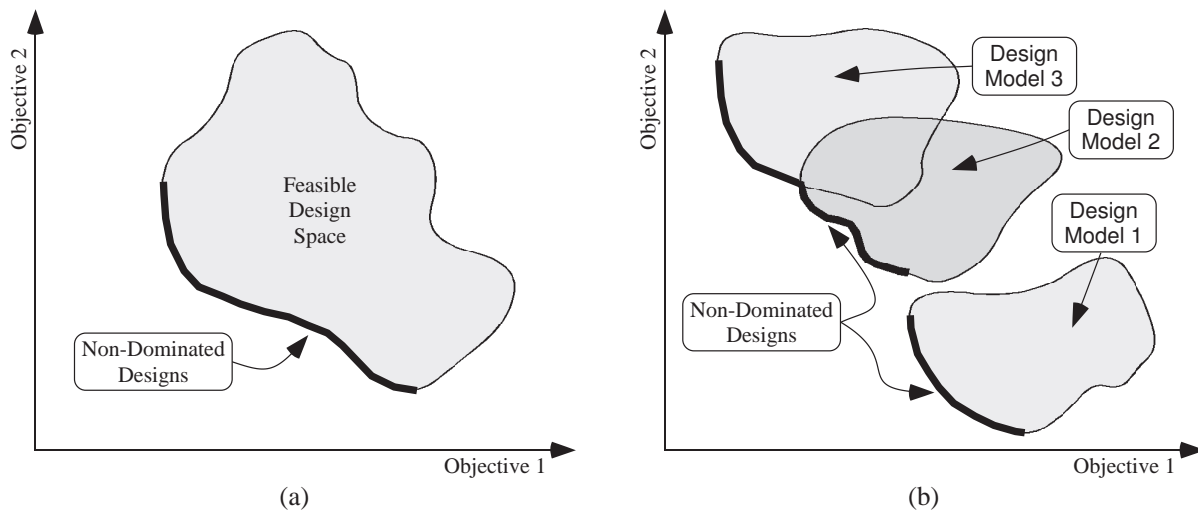


Figure 1.3: Illustration of the concept of non-dominated designs (bold line) within feasible design spaces (shaded regions) based upon minimizing two design objectives, where (a) illustrates a *single* design model and (b) illustrates *multiple* design models.

The novel intent of the method presented herein is to design a product that adapts to changing consumer needs by moving from one non-dominated design to another through the addition of a module and/or through reconfiguration. Figure 1.4 provides a graphical illustration of the intent of the method in the context of the feasible space and non-dominated designs (commonly referred to as the *Pareto frontier*) of Figure 1.3(a). From this figure it is observed that in the development

of the method presented in this thesis there are two tenants which the method is based upon: (i) In the selection of a design, non-dominated designs are preferred above any other feasible designs within the Design Space, and (ii) The current and future needs of a product represent individual designs from among the set of non-dominated designs.

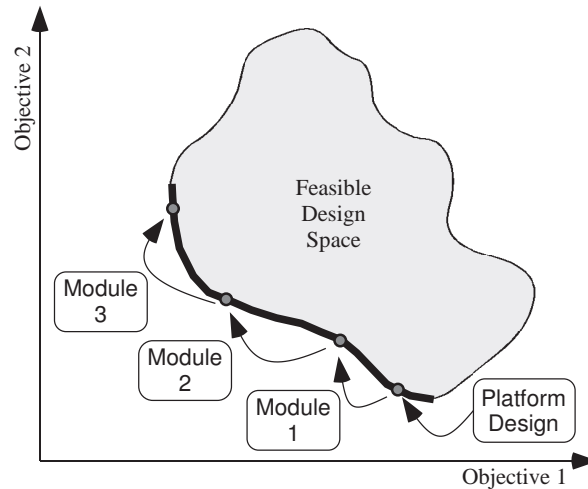


Figure 1.4: Graphical representation of the intent of the method developed in this thesis to provide a product that expands from one non-dominated design to another through the addition of modules.

Similar to traditional product family design approaches, the method presented herein uses commonality to identify an optimal product platform and module designs. Although the presented method differs from traditional product family design approaches through the process/method used to obtain the platform and module designs, the presented non-dominated design based approach for developing products that are both adaptable and reconfigurable can still be used for product family design.

1.3 Outline

The remainder of this thesis is presented as follows: A review of literature forming an enabling foundation for the developments presented herein is included in Chapter 2. In Chapter 3, the first phase (for *single* Pareto frontier cases) of the theoretical development is presented. In Chapter 4, the design of a simple unmanned air vehicle is used to demonstrate application of the method for a single Pareto frontier case. In Chapter 5, the method presented in Chapter 3 is expanded to provide

s-Pareto capabilities (multiple Pareto frontier cases). In Chapter 6, the design of a manual irrigation pump is used to demonstrate application of the method for a s-Pareto frontier case. Concluding remarks and a discussion of future work are provided in Chapter 7.

CHAPTER 2. LITERATURE SURVEY

This chapter provides a review of previous research, and establishes a foundation for the presented method of designing module-based products. The technologies that form an enabling foundation for the methodology are (i) multiobjective optimization and (ii) product modularity and adaptability.

2.1 Multiobjective Optimization

Consumer needs, some of which are expressed as design objectives $(\mu_1, \dots, \mu_{n_\mu})$, are often competing and change over time. Thus, within the context of this thesis, the ability of multiobjective optimization to balance competing objectives [10–18] – balance the competing nature of present consumer needs against future needs – represents a fundamental part of the method developed herein. Figure 2.1 demonstrates a generic characterization of trade-offs between objectives through the identification of a Pareto frontier – a set of non-dominated optimal solutions – assuming that μ_1 and μ_2 are to be minimized. Each solution comprising the frontier, graphically demonstrated in Figure 2.1, is said to be *Pareto optimal* – no other designs better satisfy *all* design objectives [19–22]. These Pareto solutions are generally sought because they indicate that objectives have been improved as much as possible without sacrificing another design objective’s performance. In addition, each solution represents the optimal balance of design objectives according to the consumer needs at a specific instance.

A generic multiobjective optimization problem (MOP) formulation yielding a set of optimal solutions – those belonging to the Pareto frontier – is presented as follows:

Problem 2.1: Generic multiobjective optimization problem statement

$$D := \{(x_1^*, x_2^*, \dots, x_{n_x}^*)\} \quad (2.1)$$

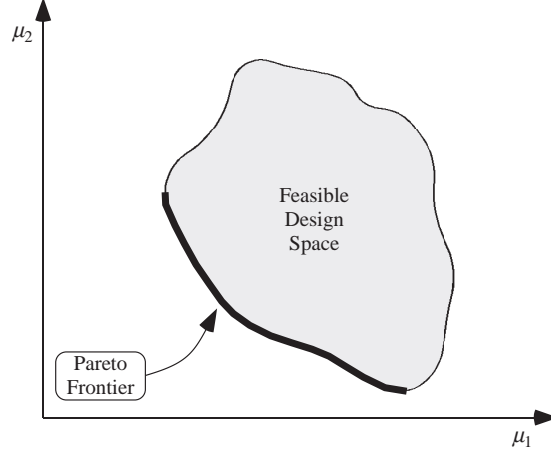


Figure 2.1: A feasible design space (shaded) for objectives 1 and 2. The Pareto frontier (bold line) represents the most desirable set of solutions in the feasible space for this minimization-minimization problem.

x^* defined by:

$$\min_x \{ \mu_1(x, p), \mu_2(x, p), \dots, \mu_{n_\mu}(x, p) \} \quad (n_\mu \geq 2) \quad (2.2)$$

subject to:

$$g_q(x, p) \leq 0 \quad \forall q \in \{1, \dots, n_g\} \quad (2.3)$$

$$h_v(x, p) = 0 \quad \forall v \in \{1, \dots, n_h\} \quad (2.4)$$

$$x_{jl} \leq x_j \leq x_{ju} \quad \forall j \in \{1, \dots, n_x\} \quad (2.5)$$

where D is a set containing all values of x^* for each Pareto-optimal design obtained through the evaluation of the MOP; μ_i denotes the i -th generic design objective; x is a vector of design variables; and p is a vector of design parameters.

For multiobjective optimization approaches, the decision of which Pareto-optimal solution is to be used comes through the inclusion of objective function parameters, and sometimes constraints that capture consumer needs or preferences for a single instance in time. As indicated in Problem 2.1 above, the current formulation of the MOP yields a set of solutions. In order to obtain a single optimal solution, the set of objectives in Equation 2.2 is often replaced by a scalar function that is optimized. This scalar function is referred to in the literature as an aggregate objective function [14, 23].

The concept of Pareto optimality is central to multiobjective optimization, [19, 21, 22, 24] and within the present method there is a need to balance the competing nature of the Pareto frontiers of multiple design models. Within the literature, this balance is addressed through the use of Pareto filters that either reduce the set of Pareto optimal solutions, [11, 25–27] or eliminate non-Pareto and locally Pareto solutions [14, 28–31]. In particular, the concept of generating an s-Pareto frontier – reduction of the Pareto frontiers from various disparate design models into a single Pareto frontier – presented in Mattson et al [14] has direct application to the balancing of the tradeoffs of a set of multiple design models needed within the proposed method. Figure 2.2 demonstrates the characterization of trade-offs between objectives through the creation of a s-Pareto frontier. Similar to a Pareto frontier, each solution comprising the s-Pareto frontier, graphically demonstrated in Figure 2.2, is said to be *s-Pareto-optimal* [14].

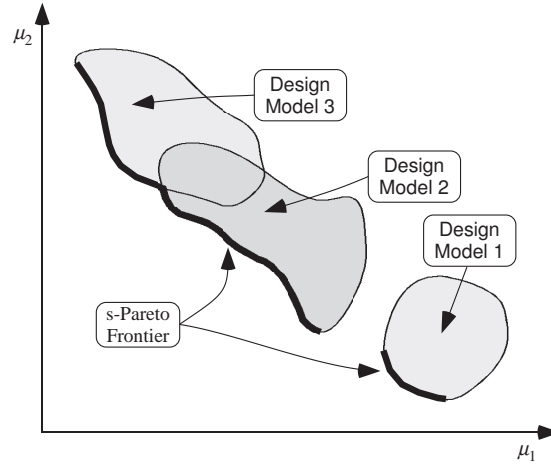


Figure 2.2: A feasible design space (shaded) for objectives 1 and 2. The s-Pareto frontier (bold line) represents the most desirable set of solutions in the feasible space for this minimization-minimization problem with three possible design models.

A generic MOP formulation yielding a set of optimal solutions – those belonging to a s-Pareto frontier – is presented as follows:

Problem 2.1: Generic s-Pareto multiobjective optimization problem statement

$$D := \{(x_1^{(k)*}, x_2^{(k)*}, \dots, x_{n_x^{(k)}}^{(k)*})\} \quad (2.6)$$

$x^{(k)*}$ defined by:

$$\min_k \left\{ \min_{x^{(k)}} \left\{ \mu_1^{(k)}(x^{(k)}, p^{(k)}), \mu_2^{(k)}(x^{(k)}, p^{(k)}), \dots, \mu_{n_\mu}^{(k)}(x^{(k)}, p^{(k)}) \right\} \quad (n_\mu \geq 2) \right\} \quad (2.7)$$

subject to:

$$g_q^{(k)}(x^{(k)}, p^{(k)}) \leq 0 \quad \forall q \in \{1, \dots, n_g^{(k)}\} \quad (2.8)$$

$$h_v^{(k)}(x^{(k)}, p^{(k)}) = 0 \quad \forall v \in \{1, \dots, n_h^{(k)}\} \quad (2.9)$$

$$x_{jl}^{(k)} \leq x_j^{(k)} \leq x_{ju}^{(k)} \quad \forall j \in \{1, \dots, n_x^{(k)}\} \quad (2.10)$$

where k denotes the k -th design model, D is now a set containing all values of $x^{(k)*}$ for each s -Pareto-optimal design obtained through the evaluation of the MOP; $\mu_i^{(k)}$ denotes the i -th generic design objective; $x^{(k)}$ is a vector of design variables for the k -th design model; and $p^{(k)}$ is a vector of design parameters for the k -th design model. It should be noted that, once again, the above MOP does not yield a unique solution. Where once again, the common method to obtain a single optimal solution is to replace the right-hand-side of Equation 2.7 with an aggregate objective function [14].

It should be noted that although Figures 2.1 and 2.2 only provide 2-dimensional (2-objective) representations of the solutions to Problem 2.1 and Problem 2.2 respectively, the MOP formulations provided in Problems 2.1 and 2.2 are not limited to 2-dimensional cases. For problems where $n_{n_\mu} = 3$, the results of Problems 2.1 and 2.2 result in 3-dimensional surfaces. Figure 2.3 demonstrates this result for Problem 2.1, where Figure 2.3(a) illustrates a 3-dimensional feasible space, and Figure 2.3(b) illustrates the resulting 3-dimensional Pareto surface for a minimization-minimization-minimization problem. For problems where $n_{n_\mu} > 3$, the results of Problems 2.1 and 2.2 are hyper-surfaces, and are therefore ill-suited for graphical representation. For this reason, although the MOP formulations that will be provided in this thesis will be applicable in n -dimensions, all graphical representations of generic MOP formulations provided hereafter will be represented in 2-dimensions in order to remain consistent with Figures 2.1 and 2.2.

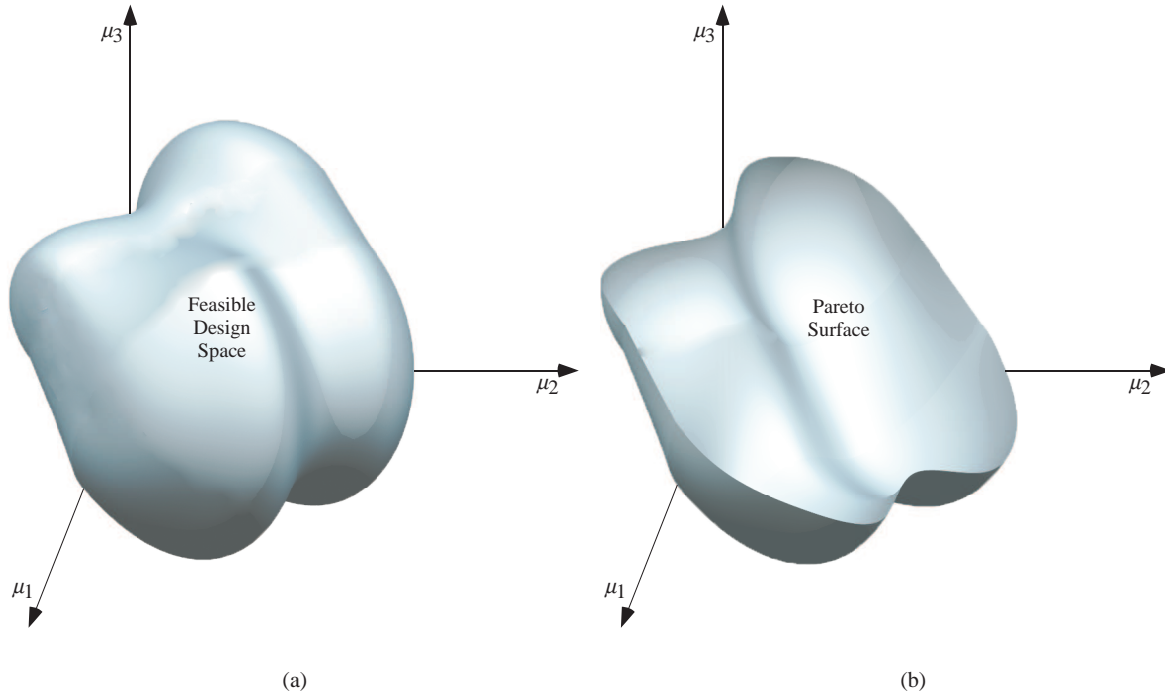


Figure 2.3: Representation of a 3-dimensional (3-objective) evaluation of Problem 2.1, where (a) illustrates a 3-dimensional feasible space, and (b) illustrates the resulting 3-dimensional Pareto surface for a minimization problem.

2.2 Product Modularity and Adaptability

Under the presented methodology, and as identified within the literature, there is a need for strategic module designs that make product platform designs progressively expandable [32–34]. To this end, previous work in the areas of product family and modular product design serve as a starting point [6, 8, 32, 33, 35, 36]. A module-based product family is a group of related products derived from independent functional or geometric units [37–39] that differ through the addition or subtraction of modules [35, 38, 39]. In the literature three types of modularity are identified: (i) *Slot-modular architecture*, (ii) *Bus-modular architecture*, and (iii) *Sectional-modular architecture* [39–41].

The conceptual differences between the three modularity architecture types is illustrated in Figure 2.4.

A slot-modular architecture provides each module with a unique interface in order to eliminate improper assembly [39–41]. Bus-modular architecture implements interfacing that is the same for all modules, thus making the platform design behave as a common connection platform for all modules [39]. Sectional-modular architecture is similar to bus-modular in that all modules contain

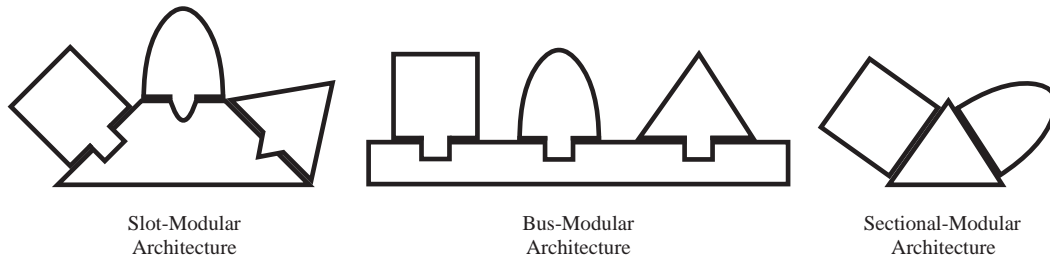


Figure 2.4: Representation of three architecture types as presented in Ulrich and Eppinger [39].

the same interface, but in this architecture no single element is identified as the platform to which all modules attach [39, 40]. Building on these foundational elements, the method presented in this thesis uses these definitions of modular architectures to specify the approach needed to develop module designs according to the a desired architecture type.

Recent developments in the literature show that a desirable product family can be identified from among the designs comprising the Pareto frontier [6, 8, 14] obtained through the evaluation of an MOP (see Section 2.1). These previous developments evaluate and select product family members from among the set of Pareto designs by considering the design’s unique performance *and* common features compared to other designs in the product family (a critical part of product family design). In addition, one method of identifying module and platform variables is accomplished through the use of Pareto-filtering methods that explore the effects of each variable on the objective space performance [6, 8].

2.3 Research Needs

While there exists useful elements in the literature on the subjects of multiobjective optimization and product modularity and adaptability, *a design methodology for finding balance in the context of changing consumer needs* is needed to fulfill the objective of this thesis identified in Section 1.1. Similar to traditional multiobjective optimization approaches, the method presented in this thesis seeks s-Pareto solutions, but expands upon traditional approaches to provide the desired unity and balance by using a series of strategically constructed MOP formulations to select solutions, within anticipated regions of interest, based on the solution’s ability to (i) be implemented by a module-based product, and (ii) expand/adapt to satisfy known changes in consumer needs over time. In

addition, in the present method a selection criteria based on known changes in consumer needs over time is added to the evaluation – to ensure that a progression from one design on the s-Pareto frontier to another can be done through the addition of a module.

In reference to the method presented in this thesis, it is also important to remember that, like most areas of engineering, research in the area of multiobjective optimization has experienced stages of evolution [42]. The first generation of research in multiobjective optimization focused on the development of theory and algorithms [43–45]. The second generation focused on the development of methods of using these algorithms to support general engineering [14,31,46]. The third (current) generation is using these methods and algorithms, and combining or expanding them in ways that improve design development [6,8,47–49] (e.g. making products more difficult to reverse engineer). Therefore, the purpose of this thesis is not to develop a new algorithm for multiobjective optimization, but instead will show how a series of optimization routines can be used to provide a method of developing a *single* product that is capable of traversing a s-Pareto frontier through the addition of modules.

CHAPTER 3. PHASE 1 METHOD DEVELOPMENT

In this chapter, the first phase in the development of a multiobjective optimization design method providing a Pareto-optimal product and module designs capable of satisfying changes in consumer needs over time is described for *single* Pareto frontier design cases.

3.1 Identification of Platforms and Modules that Account for Changing Needs

By its nature, a Pareto frontier contains many optimal, yet functionally different, designs representing all optimal product candidates. To satisfy changes in consumer needs over time through the addition of modules requires the strategic selection of these Pareto-optimal designs based on their ability to facilitate adaptability. Figure 3.1 illustrates the intent of the method to satisfy changing consumer needs by selecting a Pareto-optimal product platform design which, through the addition of modules, expands to other Pareto-optimal designs. The first (μ_1) and second (μ_2) objectives are represented along the horizontal and vertical axis, respectively.

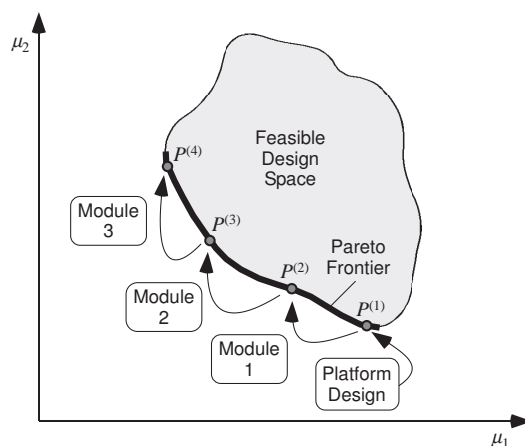


Figure 3.1: Graphical representation of the intent of the method to provide a product that expands from one Pareto-optimal design to another through the addition of modules.

Through further examination of Figure 3.1, it is seen that the platform design, shown as $P^{(1)}$, adapts to become $P^{(2)}$ through the addition of Module 1. Through this approach, the platform and subsequent modules, provide the desired product performance resulting from the changing consumer needs as represented by $P^{(1)}$, $P^{(2)}$, $P^{(3)}$, and $P^{(4)}$. Figure 3.2 provides a flow chart that illustrates the five primary steps of the multiobjective optimization design method developed in this chapter in response to the identified research need from Section 2.3 calling for a design methodology for finding balance between multiobjective optimization and product modularity and adaptability in the context of changing consumer needs. Each of these steps is described in the following sections.

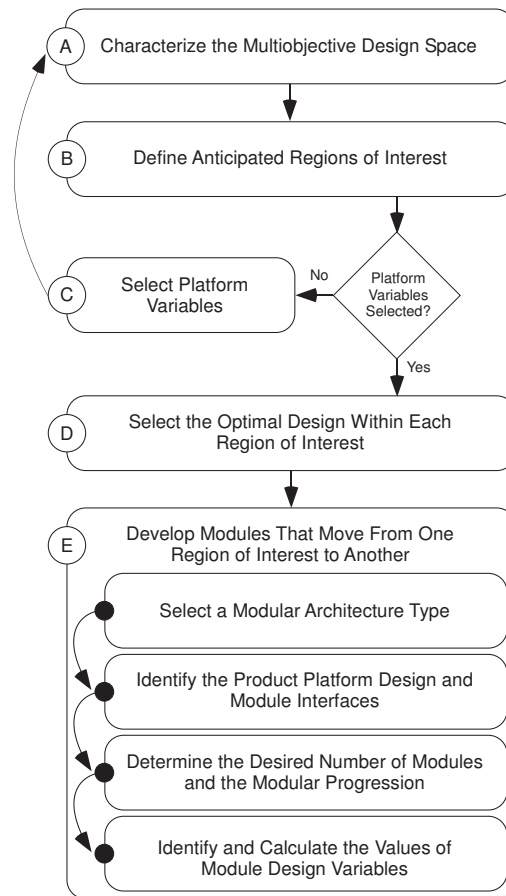


Figure 3.2: Flow chart describing the five-step multiobjective optimization design method developed in this section.

3.2 Step A: Characterize the Multiobjective Design Space

The first step of the method explores the multiobjective design space to evaluate and characterize the effects of each design variable on the objective space performance, and is accomplished through the evaluation of an MOP as described in Section 2.1.

3.3 Step B: Define Anticipated Regions of Interest

The second step of the method captures the predicted changes in consumer needs over time and enhances the ability of an optimizer to select the designs that are optimal for adaptation, as illustrated in Figure 3.1, by identifying designs within *Anticipated Regions of Interest*. Although one of the assumptions used in developing this method is that these anticipated regions of interest are known, potential methods of identifying these regions could include the use of focus groups, surveys, market observation (i.e. identification of a series of current benchmark products), etc.

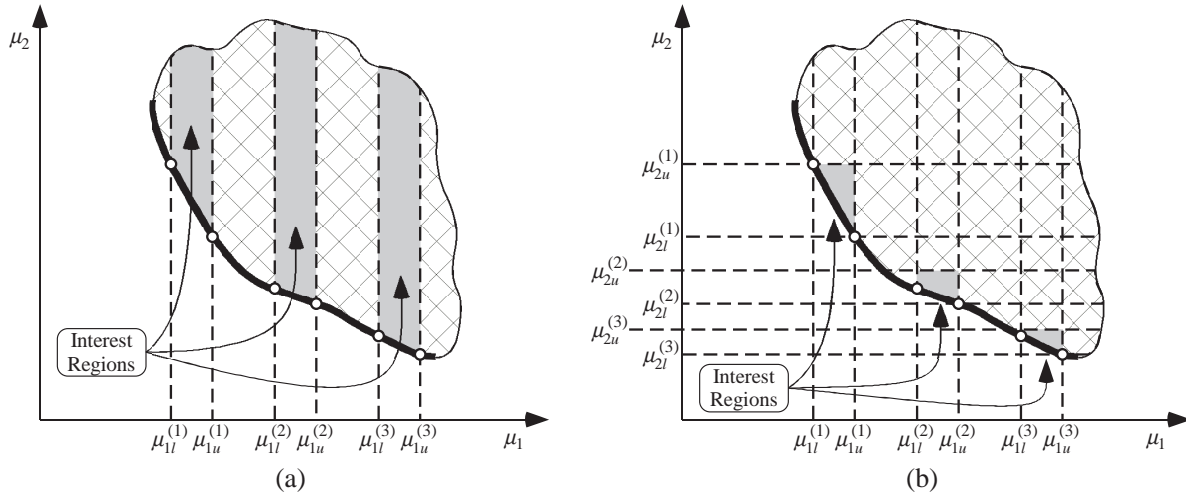


Figure 3.3: Representation of the construction of Anticipated Regions of Interest for known changes in consumer needs for three intervals. The anticipated regions of interest in (a) provide inequality constraints for μ_1 . The anticipated regions of interest in (b) provide inequality constraints for μ_1 , and μ_2 .

For each anticipated region of interest presented in Figure 3.3, a new MOP, with a reduced design space, is defined by additional objective constraints based on known changes in consumer needs. For example, for the left most region of interest in Figure 3.3(b) the objective μ_1 is constrained by $\mu_{1,l}^{(1)} \leq \mu_1 \leq \mu_{1,u}^{(1)}$, where $\mu_{1,l}^{(1)}$ and $\mu_{1,u}^{(1)}$ are prescribed. The result is the bounding of

the MOP to search the design space within the geometric shape of the anticipated region of interest. Further definition of the anticipated region of interest is unnecessary due to the function of a MOP of finding solutions along the Pareto frontier. For the examples presented in Figure 3.3, the information capturing the changes in consumer needs over time for each design in the set would be expressed as additional boundary constraints for the acceptable values of μ_1 and μ_2 . In the event that the anticipated region of interest restricts the optimizer to an infeasible space, a compromise in the acceptable range of the objectives for the infeasible region of interest is required, or a new design model must be considered which provides feasible solutions within the desired region of interest.

3.4 Step C: Select Platform Variables

The third step of the method uses the Pareto frontier *within* the regions of interest identified previously to identify those variables which are best suited as platform variables (x_p). This may be accomplished through the use of Pareto-filtering methods as described in Section 2.2 or any other suitable method. In cases where a designer knows which variables are best suited as platform variables, this step simplifies to the providing of that information for the remaining steps of the method. In addition, as is illustrated in Figure 3.4, by selecting platform variables, it is likely that the Pareto frontier will shift. This shift represents a loss in the best possible performance due to the restricting of design variable values. To ensure that the resulting shift in the Pareto frontier has not produced a shift that places an anticipated region of interest in what is now infeasible space, Steps A and B of the method must be repeated as shown in Figure 3.2.

3.5 Step D: Select the Optimal Design Within Each Region of Interest

The fourth step of the method is to develop the n -dimensional optimization routine used to select the optimal design in each anticipated region of interest and identify the accompanying design variable values. The resulting optimal design set (D_a) containing all variable values is obtained through the following MOP formulation:

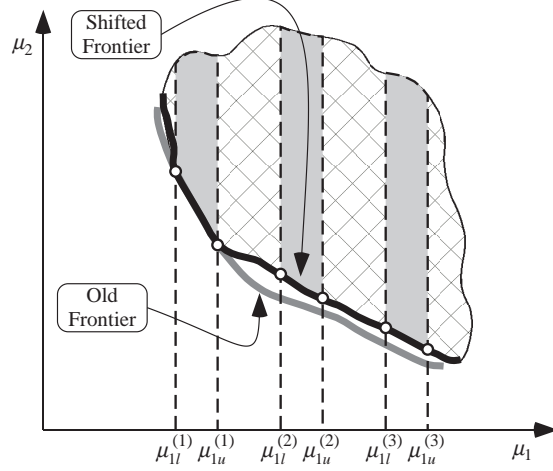


Figure 3.4: Illustration of the expected shift in the Pareto frontier from Figure 3.3 due to the selection of platform variables. The anticipated regions of interest presented in Figure 3.3(a) are also shown.

Problem 3.1: MOP Formulation for Optimal Adaptive Product Identification

$$D_a := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_{x_p}}^*, x_{a,1}^{(i)*}, x_{a,2}^{(i)*}, \dots, x_{a,n_{x_a}}^{(i)*}) \mid \forall i \in \{1, 2, \dots, n_d\}\} \quad (3.1)$$

$x_p^*, x_a^{(i)*}$ defined by:

$$\min_{x_p, x_a^{(i)}} \left\{ \frac{1}{n_d} \sum_{i=1}^{n_d} J^{(i)}(x_a^{(i)}, x_p, p^{(i)}) \right\} \quad (3.2)$$

where:

$$J^{(i)} = w_1^{(i)} \cdot \mu_1(x_a^{(i)}, x_p, p^{(i)})^{m_1} + \dots + w_{n_\mu}^{(i)} \cdot \mu_{n_\mu}(x_a^{(i)}, x_p, p^{(i)})^{m_{n_\mu}} \quad (n_\mu \geq 2) \quad (3.3)$$

subject to:

$$g_q^{(i)}(x_a^{(i)}, x_p, p^{(i)}) \leq 0 \quad \forall q \in \{1, \dots, n_g^{(i)}\} \quad (3.4)$$

$$h_v^{(i)}(x_a^{(i)}, x_p, p^{(i)}) = 0 \quad \forall v \in \{1, \dots, n_h^{(i)}\} \quad (3.5)$$

$$x_{a,j,l} \leq x_{a,j} \leq x_{a,j,u} \quad \forall j \in \{1, \dots, n_{x_a}\} \quad (3.6)$$

$$x_{p,r,l} \leq x_{p,r} \leq x_{p,r,u} \quad \forall r \in \{1, \dots, n_{x_p}\} \quad (3.7)$$

$$\mu_{y,l}^{(i)} \leq \mu_y^{(i)} \leq \mu_{y,u}^{(i)} \quad \forall y \in \{1, \dots, n_\mu^{(i)}\} \quad (3.8)$$

where the adjustable variables (x_a) represent all non-platform design variables (variables that are either scaled or discretely adjusted); m is a compromise programming power [23]; $w_1^{(i)}, w_2^{(i)}, \dots, w_{n_\mu}^{(i)}$ are weights associated with the local preference within each region of interest; the set D_a now represents the set of all design variable values of x_a^* and x_p^* obtained through the evaluation of the MOP; the subscript n_μ in Equation 3.8 indicates the additional objective constraints needed to define the anticipated regions of interest; and the superscript (i) on p , g , and h indicates the possibility that parameters and constraints are different (non-constant) for each design in the set D_a . It is important to note that Problem 3.1 will result in a single solution within each region of interest.

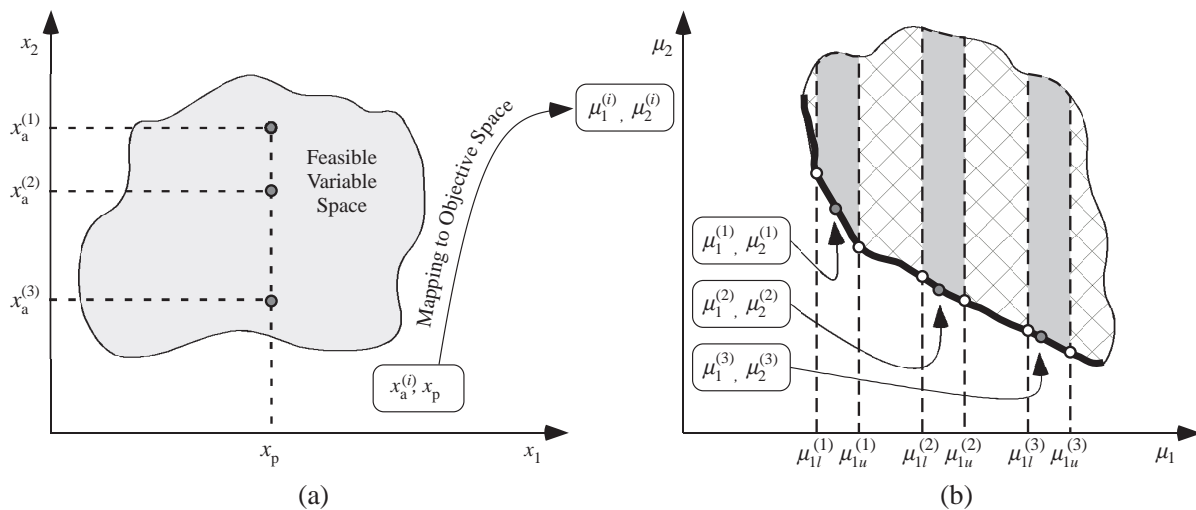


Figure 3.5: Theoretical identification of the values of x_p and $x_a^{(i)}$ for the set of anticipated regions of interest and shifted Pareto frontier from Figure 3.4 and the MOP formulation presented in Problem 3.1.

From the MOP presented in Problem 3.1, it is seen that for each design – indicated by the superscript (i) – in the set D_a , the values of x_p^* are required to be the same for all $D_a^{(i)}$, while the values of $x_a^{(i)*}$ are not. In addition, the solution of Problem 3.1 will result in a set of designs that are located along the Pareto frontier within each region of interest.

Figure 3.5 is a representation of how the solution to Problem 3.1 for the set of anticipated regions of interest and the shifted Pareto frontier are used to identify the values of x_p^* and $x_a^{(i)*}$. In addition, Figure 3.5 shows how the intent of the proposed method to strategically select Pareto-optimal designs based on their ability to facilitate adaptability is satisfied through the implementation of Problem 3.1.

3.6 Step E: Develop Modules That Move From One Region of Interest to Another

By this step in the process, the set D_a now contains all variable values that can be used to develop the module designs. Developing these designs is now a matter of constrained module design – modules are designed in a manner that constrains them to provide a specified progression in product performance when added to a specific embodiment of the product while only using the variable values from set D_a . To complete this final step of the method and obtain the module designs requires the following: (i) *Select a modular architecture type*, (ii) *Identify the platform design and module interfaces*, (iii) *Determine the desired number of modules and modular progression*, and (iv) *Identify and calculate the values of module design variables*. Each of these four parts is briefly discussed.

Select a modular architecture type: Of the three types of modularity identified in the literature (see Section 2.2), *Slot-modular architecture* and *Bus-modular architecture* are best suited for implementation in the present method due to the use of platform designs. The decision of which architecture type to be used depends on the desired functionality of the product and modules as a whole.

Identify the product platform design and module interfaces: Prior to the identification of modules, one of the designs in set D_a must be identified as the product platform design. In order to facilitate adaptability, the platform design is generally identified as the design contained in D_a with the most commonality. In addition, the module interfaces must be specified according to the modular architecture type selected previously, and any other related interfacing design activities must be performed.

Determine the desired number of modules and the modular progression: With a knowledge of the modular architecture type that is desired, it is now possible to determine the number of modules (n_m) that are desired. The identification of n_m requires a knowledge of the manner in which the product is intended to expand. For the slot/bus-modular cases the maximum and minimum values for n_m obtained for all possible module progression sequences are identified as follows.

$$n_{m,\max} = \sum_{n=1}^{n_d-1} n \quad (3.9)$$

$$n_{m,\min} = n_d - 1 \quad (3.10)$$

it follows that the selected value of n_m is an integer satisfying the condition:

$$n_{m,\min} \leq n_m \leq n_{m,\max}$$

Using the integer value of n_m that is desired, it is now possible to create an n_m -by-2 matrix (δ) dictating the desired progression from one design contained in set D_a to another. As a note, the first entry of the δ matrix ($\delta_{1,1}$) is generally the platform design from set D_a identified in the previous section. A generic construction of a δ matrix is presented as follows:

$$\delta = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \vdots & \vdots \\ \alpha_{n_m} & \beta_{n_m} \end{bmatrix} \quad (3.11)$$

where α and β respectively refer to the starting and the ending designs of the set D_a that each module is bridging. This information is used in the final step to refer to the values of x_a needed to design each module.

Identify and calculate the values of module design variables: The identification of module designs first requires that those variables that are best suited to characterize the modules be identified – module variables (x_m). This identification of module variables can be performed using the same methods described previously for identifying platform variables. In cases where a designer knows which variables are best suited as module variables for manufacturing a modular product, this process of variable identification simplifies to the providing of that information for

the module design routine presented below. Using this information, and the information provided in δ and D_a , a generic n -dimensional constrained module design routine is presented below.

Problem 3.2: Optimization Problem Formulation for Constrained Module Design

$$D_m := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_{x_p}}^*, x_{m,1}^{(i)*}, x_{m,2}^{(i)*}, \dots, x_{m,n_{x_m}}^{(i)*}) \mid \forall i \in \{1, 2, \dots, n_m\}\} \quad (3.12)$$

x_m^* is defined by:

$$\min_{x_m} J^{(i)} = \left(P^{(\beta)} - \bar{P}^{(i)} \right)^2 \quad (3.13)$$

where:

$$\alpha = \delta_{i,1} \quad (3.14)$$

$$\beta = \delta_{i,2} \quad (3.15)$$

$$\bar{P}^{(i)} = P^{(\alpha)} + \Delta P^{(i)} \quad (3.16)$$

defined by:

$$P^{(\alpha)} = \left(\mu_1 \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}}, \mu_2 \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}}, \dots, \mu_{n_\mu} \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}} \right) \quad (n_\mu \geq 2) \quad (3.17)$$

$$P^{(\beta)} = \left(\mu_1 \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}}, \mu_2 \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}}, \dots, \mu_{n_\mu} \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}} \right) \quad (n_\mu \geq 2) \quad (3.18)$$

$$\Delta P^{(i)} = \left(\Delta \mu_1(x_m^{(i)}, x_p^*, \hat{p}^{(i)}), \Delta \mu_2(x_m^{(i)}, x_p^*, \hat{p}^{(i)}), \dots, \Delta \mu_{n_\mu}(x_m^{(i)}, x_p^*, \hat{p}^{(i)}) \right) \quad (n_\mu \geq 2) \quad (3.19)$$

where D_m is the set of values and variables of x_p^* and x_m^* for each module design; $P^{(\alpha)}$ and $P^{(\beta)}$ characterize the objective space performance of the base (α) and target (β) designs; $\bar{P}^{(i)}$ represents the objective space performance of design α when used in conjunction with the i -th module; $\Delta P^{(i)}$ represents the change in objective space performance from design α to $\bar{P}^{(i)}$; and x_m^* represents the value(s) and variable(s) that characterize ΔP .

In examining Problem 3.2 it should be noted that for each design in the set D_m , the values of x_p^* are the same as those contained in set D_a . Also, if the variables contained in x_a^* are geometric (i.e., lengths, widths, heights), x_m^* represents the change of the geometric values of the variables

that produce the desired $\Delta P^{(i)}$. If the variables contained in x_a^* are non-geometric (i.e., technology selection, hardware selection, software selection), x_m^* provides the information needed to create the bridge between x_a^* from design α to design β and provide the desired $\Delta P^{(i)}$.

With completion of the constrained module design process, a product capable of adapting to changes in consumer needs over time through the addition of modules is achieved. In addition, each iteration of the product obtained through the addition of modules provides the optimal performance according to the objectives provided in Problem 3.1 (see Section 3.5).

CHAPTER 4. PHASE 1 EXAMPLE: UNMANNED AIR VEHICLE (UAV) DESIGN

The example that follows shows the application of the multiobjective optimization design method presented in Chapter 3 in the creation of a small (wingspan (b) ≤ 2.5 meters) modular UAV and demonstrates the ability of the method to provide a modular product capable of satisfying 3 different operating conditions and parameters representing the changing consumer needs over time. For this scale of aircraft, design traditionally involves the optimization of performance objectives for a set of operating conditions and mission parameters, traditionally expressed through a mission profile. Drive for the development of this modular UAV stems from the need to have a fleet of aircraft that meet the needs of a diverse range of mission profiles, or accept losses in performance of the aircraft due to changes in operating conditions and parameters when the aircraft is used for missions other than it was designed for. To overcome this disparity, a concept wing design for a modular UAV is developed (see Figure 4.1) to provide the ability to optimally expand the design of the aircraft (see Figure 4.2) between missions through the addition of modules. The intent of this concept is not to provide complete details that would be required in the design of a flight worthy UAV, but to simply provide an idea of how a UAV could be optimally expanded through modules.

Chapter Nomenclature:

D_{turn}	Distance traveled in a complete 360° UAV turn (m)
\hat{D}_{turn}	Distance traveled per degree of turn (m/°)
E	Surveillance elevation of a UAV mission profile (m)
\hat{g}	Acceleration due to gravity (m/s ²)
η_{max}	Maximum load factor
θ	Degree of UAV turn (°)
L	Temperature lapse rate (K/m)
L_m	Module wing extension lengths (m)
M	Molar mass of dry air (kg/mol)

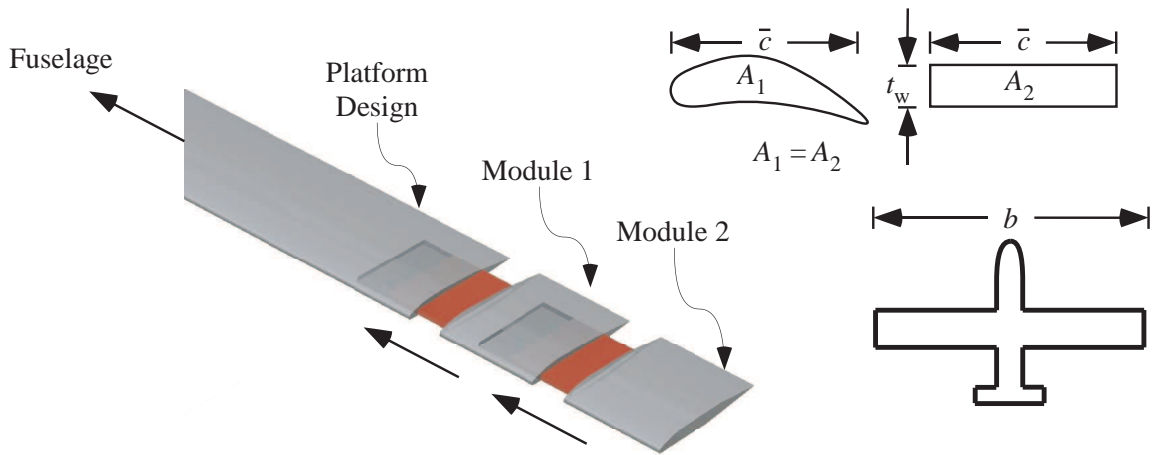


Figure 4.1: Schematic of a concept modular UAV wing design that provides the ability to optimally satisfy different mission profiles.

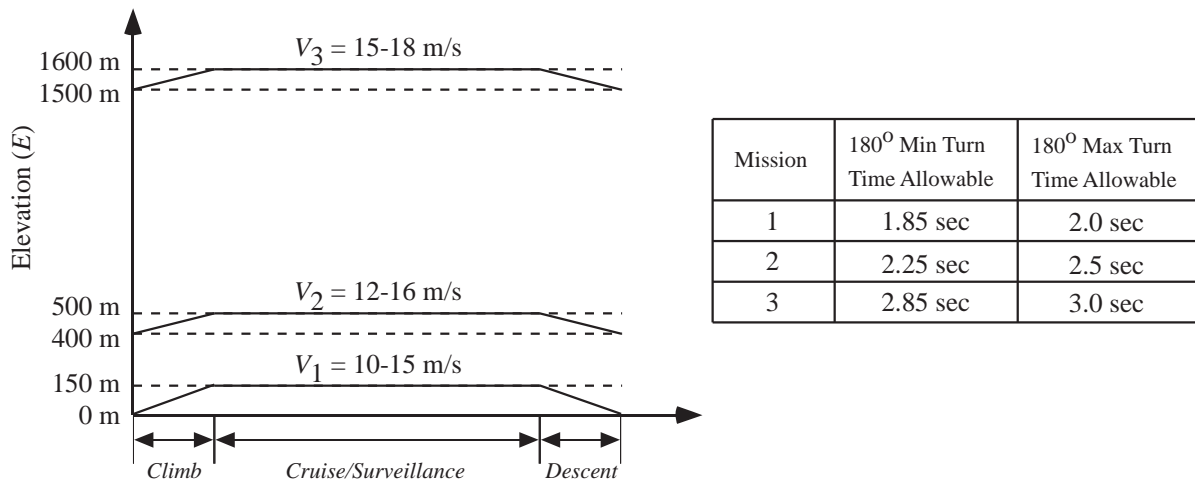


Figure 4.2: Schematic of the three mission profiles used in the UAV example.

- m_t Total mass of the UAV (kg)
- m_e Mass of onboard equipment like cameras, batteries, computers, etc. (kg)
- m_f Mass of the UAV fuselage (kg)
- m_w Mass of the UAV wings (kg)
- \dot{p}_0 Sea level standard atmospheric pressure (Pa)
- \dot{p} Atmospheric pressure at E (Pa)

ρ	Density of air at E (kg/m ³)
R	Universal gas constant (J/(mol·K))
R_{turn}	Minimum radius of turn of the UAV (m)
S_{ref}	Reference wing area (m ²)
T	Temperature of air at E (K)
T_0	Sea level standard temperature (K)
T_{lost}	Time lost in a 180° UAV turn (s)
\hat{T}_{lost}	Time lost per degree of turn (s/°)
$\hat{T}_{\text{lost,max}}$	Maximum time lost allowed per degree of turn (s/°)
t_w	Equivalent thickness of a rectangular cross-section wing (m)
V	Mission cruise velocity (m/s)

Inspection of Figures 4.1 and 4.2 reveal the platform variable best suited for manufacturing this concept design as the average cord length (\bar{c}). Assumptions made in this example are as follows: (1) During surveillance operations of the mission profiles, the UAV flies at constant altitude. (2) The aircraft has sufficient thrust for a sustained turn [50]. (3) The coefficients of lift ($C_{L_{\text{max}}}$) and thrust ($C_{T_{\text{max}}}$) are constant and equal 1.2 and 0.1 respectively [50]. (4) The UAV is being designed for surveillance operations where useful data is not captured while executing a turn – thus the inclusion of minimizing \hat{T}_{lost} as a design objective. (5) Density (ρ_w) of the wing material does not change with changes in b . As a note, the wing material is assumed to be 1.9 lb EPP foam. Current design approaches increase wing density where the wing connects to the fuselage as the wings lengthen to provide more strength and reduce the weight of the wings [51]. In the concept presented in Figure 4.1, this final assumption requires that the connection between the fuselage and wings be designed for the maximum wing span possible. The mass associated with this connection is accounted for in m_f . The complete formulation of the MOP and identification of anticipated regions of interest based upon the information provided in Figure 4.2 for this example is as follows.

Problem 4.1: UAV Example – MOP Formulation

$$D_a := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_xp}^*, x_{a,1}^{(i)*}, x_{a,2}^{(i)*}, \dots, x_{a,n_xa}^{(i)*}) \mid \forall i \in \{1, 2, 3\}\} \quad (4.1)$$

x_p^*, x_a^* defined by:

$$\min_{x_a^{(i)}, x_p} \left(\frac{1}{3} \right) \sum_{i=1}^3 \left(\hat{T}_{\text{lost}}(x_a^{(i)}, x_p, p^{(i)}) + m_t(x_a^{(i)}, x_p, p^{(i)}) \right) \quad (4.2)$$

where:

$$x_p = \{ \bar{c} \} \quad (4.3)$$

$$x_a^{(i)} = \left\{ V^{(i)} \quad b^{(i)} \right\} \quad (4.4)$$

$$p^{(i)} = \left\{ m_e \quad m_f \quad \rho_w \quad t_w \quad T_0 \quad L \quad \dot{p}_0 \quad R \quad M \quad E^{(i)} \right\} \quad (4.5)$$

subject to:

$$\hat{T}_{\text{lost}}^{(i)} - \hat{T}_{\text{lost, max}}^{(i)} \leq 0 \quad (4.6)$$

$$\hat{T}_{\text{lost, min}}^{(i)} - \hat{T}_{\text{lost}}^{(i)} \leq 0 \quad (4.7)$$

$$V_l^{(i)} \leq V^{(i)} \leq V_u \quad (4.8)$$

$$1.1\text{m} \leq b^{(i)} \leq 2.5\text{m} \quad (4.9)$$

$$0.09\text{m} \leq \bar{c} \leq 0.17\text{m} \quad (4.10)$$

where:

$$\hat{T}_{\text{lost}}^{(i)} = \frac{\hat{D}_{\text{turn}}^{(i)}}{V^{(i)}} \quad (4.11)$$

$$m_t^{(i)} = m_e + m_f + m_w^{(i)} \quad (4.12)$$

with supporting equations:

$$m_w^{(i)} = \rho_w S_{\text{ref}}^{(i)} t_w \quad (4.13)$$

$$S_{\text{ref}}^{(i)} = \bar{c} b^{(i)} \quad (4.14)$$

$$T^{(i)} = T_0 - LE^{(i)} \quad (4.15)$$

$$\dot{p}^{(i)} = \dot{p}_0 \left(1 - \frac{LE^{(i)}}{T_0} \right)^{\left(\frac{\hat{e}M}{RL} \right)} \quad (4.16)$$

$$\rho^{(i)} = \frac{\dot{p}^{(i)} M}{RT^{(i)}} \quad (4.17)$$

Table 4.1: Values of the constant parameters of $p^{(i)}$ needed to evaluate Problem 4.1.

Constant Parameters								
m_e (kg)	m_f (kg)	ρ_w (kg/m ³)	t_w (m)	T_0 (K)	L (K/m)	\dot{p}_0 (Pa)	R (J/(mol·K))	M (kg/mol)
0.25	1.5	30.435	0.06	288.5	0.0065	101325	8.31447	0.0289644

Table 4.2: Values of the non-constant objective limits, adjustable variable limits, and parameters of $p^{(i)}$ needed in Problem 4.1 to obtain the i -th design of set D_a .

Adjustable Variable Limits			Objective Limits		Parameters
i	V_l (m/s)	V_u (m/s)	$\hat{T}_{\text{lost, min}}$ (s/°)	$\hat{T}_{\text{lost, max}}$ (s/°)	E (m)
1	10	15	1.85/180	2.0/180	150
2	12	16	2.25/180	2.5/180	500
3	15	18	2.85/180	3.0/180	1600

$$\eta_{\text{max}}^{(i)} = \frac{\rho^{(i)} (V^{(i)})^2 S_{\text{ref}}^{(i)} C_{L\text{max}}}{2m_t^{(i)} \hat{g}} \quad (4.18)$$

$$R_{\text{turn}}^{(i)} = \frac{(V^{(i)})^2}{\hat{g} \sqrt{(\eta_{\text{max}}^{(i)})^2 - 1}} \quad (4.19)$$

$$D_{\text{turn}}^{(i)} = \frac{\pi R_{\text{turn}}^{(i)} \theta}{180} \quad (4.20)$$

$$\hat{D}_{\text{turn}}^{(i)} = \frac{D_{\text{turn}}^{(i)}}{\theta} \quad (4.21)$$

$$T_{\text{lost}}^{(i)} = \frac{D_{\text{turn}}^{(i)}}{V^{(i)}} \quad (4.22)$$

where all variables in the preceding equations are defined in the Nomenclature section of this Chapter.

Values of the elevation (E), $\hat{T}_{\text{lost, max}}$, and $\hat{T}_{\text{lost, min}}$, along with the lower (V_l) and upper (V_u) limits of the mission cruise velocities for the different designs presented in Tables 4.1 and 4.2 are obtained from the mission profiles presented in Figure 4.2. Values of T_0 , L , \dot{p}_0 , R , and M presented in Table 4.1 come from the 1976 International Standard Atmosphere document [52]. The variable t_w represents an equivalent thickness of the wings – approximates the wing cross sectional area as a

Table 4.3: Variable and objective values obtained through evaluation of Problem 4.1 for the i -th design of set D_a .

i	Variables			Objectives		
	\bar{c}^* (m)	V^* (m/s)	b^* (m)	m_t^* (kg)	\hat{T}_{lost}^* (s/°)	$\Rightarrow T_{\text{lost}}$ (s)
1	0.17	15	2.4635	2.5147	0.0103	1.8617
2	0.17	16	1.8259	2.3168	0.0127	2.2923
3	0.17	18	1.3857	2.1802	0.0159	2.8708

rectangle (See Figure 4.1). Equations used to evaluate the unmanned air vehicle's objective space performance (see Eqs. 4.11 and 4.12) are derived from equations presented in Nigam et al (see Eqs. 4.18 and 4.19) [50]. Equations used to calculate the density of air (see Eqs. 4.16–4.18 above) as a function of E are obtained using the ideal gas law assumption [53]. Evaluation of Problem 4.1 was performed using a Genetic Algorithm, and complete results indicating the variable and objective values of each design are presented in Table 4.3.

Through the evaluation of Problem 4.1 above, the set D_a now contains all variable values needed to develop the module designs (see Table 4.3). Prior to developing the module designs, information on the type, number, and desired progression of modules that are to be used to obtain the Pareto-optimal designs contained within set D_a is needed. Using the information provided in Figure 4.1, it can be seen that a Bus-modular approach was selected for this example. Examination of the nature of the x_a variables reveals that the differences in the variable b for each design in D_a is geometric, and therefore the design with the most commonality is the design with the smallest length of b^* ($D_a^{(3)}$). Using this information, the desired number of modules to be developed (n_m) is chosen to be two, and the δ matrix is constructed in the following equation:

$$\delta = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \quad (4.23)$$

Formulation of a constrained module design routine of the form presented in Problem 4.1 is presented as follows.

Problem 4.2: UAV Example – Constrained Module Design

$$D_m := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_x}^*, x_{m,1}^{(i)*}, x_{m,2}^{(i)*}, \dots, x_{m,n_x}^{(i)*}) \mid \forall i \in \{1, 2, \dots, n_m\}\} \quad (4.24)$$

x_m^* is defined by:

$$\min_{x_m} J^{(i)} = \left(P^{(\beta)} - \bar{P}^{(i)} \right)^2 \quad (4.25)$$

defined by:

$$\bar{P}^{(i)} = P^{(\alpha)} + \Delta P^{(i)} \quad (4.26)$$

$$P^{(\alpha)} = \left(\hat{T}_{\text{lost}}^{(\alpha)} \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}}, m_t^{(\alpha)} \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}} \right) \quad (4.27)$$

$$P^{(\beta)} = \left(\hat{T}_{\text{lost}}^{(\beta)} \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}}, m_t^{(\beta)} \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}} \right) \quad (4.28)$$

$$\Delta P^{(i)} = \left(\Delta \hat{T}_{\text{lost}}^{(i)}, \Delta m_t^{(i)} \right) \quad (4.29)$$

where:

$$x_m^{(i)} = \{L_m^{(i)}, V^{(\beta)*}\} \quad (4.30)$$

$$x_p^* = \{\bar{c}^*\} \quad (4.31)$$

$$\alpha = \delta_{i,1} \quad (4.32)$$

$$\beta = \delta_{i,2} \quad (4.33)$$

$$\Delta \hat{T}_{\text{lost}}^{(i)} = \hat{T}_{\text{lost}} \left((b^{(\alpha)*} + 2L_m^{(i)}), V^{(\beta)*}, \bar{c}^*, p^{(\beta)} \right) - \hat{T}_{\text{lost}} \left(b^{(\alpha)*}, V^{(\alpha)*}, \bar{c}^*, p^{(\alpha)} \right) \quad (4.34)$$

$$\Delta m_t^{(i)} = m_t \left((b^{(\alpha)*} + 2L_m^{(i)}), V^{(\beta)*}, \bar{c}^*, p^{(\beta)} \right) - m_t \left(b^{(\alpha)*}, V^{(\alpha)*}, \bar{c}^*, p^{(\alpha)} \right) \quad (4.35)$$

$$L_m^{(i)} = 0.5(b^{(\beta)*} - b^{(\alpha)*}) \quad (4.36)$$

where all variables in the preceding equations are defined in the Nomenclature section of this Chapter..

Results of the evaluation of Problem 4.2, as well as the variable values of the Platform Design are presented in Table 4.4.

Table 4.4: Variable values of the platform and module designs obtained through evaluation of Problem 4.2.

Platform Design			Module Designs			
Variables	Values	Units	Variables	Module # 1	Module # 2	Units
\bar{c}	0.17	(m)	L_m	0.2201	0.3188	(m)
b	1.3857	(m)	V	16	15	(m/s)
V	18	(m/s)				

Results of the evaluation of Problem 4.2 above (see Table 4.4 for complete summary) provide the variable values needed to describe the module designs. With completion of the constrained module design process, a UAV capable of adapting to three different mission profiles through the addition of modules is achieved. Figure 4.3 provides a graphical representation of the Pareto frontier for the 3 regions of interest defined in Problem 4.1, along with the objective values for the solutions to Problem 4.1 (indicated by the symbol “o”) and Problem 4.2 (indicated by the symbol “x”). In addition, Figure 4.3 illustrates the ability of each iteration of the UAV obtained through the addition of modules to provide the desired Pareto-optimal performance according to the objectives, parameters, and constraints provided in Problem 4.1, and thus satisfy the intent of the design method. The side of the Pareto frontier representing feasible solutions is indicated by the direction of the \wedge symbols placed along the frontier.

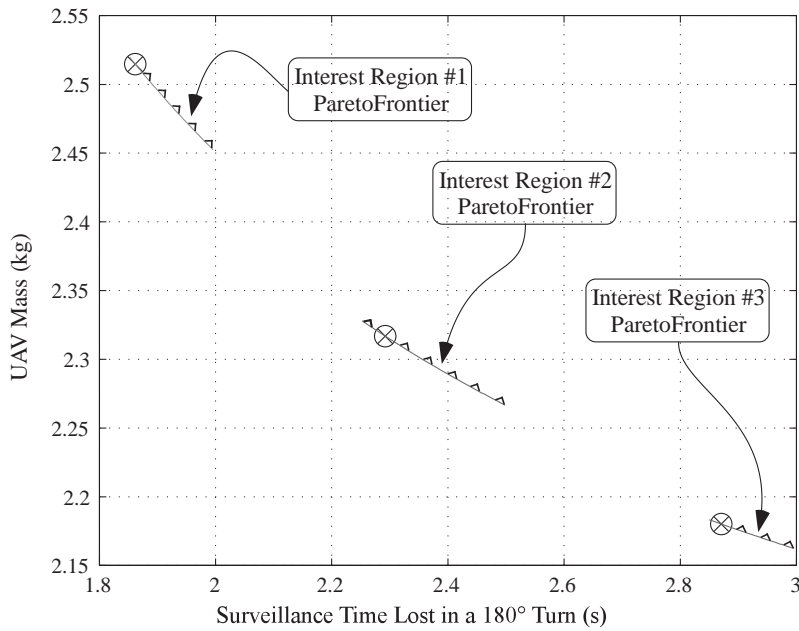


Figure 4.3: Graphical representation of the Pareto frontier for the 3 regions of interest defined in Problem 4.1, along with the plotted solutions to Problem 4.1 (indicated by the symbol “o”) and Problem 4.2 (indicated by the symbol “x”). The plot shows that each iteration of the UAV obtained through the addition of modules provides the desired Pareto-optimal performance from Problem 4.1. The side of the Pareto frontier representing feasible solutions is indicated by the direction of the \wedge symbols placed along the frontier.

CHAPTER 5. PHASE 2 METHOD DEVELOPMENT

In this chapter, the second phase in the development of a multiobjective optimization design method providing a Pareto-optimal product and module designs capable of satisfying changes in consumer needs over time is described for s-Pareto frontier design cases.

5.1 Accounting for Changing Needs With Multiple Design Models

Similar to the Pareto frontier, an s-Pareto frontier contains many optimal, yet functionally different, designs representing all optimal product candidates. However, an s-Pareto frontier represents the optimal product candidates from disparate design models. Recalling from Chapter 3 that to satisfy changes in consumer needs over time through the addition of modules requires the strategic selection of Pareto designs based on their ability to facilitate adaptability, the focus of this chapter is to demonstrate how the method presented in Chapter 3 can be expanded to incorporate the strategic selection of s-Pareto designs. Figure 5.1 illustrates the intent of the expanded method to satisfy changing consumer needs by selecting an *s-Pareto-optimal* product platform design which, through the addition of modules, expands to other s-Pareto designs.

As was the case in Figure 3.1 shown in Chapter 3, the first (μ_1) and second (μ_2) objectives are represented along the horizontal and vertical axis, respectively. From Figure 5.1 it is seen that the platform design, shown as $P^{(1)}$, adapts to become $P^{(2)}$ through the addition of Module 1. Through this approach, the platform and subsequent modules, provide the desired product performance resulting from the changing consumer needs as represented by $P^{(1)}$, $P^{(2)}$, $P^{(3)}$, and $P^{(4)}$. Figure 5.2 provides a flow chart that illustrates the six primary steps of the expanded multiobjective optimization design method developed in this chapter in response to the identified research need from Section 2.3 calling for a design methodology for finding balance between multiobjective optimization and product modularity and adaptability in the context of changing consumer needs. By comparing Figures 3.2 and 5.2 it can be seen that the major difference between Figure 5.2 and

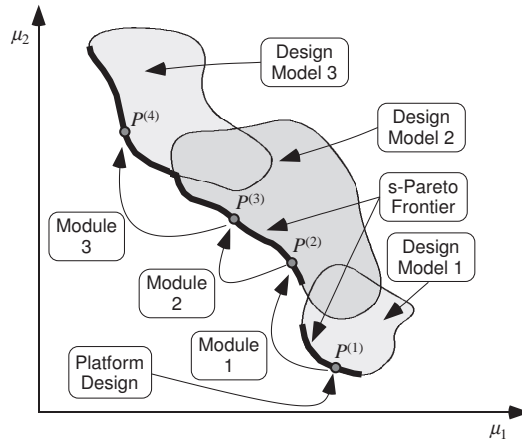


Figure 5.1: Graphical representation of the intent of the expanded method to provide a product that expands from one s-Pareto-optimal design to another through the addition of modules.

Figure 3.2, is the insertion of an additional step between Steps C and D of Figure 3.2 in which the s-Pareto frontier within the anticipated regions of interest is assembled. Although Steps A-C and E-F of Figure 5.2 appear to be identical to Steps A-E of Figure 3.2, due to the use of multiple design models, the functions of, and MOP formulations required in these steps, must necessarily change. The expanded function of each of these steps is described in the following sections.

5.2 Step A: Characterize the Multiobjective Design Space

The first step of the method explores the multiobjective design space to evaluate and characterize the effects of each design variable on the objective space performance. As presented in Figure 5.3, when multiple design models are needed to satisfy the future product needs, this step of the method requires that an MOP for each design model be evaluated and represented in the same design space.

5.3 Step B: Define Anticipated Regions of Interest

The second step of the method captures the predicted changes in consumer needs over time and enhances the ability of an optimizer to select the designs that are optimal for adaptation, as illustrated in Figure 5.1, by identifying designs within *Anticipated Regions of Interest*. Once again, although one of the assumptions used in developing this method is that these anticipated regions of interest

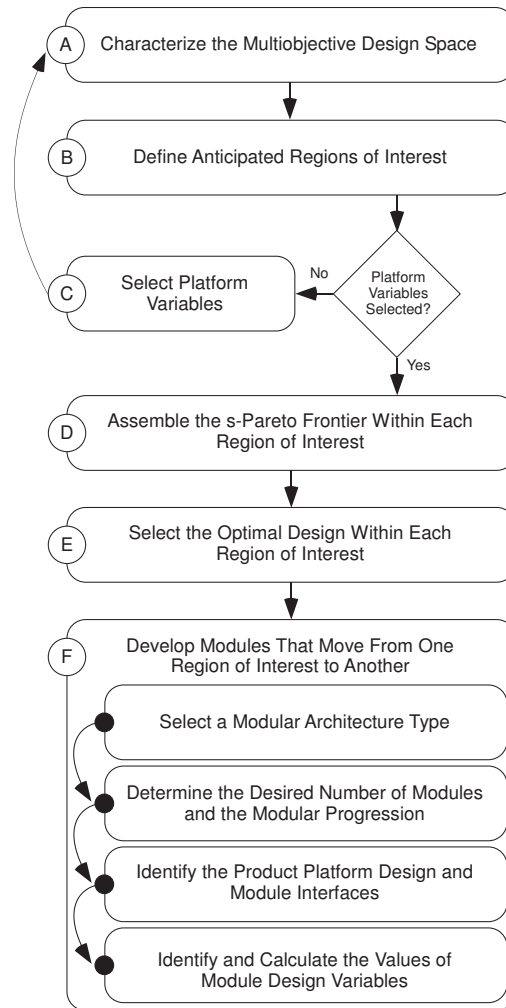


Figure 5.2: Flow chart describing the six-step multiobjective optimization design method developed in this chapter.

are known, potential methods of identifying these regions could include the use of focus groups, surveys, market observation (i.e. identification of a series of current benchmark products), etc.

For each anticipated region of interest presented in Figure 5.4, a new MOP, with a reduced design space, for the corresponding design models is defined by additional objective constraints based on known changes in consumer needs. Further definition of the anticipated region of interest is unnecessary due to the function of a MOP of finding solutions along the s-Pareto frontier. For the examples presented in Figure 5.4, as was the case for the examples provided in Figure 3.3 from Chapter 3, the information capturing the changes in consumer needs over time for each design in the set is expressed as additional boundary constraints for the acceptable values of μ_1 and μ_2 .

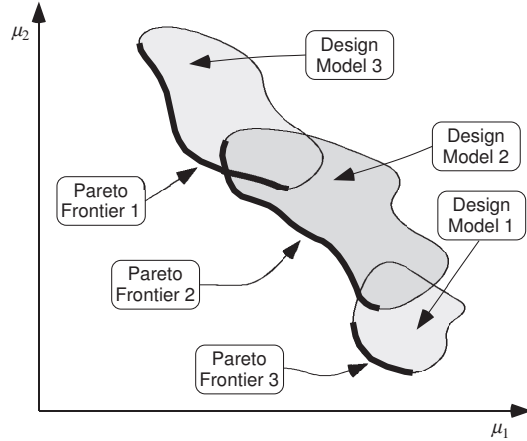


Figure 5.3: Representation of the characterization of three design models within the same design space.

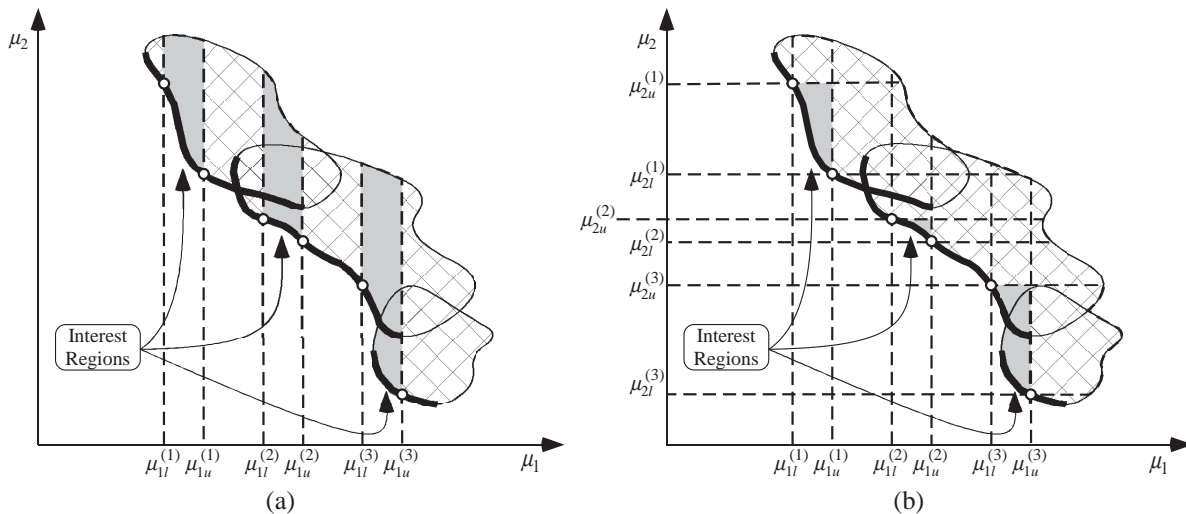


Figure 5.4: Representation of the construction of Anticipated Regions of Interest for known changes in consumer needs for three intervals. The anticipated regions of interest in (a) provide inequality constraints for μ_1 . The anticipated regions of interest in (b) provide inequality constraints for μ_1 , and μ_2 .

5.4 Step C: Select Platform Variables

The third step of the method uses the disparate Pareto frontiers *within the regions of interest* identified previously to identify those variables which are best suited as platform variables (x_p). Once again, this may be accomplished through the use of Pareto-filtering methods as described in Section 2.2 or any other suitable method. In addition, as is illustrated in Figure 5.5, by selecting platform variables, it is likely that the Pareto frontier of the different design models will shift. To insure that

the resulting shift in the Pareto frontiers has not placed an anticipated region of interest in what is now infeasible space, Steps A and B of the method must be repeated as shown in Figure 5.2.

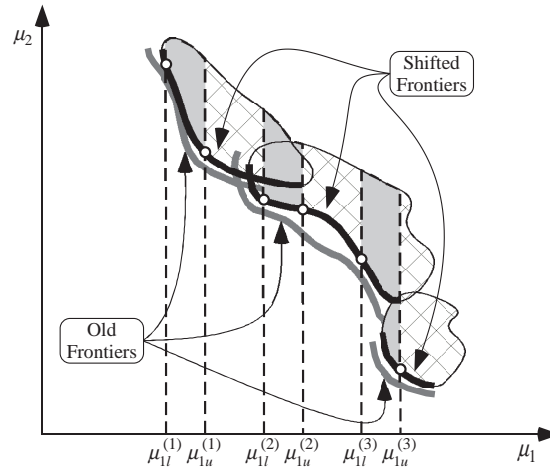


Figure 5.5: Illustration of the expected shift in the Pareto frontiers from Figure 5.4 due to the selection of platform variables. The anticipated regions of interest presented in Figure 5.4(a) are also shown.

5.5 Step D: Assemble the s-Pareto Frontier Within Each Region of Interest

The fourth step of the method identifies the Pareto-optimal solutions from the various design models *within each region of interest* which are best suited as s-Pareto – globally optimal – solutions. As described in Section 5.2 above, in step A of the method a characterization of the Pareto frontier of each design model was obtained. Thus, the current step may be accomplished through the use of Pareto-filtering methods as described in Section 2.1 or any other suitable method. Figure 5.6 illustrates the result of this step of the method.

5.6 Step E: Select the Optimal Design Within Each Region of Interest

The fifth step of the method is to develop the n -dimensional optimization routine used to select the optimal design in each anticipated region of interest and identify the accompanying design variable values. The resulting optimal design set (D_a) containing all variable values is obtained through the following MOP formulation:

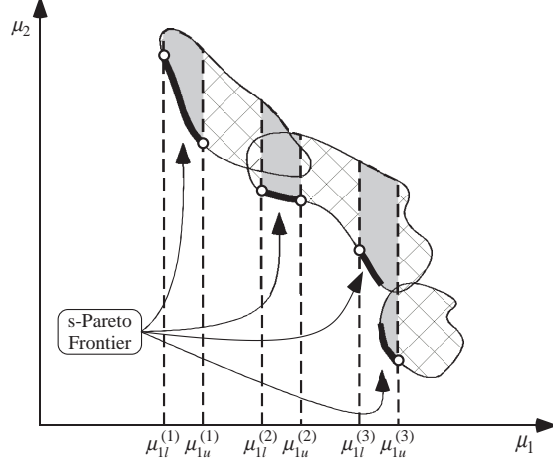


Figure 5.6: Illustration of the resulting s-Pareto frontier within the anticipated regions of interest from Figure 5.4(a).

Problem 5.1: s-Pareto MOP Formulation for Optimal Adaptive Product Identification

$$D_a := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_{x_p}}^*, x_{a,1}^{(i)*}, x_{a,2}^{(i)*}, \dots, x_{a,n_{x_a}}^{(i)*}) \mid \forall i \in \{1, 2, \dots, n_d\}\} \quad (5.1)$$

x_p^* , $x_a^{(i)*}$, and $n_{x_a}^{(i)}$ defined by:

$$\min_{x_p, x_a^{(i)}} \left\{ \frac{1}{n_d} \sum_{n=1}^{n_d} J^{(i)}(x_a^{(i)}, x_p) \right\} \quad (5.2)$$

where:

$$J^{(i)}(x_a^{(i)}, x_p) = \min_k \left\{ \min_{x_a^{(k)}, x_p} J^{(k)}(x_a^{(k)}, x_p) \right\} \quad (5.3)$$

$$J^{(k)}(x_a^{(k)}, x_p) = w_1^{(i)} \cdot \mu_1(x_a^{(k)}, x_p, p^{(k)})^{m_+} + \dots + w_{n_\mu}^{(i)} \cdot \mu_{n_\mu}(x_a^{(k)}, x_p, p^{(k)})^{m_-} \quad (n_\mu \geq 2) \quad (5.4)$$

subject to:

$$g_q^{(k)}(x_a^{(k)}, x_p, p^{(k)}) \leq 0 \quad \forall q \in \{1, \dots, n_g^{(k)}\} \quad (5.5)$$

$$h_v^{(k)}(x_a^{(k)}, x_p, p^{(k)}) = 0 \quad \forall v \in \{1, \dots, n_h^{(k)}\} \quad (5.6)$$

$$x_{a,j,l} \leq x_{a,j} \leq x_{a,j,u} \quad \forall j \in \{1, \dots, n_{x_a}^{(k)}\} \quad (5.7)$$

$$x_{p,r,l} \leq x_{p,r} \leq x_{p,r,u} \quad \forall r \in \{1, \dots, n_{x_p}\} \quad (5.8)$$

$$\mu_{y,l}^{(k)} \leq \mu_y^{(k)} \leq \mu_{y,u}^{(k)} \quad \forall y \in \{1, \dots, n_{\hat{\mu}}^{(k)}\} \quad (5.9)$$

where the adjustable variables (x_a) represent all non-platform design variables (variables that are either scaled or discretely adjusted) for each design model; k , $1 \leq k \leq n_{dm}$, denotes the k -th design model; m is a compromise programming power; [23] $w_1^{(i)}, \dots, w_{n_\mu}^{(i)}$ are weights associated with the local preference within the i -th region of interest; the set D_a now represents the set of all design variable values of x_a^* and x_p^* obtained through the evaluation of the MOP; and the superscript (k) on p , g , and h indicate the possibility that parameters and constraints are different (non-constant) for each design model. It is important to note that Problem 5.1 will result in a single solution within each region of interest.

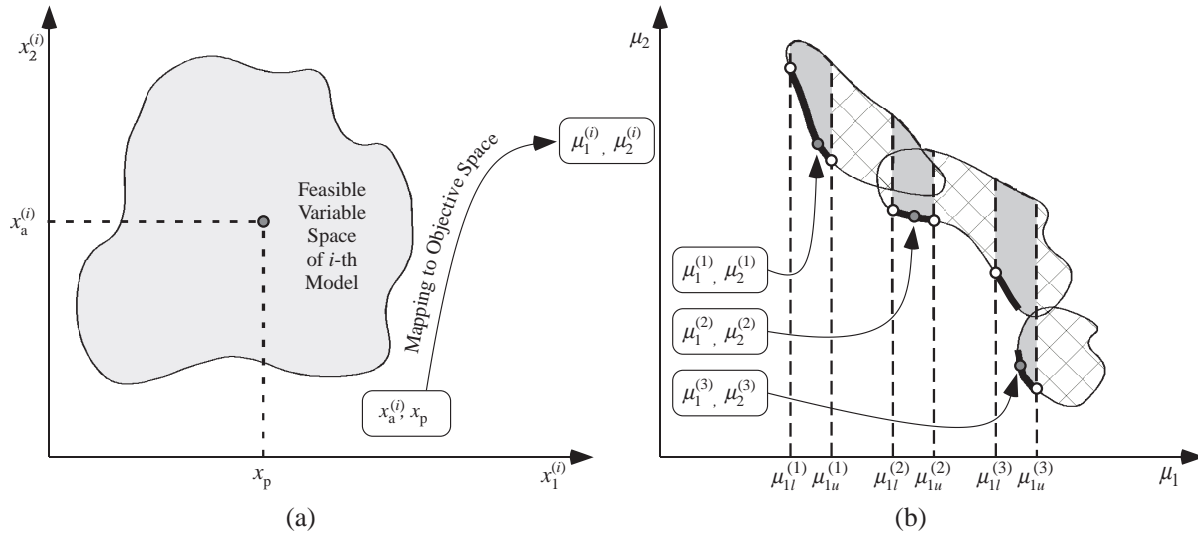


Figure 5.7: Theoretical identification of the values of x_p and $x_a^{(i)}$ for a set of three anticipated regions of interest and s-Pareto frontier from the MOP formulation presented in Problem 5.1.

Figure 5.7 is a representation of how the solution to Problem 5.1 for a set of three anticipated regions of interest and the corresponding s-Pareto frontiers are used to identify the values of x_p^* and $x_a^{(i)*}$. In addition, Figure 5.7 shows how the intent of the proposed method to strategically select s-Pareto-optimal designs based on their ability to facilitate adaptability is satisfied through the implementation of Problem 5.1.

5.7 Step F: Develop Modules That Move From One Region of Interest to Another

By this step in the process, the set D_a now contains all variable values that can be used to develop the module designs. Developing these designs is now, as was described in Section 3.6 of Chapter 3, a matter of constrained module design. To complete this final step of the method and obtain the module designs requires the following: (i) *Select a modular architecture type*, (ii) *Identify the product platform design and module interfaces*, (iii) *Determine the desired number of modules and modular progression*, and (iv) *Identify and calculate the values of module design variables*. Detailed information on each of these four parts was previously provided in Section 3.6 of Chapter 3 and therefore is not repeated here. The expanded n -dimensional optimization problem formulation for constrained module design is presented as follows:

Problem 5.2: s -Pareto Optimization Problem Formulation for Constrained Module Design

$$D_m := \{ (x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_x}^*, x_{m,1}^{(i)*}, x_{m,2}^{(i)*}, \dots, x_{m,n_{x_m}^{(i)*}}) \mid \forall i \in \{1, 2, \dots, n_m\} \} \quad (5.10)$$

x_m^* is defined by:

$$\min_{x_m} J^{(i)} = \left(P^{(\beta)} - \bar{P}^{(i)} \right)^2 \quad (5.11)$$

where:

$$\alpha = \delta_{i,1} \quad (5.12)$$

$$\beta = \delta_{i,2} \quad (5.13)$$

$$\bar{P}^{(i)} = P^{(\alpha)} + \Delta P^{(i)} \quad (5.14)$$

defined by:

$$P^{(\alpha)} = \left(\mu_1 \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}}, \mu_2 \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}}, \dots, \mu_{n_\mu} \Big|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}} \right) \quad (n_\mu \geq 2) \quad (5.15)$$

$$P^{(\beta)} = \left(\mu_1 \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}}, \mu_2 \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}}, \dots, \mu_{n_\mu} \Big|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}} \right) \quad (n_\mu \geq 2) \quad (5.16)$$

$$\Delta P^{(i)} = \left(\Delta \mu_1(x_m^{(i)}, x_p^*, \hat{p}^{(i)}), \Delta \mu_2(x_m^{(i)}, x_p^*, \hat{p}^{(i)}), \dots, \Delta \mu_{n_\mu}(x_m^{(i)}, x_p^*, \hat{p}^{(i)}) \right) \quad (n_\mu \geq 2) \quad (5.17)$$

where all variables and functions are as described in Section 3.6 of Chapter 3. It should also be noted that the current formulation now allows the variables contained in x_m to be different for each module designed (See Equation 5.10).

With completion of the constrained module design process, a product capable of adapting to changes in consumer needs over time through the addition of modules is achieved. In addition, each iteration of the product obtained through the addition of modules provides the optimal performance according to the objectives provided in Problem 5.1 (see Section 5.6).

CHAPTER 6. PHASE 2 EXAMPLE: MANUAL IRRIGATION PUMP DESIGN

There is increasing evidence that one of the most sustainable ways to help those living in extreme poverty (20.5% of the world's population who live on less than ~\$1 a day) is through a market-based approach – where all in the supply chain benefit financially, including the poor [54–57]. Among the most promising methods of producing profit for all in the supply chain is the development of products that increase the earning power of those that are living in extreme poverty [55, 58, 59]. Products such as treadle pumps, water drip irrigation kits, and coconut oil presses have generated millions of dollars in profit for poverty stricken countries and helped over 12 million people sustainably escape poverty [55, 58, 59]. However, millions of other impoverished people throughout the world are unwilling to invest in relatively costly income-generating products (~\$100 – or 3 months of income) because of the high perceived and actual financial risk involved [55, 56, 60]. Additionally, a majority of the population cannot afford the investment under the traditional approaches, and therefore remain unaided by these poverty alleviating technologies.

The example that follows shows the application of the methodology presented in Chapter 5 in the creation of a modular, manually operated irrigation pump. In addition, the example demonstrates the ability of the method to provide a modular income generating product that allows the purchaser to make a *four-stage* investment to purchase a product that would otherwise be considered unaffordable.

Chapter Nomenclature:

A_c	Cross sectional flow area of the cylinder (m^2)
A_p	Cross sectional flow area of the pipe (m^2)
C_{base}	Manufacturing cost of the base structure of the pump (\$)
C_{bike}	Manufacturing cost of the bike and rear sprocket (\$)
C_{crank}	Manufacturing cost of the rear axel crank (\$)

C_{cyl}	Manufacturing cost of the pump cylinder(s) (\$)
C_{handle}	Manufacturing cost of the pump handle (\$)
C_{link}	Manufacturing cost of the link connecting the rear axel crank to the treadles (\$)
C_{pipe}	Manufacturing cost of the inlet and outlet piping of the pump (\$)
C_{piston}	Manufacturing cost of the cylinder piston (\$)
C_{sup}	Manufacturing cost of the bike support structure (\$)
$C_{treadle}$	Manufacturing cost of the treadles (\$)
C_{vb}	Manufacturing cost of the valve-box(es) (\$)
d_{cr}	Diameter of the crank gear (m)
d_c	Diameter of the pump cylinder (m)
d_p	Diameter of the inlet/outlet pipes (m)
d_s	Diameter of the rear sprocket (m)
ϵ	Surface roughness for the pipe/pump cylinder
F_h	Force applied by the pump operator during hand operation of the pump (N)
F_{in}	Force applied at the cylinder head (N)
\hat{F}_{in}	Average force input to the treadles when connected to the bike (N)
F_l	Force applied by the pump operator during leg operation of the pump (N)
f_c	Friction coefficient for flow in the pump cylinder
f_p	Friction coefficients for flow in the inlet/outlet pipes
h_c	Distance traveled by the cylinder piston head (m)
h_L	Head loss in the pump system (m)
h_p	Height of the pivot (m)
l_c	Distance from the pivot to the pump cylinder (m)
l_{cr}	Length of the crank arm of the bike (m)
l_l	Length of the link connecting the rear axel crank to the treadles (m)
l_o	Distance from the pivot to the operator (m)
$l_{p,in}$	Length of the inlet pipe (m)
$l_{p,out}$	Length of the outlet pipe (m)
$l_{pa,x}$	Horizontal distance from the pivot to the bike rear axel (m)
$l_{pa,y}$	Vertical distance from the pivot to the bike rear axel (m)

l_r	Length of the rear axel crank (m)
l_s	Length of the operator stroke (m)
\hat{l}_s	Length of the treadle stroke when connected to the bike (m)
l_t	Length of the treadle extensions (m)
M_d	Distributor mark-up (5 %)
M_m	Manufacturing mark-up (25 %)
M_s	Sales mark-up (3 %)
m_w	Mass of the water in the entire pumping system (kg)
n_c	Number of cylinders
\hat{n}_c	Number of cylinders added by a module
n_{ct}	Number of crank gear teeth
n_{st}	Number of sprocket teeth
η	Force transmission efficiency (%)
p_d	Gear pitch diameter (m)
Q	Actual predicted water flow rate (L/s)
\hat{Q}	Potential water flow rate in the system assuming a constant flow (L/s)
S	Pump sales price (\$)
t_s	Stroke time of the operator (s)
V_p	Average flow velocity in the pipes (m/s)
V_c	Average flow velocity in the cylinders (m/s)
φ	Price scaling coefficient
w_{pa}	Width of the pump assembly (m)
ψ_{tube}	Unit material prices for the treadle/structural rectangular tubing (\$/m)
ψ_{plate}	Unit material prices for the plate material (\$/m ²)
$\psi_{p,st}$	Unit material prices for the structural pipe (\$/m)
$\psi_{p,w}$	Unit material prices for the inlet/outlet pipe (\$/m)
ψ_{seal}	Unit material prices for the piston seal (\$/m)
ψ_{cyl}	Unit material prices for the piston cylinder(s) (\$/m)
ψ_{plank}	Unit material prices for the wood base plank (\$/m ²)
ψ_{sp}	Unit material prices for the rear sprocket (\$/gear)

z_{in}	Vertical distance from the pump to the water source (m)
z_{out}	Vertical distance from the pump to the pipe outlet (m)

Drive for the development of this modular pump is best illustrated through the plot provided in Figure 6.1. This figure provides a comparison of three non-modular irrigation pumps that are sold on the market today based on their sales price, S (horizontal axis), and potential water flow rate, Q (vertical axis). From Figure 6.1 it is seen that products are available to satisfy a range of current views of what is considered affordable, but none of these products are capable of expanding as an individual's view of affordability changes due to increases in income potential (i.e. a Hip Pump cannot become a Super MoneyMaker). In short, drive for the development of a modular irrigation pump stems from the need to reduce the high perceived and actual financial risks involved with purchasing traditional irrigation pumps [55, 56, 60], while still providing the needed pump performance that will increase the purchasers income. To overcome this disparity, preliminary analytical models of the fluid and financial aspects of four different irrigation pump designs are developed to predict the behavior of a pump design based on a set of model inputs. These models are characterized by the following configuration descriptions: (1) Hand actuated with single cylinder, (2) Foot actuated with single cylinder, (3) Foot actuated with two cylinders, and (4) Cyclically actuated with two cylinders.

Preliminary assumptions made in the development of the disparate analytical financial and fluid models are as follows: (1) Water flow will always be turbulent. (2) The corresponding friction coefficients for flow in the pump cylinder (f_c) and pipes (f_p) are approximated by the average friction value for the expected flow speeds and the ratios of the surface roughness (ϵ) to pipe/cylinder diameter (d_p and d_c respectively). (3) The force transmission efficiency of the pump (η) is assumed to be constant and equal to 80%. (4) During **leg** operation of the pump, the force applied by the pump operator (F_l) is assumed to be constant and equal to 889.6 N. (5) During **hand** operation of the pump, the force applied by the pump operator (F_h) is assumed to be constant and equal to 70% of F_l (622.72 N). (6) The design variable best suited for manufacturing a modular irrigation pump is the piston cylinder diameter (d_c). (7) The pump is being designed to pull water from a water source that is three meters below the pump and then discharge it into a ditch or furrow one meter

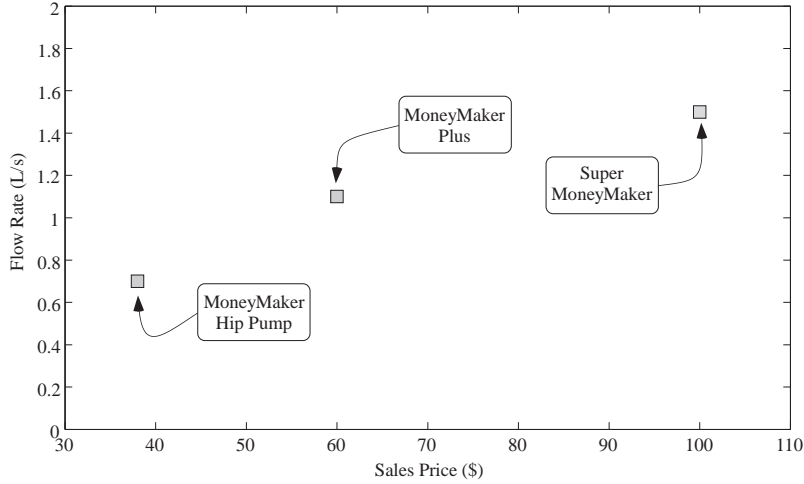


Figure 6.1: Graphical comparison of three non-modular water pumps that are currently sold on the market. The horizontal axis represents the sales price (S in US dollars, and the vertical axis represents the potential water flow rate (Q) in liters per second.

below the pump. (8) Due to the need to satisfy the consumers extreme view of affordability and desire for high performance, the objectives for this example are to minimize the sales price (S) and maximize the water flow rate (Q) of the pump.

Using the information provided in Figure 6.1 and the knowledge of the assumptions made in developing the analytical models, the four anticipated regions of interest within the design space are developed. The limits describing the four anticipated regions of interest within the design space of Q and S are provided in Table 6.1. Values of the limits, in terms of S and Q , for the i -th region are based on the performance of the MoneyMaker Hip Pump ($i = 1$), MoneyMaker Plus ($i = 2$), the Super MoneyMaker ($i = 3$), and the assumption that any product that exhibits improved performance in Q beyond that of the Super MoneyMaker must have a single product purchase price between \$110 and \$180 [55, 61–63].

The complete formulation of the MOP, of the form presented in Section 5.6, incorporating the limits describing the anticipated regions of interest provided in Table 6.1 is presented as follows.

Table 6.1: Limits describing the four anticipated regions of interest within the design space of Q and S . Values of the limits, in terms of S and Q , for the i -th region are based on the performance of the MoneyMaker Hip Pump ($i = 1$), MoneyMaker Plus ($i = 2$), the Super MoneyMaker ($i = 3$), and the assumption that any product that exhibits improved performance in Q above that of the Super MoneyMaker must have a single product purchase price between \$110 and \$180 [55, 61–63].

i	S_{\min} (\$)	S_{\max} (\$)	Q_{\min} (L/s)	Q_{\max} (L/s)
1	20	40	0.25	1.0
2	40	70	0.80	1.3
3	80	110	1.35	2.0
4	110	180	1.85	8.0

Problem 6.1: Irrigation Pump Example – MOP Formulation

$$D_a := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_{x_p}}^*, x_{a,1}^{(i)*}, x_{a,2}^{(i)*}, \dots, x_{a,n_{x_a}^{(i)*}}^{(i)*}) \mid \forall i \in \{1, 2, 3, 4\}\} \quad (6.1)$$

x_p^*, x_a^* defined by:

$$\min_{x_a^{(i)}, x_p} \frac{1}{4} \sum_{n=0}^4 J^{(i)}(x_a^{(i)}, x_p) \quad (6.2)$$

where:

$$J^{(i)} = \min_k \left\{ \min_{x_a^{(k)}, x_p} -Q(x_a^{(k)}, x_p, p) + .1 \cdot S(x_a^{(k)}, x_p, p) \right\} \quad (6.3)$$

$$x_p = \left\{ d_c \quad l_c \right\} \quad (6.4)$$

$$x_a^{(k)} = \begin{cases} \left\{ l_o^{(k)} \quad n_c^{(k)} \right\} & , k \leq 3 \\ \left\{ l_o^{(k)} \quad n_c^{(k)} \quad l_{pa,x}^{(k)} \quad l_r^{(k)} \quad l_1^{(k)} \quad n_{st}^{(k)} \right\} & , k = 4 \end{cases} \quad (6.5)$$

$$p = \left\{ \begin{array}{cccccccccccccc} g & \rho_w & \varepsilon & \eta & F_l & F_h & l_s & d_p & l_{p,in} & l_{p,out} & z_{in} & z_{out} & \dots \\ f_p & f_c & n_{ct} & l_{pa,y} & d_{cr} & l_{cr} & \varphi & w_{pa} & h_p & \Psi_{tube} & \Psi_{plate} & \Psi_{p,st} & \dots \\ & & & \dots & \Psi_{p,w} & \Psi_{seal} & \Psi_{cyl} & \Psi_{plank} & \Psi_{sp} & & & & \dots \end{array} \right\} \quad (6.6)$$

subject to:

$$Q_{\min}^{(i)} \leq Q^{(i)} \leq Q_{\max}^{(i)} \quad (6.7)$$

$$S_{\min}^{(i)} \leq S^{(i)} \leq S_{\max}^{(i)} \quad (6.8)$$

$$\begin{cases} 0.5 & , k \leq 3 \\ 0.75 & , k = 4 \end{cases} \leq t_s^{(k)} \leq \begin{cases} 6.0 & , k \leq 3 \\ 4.0 & , k = 4 \end{cases} \quad (6.9)$$

$$\begin{cases} 0.3 & , i \leq 2 \\ l_o^{(i-1)} & , i > 2 \end{cases} \leq l_o^{(i)} \leq 1.5 \quad (6.10)$$

$$l_o^{(k)} \leq l_{pa,x}^{(k)} \leq 1.5 \quad , k = 4 \quad (6.11)$$

$$l_o^{(k)} + l_r^{(k)} + l_1^{(k)} < l_{pa}^{(k)} \quad (6.12)$$

$$l_r^{(k)} + l_{pa}^{(k)} < l_o^{(k)} + l_1^{(k)} \quad (6.13)$$

with d_c , $n_c^{(k)}$, and $n_{st}^{(k)}$ assuming discrete values according to:

$$d_c = \{ 0.0525, 0.0627, 0.0779, 0.1023, 0.1541, 0.2027 \} \quad (6.14)$$

$$n_c^{(k)} = \begin{cases} 1 & , k \leq 2 \\ 2 & , k > 2 \end{cases} \quad (6.15)$$

$$n_{st}^{(k)} = \{ 16 \ 17 \ 18 \} \quad (6.16)$$

where:

$$Q^{(k)} = \begin{cases} 0.5 \cdot \hat{Q}^{(k)} & , n_c^{(k)} = 1 \\ \hat{Q}^{(k)} & , \text{otherwise} \end{cases} \quad (6.17)$$

$$S^{(k)} = \left(\sum_{j=0}^{n_{\text{comp}}^{(k)}} C_j(x_a^{(k)}, x_p, p) \right) \cdot \left(\frac{1 + M_m + M_d + M_s}{\varphi} \right) \quad (6.18)$$

with fluid model supporting equations for $k = 4$:

$$l_{pa}^{(k)} = \sqrt{\left(l_{pa,x}^{(k)}\right)^2 + \left(l_{pa,y}\right)^2} \quad (6.19)$$

$$p_d = \frac{n_{ct}}{d_{cr}} \quad (6.20)$$

$$d_s^{(k)} = \frac{n_{st}^{(k)}}{p_d} \quad (6.21)$$

$$M_s^{(k)} = F_h \cdot \left(\frac{l_{cr} \cdot d_s^{(k)}}{d_{cr}}\right) \quad (6.22)$$

$$\hat{F}_{in}^{(k)} = \left(\frac{M_s^{(k)}}{l_r^{(k)}}\right) \cdot \text{Ave} \left(\left| \frac{\sin(\theta_{3,v} - \theta_{4,v})}{\sin(\theta_{3,v} - \theta_{2,v})} \right|, \dots, \left| \frac{\sin(\theta_{3,n_v} - \theta_{4,n_v})}{\sin(\theta_{3,n_v} - \theta_{2,n_v})} \right| \right), \forall v \in \{0 \dots 180^\circ\} \quad (6.23)$$

$$\hat{l}_s^{(k)} = l_o^{(k)} \cdot |\sin(\theta_{4,max}) - \sin(\theta_{4,min})| \quad (6.24)$$

with general fluid model supporting equations:

$$h_c^{(k)} = \left(\frac{l_c}{l_o^{(k)}}\right) \cdot \left(\begin{cases} l_s & , k \leq 3 \\ \hat{l}_s^{(k)} & , k = 4 \end{cases} \right) \quad (6.25)$$

$$A_p = \pi \cdot \frac{(d_p)^2}{4} \quad (6.26)$$

$$A_c = \pi \cdot \frac{(d_c)^2}{4} \quad (6.27)$$

$$m_w^{(k)} = \begin{cases} \rho \cdot (A_c \cdot h_c^{(k)} + A_p \cdot l_{p,in}) & , n_c^{(k)} = 1 \\ \rho \cdot (A_c \cdot h_c^{(k)} + A_p \cdot (l_{p,in} + l_{p,out})) & , \text{otherwise} \end{cases} \quad (6.28)$$

$$h_{L,major}^{(k)} = \begin{cases} \frac{f_c \cdot h_c^{(k)} \cdot d_p^4}{(d_c)^5} + \frac{f_p \cdot l_{p,in}}{d_p} & , n_c^{(k)} = 1 \\ \frac{f_c \cdot h_c^{(k)} \cdot d_p^4}{(d_c)^5} + \frac{f_p \cdot (l_{p,in} + l_{p,out})}{d_p} & , \text{otherwise} \end{cases} \quad (6.29)$$

$$h_{L,minor}^{(k)} = \begin{cases} K_{L,1} + K_{L,3} & , n_c^{(k)} = 1 \\ K_{L,1} + K_{L,2} + K_{L,3} & , \text{otherwise} \end{cases} \quad (6.30)$$

$$K_{L,1} = 0.5 \quad (6.31)$$

$$K_{L,2} = 0.45 - 0.625 \cdot \left(\frac{d_p}{d_c}\right) \quad (6.32)$$

$$K_{L,3} = \left(1 - \frac{d_p}{d_c}\right)^2 \quad (6.33)$$

$$\lambda^{(k)} = 1 + h_{L,major}^{(k)} + h_{L,minor}^{(k)} \quad (6.34)$$

$$F_{\text{in}}^{(k)} = \eta \cdot \left(\frac{l_o^{(k)}}{l_c} \right) \left(\begin{cases} F_h & , k = 1 \\ F_l & , k = 2, 3 \\ \hat{F}_{\text{in}}^{(k)} & , k = 4 \end{cases} \right) \quad (6.35)$$

$$V_p^{(k)} = \begin{cases} \sqrt{2 \cdot \left(\frac{F_{\text{in}}^{(k)} \cdot h_c^{(k)}}{\lambda^{(k)} \cdot m_w^{(k)}} + \frac{g}{\lambda^{(k)}} \cdot z_{\text{in}} \right)} & , n_c^{(i)} = 1 \\ \sqrt{2 \cdot \left(\frac{F_{\text{in}}^{(k)} \cdot h_c^{(k)}}{\lambda^{(k)} \cdot m_w^{(k)}} + \frac{g}{\lambda^{(k)}} \cdot (z_{\text{in}} - z_{\text{out}}) \right)} & , \text{otherwise} \end{cases} \quad (6.36)$$

$$V_c^{(k)} = V_p^{(k)} \cdot \left(\frac{d_p}{d_c} \right)^2 \quad (6.37)$$

$$t_s^{(k)} = \frac{h_c^{(k)}}{V_c^{(k)}} \quad (6.38)$$

$$\hat{Q}^{(k)} = 1000 \cdot V_p^{(k)} \cdot A_p \quad (6.39)$$

with financial model supporting equations for $k = 4$:

$$C_{\text{sup}}^{(k)} = 2 \left(l_{\text{pa,y}}^{(k)} \cdot \psi_{\text{tube}} + 0.375 \cdot l_{\text{pa,y}}^2 \cdot \psi_{\text{plate}} \right) \quad (6.40)$$

$$C_{\text{crank}}^{(k)} = \left(0.3556 + 2 \cdot l_r^{(k)} \right) \cdot \psi_{\text{pipe,st}} \quad (6.41)$$

$$C_{\text{link}}^{(k)} = 2 \cdot 0.0127 \cdot l_1^{(k)} \cdot \psi_{\text{plate}} \quad (6.42)$$

$$C_{\text{bike}}^{(k)} = 225 + \psi_{\text{sp}} \quad (6.43)$$

with general financial model supporting equations:

$$C_{\text{base}}^{(k)} = w_{\text{pa}} \cdot \left(l_o^{(k)} + 1.524 \right) \cdot \psi_{\text{plank}} + \left(w_{\text{pa}} \cdot l_o^{(k)} + h_{\text{pivot}} \cdot 0.1524 \right) \cdot \psi_{\text{plate}} + w_{\text{pa}} \cdot \psi_{\text{p,st}} \quad (6.44)$$

$$C_{\text{cyl}}^{(k)} = n_c^{(k)} \cdot \left(h_c^{(k)} + 0.03635 \right) \cdot \psi_{\text{cyl}} \quad (6.45)$$

$$C_{\text{vb}}^{(k)} = n_c^{(k)} \cdot \psi_{\text{vb}} \quad (6.46)$$

$$C_{\text{piston}}^{(k)} = n_c^{(k)} \cdot \left(0.8 \cdot A_c^{(k)} \cdot \psi_{\text{plate}} + \pi \cdot d_c^{(k)} \cdot \psi_{\text{seal}} \right) \quad (6.47)$$

$$C_{\text{pipe}}^{(k)} = (l_{\text{p,in}} + l_{\text{p,out}}) \cdot \psi_{\text{p,w}} \quad (6.48)$$

$$C_{\text{treadle}}^{(k)} = 2 \cdot \left(l_o^{(k)} + 0.1524 \right) \cdot \psi_{\text{tube}} \quad (6.49)$$

$$C_{\text{handle}}^{(k)} = l_c^{(k)} + 0.762 + \left((l_o^{(k)} - l_c^{(k)} - 0.762)^2 + (1.2192 - h_p)^2 \right) \cdot \psi_{\text{tube}} \quad (6.50)$$

Table 6.2: Values of the limits and step sizes for the discrete variables l_o , l_p , $l_{pa,x}$, l_r , and l_l needed to evaluate Problem 6.1.

Variable (units)	Lower Limit	Upper Limit	Step Size
l_o (m)	0.30	1.5	0.01
l_c (m)	0.20	0.4	0.01
$l_{pa,x}$ (m)	0.30	1.5	0.001
l_r (m)	0.01	0.20	0.001
l_l (m)	0.01	1.50	0.001

Table 6.3: Possible values of the parameter ψ_{cyl} for the corresponding value of d_c needed to evaluate Problem 6.1.

ψ_{cyl} (\$/m)	d_c (m)
36.22	0.0525
61.42	0.0627
86.22	0.0779
105.12	0.1023
223.62	0.1541
314.21	0.2027

where θ_1 , θ_2 , θ_3 , and θ_4 are the angles of l_{pa} , l_r , l_l , and l_o respectively obtained through four-bar position analysis [64]; and M_s is the moment (N-m) applied to the rear axel crank obtained using the principle of virtual work [64]. All other variables in the preceding equations are defined in the Nomenclature section of this Chapter. The selected objectives for this problem are to maximize the predicted flow rate ($Q^{(k)}$) and minimize the predicted sales price ($S^{(k)}$) (see Equations 6.2-6.3).

It should be noted that, as was previously presented in Equations 6.14-6.16 for the possible variable values of d_c , n_c , and n_{st} , the variables contained within x_p and x_a are defined as discrete variables. The ranges and value step sizes of l_o , l_p , $l_{pa,x}$, l_r , and l_l are given in Table 6.2. In Table 6.3 the possible values of the parameter ψ_{cyl} for the corresponding value of d_c are presented. The values of the remaining fixed parameters contained in p are provided in Table 6.4.

Values for the variables $l_{p,in}$, $l_{p,out}$, z_{in} , and z_{out} presented in Table 6.4 indicate that the pump is being designed to pull water from a water source that is three meters below the pump and then discharge it into a ditch or furrow one meter below the pump. The equations used to evaluate

Table 6.4: Values of the fixed parameters of p needed to evaluate Problem 6.1.

Variable (units)	Value	Variable (units)	Value
g (m/s ²)	9.80665	ρ_w (kg/m ³)	1000
ε (m)	0.0015	η (%)	80
F_l (N)	889.6	F_h (N)	622.72
l_s (m)	0.3048	d_p (m)	0.0254
$l_{p,in}$ (m)	3.0	$l_{p,out}$ (m)	1.0
z_{in} (m)	-2.0	z_{out} (m)	-1.0
f_c	0.05	f_p	0.075
n_{ct}	44	$l_{pa,y}$ (m)	-0.019
d_{cr} (m)	0.11	l_{cr} (m)	0.17
φ	4.5	w_{pa} (m)	0.4064
h_p (m)	0.4	ψ_{tube} (\$/m)	25.591
ψ_{plate} (\$/m ²)	201.5	$\psi_{p,st}$ (\$/m)	6.299
$\psi_{p,w}$ (\$/m)	4.362	ψ_{seal} (\$/m)	2.625
ψ_{plank} (\$/m ²)	10.629	ψ_{sp} (\$/gear)	22.95

the pumps's performance with respect to the objective Q (see Equation 6.17) are derived from the Energy Equation of the First Law of Thermodynamics presented in Munson et al (see Equations 6.29–6.39 above) [65].

Results of the variable and objective values of the optimal design selected within each region of interest resulting from the evaluation of Problem 6.1 are presented in Table 6.5. It should be noted that these pump designs do not represent platform and module designs. Instead, they represent the non-modular product designs chosen by the method to be the best suited for conversion into platform and module designs while simultaneously providing the best average objective performance. Results provided in Table 6.5 were obtained through the use of a genetic algorithm.

Prior to developing the module designs, information on the type, number, and desired progression of modules that are to be used to obtain the objective space performance of the Pareto designs presented in Table 6.5 is needed. In order to limit the potential of operator assembly errors, a slot modular approach is selected. Examination of the nature of the x_a variables reveals that the differences in the variable values for each design in the set D_a is geometric, and therefore the design with the most commonality is the design with the smallest value of n_c^* and k (see row $i = 1$

Table 6.5: Variable and objective values of the optimal design and design model (column 2) selected within the i -th region of interest (column 1) obtained through the evaluation of Problem 6.1. In addition, the design model corresponding to the design selected within each region is provided in column 2.

		Variables								Objectives	
i	k	d_c^* (m)	l_o^* (m)	l_c^* (m)	n_c^* (m)	$l_{pa,x}^*$ (m)	l_r^* (m)	l_l^* (m)	n_{st}^*	Q^* (L/s)	S^* (\$)
1	1	0.1541	0.45	0.21	1					0.4415	38.86
2	2	0.1541	0.96	0.21	1					0.8270	60.57
3	3	0.1541	1.44	0.21	2					1.5384	85.36
4	4	0.1541	1.44	0.21	2	1.461	0.175	0.296	18	1.9766	168.87

of Table 6.5). Using this information, the desired number of modules to be developed is chosen to be three ($n_m = 3$), and the δ matrix is constructed as follows:

$$\delta = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix} \quad (6.51)$$

The formulation of a constrained module design routine of the form presented in the Section 5.7 is now provided.

Problem 6.2: Irrigation Pump Example – Constrained Module Design

$$D_m := \{(x_{p,1}^*, x_{p,2}^*, \dots, x_{p,n_{xp}}^*, x_{m,1}^{(i)*}, x_{m,2}^{(i)*}, \dots, x_{m,n_{xm}}^{(i)*}) \mid \forall i \in \{1, 2\}\} \quad (6.52)$$

x_m^* is defined by:

$$\min_{x_m} J^{(i)} = \left(P^{(\beta)} - \bar{P}^{(i)} \right)^2 \quad (6.53)$$

defined by:

$$\bar{P}^{(i)} = P^{(\alpha)} + \Delta P^{(i)} \quad (6.54)$$

$$P^{(\alpha)} = \left(Q|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}}, S|_{x_a^{(\alpha)*}, x_p^*, p^{(\alpha)}} \right) \quad (6.55)$$

$$P^{(\beta)} = \left(Q|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}}, S|_{x_a^{(\beta)*}, x_p^*, p^{(\beta)}} \right) \quad (6.56)$$

$$\Delta P^{(i)} = \left(\Delta Q^{(i)}, \Delta S^{(i)} \right) \quad (6.57)$$

where:

$$x_m^{(i)} = \begin{cases} \left\{ l_t^{(i)} \hat{n}_c^{(i)} \right\} & , i \leq 3 \\ \left\{ l_t^{(i)} \hat{n}_c^{(i)} l_{pa,x}^{(i)} l_r^{(i)} l_1^{(i)} n_{st}^{(i)} \right\} & , i = 4 \end{cases} \quad (6.58)$$

$$x_p^* = \{d_c^*, l_c^*\} \quad (6.59)$$

$$\alpha = \delta_{i,1} \quad (6.60)$$

$$\beta = \delta_{i,2} \quad (6.61)$$

$$\Delta Q^{(i)} = \begin{cases} Q(d_c^*, (l_o^{(\alpha)*} + l_t^{(i)}), l_c^*, (n_c^{(\alpha)*} + \hat{n}_c^{(i)}), p) - Q(x_a^{(\alpha)*}, x_p^*, p) & , i \leq 3 \\ Q(d_c^*, (l_o^{(\alpha)*} + l_t^{(i)}), l_c^*, (n_c^{(\alpha)*} + \hat{n}_c^{(i)}), l_{pa,x}^{(i)}, l_r^{(i)}, l_1^{(i)}, n_{st}^{(i)}, p) - Q(x_a^{(\alpha)*}, x_p^*, p) & , i = 4 \end{cases} \quad (6.62)$$

$$\Delta S^{(i)} = \begin{cases} S(d_c^*, (l_o^{(\alpha)*} + l_t^{(i)}), l_c^*, (n_c^{(\alpha)*} + \hat{n}_c^{(i)}), p) - S(x_a^{(\alpha)*}, x_p^*, p) & , i \leq 3 \\ S(d_c^*, (l_o^{(\alpha)*} + l_t^{(i)}), l_c^*, (n_c^{(\alpha)*} + \hat{n}_c^{(i)}), l_{pa,x}^{(i)}, l_r^{(i)}, l_1^{(i)}, n_{st}^{(i)}, p) - S(x_a^{(\alpha)*}, x_p^*, p) & , i = 4 \end{cases} \quad (6.63)$$

$$\hat{n}_c^{(i)} = n_c^{(\beta)*} - n_c^{(\alpha)*} \quad (6.64)$$

where the values and variables of x_p are the same as those obtained through the evaluation of Problem 6.1; and l_t , $0.01 \leq l_t \leq 1.5$. All other variables in the preceding equations are defined in the Nomenclature section of this Chapter.

The variable values of the Platform Design and the module designs obtained through evaluation of the constrained module design optimization formulation presented in Problem 6.2 are provided in Tables 6.6 and 6.7 respectively. In addition, It should be noted that the values and variables of x_p are the same as those presented in Table 6.5.

In order to visually validate that the method has provided the optimal set of platform and module designs, Figure 6.2 is provided. Contained in this figure is a collection of plots that summarize the progression of the method as implemented in this chapter. Figure 6.2(a) provides an approximation of the feasible design space of each of the four candidate pump design models within the regions of interest, assuming that all variables are allowed to vary (i.e. no platform variables are selected) along with the graphical representations of the benchmark products provided in

Table 6.6: Variable and objective values of the platform design obtained through evaluation of Problem 6.1.

Variables				Objectives	
d_c (m)	l_o (m)	l_c (m)	n_c (m)	Q (L/s)	S (\$)
0.1541	0.45	0.21	1	0.4415	38.86

Table 6.7: Variable and objective values of the module designs (i) obtained through evaluation of Problem 6.2.

i	Variables							Objectives		
	d_c (m)	l_t (m)	l_c (m)	\hat{n}_c (m)	$l_{pa,x}$ (m)	l_r (m)	l_l (m)	n_{st}	Q (L/s)	S_m (\$)
1	0.1541	0.96	0.21	0					0.8270	60.57
2	0.1541	0.48	0.21	1					1.5384	85.36
3	0.1541	0	0.21	0	1.461	0.175	0.296	18	1.9766	168.87

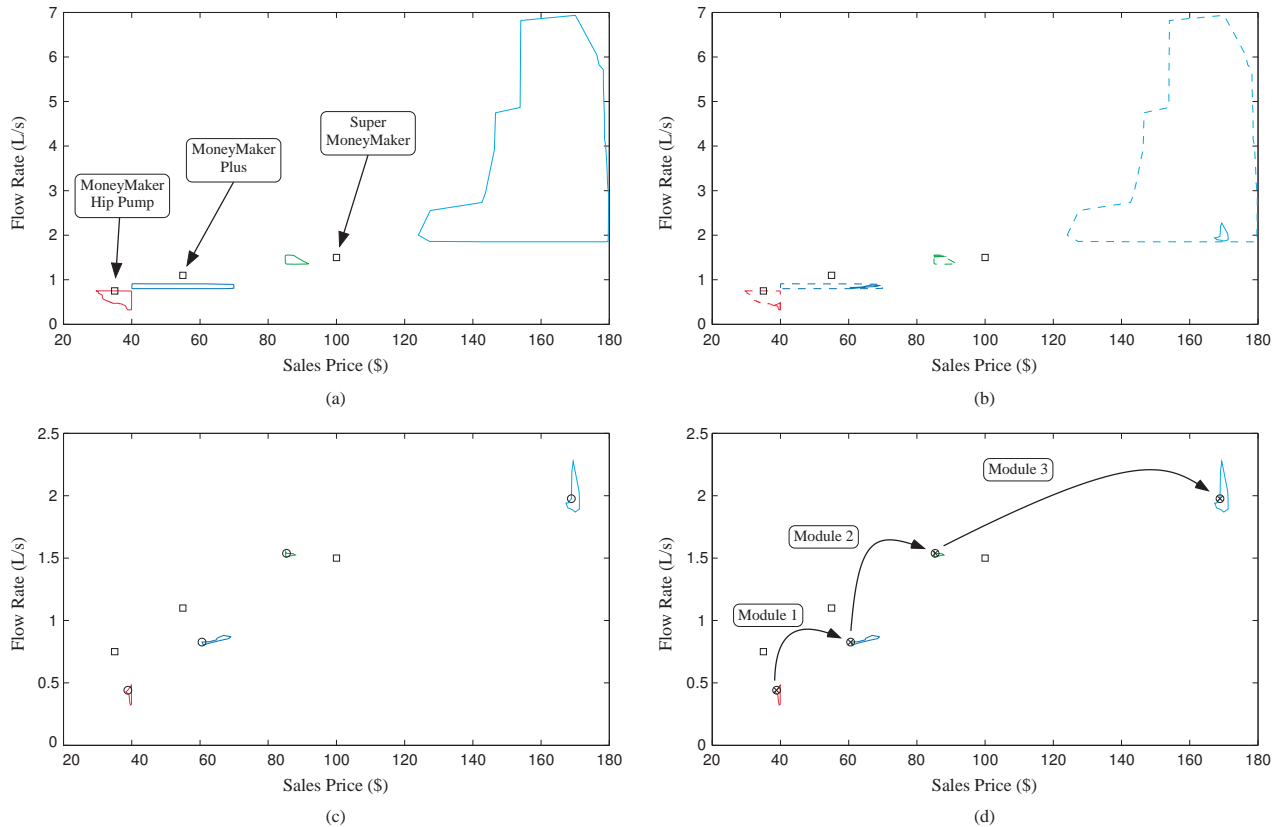


Figure 6.2: Series of plots that visually validate the method results obtained through Problems 6.1 and 6.2 using a Genetic Algorithm.

Figure 6.1. From this plot it is seen that the feasible space for pump configuration 2 within the region of interest located second from the left axis does not provide a performance that is equivalent to the benchmark design. This difference is due to differences in the design and overall function of the design model represented by the feasible space. Figure 6.2(b) shows the feasible design spaces from Figure 6.2(a) (dashed lines) and the shifted/reduced feasible regions (solid lines) that result from selecting d_c and l_c as the platform design variables. Figure 6.2(c) shows the reduced feasible regions from Figure 6.2(b) and the Pareto designs selected through evaluation of Problem 6.1 by a Genetic Algorithm (indicated by the symbol “o”). From this plot it is observed that, based on the objectives to minimize S and maximize Q , the designs are located on the optimal boundary of the reduced regions of the feasible design space. Finally, Figure 6.2(d) provides the same plot as shown in Figure 6.2(c), except that the platform and module designs (indicated by the symbol “x”) obtained through the evaluation of Problem 6.2 are also shown. From this series of plots it is seen that the method is capable of selecting a set of designs that provides the best average objective performance as well as providing the platform and module designs that allow the product to provide the desired modularity that was previously unattainable.

Having verified that the method has provided the optimal set of platform and module designs, 3D solid CAD models of the irrigation pump are developed. Renderings of these models are provided in Figure 6.3. Inspection of Figure 6.3 shows that the intended progression of the pump, as identified through Problem 6.1 and 6.2 above, is to begin by providing a platform pump design that is hand operated and only provides one cylinder (see Figure 6.3(a)). The first module requires reconfiguration of the pump by attaching two new levers (treadles) that the user can step on, and reconfiguring the handle to provide balance while operating the pump (see Figure 6.3(b)). The second module requires additional reconfiguration of the pump through the attaching of one additional cylinder, extensions for the treadles, and the necessary hardware to ensure proper pump function (see Figure 6.3(c)). The third module requires the addition of a support structure for the rear axle of the bike, and the needed links that connect the sprocket attached to the rear axle support structure to the treadles (see Figure 6.3(d)). From these illustrations it is seen that the goal of providing an income generating product that allows for a four-stage investment to incrementally increase the performance of the product is realized. In addition, each configuration of the product

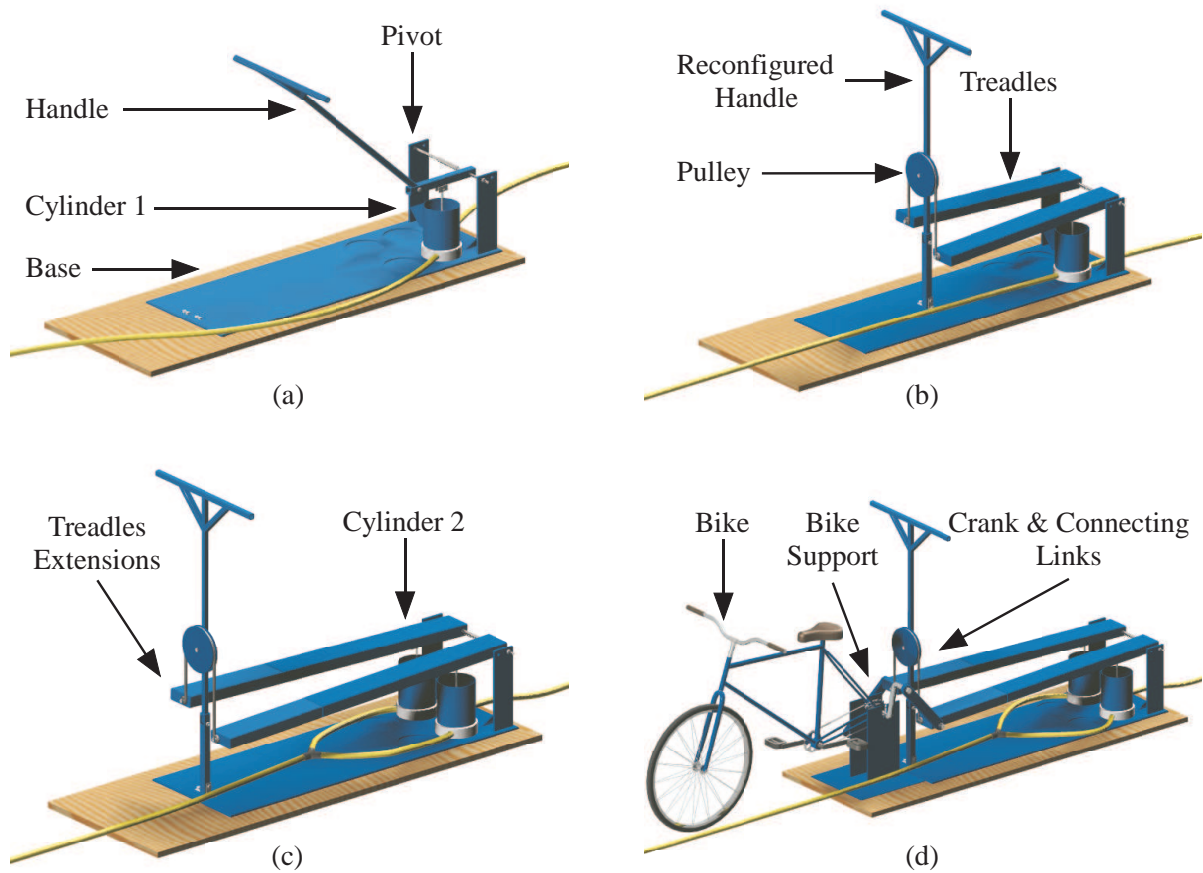


Figure 6.3: Renderings of 3D solid CAD models depicting the progression of the modular irrigation pump design developed in this chapter.

achieved through the addition of a module accounts for changes in what is considered affordable due to increases in income potential.

CHAPTER 7. CONCLUSIONS

This thesis has addressed an important limitation of current methods of module-based product design in accounting for significant, natural changes in consumer needs over time. In response to this limitation, a multiobjective optimization design method has been developed and demonstrated. The presented approach involves the strategic use of a series of optimization formulations that ultimately result in modular products that can adapt to changing consumer needs by moving from one design on the s-Pareto frontier to another through the addition of a module. While more traditional approaches focus on changes in consumer needs across market segments, the exclusion of the effects of future needs of the various market segments, represented by the movement of consumers from one market segment to another or the emergence of new market segments, is an important limitation of current design methods. Thus, by overcoming these limitations the present approach enables the design of a new kind of product that is based on natural changes in consumer needs over time.

Development of the method presented in this thesis was separated into two phases. The first phase of these developments focused on design cases where there is a *single* Pareto frontier, and resulted in a five-step design process. An example implementation of this first phase in the method development was provided through the design of a modular UAV. This example illustrates the ability of the method to identify platform and module designs that provide the desired Pareto-optimal performance according to the changing objectives, parameters, and constraints over time identified within the problem description. The second phase in the method developments focused on the changes required to adapt the five-step process for single Pareto frontier cases to *multiple* Pareto frontier design cases. As a result, a six-step design process which implements the identification of a s-Pareto frontier was provided. To illustrate implementation of this second phase in the method development, the design of a modular manual irrigation pump was provided. Similar to the example of the UAV, the modular pump example demonstrates the ability of the method to design a

product that is capable of traversing the s-Pareto frontier over time through the addition of modules based on changes in the target consumers view of affordability and desired pump performance.

Through the examples of the UAV and manual irrigation pumps presented in this thesis, it is seen that the method developed herein is broadly applicable to diverse applications. In the case of the UAV, the method successfully provides designs based on known changes in mission profiles. In the case of developing an inexpensive income-generating modular irrigation pump, the method can be used to provide designs based on changes in the target consumers view of affordability and desired pump performance.

Recognizing that one of the fundamental assumptions of the method developed herein is that the changes in consumer needs over time are known, future developments related to this thesis include the identification of methods for determining and quantifying the future needs of a product, and the incorporation of uncertainty analysis in the selection of platform and module designs.

In current approaches of product development there are many methods available for determining the *present* consumer needs of a product. The benefit of these methods comes in the ability of the designer to characterize these needs and translate them into performance specifications and attributes that are used to guide the design of the product. However, as has been demonstrated in this thesis, through the development of methods for determining and quantifying the *future* needs of a product, it is possible to translate these needs into performance specifications and attributes that are used to guide the design of products that adapt to satisfy changing needs over time. In the development of these methods of determining future consumer needs, it is anticipated that many of the methods currently used for determining the current consumer needs (e.g. focus groups, surveys, observation) could be adapted to provide the desired outcomes. In addition, more mathematical studies of the movement of consumers between market segments and the emergence of additional market segments could be used to develop gradient based methods that would use previous information detailing the past changes in consumer needs to forecast the future consumer needs.

Recognizing that by using *predicted* changes in consumer needs to develop products that adapt to these changes through the addition of modules involves a degree of uncertainty in the information provided, the incorporation of uncertainty analysis in the selection of platform and

module designs will serve to mitigate the resulting negative impacts due to errors in the provided information. In addition, uncertainties caused by variations in consumer perception, available market data, material properties, manufacturing precision, and other sources can – and should – significantly affect the selection of platform and module designs. In the literature are found two broad categories of approaches to determining the level of uncertainty in decision making: (i) *reliability-based design methods* [66–69] which focus on assessing the probability of design failure, and seeks to reduce such probabilities by shifting the mean performance away from constraint limits [69] and (ii) *robust design based methods* [23, 70–75] which focus on optimizing the mean performance, and minimizing performance variation, while maintaining feasibility with probabilistic constraints [73, 76, 77]. In the context of future work related to this thesis, these methods of uncertainty analysis would be applied and expanded where necessary to provide the needed capabilities to analyze uncertainty in the prediction and implementation of future needs in the development of platform and module designs.

Additionally, future work related to this thesis includes the exploration and characterization of the effects of alternative formulations of the aggregate objective functions of Equations 3.2, 3.3, 5.2, and 5.3. Currently, the aggregate objective space performance of each design in the adaptive set is given equal importance (See Equations 3.2 and 5.2), and results in the set of designs with the best average for the aggregate objective functions of Equations 3.3 and 5.3 being selected. The resulting set of designs naturally sacrifices performance in one (or multiple) region(s) of interest to obtain the best average performance, but there is no active control of which regions of interest may be of greater importance. Therefore, the anticipated benefit of this exploration would be manifest through increased control in the selection of the optimal adaptive design set identified through the design method.

REFERENCES

- [1] Trope, Y., and Liberman, N., 2000. “Temporal construal and time-dependent changes in preference.” *Journal of Personality and Social Psychology*, **79**(6), pp. 876–889.
- [2] Tseng, M. M., Jiao, J., and Merchant, M. E., 1996. “Design for mass customization.” *CIRP Annals - Manufacturing Technology*, **45**(1), pp. 153–156.
- [3] Simpson, T. W., 1998. “Product platform design and optimization: Status and promise.” *Artificial Intelligence for Engineering Design, Analysis and Manufacturing*, **18**(1), pp. 3–20.
- [4] Li, Y., Xue, D., and Gu, P., 2008. “Design for product adaptability.” *Concurrent Engineering*, **16**(3), pp. 221–232.
- [5] Tseng, M. M., and Jiao, J., 1998. “Design for mass customization by developing product family architecture.” In *ASME International Design Engineering Technical Conference & Computers and Information in Engineering Conference*, no. DETC98/DFM-5717.
- [6] Yearsley, J. D., and Mattson, C. A., 2008. “Interactive design of combined scale-based and module-based product family platforms.” In *Proceedings of 12th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, no. AIAA-2008-5819.
- [7] Yang, T. G., Beiter, K. A., and Ishii, K., 2004. “Product platform development: An approach for products in the conceptual stages of design.” In *2004 ASME International Mechanical Engineering Congress and RDD Expo*, no. IMECE2004-62171.
- [8] Yearsley, J. D., and Mattson, C. A., 2008. “Product family member and platform identification with concurrent variable and objective space smart pareto filtering.” In *4th AIAA Multidisciplinary Design Optimization Specialist Conference*, no. AIAA-2008-2220.
- [9] Meyer, M. H., 1997. “Revitalize your product lines through continuous platform renewal.” *Research Technology Management*, **40**(2), pp. 17–28.
- [10] Balling, R. J., 2000. “Pareto sets in decision-based design.” *Engineering Valuation and Cost Analysis*, **3**, pp. 189–198.
- [11] Das, I., 1999. “A preference ordering among various pareto optimal alternatives.” *Structural and Multidisciplinary Optimization*, **18**(1), Aug., pp. 30–35.
- [12] Kasprzak, E. M., and Lewis, K. E., 2000. “An approach to facilitate decision tradeoffs in pareto solution sets.” *Engineering Valuation and Cost Analysis*, **3**, pp. 173–187.
- [13] Li, Y., Fadel, G. M., and Wiecek, M. M., 1998. “Approximating pareto curves using the hyper-ellipse.” In *Proceedings of the 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, no. AIAA-98-4961.

- [14] Mattson, C. A., and Messac, A., 2003. "Concept selection using s-pareto frontiers." *AIAA Journal*, **41**(6), pp. 1190–1204.
- [15] Mattson, C. A., and Messac, A., 2005. "Pareto frontier based concept selection under uncertainty, with visualization." *OPTE: Optimization and Engineering*, **6**(1), pp. 85–115.
- [16] Park, K. W., and Grierson, D. E., 1999. "Pareto-optimal conceptual design of the structural layout of buildings using a multicriteria genetic algorithm." *Computer-Aided Civil and Infrastructure Engineering*, **14**, pp. 163–170.
- [17] Tappeta, R. V., Renaud, J. E., Messac, A., and Sundaraj, G. J., 2000. "Interactive physical programming: Tradeoff analysis and decision making in multidisciplinary optimization." *AIAA Journal*, **38**(5), May, pp. 917–926.
- [18] Wu, J., and Azarm, S., 2001. "Metrics for quality assessment of a multiobjective design optimization solution set." *ASME Journal of Mechanical Design*, **123**, March, pp. 18–25.
- [19] Miettinen, K. M., 1999. *Nonlinear Multiobjective Optimization*. International Series in Operations Research & Management Science. Kluwer Academic Publishers.
- [20] Messac, A., and Mattson, C. A., 2002. "Generating well-distributed sets of pareto points for engineering design using physical programming." *Optimization and Engineering*, **3**(4), pp. 431–450 Kluwer Academic Publishers.
- [21] Steuer, R. E., 1986. *Multiple Criteria Optimization, Theory Computations and Applications*. John Wiley & Sons, Inc., New York.
- [22] Belegundu, A., and Chandrupatla, T., 1999. *Optimization Concepts and Applications in Engineering*. Prentice Hall, New Jersey.
- [23] Chen, W., Wiecek, M. M., and Zhang, J., 1999. "Quality utility - a compromise programming approach to robust design." *ASME Journal of Mechanical Design*, **121**, June.
- [24] Pareto, V., 1964. *Cour dEconomie Politique.*, the first edition in 1896 ed. Librairie Droz, Geneve.
- [25] Das, I., 1999. "On characterizing the knee of the pareto curve based on normal-boundary intersection." *Structural Optimization*, **18**(2/3), pp. 107–115.
- [26] Di Barba, P., 2001. "A fast evolutionary method for identifying non-inferior solutions in multicriteria shape optimization of a shielded reactor." *International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, **20**(3), pp. 762–776.
- [27] Mattson, C. A., Mullur, A. A., and Messac, A., 2004. "Smart pareto filter: Obtaining a minimal representation of multiobjective design space." *Engineering Optimization*, **36**(4), pp. 721–740.
- [28] Montusiewicz, J., and Osyczka, A., 1990. "A decomposition strategy for multicriteria optimization with application to machine tool design." *Engineering Costs and Production Economics*, **20**, pp. 191–202.

- [29] Abraham, S. G., and Schriber, B. R. R., 2000. Fast design space exploration through validity and quality filtering of subsystem designs Tech. Rep. HPL-2000-98, Hewlett Packard.
- [30] Cheng, F. Y., and Li, D., 1998. "Genetic algorithm development for multiobjective optimization of structures." *AIAA Journal*, **36**(6), pp. 1105–1112.
- [31] Messac, A., Ismail-Yahaya, A., and Mattson, C. A., 2003. "The normalized normal constraint method for generating the pareto frontier." *Structural and Multidisciplinary Optimization*, **25**(2), pp. 86–98.
- [32] Mattson, C. A., and Magleby, S. P., 2001. "The influence of product modularity during concept selection of consumer products." In *Proceedings of the ASME DETC'01, ASME 2001 Design Engineering Technical Conferences, and Computers and Information in Engineering Conference*, no. DETC2001/DTM21712.
- [33] Gonzalez-Zugasti, J., and Otto, K., 2000. "Modular platform-based product family design." In *Proceedings of the ASME Design Automation Conference*, no. DETC2000/DAC-14238.
- [34] Magrab, E., 1997. *Integrated Product and Process Design and Development*. CRC Press.
- [35] Yearsley, J. D., and Mattson, C. A., 2008. "Product family design using a smart pareto filter." In *46th AIAA Aerospace Sciences Meeting and Exhibit*, no. AIAA 2008-909.
- [36] Wie, M. V., Greer, J., Campbell, M., Stone, R., and Wood, K., 2001. "Interfaces and product architecture." In *Proceedings of the ASME Design Theory and Methodology Conference*, no. DETC2001/DTM-21689.
- [37] Baguley, P., Bramall, D., Maropoulos, P., and Page, T., 2004. "The automation of sustainability via re-use, modularization, and fuzzy logic." In *Design and Manufacturing for Sustainable Development 2004*, Professional Engineering Publishing.
- [38] Pahl, G., and Beitz, W., 2007. *Engineering Design: A Systematic Approach*. Springer-Verlag.
- [39] Ulrich, K., and Eppinger, S., 2004. *Product Design and Development*, 4th ed. McGraw-Hill.
- [40] Oyebode, A., 2004. "Modularity and quality." In *Proceedings From the 2nd Seminar on Development of Modular Products*, G. Erixon and P. Kenger, eds., pp. 19–24.
- [41] Padamat, M., 2004. "Methods for modularisation." In *Proceedings From the 2nd Seminar on Development of Modular Products*, G. Erixon and P. Kenger, eds., pp. 121–126.
- [42] Fonseca, C. M., and Fleming, P. J., 1995. "An overview of evolutionary algorithms in multiobjective optimization." *Evolutionary Computation*, **3**(1), pp. 1–16.
- [43] Rosenbrock, H. H., 1960. "An automatic method for finding the greatest or least value of a function." *The Computer Journal*, **3**, p. 175.
- [44] Fletcher, R., 1981. *Practical Methods of Optimization: Vol. 2: Constrained Optimization*. John Wiley & Sons.
- [45] Box, M. J., 1966. "A comparison of several current optimization methods, and the use of transformations in constrained problems." *The Computer Journal*, **9**(1), pp. 67–77.

- [46] Fischer, A., 1992. “A special newton-type optimization method.” *Optimization*, **24**(3 & 4), pp. 269–284.
- [47] Lewis, P. K., Murray, V. R., and Mattson, C. A., 2010. “Identification of modular product platforms and modules that account for changing consumer needs.” In *6th AIAA Multidisciplinary Design Optimization Specialist Conference*.
- [48] Lewis, P. K., Murray, V. R., and Mattson, C. A., 2010. “An engineering design strategy for reconfigurable products that support poverty alleviation.” In *the ASME 2010 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, no. DETC2010-28739.
- [49] Lewis, P. K., Murray, V. R., and Mattson, C. A., 2010. “Accounting for changing consumer needs with s-pareto frontiers.” In *the 13th AIAA/ISSMO Multidisciplinary Analysis Optimization Conference*.
- [50] Nigam, N., and Kroo, I., 2008. “Control and design of multiple unmanned air vehicles for a persistent surveillance task.” In *12th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, no. AIAA-2008-5913.
- [51] Wakayama, S., and Kroo, I., 1995. “Subsonic wing planform design using multidisciplinary optimization.” *Journal of Aircraft*, **32**(4), July–August, pp. 746–753.
- [52] International Organization for Standardization (ISO), 1975. “International standard atmosphere.” No. ISO 2533:1975.
- [53] Shelquist Engineering, 2009. “Air density and density altitude calculations.” http://wahiduddin.net/calc/density_altitude.htm, June.
- [54] International Development Enterprises, 2008. *Rural Wealth Creation: Strategic Directions for IDE*.
- [55] Fisher, M., 2006. “Income is development.” *Innovations Journal*, **Winter 2006**, pp. 9–30.
- [56] World Resources Institute and International Finance Corporation, 2007. The next 4 billion: Market size and business strategy at the base of the pyramid Tech. rep., World Bank, June <http://rru.worldbank.org/thenext4billion>.
- [57] Lehr, D., and Greenstadt, D., 2009. “Poor markets make good cents - phones, finance and innovation at the base of the pyramid.” *Receiver Magazine* <http://www.receiver.vodafone.com/poor-markets-make-good-cents>.
- [58] Airhart, M., 2008. “Expandable irrigation systems help poor.” *Earth & Sky* <http://www.earthsky.org/radioshows/49115/expandable-irrigation-systems-help-poor>.
- [59] Polak, P., 2005. “The big potential of small farms.” *Scientific American*, **293**(3).
- [60] Johnson, N. G., Hallam, A., Bryden, M., and Conway, S., 2006. “Sustainable and market-based analysis of cooking technologies in developing countries.” In *Proceeding of the ASME International Mechanical Engineering Congress and Exposition*, no. IMECE2006-15375.

- [61] KickStart, 2009. “MoneyMaker Hip Pump.” <http://www.kickstart.org/products/moneymaker-hip-pump/>.
- [62] Copper-Hewitt : National Design Museum “MoneyMaker Hip Pump.” <http://other90.cooperhewitt.org/Design/moneymaker-hip-pump>.
- [63] Wu, P., and Doshi, S., 2008. “What is a MoneyMaker Hip Pump?.” *Design for Development* – <http://design4dev.wetpaint.com/page/Money+Maker+Pump>.
- [64] Howell, L. L., 2001. *Compliant Mechanisms*. John Wiley & Sons, Inc.
- [65] Munson, B. R., Young, D. F., and Okiishi, T. H., 2006. *Fundamentals of Fluid Mechanics*, fifth ed. John Wiley & Sons.
- [66] Kiureghian, A. D., 1996. “Structural reliability methods for seismic safety assessment: A review.” *Engineering Structures*, **18**(6), p. Engineering Structures.
- [67] Frangopol, D. M., and Corotis, R. B., 1996. “Reliability-based structural system optimization: State-of-the-art versus state-of-the-practice.” *Proceedings of the Twelfth Conference on Analysis and Computation*, April, pp. 67–78.
- [68] Thanedar, P. B., and Kodiyalam, S., 1991. “Structural optimization using probabilistic constraints.” In *AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference*, no. AIAA-91-0922-CP.
- [69] Melchers, R. E., 1999. *Structural Reliability: Analysis and Prediction*. Ellis Horwood Series in Civil Engineering. John Wiley & Sons, New York.
- [70] Parkinson, A., Sorensen, C., and Pourhassan, N., 1995. “A general approach for robust optimal design.” *ASME Journal of Mechanical Design*, **115**, pp. 74–80.
- [71] Chen, W., Sahai, A., Messac, A., and Sundaraj, G. J., 2000. “Exploring the effectiveness of physical programming in robust design.” *ASME Journal of Mechanical Design*, **122**(2), June, pp. 155–163.
- [72] Su, J., and Renaud, J. E., 1997. “Automatic differentiation in robust optimization.” *AIAA Journal*, **35**(6), pp. 1072–1079.
- [73] Taguchi, G., 1993. *Taguchi on Robust Technology Development: Bringing Quality Engineering Upstream*. ASME Press, New York.
- [74] Messac, A., and Ismail-Yahaya, A., 2002. “Multiobjective robust design using physical programming.” *Structural and Multidisciplinary Optimization*, **23**(5), pp. 357–371.
- [75] Chen, W., and Wassenaar, H. J., 2001. “An approach to decision-based design.” In *Proceedings of DETC’01, ASME 2001 Design Engineering Technical Conference and Computers and Information in Engineering Conference*, no. DETC2001/DMT-21683.
- [76] DeVor, R. E., Chang, T. H., and Sutherland, J. W., 1992. *Statistical Quality Design and Control: Contemporary Concepts and Methods*. Prentice Hall, New Jersey, pp. 525–535.

- [77] Koch, P. N., 2002. “Probabilistic design: Optimizing for six sigma quality.” In *AIAA 43rd AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference*, no. AIAA-2002-1471.