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# Optimal Interest Rate for a Borrower with Estimated Default and Prepayment Risk

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OPTIMAL INTEREST RATE FOR A BORROWER WITH ESTIMATED  
DEFAULT AND PREPAYMENT RISK

by

Scott T. Howard

A project submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Master of Science

Department of Statistics

Brigham Young University

August 2008

BRIGHAM YOUNG UNIVERSITY

GRADUATE COMMITTEE APPROVAL

of a project submitted by

Scott T. Howard

This project has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

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BRIGHAM YOUNG UNIVERSITY

As chair of the candidate's graduate committee, I have read the project of Scott T. Howard in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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## ABSTRACT

### OPTIMAL INTEREST RATE FOR A BORROWER WITH ESTIMATED DEFAULT AND PREPAYMENT RISK

Scott T. Howard

Department of Statistics

Master of Science

Today's mortgage industry is constantly changing, with adjustable rate mortgages (ARM), loans originated to the so-called "subprime" market, and volatile interest rates. Amid the changes and controversy, lenders continue to originate loans because the interest paid over the loan lifetime is profitable.

Measuring the profitability of those loans, along with return on investment to the lender is assessed using Actuarial Present Value (APV), which incorporates the uncertainty that exists in the mortgage industry today, with many loans defaulting and prepaying. The hazard function, or instantaneous failure rate, is used as a measure of probability of failure to make a payment. Using a logit model, the default and prepayment risks are estimated as a function of interest rate. The "optimal" interest rate can be found where the profitability is maximized to the lender.

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## 1. INTRODUCTION

The mortgage industry in today's society is constantly changing, with adjustable rate mortgages (ARM) providing creative borrower opportunities and lenders originating loans to subprime borrowers with credit history problems and volatile interest rates. Amid changes and controversy, lenders continue to originate loans because the interest paid over the lifetime of the loan is profitable. Measuring the profitability of mortgage loans will be discussed in detail in the sections that follow.

### 1.1 Return on Capital

One measure of profitability is return on capital (ROC). In a general sense, ROC can be defined as the ratio of Net Income to Capital. As applied to a mortgage, the capital is the loan amount and the net income is the interest paid. That is,

$$\text{ROC} = \frac{\sum(PV(\text{Interest Payments}))}{\text{Capital}} = \frac{\sum PV(\text{Payments}) - \text{Capital}}{\text{Capital}},$$

where  $PV$  denotes the present value, which adjusts for the time value of money, formally defined below.

Difficulty arises when measuring the profitability of loans since the cash flows are spread out over different economic cycles. For that reason, it is necessary to take the present value of the cash flows so as to adjust for inflation as the value changes over time. Another adjustment such as credit spread, which is the difference between revenue and cost of funds, could be used as well. For this project, inflation rate will be used. Inflation fluctuates, but on average is around 3% per year. (InflationData.com 2008) This percentage will be used as the annual rate in this project. Treating a fixed-rate mortgage as an annuity, the present value (Brigham and Houston 2002) is

calculated as

$$PV(\text{annuity}) = \frac{1 - (1 + r)^{-n}}{r} \cdot y,$$

where  $y$  represents the monthly payment,  $r$  represents the inflation rate (expressed monthly) and  $n$  represents the number of months in which a payment was received.

## 1.2 Loan Payback Situations

Consider the profitability of three examples of borrower repayment. First, a complete payback scenario, in which a borrower makes all payments for the loan term. Second, a default scenario, in which a borrower makes payments for a period of time, then stops paying. The lender receives the collateral from the borrower. Third, a prepayment scenario, in which a borrower makes payments until a certain point and then pays the outstanding balance with a single payment.

### 1.2.1 Complete Loan Payback

The most common mortgage is for a fixed principal amount, with a specified term and interest rate. The promissory note is agreed upon by the lender and borrower after loan approval.

Consider a loan for \$100,000 with a 30-year term fixed interest rate of 6% APR. Using the  $PV$  equation, the monthly payment will be

$$y = 100000 / \frac{1 - (1 + (.06/12))^{-360}}{.06/12} = 599.55.$$

Using the  $PV$  equation with inflation at 3.0 APR,

$$PV(\text{Payments}) = \frac{1 - (1 + (.03/12))^{-360}}{(.03/12)} \cdot \$599.55 = \$142,207.$$

That is, from a return on capital perspective,  $\$142,207 - \$100,000 = \$42,207$  (in today's dollars) or 42% return on the \$100,000 invested capital.

### 1.2.2 Default

When a borrower violates the terms of the mortgage by failing to make a payment, the borrower is in default. Defaults may be due to the inability to pay the loan. Accidents, sickness, death, and termination of employment are a few situations which may lead to inability to pay. A borrower with the ability to pay may default if the collateral of the loan is worth less than the outstanding balance. For example, consider a \$100,000 loan whose collateral is a home with an estimated value of \$150,000. As long as the value of the home is greater than the outstanding balance, a borrower who could not make the monthly payments could avoid default by selling the house and paying the outstanding balance. But, if after 5 or 10 years the value of the home decreased to \$60,000 or \$40,000, then paying for the full term of the loan would be paying more for the home than it is worth, so a borrower may choose to default. The return on capital is affected not only by the reduced number of payments but also by the loss incurred because the collateral sold through foreclosure is worth less than the outstanding balance.

The return on capital of a defaulted loan requires the present value of the payments and the present value of the loss incurred at default. When a borrower defaults, the collateral is repossessed by the lender and sold, almost always for less than the outstanding balance. Return on capital (ROC) could then be calculated as follows:

$$\begin{aligned} \text{ROC} &= \frac{\sum PV(\text{Payments up to Default}) + PV(\text{Collateral}) - \text{Capital}}{\text{Capital}} \\ &= \frac{PV(y_1, y_2, \dots, y_{t-1}) + PV(\Delta_t) - \text{Capital}}{\text{Capital}}, \end{aligned}$$

where  $y_1, y_2, \dots, y_{t-1}$  denote monthly payments made prior to default and  $\Delta_t$  is the value of the collateral in month  $t$ .

In this example, consider a borrower that defaulted after 10 years. To demonstrate the effect that collateral value has on ROC, two scenarios with different values

for  $\Delta_t$  are presented. For the first scenario, let  $\Delta_{120} = 60,000$ . In other words, the collateral at default is valued at \$60,000. After 120 months,  $PV(\Delta_{120}) = 44,465.74$ . For the second scenario, let  $\Delta_{120} = 40,000$ . In other words, the collateral at default can be sold for \$40,000. For this scenario,  $PV(\Delta_{120}) = 29,643.82$  after 120 months. The respective ROC with these two specified values for  $\Delta_{120}$  after 10 years of paying \$599.55 per month, at 3% inflation are

$$\begin{aligned} \text{ROC}_1 &= \frac{PV(y_1, y_2, \dots, y_{t-1}) + PV(\Delta_{120}) - \$100,000}{\$100,000} \\ &= \frac{\$62,090.45 + \$44,465.74 - \$100,000}{\$100,000} = 6.6\% \end{aligned}$$

and

$$\begin{aligned} \text{ROC}_2 &= \frac{PV(y_1, y_2, \dots, y_{t-1}) + PV(\Delta_{120}) - \$100,000}{\$100,000} \\ &= \frac{\$62,090.45 + \$29,643.82 - \$100,000}{\$100,000} = -8.3\%. \end{aligned}$$

As expected, borrowers who default are very costly, as the lender is not receiving the revenue that was agreed to upon origination of the loan. There is an obvious benefit to the lender if they can identify the borrowers who are more likely to default and not originate the loans to these borrowers.

### 1.2.3 Prepayment

In the current mortgage market, few borrowers follow the complete loan payback described in Section 1.2.1. Instead, a borrower might decide, after a certain period of time has passed, that the conditions set out when their loan was originated (i.e. payment amount, interest rate, etc.) are not ideal for them. They choose to pay off their current loan with the proceeds from another loan using the same property or collateral as security. Occasionally, a borrower might pay off their current loan with cash on hand rather than with the proceeds from another loan. This process is called prepaying, and is quite popular in a declining interest rate environment.

Consider a borrower who chose to pay off their loan earlier than anticipated. For this example, consider prepayment at 5 years (60 months) and 10 years (120 months).

Calculating the return on capital for a borrower that prepays is easier than for a default scenario, since the borrower pays the exact outstanding balance.

$$\begin{aligned} \text{ROC} &= \frac{PV(\text{Payments up to Prepayment}) + PV(\text{Outstanding Balance}) - \text{Capital}}{\text{Capital}} \\ &= \frac{\sum PV(\text{Interest Payments up to Prepayment})}{\text{Capital}} \\ &= \frac{PV(y_1, y_2, \dots, y_{t-1}) + PV(B_t) - \text{Capital}}{\text{Capital}}, \end{aligned}$$

where  $y_1, y_2, \dots, y_{t-1}$  denote monthly payments up to prepayment and  $B_t$  is the outstanding balance in month  $t$ .

In the case of a borrower who prepays after 5 years,

$$\text{ROC}_5 = \frac{\$33,366.37 + \$80,107.62 - \$100,000}{\$100,000} = 13.5\%.$$

If the borrower were to prepay after 10 years,

$$\text{ROC}_{10} = \frac{\$62,090.45 + \$62,019.12 - \$100,000}{\$100,000} = 24.1\%.$$

Since the capital is all paid back, just earlier than expected, the return on capital in the situation in which the borrower prepays is still positive, but not as high as the the ROC for complete loan payback.

### 1.3 Actuarial Present Value (APV)

The previous examples have assumed known times of payment or default. If a lender knew that a borrower was going to default after 10 years, it is obvious that the loan to that specific borrower would never have been originated because the ROC is a loss. Uncertainty of default exists when a borrower applies for a loan because each

borrower has some positive probability of defaulting associated with their loan and circumstances.

Present value depends on outcome; that is, whether a borrower defaults, pre-pays, or pays the loan in full and the timing of when this outcome might occur. Accounting for the uncertainty associated with borrower outcome and timing provides a more realistic measure of return on capital.

The Actuarial Present Value (APV) is defined as the present value of a contingent event; that is, the sum of the present value of the monthly payments under a given outcome multiplied by the probabilities associated with that outcome (full payment, prepayment, and default). That is,

$$APV = \sum_{\substack{\text{All} \\ \text{possible} \\ \text{outcomes}}} PV(\text{Outcome}) \cdot P(\text{Outcome}).$$

The general idea is finding the present value of the payments up to a certain point (contingent on a certain outcome) and then multiplying by the probability of the outcome (full payment, prepayment, or default).

In survival analysis, the hazard function, or instantaneous failure rate, is used as a measure of this probability of time to failure (Lee and Wang 2003). In the context of borrower repayment behavior, the hazard function,  $h(t)$ , is the conditional probability of missing the  $t^{th}$  payment given the borrower pays their loan up until month  $t - 1$ . That is,

$$h(t) = P[\text{no payment in month } t \mid \text{paid in } 1, \dots, t - 1]$$

and probability that the borrower makes the  $t^{th}$  payment is

$$1 - h(t) = P[\text{paying in month } t \mid \text{paid in month } 1, \dots, t - 1].$$



Failure to pay is the consequence of either default or prepayment. The hazard function can therefore be expressed as the hazard function of two competing risks:

$$\begin{aligned}
h(t) &= P[\text{default} \cup \text{prepay in month } t \mid \text{paid in month } 1, \dots, t-1] \\
&= P[\text{default} \mid \text{paid in month } 1, \dots, t-1] + P[\text{prepay} \mid \text{paid in month } 1, \dots, t-1] \\
&= h_d(t) + h_p(t),
\end{aligned}$$

where  $h_d(t)$  and  $h_p(t)$  are the hazard functions of default and prepayment.

The probabilities for all possible outcomes in APV can be expressed using  $h_d(t)$  and  $h_p(t)$ . For example, the probability of a default in the 2nd month is

$$\begin{aligned}
&P[\text{paid in month 1, default in month 2}] \\
&= P[\text{paid in month 1}] P[\text{default in month 2} \mid \text{paid in month 1}] \\
&= [1 - h(1)] h_d(2).
\end{aligned}$$

The contribution to APV for default in month 2 is

$$\begin{aligned}
&[PV(\text{Payments up to Default}) + PV(\text{Collateral} - \text{Outstanding Balance})] \\
&\cdot P[\text{paid in month 1, default in month 2}] \\
&= [PV(y_1) + PV(\Delta_2)] \cdot [1 - h(1)] h_d(2),
\end{aligned}$$

where  $y_1$  is the monthly payment received and  $\Delta_2$  is the value of the collateral at month 2. Similarly, the contribution to the APV for a default in month 3 is

$$[PV(y_1, y_2) + PV(\Delta_3)] [1 - h(1)][1 - h(2)] h_d(3),$$

where  $y_1$  and  $y_2$  are the monthly payments received and  $\Delta_3$  is the value of the collateral at month 3. In general, for any default outcome, the contribution to the APV is

$$[PV(y_1, y_2, \dots, y_{t-1}) + PV(\Delta_t)] [1 - h(1)][1 - h(2)] \cdots [1 - h(t-1)] h_d(t), \quad t = 1, \dots, 359.$$

For the prepayment outcome, the probability of a prepayment in the 2nd month is

$$\begin{aligned}
 & P[\text{paid in month 1, prepay in month 2}] \\
 &= P[\text{paid in month 1}] P[\text{prepay in month 2} \mid \text{paid in month 1}] \\
 &= [1 - h(1)] h_p(2).
 \end{aligned}$$

The contribution to APV for prepayment in month 2 is

$$\begin{aligned}
 & [PV(\text{Payments up to Prepayment}) + PV(\text{Outstanding Balance})] \\
 & \cdot P[\text{paid in month 1, prepay in month 2}] \\
 &= [PV(y_1) + PV(B_2)] \cdot [1 - h(1)] h_p(2),
 \end{aligned}$$

where  $y_1$  is the monthly payment received and  $B_2$  is the outstanding balance at month 2. Similarly, the contribution to the APV for a prepayment in month 3 is

$$[PV(y_1, y_2) + PV(B_3)] [1 - h(1)][1 - h(2)] h_p(3),$$

where  $y_1$  and  $y_2$  are the monthly payments received and  $B_3$  is the outstanding balance at month 3. In general, for any prepayment outcome, the contribution to the APV is

$$[PV(y_1, y_2, \dots, y_{t-1}) + PV(B_t)] [1 - h(1)][1 - h(2)] \cdots [1 - h(t-1)] h_p(t), \quad t = 1, \dots, 359.$$

The APV for a mortgage combines the possible default and prepayment outcomes with the full payment outcome, shown in the equation on the following page. It is assumed that the probability that a borrower misses a payment in the first month ( $h(t)$ ) is zero. First payment defaults are considered fraud.

$$\begin{aligned}
APV &= \sum_{\substack{\text{All possible} \\ \text{default} \\ \text{outcomes}}} PV(\text{Outcome}) \cdot P(\text{Outcome}) \\
&+ \sum_{\substack{\text{All possible} \\ \text{prepay} \\ \text{outcomes}}} PV(\text{Outcome}) \cdot P(\text{Outcome}) \\
&+ PV(\text{Monthly payments}) \cdot P(\text{Complete loan payback}) \\
&= [(PV(y_1, \Delta_2) [1 - h(1)] h_d(2) + PV(y_1, y_2, \Delta_3) [1 - h(1)][1 - h(2)] h_d(3) \\
&\quad + \dots + PV(y_1, y_2, \dots, y_{359}, \Delta_{360}) [1 - h(1)][1 - h(2)] \dots [1 - h(359)] h_d(360)) \\
&\quad + (PV(y_1, B_2) [1 - h(1)] h_p(2) + PV(y_1, y_2, B_3) [1 - h(1)][1 - h(2)] h_p(3) \\
&\quad + \dots + PV(y_1, y_2, \dots, y_{359}, B_{360}) [1 - h(1)][1 - h(2)] \dots [1 - h(359)] h_p(360)) \\
&\quad + (PV(y_1, y_2, \dots, y_{360}) \prod_{t=1}^{360} [1 - h(t)])].
\end{aligned}$$

## 2. DEFAULT HAZARD FUNCTION ESTIMATION

Calculating Actuarial Present Value requires the probability of missing a payment,  $h(t)$ . This chapter describes how to estimate  $h_d(t)$ , the default hazard function using data.

### 2.1 Data Description

The loans in this data set are from a \$7 billion portfolio in the subprime home equity market. “Subprime” borrowers generally have weak or damaged credit, which prevents them from qualifying for loans in the prime market. Not surprisingly, loss rates in the subprime sector are greater than in the prime sector. These loans are fixed-rate, with first liens secured by residential real estate originated between 1994 and 2002. Table 2.1 contains summary statistics on these 97,124 loans. Repayment behavior on these loans is observed between March 2001 and February 2002. This time window is narrow because many loans in the portfolio are purchased from other lenders without data on earlier repayment history.

Table 2.1: Summary statistics of a \$7 billion portfolio of subprime home equity loans secured by residential real estate.

	Minimum	Q1	Median	Q3	Maximum
Interest Rate (%)	6.00	7.50	8.75	9.5	18.50
Loan Amount (\$)	25,800	55,933	71,519	88,725	331,015
Loan-to-Value Ratio (%)	1.00	70.33	90.39	94.94	100.00
Proprietary Credit Score (0=bad, 1=good)	0.00	0.52	0.64	0.76	1.00

#### Percentage of Credit Report Derogatories

12.86	Filed for Bankruptcy
11.72	At Least One NSF Check
41.24	At Least One Major Derogatory

## 2.2 Methodology

The probability of default is modeled using a logit model. The response variable is the incidence of default where  $y = 1$  in the month when default was observed and  $y = 0$  otherwise. The data for the 97,124 loans are “exploded” by examining the default behavior each month. For the default data set, if a borrower’s status was not default (i.e., delinquent less than 180 days) in month  $t$ , then the response (incidence of default) is  $y = 0$ . If a borrower defaults (i.e., first occurrence of 180 days delinquent) in month  $t$ , then the response is  $y = 1$ . There are no more observations for a delinquent borrower, as the assumption is made that a borrower cannot transition from default back to a current state. If a borrower were to prepay in month  $t$ , then  $y = 0$  since the borrower did not default. Consider a borrower that defaults in month  $t = 10$ . Then  $y_1, \dots, y_9 = 0$  and  $y_{10} = 1$ .

When a borrower files a loan application, the lender receives information from the application and obtains the borrower’s credit bureau report. These reports include variables such as the number of non-sufficient funds, the difference between credit limit and outstanding balance, the number of derogatories, and so forth. The list of possible covariates for predicting default is large. Many companies have found value in creating a proprietary or custom score that incorporates credit bureau information and measurements of the borrower’s behavior with the bank. A proprietary score was used in this project.

The estimated probability of default changes over time. The rate at which these probabilities change also varies. To allow for a flexible model, consider piecewise linear splines for time with possible knots at 12–month intervals from 0 to 360. Using stepwise variable selection, the significant knots were found to be at months 12, 24, 36, and 60. To avoid too many knots, the knots chosen were at months 12, 36, and 60. Linear extrapolation is used for months greater than the window of loans in the data set (months 100+).

The default probabilities are estimated using logistic regression. The default model has the standardized custom score (Score), interest rate (APR), and time (t), where

$$\begin{aligned} \text{logit}(p) = \log\left(\frac{p}{1-p}\right) = & \beta_0 + \beta_1 \cdot \text{Score} + \beta_2 \cdot \text{APR} + \beta_3 \cdot t + \beta_4 \cdot (t - 12) \cdot I(t > 12) \\ & + \beta_5 \cdot (t - 36) \cdot I(t > 36) + \beta_6 \cdot (t - 60) \cdot I(t > 60). \end{aligned}$$

The coefficients can be interpreted as  $\beta_1$  being the increase in the log-odds for a one unit increase in the standardized custom score;  $\beta_2$  being the increase in the log-odds for a one-percent increase in interest rate, holding all else constant; and  $\beta_3, \beta_4, \beta_5$  and  $\beta_6$  being coefficients in a linear spline for time with knots at 12, 36 and 60 months. Estimates for  $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ , and  $\beta_6$  are given in Table 2.2.

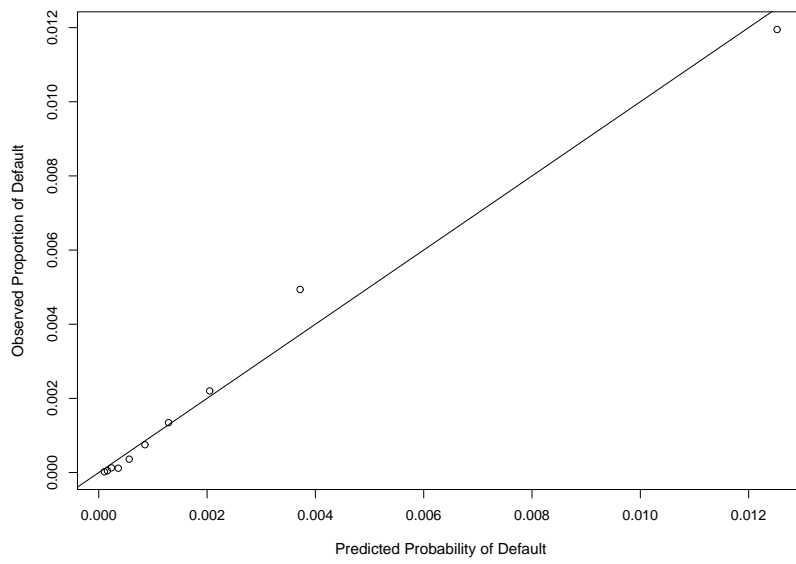
Table 2.2: Maximum Likelihood Logistic Regression Coefficient Estimates for Default

	Default
$\beta_0$	-4.7181
$\beta_1$	-7.8543
$\beta_2$	0.0958
$\beta_3$	0.1207
$\beta_4$	-0.1129
$\beta_5$	-0.0150
$\beta_6$	0.000997

A Decile plot is constructed as a measure of the prediction performance of the model. The Decile plot is constructed by calculating the estimated probability of default and grouping into deciles. In each decile, the observed proportion of loans defaulting is computed. Figure 2.1 shows the Decile plot comparing the estimated probability of default and observed proportion of accounts defaulting. The actual probabilities appear well predicted because the values fall closely on a unit slope through the origin.

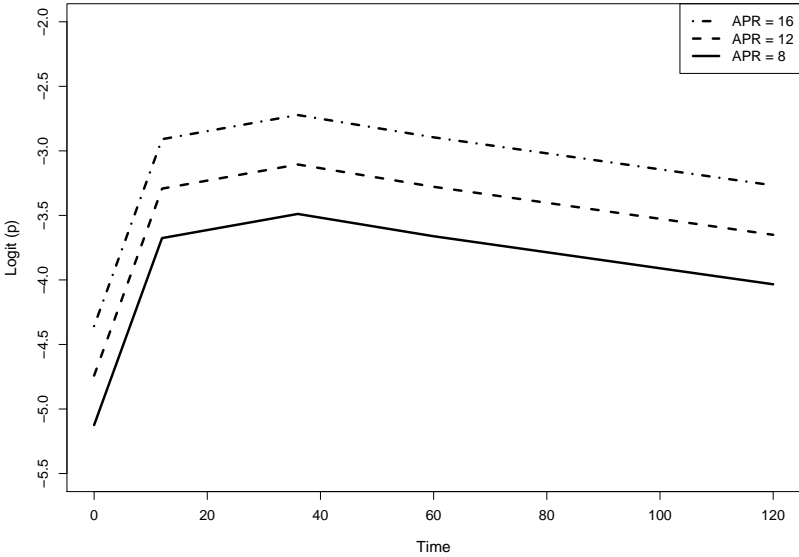
To demonstrate the effect of interest rate on  $h_d(t)$ , the logistic regression coefficient estimates from Table 2.2 are plotted for interest rates of 8, 12, and 16% in the

Figure 2.1: Default Decile Plot demonstrating prediction performance of the logistic model.



partial logit plot in Figure 2.2. Similar partial logit curves could be plotted for other interest rates. A clear difference exists between loans with interest rates of 8, 12 and 16%, as the  $\text{logit}(p)$  for a borrower with 16% interest is more than double that for a borrower with 8% interest.

Figure 2.2: Partial Default Logit Plot. Plot of maximum likelihood estimates at 8%, 12%, and 16% interest for months 1, . . . , 120.

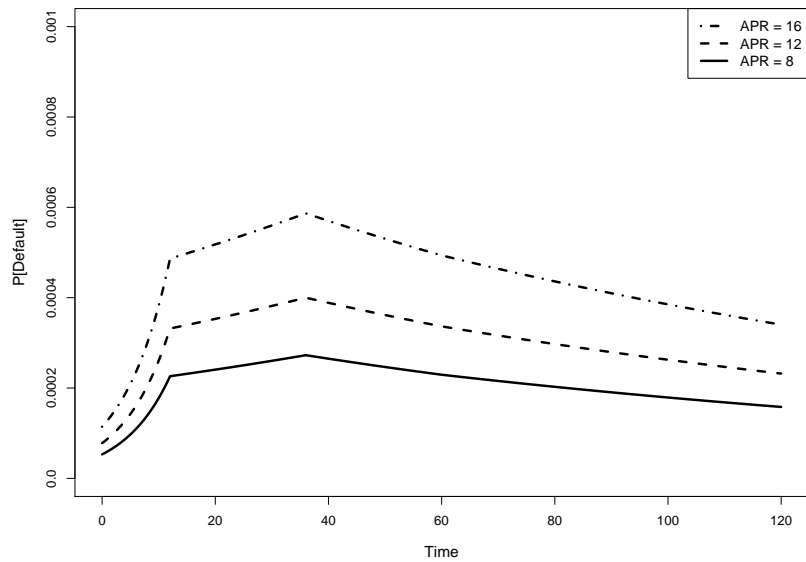


The logit coefficient estimate for  $\beta_1$  is negative, indicating that the higher the standardized custom score, the lower the estimated probability of default. The logit coefficient estimates for  $\beta_4$  and  $\beta_5$  (piecewise linear splines) are negative, indicating that the rate at which the log-odds and probability of default increases begins to slow after month 12 and 36 compared to the previous months. The positive coefficient estimate for  $\beta_6$  indicates that the rate at which the log-odds and probability of default decreases begins to slow after month 60.

The impact of the negative spline coefficients is magnified in the default probability plot (Figure 2.3). The estimated probabilities of default for a borrower with a score of 0.75 are increasing for months  $t = 1, \dots, 36$ , but these probabilities decrease



Figure 2.3: Default Probability Curves. Plot of probability of default for months  $1, \dots, 120$  at 8%, 12%, and 16% interest using maximum likelihood estimates from logistic regression for a borrower with a proprietary score of 0.75.



after month 36. The rate at which these probabilities increase is greater for months  $t = 1, \dots, 12$  than it is for months after 12. One plausible explanation for this effect is that borrowers originate a mortgage, but after a short period of time, they realize that they are in over their heads and default on their loan.

Although small, the probability of default, as shown in Figure 2.3, is much greater for a loan with an interest rate of 16% than for a loan with 8 or 12% interest. From a business side, if a lender had the choice to charge a borrower a high interest rate, it would seem like the most profitable return on investment, since this would maximize interest return. A lender might, or should, be hesitant to originate loans with extremely high interest rates, as the probabilities of default are greater for loans with these interest rates (Figure 2.3). The APV should reflect this fact, and it is possible for the APV to decrease if interest rate increases.

### 3. PREPAYMENT HAZARD FUNCTION ESTIMATION

Calculating Actuarial Present Value requires the probability of missing a payment,  $h(t)$ . This chapter describes how to estimate  $h_p(t)$ , the prepayment hazard function using data.

The probability of prepayment is modeled using a logit model. The response variable is the incidence of prepayment, where  $y = 1$  the month prepayment was observed and  $y = 0$  otherwise. The data for the 97,124 loans are “exploded” by examining the prepayment behavior each month. For the prepayment data set, if a borrower did not prepay in month  $t$ , then the response (incidence of prepayment) is  $y = 0$ . If a borrower prepays in month  $t$ , then the response is  $y = 1$ . Consider a borrower that prepays in month  $t = 10$ . Then  $y_1, \dots, y_9 = 0$  and  $y_{10} = 1$ .

The prepayment probability changes over time. To allow for a flexible model, consider piecewise linear splines with possible knots at 12 month intervals from 0 to 360. Using stepwise variable selection, the significant knots were found to be at months 36 and 60.

The prepayment hazard function is estimated using logistic regression. The prepayment model has the standardized custom score (Score), interest rate (APR), time (t), and linear splines with knots at 36 and 60 months. In other words, with  $p = P[\text{prepayment} | \text{Score}, \text{APR}, t]$ , the logit model for prepayment is

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 \cdot \text{Score} + \beta_2 \cdot \text{APR} + \beta_3 \cdot t + \beta_4 \cdot (t-36) \cdot I(t > 36) + \beta_5 \cdot (t-60) \cdot I(t > 60).$$

The coefficients can be interpreted as  $\beta_1$  being the increase in the log-odds for a one unit increase in the standardized custom score;  $\beta_2$  being the increase in the log-odds for a one-percent increase in interest rate, holding all else constant; and  $\beta_3$ ,  $\beta_4$  and  $\beta_5$  being coefficients in a linear spline for time with a knot at 36 and 60 months. Estimates for  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ ,  $\beta_4$ , and  $\beta_5$  are given in Table 3.1.

A Decile plot is constructed as a measure of the prediction performance of

Table 3.1: Maximum Likelihood Logistic Regression Coefficient Estimates for Prepayment

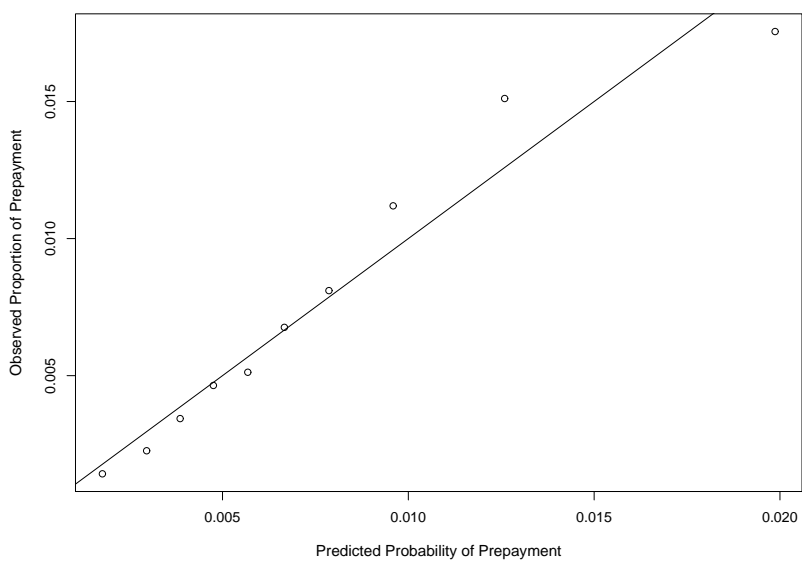
	Prepayment
$\beta_0$	-9.9756
$\beta_1$	2.9684
$\beta_2$	0.2760
$\beta_3$	0.0180
$\beta_4$	-0.00399
$\beta_5$	-0.0157

the model. The Decile plot is constructed by calculating the estimated probability of prepayment and grouping into deciles. In each decile, the observed proportion of loans prepaying is computed. Figure 3.1 shows the Decile plot comparing the estimated probability of prepayment and observed proportion of accounts defaulting. The actual probabilities appear well predicted since the values fall closely on a unit slope through the origin. An interesting feature in the prepayment Decile plot is the large spread of average probability among deciles indicating a very high risk in the population for prepaying.

To demonstrate the effect of interest rate on  $h_p(t)$ , the logistic regression coefficient estimates from Table 3.1 are plotted for interest rates of 8, 12 and 16% in the partial logit plot in Figure 3.2. A clear difference exists between loans with interest rates of 8, 12 and 16%, as the  $\text{logit}(p)$  for a borrower with 16% interest is more than double that for a borrower with 8% interest.

The logit coefficient estimate for  $\beta_1$  is positive, indicating that the higher the standardized custom score, the higher the estimated probability of prepaying. If a borrower were to have a high custom score, they would be able to receive credit from another lender, perhaps at a lower rate, enabling them to prepay. The logit coefficient estimates for  $\beta_4$  and  $\beta_5$  (piecewise linear splines) are negative, indicating that the rate at which the log-odds and probability of prepayment increases begins to slow after month 36 and 60 compared to the previous months. Perhaps many borrowers originate

Figure 3.1: Prepayment Decile Plot demonstrating prediction performance of the logistic model.



loans with the intention of prepaying within a few years, especially if interest rates are “high.”

The impact of the negative spline coefficients is magnified in the prepayment probability plots (Figure 3.3). The estimated prepayment probabilities for a borrower with score of 0.75 shown in Figure 3.3 are increasing throughout months 1 through 60, then begin to decrease after month 60.

The probability of prepayment, as shown in Figure 3.3, is much greater for a loan with an interest rate of 16% than for a loan with 8 or 12% interest, and this probability grows at a higher interest rate of 16%.

From a business side, if a lender had the choice to charge a borrower a high interest rate, it would seem like the most profitable return on investment, since this would maximize interest return. A lender might, or should, be hesitant to originate loans with extremely high interest rates, as the probabilities of prepayment are greater for loans with these interest rates (Figure 3.3). Essentially, the lender is pushing the borrower to prepay on their account. The APV should reflect this fact, and it is possible for the APV to decrease as interest rate increases.

Figure 3.2: Partial Prepayment Logit Plot. Plot of maximum likelihood estimates at 8%, 12%, and 16% interest for months 1, ..., 120.

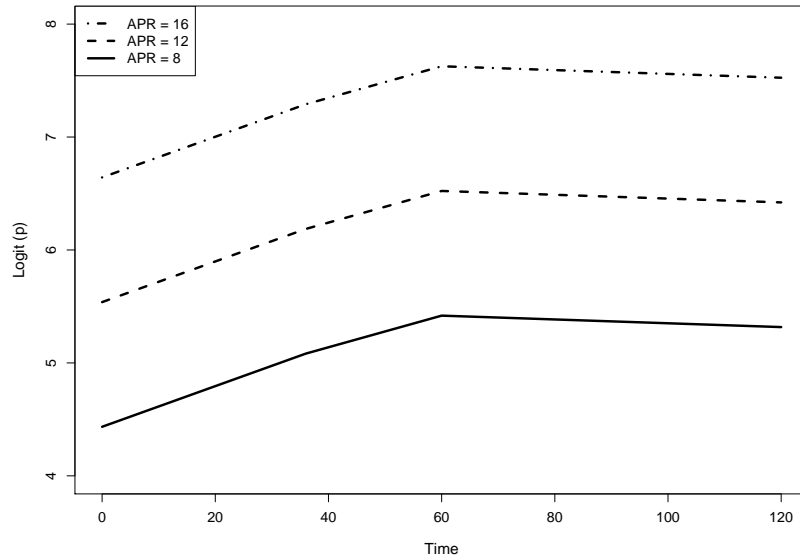
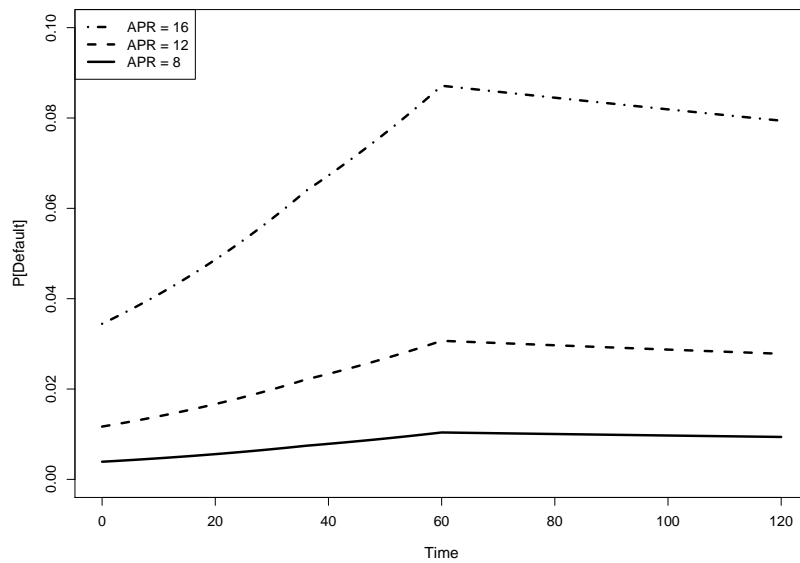


Figure 3.3: Prepayment Probability Curves. Plot of probability of prepayment for months 1, ..., 120 at 8%, 12%, and 16% interest using maximum likelihood estimates from logistic regression for a custom score of 0.75.



## 4. RESULTS

### 4.1 Actuarial Present Value Estimates

Using the APV formula presented in Section 1.3, APV estimates were obtained for a borrower with a specified proprietary credit score and interest rate. Table 4.1 shows the actuarial present value calculated for a \$100,000 subprime loan to a borrower with a 0.75 proprietary credit score using the estimated hazard functions described in Chapters 2 and 3. The APVs presented are for interest rates ranging from 6 to 18%. A proprietary credit score of 0.75 represents a borrower with worthy credit (3<sup>rd</sup> Quartile). In calculating the APV, the assumption was made that the security, or collateral, on the loan was worth \$125,000 upon origination, and that upon defaulting, the bank would receive 75% of \$125,000. Some might argue that the assumption of the lender receiving 75% of the asset's present value is generous. Therefore, the effect of different collateral values on APV should be further investigated. Also, no assumption is made as to what occurs with the capital when prepayment occurs. In other words, if a borrower prepays on a loan, these calculations do not take into account what the lender could or could not do with the capital received from the borrower.

The results from Table 4.1 are intriguing. First, one would think that the higher the interest that you charge, the more money that you make. This is not so when incorporating the uncertainty due to default and prepayment. Higher interest rates may push borrowers to prepayment if their credit scores are good enough to receive credit elsewhere.

Viewed as a function of interest rate, the APV increases as the interest rate increases from 6 to 10.25%, then decreases. The difference, in terms of return on capital (ROC) is 5% more for a loan at 12% interest when compared to the same loan at 8% interest. Also, if the borrower were to receive a 16% interest rate, the APV



associated with this loan is about 13% less than the loan at 12%. This shows that just because a lender can charge higher interest rates, a higher return is not guaranteed.

## 4.2 Solving APV for Interest Rate

Since  $\hat{h}_d(t)$  and  $\hat{h}_p(t)$  are functions of interest rate, it is possible to solve for the APV for a borrower with a given credit score applying for a loan of  $B_0$  using collateral valued at  $H_0$  at different interest rates.

For example, consider finding  $APR^*$  such that  $APV(APR^*, \mathbf{x}, \beta) = B_0$ . That is, the loan will only be profitable with interest rates greater than or equal to  $APR^*$  since  $APV(APR^*, \mathbf{x}_p, \hat{\beta}) - B_0 \geq 0$ , where  $B_0$  is the capital loaned to the borrower. This value ( $APR^*$ ) represents the interest rate at which the lender guarantees a positive return on their investment. Since lenders desire more than just a positive return, it is more interesting to solve the following equation for  $APR^*$ :

$$\frac{APV(APR^*, \mathbf{x}_p, \hat{\beta}) - B_0}{B_0} \geq \delta,$$

where  $\delta$  represents the desired return on capital (ROC).

### 4.2.1 Business

From a business perspective, solving APV for interest rate allows for the lender to set a desired ROC ( $\delta$ ). Since APV is a function of interest rate, a lender desiring a certain ROC can then use the borrower's covariates (credit history, or custom score), and originate a loan such that with interest rate  $APR^*$  will give the lender their desired return on capital, if such an  $APR^*$  exists (e.g., ROC = 35% is not possible in Table 4.1).

#### 4.2.2 Analytics

From an analytics perspective, because APV is a function of interest rate, this allows for solving for interest rate. By solving for interest rate, the desired return can be chosen, which allows the lender to find an “optimal” interest rate at which they will receive their desired return on capital. “Optimal” can also be defined as the APV at which the lender maximizes their profit. In the business setting, the latter is what matters most.

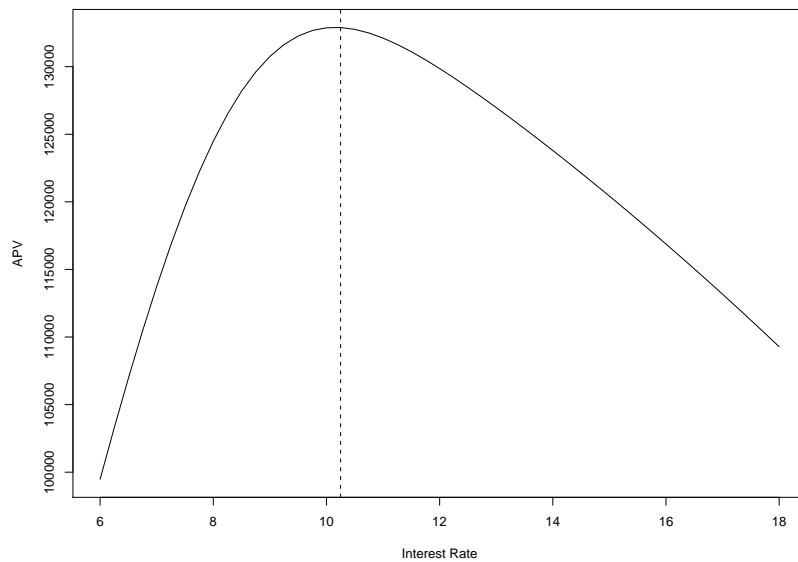
#### 4.3 Properties of APV as interest rate changes

Investigating the properties of APV for different values of interest rate, as done in Section 3.1, shows that interest rate does have an effect on APV. The simple example presented involved interest rates ranging from 6 to 18% and were for one loan. It appears that a “maximum” APV exists for a given loan. For the situation described in Section 3.1, the “optimal” interest rate that produces this maximum APV was 10.25% (See Figure 4.1). For a borrower with a standardized custom score of 0.75, with a \$100,000 loan and collateral value of \$125,000, this interest rate maximizes the lender’s risk adjusted return on investment.

#### 4.4 Further Research

This project presented APV calculations for a \$100,000 loan with a 0.75 standardized custom score, interest rates ranging from 6 to 18% and collateral value of \$125,000. These variables can be further evaluated with varied loan amounts, custom scores, and collateral values to see the effect that each of these variables has on APV. Also, this project has data for months 1 through 100, and extrapolates for months 100 through 360. For subsequent projects, a bigger window of data would be useful to evaluate.

Figure 4.1: Plot of Actuarial Present Value for a \$100,000 loan with a 0.75 standardized custom score, interest rates ranging from 6 to 18%, and collateral value of \$125,000.



Clearly, the “current” interest rate in month  $t$  and the APR at loan origination affects the prepayment hazard function. It would be beneficial to evaluate in further detail how much of an affect the “current” interest rate has on prepayment.

Finally, many investment options exist in the financial market. The analysis in this project could be compared with other investment options, such as a 30 year treasury bond. For example, a \$100,000 in a 30 year zero coupon treasury bond at 4% APR with 3% inflation has  $APV = 148,788.67$ . Research could be done to evaluate other investment options.

Table 4.1: Actuarial Present Value (APV) for a \$100,000 subprime loan with interest rates from 6 to 18%, proprietary score of 0.75, and inflation at 3.0 APR.

Interest.Rate	APV	Interest.Rate	APV
6.00	99,476.47	12.25	129,176.60
6.25	103,285.73	12.50	128,465.00
6.50	106,952.57	12.75	127,729.56
6.75	110,447.58	13.00	126,974.08
7.00	113,743.24	13.25	126,201.50
7.25	116,814.02	13.50	125,413.75
7.50	119,638.84	13.75	124,612.24
7.75	122,198.87	14.00	123,797.95
8.00	124,481.56	14.25	122,971.53
8.25	126,478.19	14.50	122,133.49
8.50	128,186.94	14.75	121,284.20
8.75	129,609.69	15.00	120,423.92
9.00	130,754.93	15.25	119,552.80
9.25	131,635.14	15.50	118,671.09
9.50	132,268.21	15.75	117,778.86
9.75	132,673.67	16.00	116,876.26
10.00	132,874.54	16.25	115,963.31
10.25	132,895.12	16.50	115,040.02
10.50	132,759.63	16.75	114,106.43
10.75	132,491.87	17.00	113,162.47
11.00	132,113.85	17.25	112,207.97
11.25	131,645.35	17.50	111,242.77
11.50	131,104.05	17.75	110,266.60
11.75	130,504.55	18.00	109,279.07
12.00	129,858.95		

## 5. CONCLUSIONS

The mortgage industry in today's society is constantly changing, with adjustable rate mortgages (ARM) under attack, controversy dealing with lenders originating loans to the so-called "subprime" market (borrowers with credit history problems), and volatile interest rates. Amid the changes and controversy, lenders continue to originate loans because the interest paid over the lifetime of the loan is profitable. One measure of profitability is return on capital (ROC), which uses present value to find the lender's overall return. However, ROC does not incorporate the uncertainty that exists in today's society, as outcomes are uncertain.

To incorporate the uncertainty that exists, actuarial present value (APV) is used. In general, APV is the sum of the present value of the monthly payments under a given outcome times the probabilities associated with that outcome (full payment, prepayment, and default). The hazard function, or instantaneous failure rate, is used as a measure of the probability associated with a given outcome. The hazard functions are estimated using logistic regression.

Once estimated, APVs are calculated for different interest rates, as different interest rates produce different APVs. A maximum APV exists, suggesting an "optimal" interest rate. The "optimal" interest rate is the point at which the lender is maximizing their risk-adjusted return on investment.

Originating loans according to the "optimal" interest rate for a given borrower allows the lender to maximize the return for a given loan, adjusting for the uncertainty that exists in today. Incorporating APV and optimizing the interest rate for each loan over an entire portfolio thus maximizes the risk-adjusted return on capital.

## 5.1 Methodology & Models

The default and prepayment hazard functions are each estimated using a logit model. The response variable is the incidence of default ( $y = 1$  in the month when default was observed, and  $y = 0$  otherwise) or the incidence of prepayment ( $y = 1$  in the month when prepayment was observed, and  $y = 0$  otherwise).

The data comes from a \$7 billion portfolio in the subprime home equity market. These loans are fixed-rate, with first liens secured by residential real estate originated between 1994 and 2002. Repayment behavior is observed between March 2001 and February 2002. The list of possible covariates for predicting default and prepayment is large. Many companies have found value in creating a proprietary or custom score that incorporates credit bureau information and measurements of the borrower's behavior with the bank. A proprietary score was used in this project.

The default probabilities are estimated using logistic regression. The default model has the standardized custom score (Score), interest rate (APR), time (t), where

$$\begin{aligned} \text{logit}(p) = \log\left(\frac{p}{1-p}\right) = & \beta_0 + \beta_1 \cdot \text{Score} + \beta_2 \cdot \text{APR} + \beta_3 \cdot t + \beta_4 \cdot (t - 12) \cdot I(t > 12) \\ & + \beta_5 \cdot (t - 36) \cdot I(t > 36) + \beta_6 \cdot (t - 60) \cdot I(t > 60). \end{aligned}$$

The coefficients can be interpreted as follows:  $\beta_1$  is the increase in the log-odds for a one unit increase in the standardized custom score;  $\beta_2$  is the increase in the log-odds for a one-percent increase in interest rate, holding all else constant;  $\beta_3$ ,  $\beta_4$ ,  $\beta_5$ , and  $\beta_6$  are coefficients in a linear spline for time with knots at 12, 36 and 60 months.

The prepayment hazard function is estimated using logistic regression. The prepayment model has the standardized custom score (Score), interest rate (APR), time (t), and linear splines with knots at 36 and 60 months. In other words, with  $p = P[\text{prepayment} | \text{Score}, \text{APR}, t]$ , the logit model for prepayment is

Table 5.1: Maximum Likelihood Logistic Regression Coefficient Estimates for Default and Prepayment

	Default		Prepayment
$\beta_0$	-4.7181	$\beta_0$	-9.9756
$\beta_1$	-7.8543	$\beta_1$	2.9684
$\beta_2$	0.0958	$\beta_2$	0.2760
$\beta_3$	0.1207	$\beta_3$	0.0180
$\beta_4$	-0.1129	$\beta_4$	-0.00399
$\beta_5$	-0.0150	$\beta_5$	-0.0157
$\beta_6$	0.000997		

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 \cdot \text{Score} + \beta_2 \cdot \text{APR} + \beta_3 \cdot t + \beta_4 \cdot (t-36) \cdot I(t > 36) + \beta_5 \cdot (t-60) \cdot I(t > 60).$$

The coefficients can be interpreted as follows:  $\beta_1$  is the increase in the log-odds for a one-unit increase in the standardized custom score;  $\beta_2$  is the increase in the log-odds for a one-percent increase in interest rate, holding all else constant;  $\beta_3$ ,  $\beta_4$ , and  $\beta_5$  are coefficients in a linear spline for time with a knot at 36 and 60 months.

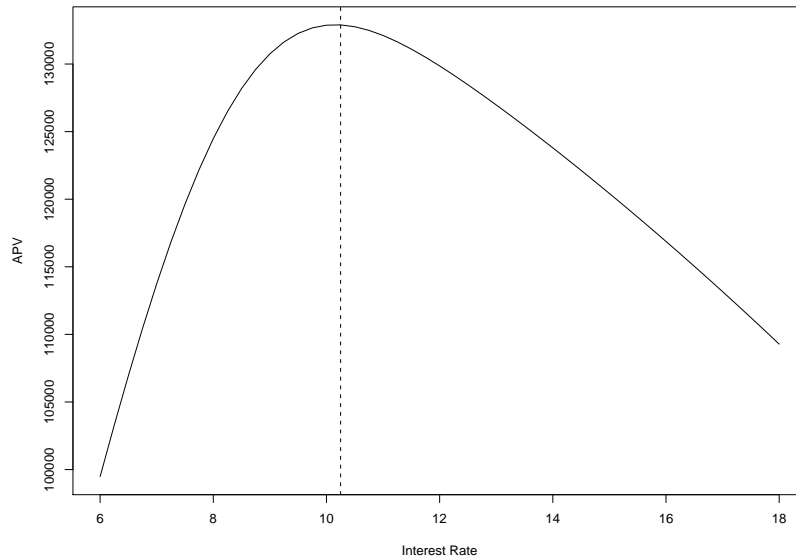
Table 5.1 displays the logit regression coefficient estimates for the default and prepayment scenarios. Interesting to note are the opposite signs for the coefficient estimate associated with the custom score. For the default scenario, the coefficient is negative, suggesting that as a borrower's custom score increases, the log-odds and probability of default decreases. For the prepayment scenario, the coefficient is positive, suggesting that as a borrower's custom score increases, the log-odds and probability of prepayment increases. Either the borrower is smarter than the default borrowers, or has the credit necessary to obtain a loan somewhere else, and prepay their current loan.

After estimating the default and prepayment hazard functions, APV is calculated for a \$100,000 loan, assuming a collateral value of \$125,000 and a standardized custom score of 0.75, and is displayed for interest rates ranging from 6 to 18%. The



APV values are displayed in Table 5.2. As can be seen in Table 5.2 and in Figure 5.1,

Figure 5.1: Plot of Actuarial Present Value for a \$100,000 loan with a 0.75 standardized custom score, interest rates ranging from 6 to 18% and collateral value of \$125,000.



a maximum exists at which the lender is receiving the maximum APV. This maximum APV occurs at an interest rate of 10.25% for the given example, suggesting that 10.25% is the “optimal” interest rate for the given borrower. Similar tables and figures could be displayed for differing loan amounts, collateral values, and standardized custom scores, but each suggests that an “optimal” interest rate exists; that is, a point exists at which the lender is maximizing their risk-adjusted return on investment.

## 5.2 Application & Use

Originating loans according to the “optimal” interest rate for a given borrower allows the lender to maximize the return for a given loan, adjusting for the uncertainty that exists in today’s society. Incorporating APV and optimizing the interest rate for each loan over an entire portfolio thus maximizes the risk-adjusted return on capital.

### 5.3 Further Research

This project presented APV calculations for a \$100,000 loan with a 0.75 standardized custom score, interest rates ranging from 6 to 18%, and collateral value of \$125,000. These variables can be further evaluated, with varied loan amounts, custom scores, and collateral values to see the effect that each of these variables has on APV. Also, this project has data for months 1 through 100, and extrapolates for months 100 through 360. For subsequent projects, a bigger window of data would be useful to evaluate.

Clearly, the “current” interest rate in month  $t$  and the APR at loan origination affects the prepayment hazard function. It would be beneficial to evaluate in further detail how much of an affect the “current” interest rate has on prepayment.

Finally, many investment options exist in the financial market. The analysis in this project could be compared with other investment options, such as a 30-year treasury bond. For example, a \$100,000 in a 30-year zero coupon treasury bond at 4% APR with 3% inflation has  $APV = 148,788.67$ . Research could be done to evaluate other investment options.

Table 5.2: Actuarial Present Value (APV) for a \$100,000 subprime loan with interest rates from 6-18%, and standardized custom score of 0.75.

Interest.Rate	APV	Interest.Rate	APV
6.00	99,476.47	12.25	129,176.60
6.25	103,285.73	12.50	128,465.00
6.50	106,952.57	12.75	127,729.56
6.75	110,447.58	13.00	126,974.08
7.00	113,743.24	13.25	126,201.50
7.25	116,814.02	13.50	125,413.75
7.50	119,638.84	13.75	124,612.24
7.75	122,198.87	14.00	123,797.95
8.00	124,481.56	14.25	122,971.53
8.25	126,478.19	14.50	122,133.49
8.50	128,186.94	14.75	121,284.20
8.75	129,609.69	15.00	120,423.92
9.00	130,754.93	15.25	119,552.80
9.25	131,635.14	15.50	118,671.09
9.50	132,268.21	15.75	117,778.86
9.75	132,673.67	16.00	116,876.26
10.00	132,874.54	16.25	115,963.31
10.25	132,895.12	16.50	115,040.02
10.50	132,759.63	16.75	114,106.43
10.75	132,491.87	17.00	113,162.47
11.00	132,113.85	17.25	112,207.97
11.25	131,645.35	17.50	111,242.77
11.50	131,104.05	17.75	110,266.60
11.75	130,504.55	18.00	109,279.07
12.00	129,858.95		

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## A. ACTUARIAL PRESENT VALUE R FUNCTION CODE

```
### Function to calculate APV
## Inputs: interest rate(6% = 6, etc.), loan term (years), Principal Amount,
## inflation rate, Collateral value, asset return %(If default, get .75 of collateral),
## custom standardized score(% from 0 to 1)

amort <- function(rate,num.years,Loan.Amount,inflation,collateral,
asset.return,cust.score){
rate.period <- rate / 100/12  ## Monthly interest rate
num.period <- num.years*12    ## Number of periods
inflation.rate <- inflation/100/12  ## Inflation rate per period

### Initializing Vectors and values used in APV calculations ###
Out.Balance <- matrix(0,num.period,1,byrow=T)
Payment <- matrix(0,num.period,1,byrow=T)
Interest <- matrix(0,num.period,1,byrow=T)
Principal <- matrix(0,num.period,1,byrow=T)
PV.pmt <- matrix(0,num.period,1,byrow=T)
PV.Out.Balance <- matrix(0,num.period,1,byrow=T)
PV.Collateral <- matrix(0,num.period,1,byrow=T)
h.def <- matrix(0,num.period,1,byrow=T)
h.prepay <- matrix(0,num.period,1,byrow=T)
prob.nopay <- matrix(0,num.period,1,byrow=T)
prob.pay <- matrix(0,num.period,1,byrow=T)
def.probs <- matrix(0,num.period,1,byrow=T)
prepay.probs <- matrix(0,num.period,1,byrow=T)
def.case <- matrix(0,num.period,1,byrow=T)
prepay.case <- matrix(0,num.period,1,byrow=T)
cumprods <- matrix(0,num.period-1,1,byrow=T)

Init.Balance <- Loan.Amount
Cum.Payment <- 0
Cum.Interest <- 0
Cum.Principal <- 0

for(i in 1:num.period){

Payment[i] <- round((rate.period*Loan.Amount*(1+rate.period)^num.period)/
((1 + rate.period)^num.period - 1),digits=2)
```

```

### First Month Values
if(i == 1){
Interest[i] <- round((Init.Balance * rate.period),digits=2)
Principal[i] <- round((Payment[i] - Interest[i]),digits=2)
Out.Balance[i] <- round((Init.Balance - Principal[i]),digits=2)
PV.pmt[i] <- round(((1- (1+ inflation.rate)^(-i))/
(inflation.rate))*Payment[i],digits=2)
PV.Out.Balance[i] <- round((Out.Balance[i]/(1+inflation.rate))
*(1/(1+inflation.rate)^i),digits=2)
PV.Collateral[i] <- round(((collateral*asset.return)/(1+inflation.rate))
*(1/(1+inflation.rate)^i),digits=2)

## Default Hazard
h.def[i] <- ((exp(-4.7181 - 7.8543*cust.score+ 0.0958*rate + 0.1207*i -
0.1129*(i-12)*(i>12) - 0.0150*(i-36)*(i>36) - 0.000997*(i-60)*(i>60)))
/(1 + exp(-4.7181 - 7.8543*cust.score+ 0.0958*rate + 0.1207*i -
0.1129*(i-12)*(i>12) - 0.0150*(i-36)*(i>36) - 0.000997*(i-60)*(i>60))))

## Prepay Hazard
h.prepay[i] <- ((exp(-9.9756 + 2.9684*cust.score+ 0.2760*rate
+ 0.0180*i - 0.00399*(i-36)*(i>36) - 0.0157*(i-60)*(i>60)))
/(1 + exp(-9.9756 + 2.9684*cust.score+ 0.2760*rate + 0.0180*i
- 0.00399*(i-36)*(i>36) - 0.0157*(i-60)*(i>60))))

prob.nopay[i] <- h.def[i]+h.prepay[i]
prob.pay[i] <- 1 - (h.def[i]+h.prepay[i])}

else{ ### Months 2, ...,359
Interest[i] <-round((Out.Balance[i-1] * rate.period),digits=2)
Principal[i] <- round((Payment[i] - Interest[i]),digits=2)
Out.Balance[i] <- round((Out.Balance[i-1] - Principal[i]),digits=2)
PV.pmt[i] <- round(((1- (1+ inflation.rate)^(-i))/
(inflation.rate))*Payment[i],digits=2)
PV.Out.Balance[i] <- round((Out.Balance[i]/(1+inflation.rate))
*(1/(1+inflation.rate)^i),digits=2)
PV.Collateral[i] <- round(((collateral*asset.return)/(1+inflation.rate))
*(1/(1+inflation.rate)^i),digits=2)

## Default Hazard
h.def[i] <- ((exp(-4.7181 - 7.8543*cust.score+ 0.0958*rate + 0.1207*i

```

```

- 0.1129*(i-12)*(i>12) - 0.0150*(i-36)*(i>36) - 0.000997*(i-60)*(i>60)))
/(1 + exp(-4.7181 - 7.8543*cust.score+ 0.0958*rate + 0.1207*i
- 0.1129*(i-12)*(i>12) - 0.0150*(i-36)*(i>36) - 0.000997*(i-60)*(i>60)))

## Prepay Hazard
h.prepay[i] <- ((exp(-9.9756 + 2.9684*cust.score+ 0.2760*rate
+ 0.0180*i - 0.00399*(i-36)*(i>36) - 0.0157*(i-60)*(i>60)))
/(1 + exp(-9.9756 + 2.9684*cust.score+ 0.2760*rate + 0.0180*i
- 0.00399*(i-36)*(i>36) - 0.0157*(i-60)*(i>60))))

prob.nopay[i] <- h.def[i]+h.prepay[i]
prob.pay[i] <- 1 - (h.def[i]+h.prepay[i])

cumprods <- cumprod(prob.pay)

def.probs[i] <- (cumprods[i-1])* h.def[i]
prepay.probs[i] <- (cumprods[i-1])* h.prepay[i]
def.case[i] <- def.probs[i]*(PV.pmt[i-1]+PV.Collateral[i-1])
prepay.case[i] <- prepay.probs[i]*(PV.pmt[i-1]+PV.Out.Balance[i-1]) }

#### Adjusting for rounding error in last month
if(i == num.period){
Interest[i] <- round((Out.Balance[i-1] * rate.period),digits=2)
Principal[i] <- round(Out.Balance[i-1],digits=2)
Payment[i] <- Principal[i] + Interest[i]
Out.Balance[i] <- 0.00
PV.pmt[i] <- round(((1- (1+ inflation.rate)^(-i))
/(inflation.rate))*Payment[i-1],digits=2)
PV.Collateral[i] <- round(((collateral*asset.return)/(1+inflation.rate))
*(1/(1+inflation.rate)^i),digits=2)
PV.Out.Balance[i] <- 0.00

## Default Hazard
h.def[i] <- ((exp(-4.7181 - 7.8543*cust.score+ 0.0958*rate + 0.1207*i
- 0.1129*(i-12)*(i>12) - 0.0150*(i-36)*(i>36) - 0.000997*(i-60)*(i>60)))
/(1 + exp(-4.7181 - 7.8543*cust.score+ 0.0958*rate + 0.1207*i
- 0.1129*(i-12)*(i>12) - 0.0150*(i-36)*(i>36) - 0.000997*(i-60)*(i>60))))

## Prepay Hazard
h.prepay[i] <- ((exp(-9.9756 + 2.9684*cust.score+ 0.2760*rate
+ 0.0180*i - 0.00399*(i-36)*(i>36) - 0.0157*(i-60)*(i>60)))

```

```

/(1 + exp(-9.9756 + 2.9684*cust.score+ 0.2760*rate + 0.0180*i
- 0.00399*(i-36)*(i>36) - 0.0157*(i-60)*(i>60)))

prob.nopay[i] <- h.def[i]+h.prepay[i]
prob.pay[i] <- 1 - (h.def[i]+h.prepay[i])

def.probs[i] <- (cumprods[i-1])*h.def[i]
prepay.probs[i] <- (cumprods[i-1])*h.prepay[i]

def.case[i] <- def.probs[i]*(PV.pmt[i-1]+PV.Collateral[i-1])
prepay.case[i] <- prepay.probs[i]*(PV.pmt[i-1]+PV.Out.Balance[i-1])

prob.fullpay <- cumprods[i]
fullpay.case <- prob.fullpay*PV.Out.Balance[i] }

### Calculating total amt. paid in Payment, Interest, Principal ####
Cum.Payment <- Cum.Payment + Payment[i]
Cum.Interest <- Cum.Interest + Interest[i]
Cum.Principal <- Cum.Principal + Principal[i] }

APV <- sum(prepay.case)+sum(def.case)+ fullpay.case
Prob.check <- sum(def.probs)+ sum(prepay.probs)+prob.fullpay
Pmt.No <- seq(1,num.period)

### Other possible values to return - (Amortization table with Cum. Totals, etc.)
## out1 <- cbind(Pmt.No,Payment,Principal,Interest,Out.Balance,PV.pmt
## ,PV.Out.Balance,PV.Collateral,h.def,h.prepay,prob.nopay,prob.pay)
## cum.Totals <- rbind(c(num.period,Cum.Payment,Cum.Principal,Cum.Interest,
## 0,0,0,0,0,0,0,0))
## out2 <- rbind(out1,cum.Totals)

return(cbind(APV))}

yy <- matrix(0,49,1,byrow=T)
xx <- seq(6,18,by=.25)
for(i in 1:length(xx)){
yy[i] <- amort(xx[i],30,100000,3,125000,.75,.75) }

out1 <- as.matrix(cbind(xx,yy))

```