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Consensus of large-scale group decision making in social network: the minimum cost model based on robust optimization

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ABSTRACT

Recently, large-scale group decision making (LSGDM) in social network comes into being. In the practical consensus of LSGDM, the unit adjustment cost of experts is difficult to obtain and may be uncertain. Therefore, the purpose of this paper is to propose a consensus model based on robust optimization. This paper focuses on LSGDM, considering the social relationship between experts. In the presented model, an expert clustering method, combining trust degree and relationship strength, is used to classify experts with similar opinions into subgroups. A consensus index, reflecting the harmony degree between experts, is devised to measure the consensus level among experts. Then, a minimum cost model based on robust optimization is proposed to solve the robust optimization consensus problem. Subsequently, a detailed consensus feedback adjustment is presented. Finally, a case study and comparative analysis are provided to verify the validity and advantage of the proposed method.

1. Introduction

With the development of information technology, such as e-democracy and peer to peer working, a large number of decision makers (experts) are involved in decision making problems. This has led to the large-scale group decision making (LSGDM) to become a hotspot [40,24,25]. Generally, LSGDM is a special type of decision making in which no fewer than 20 experts collaboratively choose the best alternative [9]. A number of practical applications [8], such as porous media [41] and petroleum engineering [27], involve LSGDM. In actual LSGDM, the experts may not be independent individuals, they may have specific relationships or be stakeholders. Consequently, their relationships may accelerate the exchange of internal information, thereby affecting the final decision results.

LSGDM in social network has received increasing attention recently [30,23]. As a new type of decision case, LSGDM in social network refers to the process in which experts rely on social relations and opinion support from close friends to make decisions. In this decision process, the trust relationship between experts can potentially affect the process of clustering, consensus and opinion gathering. Several studies have demonstrated that social network has a positive impact on missing value

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estimation [30] and achieving LSGDM [26]. For example, based on trust relationships among experts, Tian et al. [30] constructed a multi-objective programming model to estimate incomplete interval type-2 fuzzy information. Considering social trust behavior, Wu et al. [38] proposed a two-stage trust network partition algorithm to reduce the complexity of the LSGDM problem. In addition, many studies have developed various methods based on trust relations in LSGDM, such as trust-based consensus [19], trust-based conflict detection and elimination [23] and non-cooperative behavior management based on trust relations [42].

To eliminate conflicts between experts, a consensus reaching process (CRP) is often used, which aims to achieve a collective solution as close to a unanimous agreement as possible [19,30,23]. To date, several types of CRPs in group decision making (GDM) have been developed from the following five aspects: (1) CRPs with preferred representative structure [16,11,43,18], investigating various CRPs based on preference order, utility function and preference relations, (2) individual consistency in the consensus problem [39,44,21], exploring how to guarantee individual consistency in the CRPs, (3) dynamic consensus realization process [1,7,20], seeking solutions for cases where the decision elements are dynamically changed, (4) CRPs in social network [32,36,35,48], considering the social relationship between experts in CRPs, and (5) minimum cost consensus models [4,12,17]. It can be observed that all the five aspects have the same goal: to eliminate conflicts among experts and to improve the quality of decision result.

In LSGDM, however, experts come from different fields and represent different interests. This causes the LSGDM consensus to take longer and be costlier than GDM. Therefore, LSGDM consensus requires a capable moderator to devote time and resources to convincing experts to change their opinions into an acceptable level. To model this kind of consensus, the minimum cost consensus model is selected from the five models introduced. The minimum cost consensus model was first constructed by Ben-Arieh and Easton [4] using a distance measured method. On the basis of Ben-Arieh and Easton's work, Gong et al. [11] further developed the minimum cost consensus model with interval preference opinions. Recently, Labella et al. [17] developed an objective metric based on a comprehensive minimum cost model to evaluate the performance of CRPs.

In the above minimum cost consensus models, the unit adjustment cost of experts is usually considered to be fixed. In practice, however, it is always uncertain and reflected by interval values [20], a distribution uncertainty set [13] and a box set [14]. Robust optimization is a new method of investigating the uncertain optimization problem [31,10,28]. It originates in robust control theory and is a supplement to stochastic optimization. Different from stochastic optimization, robust optimization does not assume the distribution of uncertain parameters. The purpose of robust optimization is to obtain a solution that satisfies all constraints and optimizes the value of the objective function in the worst case. Many theories and methods have been developed based on robust optimization. For example, Bandi and Bertsimas [2] suggested that the option pricing problem was a robust optimization problem, and reduced the worst-case replication error to a minimum. Ji et al. [15] proposed a fuzzy robust weighted method for a bilevel game model. Yaníkoğlu et al. [45] introduced adjustable robust optimization. A guide on how to apply an adjustable robust optimization method is given in the work of Yanıkoğlu et al. [45].

The CRP in social network and the consensus models mentioned above are very insightful, but they still need to be improved to address the actual CRP problem: (1) The traditional consensus in social network only considers the trust and distrustful relationship between experts. Such a relationship is regarded as the input information in a CRP, without considering the internal diversity of social relationships. For LSGDM problems, if experts develop closer relationships, they are more likely to form similar opinions regarding an alternative. In contrast, if their relationships are weak, their opinions may display obvious differences. Therefore, when considering the relationship information among experts, how can an LSGDM consensus be achieved? (2) The existing minimum cost models mostly assume that the unit adjustment cost given by the moderator is definite and known beforehand. This certain value is used directly in the feedback process without considering the uncertain situation. However, it is hardly a guarantee that the unit adjustment cost is certain and known in the actual LSGDM problem. The reason for this is that because the number of experts that take part in the LSGDM is more than or equal to 20 [9], it is difficult for the moderator to master the compensation information of each expert. Once an uncertain situation is encountered, the feasibility and validity of the consensus process will be largely hampered for LSGDM problems. (3) To achieve consensus, the previous minimum cost models only focus on minimizing the total compensation cost provided by the moderator. However, when the unit adjustment cost is uncertain, each expert usually expects the unit adjustment cost paid them to be maximized. Thus, this case will lead the total compensation cost paid by the moderator to reach a maximum, which is the worst-case consensus problem. Therefore, facing this case, we should consider how to minimize the total compensation cost in the worst-case consensus problem.

Motivated by the limitations summarized above, we solve the LSGDM consensus in social network using the minimum cost model based on robust optimization. In the proposed LSGDM consensus, considering the relationship information among experts, a novel expert clustering is given to improve the efficiency of the LSGDM consensus. Considering the relationship strength between experts, a new consensus measure method to manage opinion difference between experts is proposed. According to the harmony degree between experts, a consensus index (CI) is developed to measure the consensus level of LSGDM in social network. To address the uncertain situation of the unit adjustment cost, an ellipsoidal set restricting the unit adjustment cost is proposed. Then, in the feedback adjustment process, a consensus model based on robust optimization is constructed to optimize the total compensation cost in the worst case. To the best of our knowledge, this is the first consensus of LSGDM in social network based on robust optimization.

The remainder of this paper is organized as follows. Section 2 reviews some preliminaries on the minimum cost consensus model, social network analysis and trust propagation in social network. In Section 3, the CRP of LSGDM in social network based on robust optimization is shown. In Section 4, a case study is discussed, which illustrates the application of the proposed consensus model in LSGDM in social network. Following this, a comparative analysis and discussion are presented in Section 5. Finally, conclusions are provided in Section 6.

2. Preliminaries

In this section, we first introduce the LSGDM problem. Then, the minimum cost consensus model is presented. Finally, the social network analysis is briefly described, along with some related definitions.

2.1. The LSGDM problem

Let $E = \{e_1, \ldots, e_i, \ldots, e_m\}$ be the set of m experts, and $O = \{o_1, \ldots, o_i, \ldots, o_m\}$ be the opinion set provided by experts. Usually, the group is referred to as a large-scale group when $m \geqslant 20$.

To understand LSGDM better, we describe the difference between GDM and LSGDM based on three points: the characteristics of experts, the clustering process and the consensus measure.

(1) The characteristics of experts

In GDM, only a few experts are usually considered. However, in LSGDM, the number of group members is greater than or equal to 20. In addition, experts in GDM are generally described as independent individuals, neglecting the objective trust relationships between them. However, because LSGDM involves so many experts, this premise rarely holds. Therefore, social relationships can be widely observed among experts in LSGDM.

(2) The clustering process

Because LSGDM is the more complicate process, it is difficult to solve LSGDM with traditional GDM methods. Therefore, expert clustering is often used to classify experts with similar opinions into one subgroup. This process can effectively reduce the necessary number of experts in LSGDM and improve the efficiency of consensus.

(3) The consensus measure

In GDM, the consensus measure is conducted orderly on the expert and group level. However, in order to accelerate the speed of the consensus measure, the consensus measure is usually implemented at the subgroup level in LSGDM. This step avoids massive calculations on the large-scale expert level, which further improves the efficiency of consensus in LSGDM.

2.2. The basic model

Based on CRP and by considering the aggregation process of expert opinions, Zhang et al. [46] constructed the minimum cost consensus model by employing an aggregation operator:

$$\begin{aligned} & Min\sum_{i=1}^{m} c_i|o_i' - o_i| \\ s.t.\ & |o_i' - o_c'| \leqslant \varepsilon, i = 1, 2, \ldots, m \\ & o_c' = \sum_{i=1}^{m} l_i o_i' \end{aligned} \tag{1}$$

where $E = \{e_1, \ldots e_i, \ldots, e_m\}$ is a set of experts. c_i is the unit adjustment cost of expert e_i and o_i is the initial opinion of expert e_i. Here, o_i is a real number, which belongs to [0,1]. o_i' signifies the consensus opinion after the initial opinion o_i is adjusted. $c_i|o_i' - o_i|$ represents the consumption cost function generated by changing opinion from o_i to o_i'. o_c' is the adjusted collective opinion through the aggregation of the consensus opinions $\{o_1', \ldots o_i', \ldots o_m'\}$. It can be obtained through aggregation operators, such as the ordered weighted average (OWA) operator or weighted average (WA) operator. In the aggregation process, $l = (l_1, l_2, \ldots, l_m)^T$ denotes the weight vector associated with the WA operator. It satisfies $\sum_{i=1}^{m} l_i = 1$ and $l_i \geqslant 0$.

Remark 1. In model (1), it is assumed that the unit adjustment cost c_i is certain and known in advance by the moderator. The consensus opinion o_i' is the only independent variable in the nonlinear optimization model (1). In order to solve o_i', a transformation process occurs in [46], which ensures that the solution process is more operational.

Theorem 1. [46] By setting $x_i = o_i - o_i'$ and $t_i = |x_i|$, model (1) can be transformed into the following linear programming model

$$\begin{aligned} & Min\sum_{i=1}^{m} c_i \times t_i \\ & s.t.\ o'_i - o'_c \leqslant \varepsilon \\ & o'_c - o'_i \leqslant \varepsilon \\ & o'_c = \sum_{i=1}^{m} l_i o'_i \\ & o_i - o'_i = x_i \\ & x_i \leqslant t_i \\ & -x_i \leqslant t_i \\ & t_i \geqslant 0, i = 1, 2, \ldots, m \end{aligned} \tag{2}$$

Theorem 1 indicates that the optimal consensus opinion o'_i can be derived by solving the linear programming model (2).

Remark 2. In minimum cost consensus models (1) and (2), the unit adjustment cost c_i is deemed to be certain and already known. However, due to the diversity of experts and the complexity of LSGDM, the unit adjustment cost c_i is difficult to derive, and it is usually uncertain. Therefore, in this paper, we consider the value of c_i to be unknown.

2.3. Social network analysis

Social network analysis investigates the relationships between social members, such as group members, enterprises or countries [26,8,29]. It can establish the relationship model between members in a group. As a useful method, social network analysis has been widely used in resource optimization and CRPs. For example, Wu et al. [37] developed a social network GDM approach to implementing water-energy-food nexus evaluation. Tian et al. [30] utilized a social network analysis based on consensus-supporting framework to determine the logistics of a park construction in Xinjiang. Moreover, Chu et al. [6] used a social network community analysis to select a suitable manufacturing system to reduce manufacturing costs.

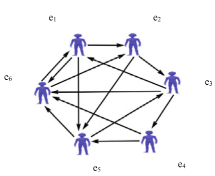
Social network analysis consists of three elements: the set of experts, the relationship between them, and the sociometric data. Detailed information is depicted in Table 1.

(1) Sociometric data: The relationship data between experts is presented in a matrix, called the sociometric matrix. In the sociometric matrix, $t_{ij} = 1$ denotes that expert e_i has a direct trust relationship with e_j. Otherwise, the value of t_{ij} is equal to 0 if expert e_i does not have a direct trust relationship with e_j.
(2) Graph: A social network is represented by a graph in which points are connected by straight lines. In the graph, $e_i \rightarrow e_j$ signifies a direct trust relationship between e_i and e_j.
(3) Algebraic representation: This representation distinguishes between several different relationships and represents a combination of relationships.

However, the above sociometric only describes the relationship between experts, it does not accurately quantify the evaluation opinion and relationship strength between experts. Therefore, we adopt trust information with relationship strength to express the relationship information between experts in social network.

Liu et al. [23] introduced the basic concept of relationship strength between experts, which is provided in Definition 1.

Table 1
Different elements of social network analysis.

Sociometric	Graph theory	Algebraic
$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$		$e_1Re_2, e_1Re_5, e_1Re_6$ e_2Re_3, e_2Re_5 $e_3Re_1, e_3Re_4, e_3Re_6$ e_4Re_5, e_4Re_6 e_5Re_3, e_5Re_6 e_6Re_1, e_6Re_2

Definition 1. [23] Trust information with relationship strength is defined in the form of a 2-tuple, and it is expressed as

$$ts_{ij} = (t_{ij}, s_{ij}) \tag{3}$$

where the first component t_{ij} denotes the trust degree from experts e_i to e_j, satisfying the condition $0 \leqslant t_{ij} \leq 1$. If t_{ij} is equal to 0, it implies that expert e_i fully distrusts e_j. In contrast, expert e_i will absolutely trust e_j if the value of t_{ij} is 1. The second component s_{ij} is the relationship strength between experts e_i and e_j. Relationship strength represents the contact frequency between experts. If the value of s_{ij} is high, the relationship between experts e_i and e_j is closer. For convenience, $TS = (ts_{ij})_{m \times m}$ is referred to as a trust matrix with relationship strength.

2.4. Trust propagation in social network

In social network, information is transitive. Thus, trust can be spread by one or more mediators. Trust information may be distorted when information is propagated through multiple mediators. So, in this paper, we only consider the case in which trust is transmitted by one mediator. Taking Fig. 1 as an example, there is no direct relationship between e_i and e_j, but trust propagation can be achieved through mediator e_f. For example, through a trust propagation path $e_i \rightarrow e_f \rightarrow e_j$, the trust evaluation from e_i to e_j can be reached.

To achieve trust propagation in social network, we first present the concept of uninorms. Then the uninorm propagation operator and other definitions are explained.

Definition 2. [34] A uninorm U is a mapping U: $[0,1]^2 \rightarrow [0,1]$, that is defined as

$$U(x,y) = \begin{cases} 0 & (x,y) \in \{(0,1),(1,0)\} \\ \frac{xy}{xy+(1-x)(1-y)} & otherwise \end{cases} \tag{4}$$

Definition 3. [36] Assume that U is a uninorm trust propagation operator defined as a mapping. Corresponding two 2-tuples (t_{if}, s_{if}) and (t_{fj}, s_{fj}), we have

$$U(t_{if}, t_{fj}) = t_{ij}^U = \begin{cases} 0 & (t_{if}, t_{fj}) \in \{(0,0)\} \\ \frac{t_{if}t_{fj}}{t_{if}t_{fj}+(1-t_{if})(1-t_{fj})} & otherwise \end{cases} \tag{5}$$

In the process of trust propagation, an expert is more likely to share information with an expert with whom he shares a close relationship. In other words, the relationship strength s_{ij} as a key factor affects the trust propagation efficiency. According to Liu et al. [23], trust propagation efficiency can be deemed a function of relationship strength.

Definition 4. [23] Assume that the relationship information of expert e_i toward expert e_j is $ts_{ij} = (t_{ij}, s_{ij})$, then the propagation efficiency from e_i to e_j is

$$p(s_{ij}) = 1 - cos\frac{\pi s_{ij}}{2} \tag{6}$$

where $p(s_{ij})$ belongs to the interval $[0,1]$. The greater the value of s_{ij}, the greater the value of $p(s_{ij})$. When the contact frequency between e_i and e_j is 0, there is no trust propagation between them. However, trust propagation efficiency between experts e_i and e_j will be greatest if their relationship strength is 1.

Based on trust propagation and propagation efficiency, according to [23], the trust value of experts e_i to e_j in Fig. 1 is calculated as

$$t_{ij} = p_{if}p_{fj}t_{ij}^U = \left(1 - cos\frac{\pi s_{if}}{2}\right)\left(1 - cos\frac{\pi s_{fj}}{2}\right)\frac{t_{if}t_{fj}}{t_{if}t_{fj} + (1-t_{if})(1-t_{fj})}$$

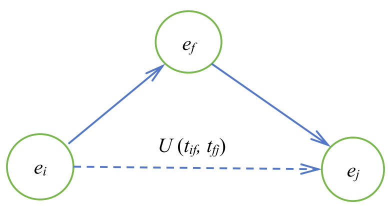


Fig. 1. Trust propagation with one mediator.

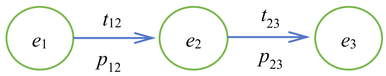


Fig. 2. Single path trust propagation.

Definition 5. [23] When there are multiple trust propagation paths, the trust propagation value from e_i to e_j is defined as

$$t_{ij} = \sum_{p=1}^{\pi} p_{ij}^{path_p} t_{ij}^{path_p} \tag{7}$$

where $p_{ij}^{path_p}$ is the overall propagation efficiency for path p, and $t_{ij}^{path_p}$ represents the fully propagated trust value of e_i to e_j.

Example 1. In Fig. 3, two paths $e_1 \to e_2 \to e_3$ and $e_1 \to e_5 \to e_3$ can simultaneously propagate trust information from e_1 to e_3. In this situation, the trust value from e_1 to e_3 is as follows:

$$\begin{aligned} t_{13} &= p_{12}p_{23}t_{13}^U + p_{15}p_{53}t_{15}^U \\ &= (1 - cos\tfrac{\pi s_{12}}{2})(1 - cos\tfrac{\pi s_{23}}{2})\tfrac{t_{12}t_{23}}{t_{12}t_{23}+(1-t_{12})(1-t_{23})} \\ &\quad +(1 - cos\tfrac{\pi s_{15}}{2})(1 - cos\tfrac{\pi s_{53}}{2})\tfrac{t_{15}t_{53}}{t_{15}t_{53}+(1-t_{15})(1-t_{53})} \\ &= 0.056 \end{aligned}$$

3. Consensus of LSGDM in social network with robust optimization

In this section, we present the proposed consensus problem with a strength relationship and uncertain unit adjustment cost. In Section 3.1, a detailed framework regarding the CRP of LSGDM with an uncertain cost is proposed. In Section 3.2, a new K-means algorithm is presented to classify experts into different subgroups. A consensus measure considering the harmony degree is given in Section 3.3. Finally, a minimum cost consensus model based on robust optimization is provided in Section 3.4.

3.1. Framework of the LSGDM consensus with an uncertain unit adjustment cost

To discuss the consensus decision problem of LSGDM in the social network environment, this paper uses the trust evaluation value and relationship strength to represent an expert's evaluation of other experts. Subsequently, a minimum cost model based on robust optimization is proposed to deal with the uncertain unit adjustment cost. The framework of the LSGDM consensus with an uncertain unit adjustment cost is shown in Fig. 4, and the detailed procedures are as follows: (1) Expert clustering: Implement Algorithm 1 to classify the experts into several subgroups based on the comprehensive evaluation among them. (2) Consensus measure: Use Eqs. (14) and (15) to calculate the consensus index of the subgroup (CIC) and the CI of the collective, respectively. (3) Feedback adjustment process: Use the minimum cost model based on robust optimization to achieve LSGDM consensus in social network.

For simplicity, the notation for this paper is as follows:

$E = \{e_1, \ldots, e_i, \ldots, e_m\}$: The set of experts.

τ: The number of subgroups.

cl_k: The center of subgroup $k, 1 \leqslant k \leqslant \tau$.

ω_k: The weight of subgroup cl_k.

n_k: The number of experts in subgroup cl_k.

d_i^k: The distance difference between expert e_i and subgroup center cl_k.

$DC = (d_i^k)_{m\times 1}$: The distance difference matrix.

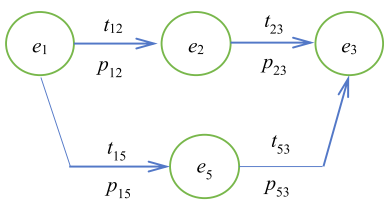


Fig. 3. Multiple paths trust propagation.

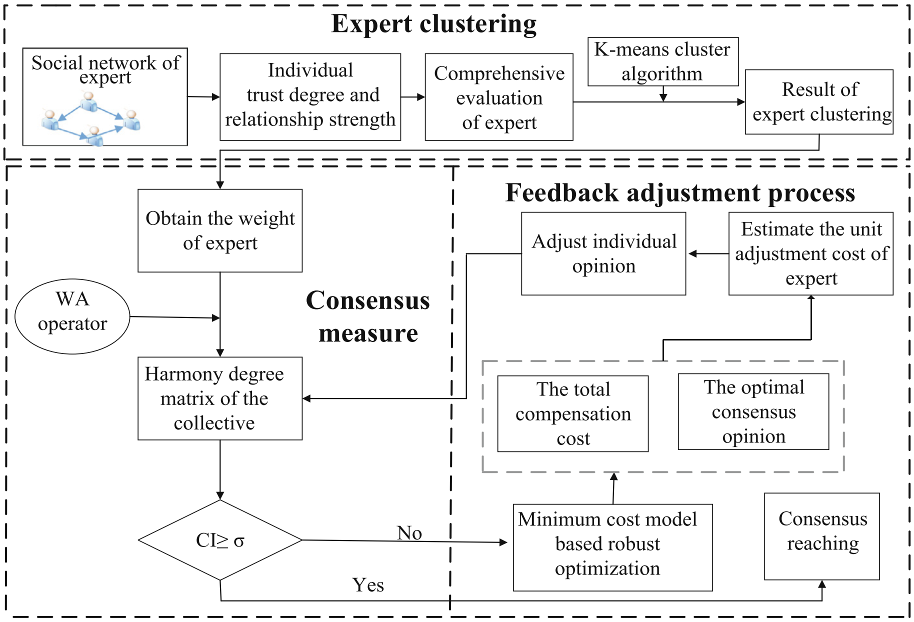


Fig. 4. The framework of consensus for LSGDM.

3.2. Expert clustering

Considering the sociality of experts, expert clustering is to classify experts with similar opinions into one subgroup, which can improve the efficiency of consensus in LSGDM when the unit adjustment of an expert is unknown. There are many different clustering methods, including fuzzy clustering methods [3] and the K-means algorithm [40]. In traditional clustering methods, such as fuzzy clustering methods [3], each expert may be assigned to many subgroups simultaneously. This does not effectively provide a convincing clustering result for LSGDM, which leaves the clustering process meaningless. However, the K-means algorithm can overcome the defect of some fuzzy clustering methods, which makes the clustering more reasonable as every expert is assigned to one and only one subgroup. Moreover, the K-means clustering method has enough robustness to determine a clustering result [40]. It has also demonstrated high efficiency in dealing with LSGDM. Motivated by these advantages, the K-means algorithm is used to implement expert clustering in this paper.

The K-means algorithm is based on the distance difference between expert and subgroup center to conduct clustering: the smaller the opinion distance between expert and subgroup center, the greater their similarity, and the more likely the expert belongs to the subgroup. Here are the basic principles of the K-means algorithm: (1) The subgroup center cl_k is initialized, then the distance value d_i^k calculated. (2) The distance between each e_i and each cl_k is compared in turn, and the e_i is assigned to the subgroup for which d_i^k has the minimum value.

Relationship strength, which indicates the intimacy or contact frequency between experts in social network, is one of the most important factors affecting human relationships. If the relationship between e_i and e_j is strong, e_i will have a greater comprehensive evaluation toward e_j. However, if the relationship between them is weak, e_i may have a lower comprehensive evaluation of e_j. Thus, relationship strength can reflect the intimacy between experts and can impact the final comprehensive evaluation among experts. Therefore, in order to consider relationship strength in the clustering process effectively, we propose an expert clustering method that combines trust degree and relationship strength.

Definition 6. Let $ts_{ij} = (t_{ij}, s_{ij})$ be the relationship information given by e_i to e_j, then the comprehensive evaluation from e_i toward e_j is defined as:

$$ce_{ij} = t_{ij} \times s_{ij} \tag{8}$$

Assume a given expert set $E = \{e_1, \ldots, e_i, \ldots, e_m\}$, every expert e_i has his comprehensive personal evaluation of other experts. Thus, the set $CE_i = (ce_{i1}, ce_{i2}, \ldots ce_{im})$ represents e_i's comprehensive evaluation of other experts in his social network. The matrix $CE = (ce_{ij})_{m\times m}$ denotes the comprehensive evaluation of all experts in the social network. When the K-means algorithm is used, initial subgroup centers are randomly generated from the matrix CE. Generally, we select one of the CE_i as the subgroup center. The similarity degree between e_i and subgroup center cl_k is expressed by Euclidean distance. It is shown as Definition 7.

Definition 7. The Euclidean distance between expert e_i and subgroup center cl_k on trust degree and relationship strength is defined as

$$d_i^k = \sqrt{\sum_{j=1, j\neq i}^{m} (ce_{ij} - ce_{kj})^2} \quad (9)$$

where ce_{ij} denotes expert e_i's comprehensive evaluation of expert $e_j, 1 \leqslant j \leqslant m, j \neq i$, and ce_{kj} denotes the comprehensive evaluation of expert e_j by subgroup center cl_k.

Remark 3. For Definition 7, the smaller the value d_i^k, the larger the similarity degree between expert e_i and subgroup cl_k. When the similarity degree is larger, expert e_i is more likely to be classified into subgroup cl_k.

Example 2. For $TS = (t_{ij}, s_{ij})_{4\times4}$, it is shown as follows. We suppose the number of τ is equal to 2.

$$TS = \begin{bmatrix} (1,0) & (0.6,0.7) & (0.4,0.9) & (0.6,0.8) \\ (0.8,0.5) & (1,0) & (0.4,0.7) & (0.5,0.6) \\ (0.7,0.4) & (0.3,0.8) & (1,0) & (0.6,0.4) \\ (0.9,0.8) & (0.4,0.5) & (0.9,0.7) & (1,0) \end{bmatrix}$$

(1) According to Definition 6, the comprehensive evaluation matrix is calculated as:

$$CE = \begin{bmatrix} 0 & 0.42 & 0.36 & 0.48 \\ 0.4 & 0 & 0.28 & 0.3 \\ 0.28 & 0.24 & 0 & 0.24 \\ 0.72 & 0.2 & 0.63 & 0 \end{bmatrix}$$

(2) Then, the initial subgroup center $CE_1 = cl_1 = (0, 0.42, 0.36, 0.48)$ and $CE_4 = cl_4 = (0.72, 0.2, 0.63, 0)$ are selected randomly.

(3) Finally, for expert e_2, the Euclidean distances of d_2^1 and d_2^4 are calculated, respectively, giving us

$$d_2^1 = \sqrt{(0-0.4)^2 + (0.42-0)^2 + (0.36-0.28)^2 + (0.48-0.3)^2} = 0.613$$

$$d_2^4 = \sqrt{(0.72-0.4)^2 + (0.2-0)^2 + (0.63-0.28)^2 + (0-0.3)^2} = 0.596$$

Because $d_2^1 > d_2^4$, expert e_2 is classified into subgroup 4.

For expert e_3, similarly, the Euclidean distances of d_3^1 and d_3^4 are also calculated, from which we obtain $d_3^1 = 0.546$ and $d_3^4 = 0.806$. Thus expert e_3 is classified into subgroup 1 because the value d_3^1 is lower than d_3^4.

(4) Finally, we obtain the clustering result. The first subgroup is $\{e_1, e_3\}$ and the second is $\{e_2, e_4\}$

The detailed expert clustering process for LSGDM in social network is expressed in Algorithm 1.

Algorithm 1 The expert clustering process for LSGDM in social network.

There are m experts involved in the LSGDM problem. Each expert has certain social relationship with other experts, and he gives relationship information about the other experts $ts_{ij} = (t_{ij}, s_{ij})$.

Input: The relationship information matrix $TS = (ts_{ij})_{m\times m}$, and the number of subgroup τ.

Step 1. Apply Eq. (8) to calculate the value of ce_{ij}.

Step 2. Initial subgroup center cl_k is selected from comprehensive evaluation matrix CE_{ij}.

Step 3. Utilize Eq. (9) to compute the Euclidean distance between expert e_i and subgroup center cl_k, and compare the value of d_i^k and d_j^k. If $d_i^k < d_j^k$, then expert e_i is classified into subgroup cl_k.

Step 4. The sample mean of each subgroup is used as the new subgroup centering.

Step 5. Repeat Step 2 and Step 3 until the subgroup center no longer changes.

Output: The final clustering result.

Remark 4. Subgroup parameter τ can be set by leaders and experts based on the actual consensus decision.

3.3. Consensus measure

Using Algorithm 1, large-scale experts in social network can be classified into τ subgroups. To facilitate the subsequent consensus process, we assume that the subgroup with a large number of experts should be assigned large weight. The weight of subgroup k can be calculated as follows:

$$\omega_k = n_k / \sum_{k=1}^{\tau} n_k \tag{10}$$

Then, taking relationship strength into account, a consensus measure is designed to calculate the consensus degree among experts.

Let o_i^k, o_j^k be the opinion of expert e_i and expert e_j in subgroup k, respectively. o_i^k and o_j^k are real numbers. Let $SM^k = (sm_{ij}^k)_{m\times m}$ be the similarity matrix of subgroup k, where sm_{ij}^k represents the similarity degree between experts e_i and e_j in subgroup k, and sm_{ij}^k is defined as

$$sm_{ij}^k = 1 - |o_i^k - o_j^k| \tag{11}$$

In addition, in social network, the relationship between experts may increase or decrease the harmony degree between them. In other words, when expert e_i has a strong relationship with expert e_j, there is a higher harmony degree between them. However, if expert e_i has a weak relationship with expert e_j, the harmony degree between them will be very low, even though there is a high similarity degree. Thus, the harmony degree between experts e_i and e_j in subgroup k is defined as

Definition 8. Let $H^k = (h_{ij}^k)_{m\times m}$ be the harmony degree matrix of subgroup k. The harmony degree between experts e_i and e_j in subgroup k is defined as:

$$h_{ij}^k = sm_{ij}^k \times s_{ij}^k \tag{12}$$

Remark 5. The larger the value of h_{ij}^k, the higher the harmony degree between e_i and e_j in subgroup k. Obviously, H^k is a symmetric matrix where the elements on the main diagonal are 0.

Definition 9. Let $HC = (hc_{ij})_{m\times m}$ be the harmony degree matrix of the collective. Using the weighted averaging(WA) operator, it can be computed as

$$HC = (hc_{ij})_{m\times m} = \sum_{k=1}^{\tau} \omega_k \times (h_{ij}^k)_{m\times m} \tag{13}$$

When the number of experts in subgroup k is $m-n$, but the number of experts in another subgroup is n (e.g., $m-n<n$), the two harmony degree matrices may have different dimensions. This situation could result in the weighted process in Definition 9 not being implemented. Therefore, for the sake of calculation, we extend the dimension of the matrix $(m-n)\times(m-n)$ to $n\times n$. The added virtual number is equal to 0.

Example 3. The three harmony degree matrices, $(h_{ij}^1)_{3\times 3}, (h_{ij}^2)_{4\times 4}$ and $(h_{ij}^3)_{5\times 5}$, are as follows:

$$(h_{ij}^1)_{3\times 3} = \begin{bmatrix} 0 & 0.1 & 0.3 \\ 0.1 & 0 & 0.8 \\ 0.3 & 0.8 & 0 \end{bmatrix}$$

$$(h_{ij}^2)_{4\times 4} = \begin{bmatrix} 0 & 0.2 & 0.6 & 0.4 \\ 0.2 & 0 & 0.8 & 0.3 \\ 0.6 & 0.8 & 0 & 0.5 \\ 0.4 & 0.3 & 0.5 & 0 \end{bmatrix}$$

$$(h_{ij}^3)_{5\times 5} = \begin{bmatrix} 0 & 0.4 & 0.7 & 0.5 & 0.8 \\ 0.4 & 0 & 0.2 & 0.5 & 0.4 \\ 0.7 & 0.2 & 0 & 0.6 & 0.4 \\ 0.5 & 0.2 & 0.6 & 0 & 0.3 \\ 0.8 & 0.4 & 0.4 & 0.3 & 0 \end{bmatrix}$$

In the aggregation process, we suppose that $\omega_1 = 0.3, \omega_2 = 0.4$ and $\omega_3 = 0.3$. Because the three matrices have different dimensions, the dimensions of the matrix $(h_{ij}^1)_{3\times 3}$ and $(h_{ij}^2)_{4\times 4}$ are extended to 5×5. Therefore, the harmony degree matrix of the collective can be calculated as

$$HC = (hc_{ij})_{5\times 5} = 0.3(h_{ij}^1)_{5\times 5} + 0.4(h_{ij}^2)_{5\times 5} + 0.3(h_{ij}^3)_{5\times 5}$$
$$= 0.3\begin{bmatrix} 0 & 0.1 & 0.3 & 0 & 0 \\ 0.1 & 0 & 0.8 & 0 & 0 \\ 0.3 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + 0.4\begin{bmatrix} 0 & 0.2 & 0.6 & 0.4 & 0 \\ 0.2 & 0 & 0.8 & 0.3 & 0 \\ 0.6 & 0.8 & 0 & 0.5 & 0 \\ 0.4 & 0.3 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$+0.3\begin{bmatrix} 0 & 0.4 & 0.7 & 0.5 & 0.8 \\ 0.4 & 0 & 0.2 & 0.5 & 0.4 \\ 0.7 & 0.2 & 0 & 0.6 & 0.4 \\ 0.5 & 0.2 & 0.6 & 0 & 0.3 \\ 0.8 & 0.4 & 0.4 & 0.3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0.23 & 0.46 & 0.39 & 0.24 \\ 0.23 & 0 & 0.62 & 0.27 & 0.12 \\ 0.54 & 0.62 & 0 & 0.38 & 0.12 \\ 0.31 & 0.18 & 0.38 & 0 & 0.09 \\ 0.24 & 0.12 & 0.12 & 0.09 & 0 \end{bmatrix}$$

Definition 10. The CIC of subgroup k is defined as:

$$CIC^k = \frac{1}{n(n-1)} \sum_{i=1}^{m} \sum_{j>i}^{m} |h_{ij}^k - hc_{ij}| \tag{14}$$

Similarly, the CI of the collective can be calculated as:

$$CI = \sum_{k=1}^{\tau} \omega_k \times CIC^k \tag{15}$$

It is obvious that $CI \in [0, 1]$. Particularly, because $CI = 1$, the large-scale experts in the social network have achieved the full consensus. A higher CI value indicates a higher consensus among experts. In this consensus process, a consensus threshold $\sigma \in [0, 1]$ is predefined to measure whether or not consensus is achieved among experts in social network. Generally, if $CI > \sigma$, an acceptable consensus is achieved among large-scale experts in social network. Otherwise, a feedback adjustment process is conducted until the CI is acceptable.

3.4. The minimum cost model based on robust optimization

In this section, we propose a feedback adjustment process that consists of three important stages: (1) construction of a model of robust optimization consensus, (2) determination of the robust counterpart of the robust optimization consensus, and (3) estimation of the unit adjustment cost of each expert.

3.4.1. The robust optimization consensus problem

When the CI among experts is unacceptable, the opinion of each expert needs to be further adjusted according to the minimum cost model. In the exiting minimum cost consensus models, it is assumed that the precise unit adjustment cost is known. In reality, however, the unit adjustment cost is normally difficult to obtain and is uncertain. When the unit adjustment cost is uncertain, experts always want to get the highest unit adjustment cost from the moderator. So, the total compensation cost paid by the moderator may be very high, which is the worst-case consensus problem. In order to deal with this problem, we propose to minimize the total compensation cost in the worst case, which is the consensus problem based on robust optimization.

In the robust optimization consensus problem, a method of dealing with this uncertain cost is proposed. Assuming that we know the nominal value of an uncertain unit cost, it is denoted as c_i^*. Rather than assuming that c_i is in an interval [20], we suppose that c_i belongs to an ellipsoidal set. Every point of the ellipsoid is a possible value of unit cost c_i. The ellipsoidal set is shown as:

$$\Gamma = \{c_i | |c_i - c_i^*| \leqslant \delta_i,\ i = 1, 2, \ldots, m\}$$

where δ_i is a parameter that represents the deviation between the uncertain unit adjustment cost and the nominal value.

In the minimum cost model (2), the unit adjustment cost c_i is certain. We mainly focus on minimizing the total compensation cost. However, in the robust optimization consensus problem, we minimize the total compensation cost based on robust optimization. Hence, the model of the robust optimization consensus problem is defined as

$$Minmax\sum_{i=1}^{m}c_i \times t_i \tag{16}$$

$$s.t.\ o'_i - o'_c \leqslant \varepsilon \tag{16-1}$$
$$o'_c - o'_i \leqslant \varepsilon \tag{16-2}$$
$$o'_c = \sum_{i=1}^{m} l_i o'_i \tag{16-3}$$
$$o_i - o'_i = x_i \tag{16-4}$$
$$x_i \leqslant t_i \tag{16-5}$$
$$-x_i \leqslant t_i \tag{16-6}$$
$$|c_i - c_i^*| \leqslant \delta_i \tag{16-7}$$
$$t_i \geqslant 0, i = 1, 2, \ldots, m \tag{16-8}$$

where the objective function represents the minimization of the total compensation cost of the robust optimization consensus. Constraints (16–1) and (16–2) that restrict the deviation between consensus opinion o'_i and adjusted collective opinion o'_c are not larger than the predetermined parameter ε. Constraint (16–3) denotes that the collective opinions of experts are aggregated using the WA operator.

Compared with the minimum cost model (2), model (16) is a robust optimization model. The *Minmax* means that the total compensation cost is minimized in the worst case. However, in model (2), the *Min* only intends to minimize the total compensation cost of the moderator, without optimizing the cost in the worst case. In addition, t_i is the only uncertain parameter in the linear model (2). In model (16), both c_i and t_i are uncertain. Therefore, model (16) is a robust optimization model that focuses on minimizing the total compensation cost in the worst case when the unit adjustment cost c_i is uncertain.

It can be observed that the proposed model is more complex than the minimum cost consensus models (2). Firstly, we consider the uncertainty of the unit adjustment cost in the proposed model. The unit adjustment cost is assumed in an ellipsoidal set. Moreover, our model is based on robust optimization where, rather than optimizing the minimum total cost, one optimizes the worst-case total cost with respect to an ellipsoidal set. Finally, unlike the linear model (2), the robust optimization model (16) is a difficult problem to solve.

3.4.2. The robust counterpart of the robust optimization consensus problem

After building the model of the robust optimization consensus problem, we next discuss how to solve this model when both unit adjustment cost and consensus opinion are unknown. To deal with this problem, a robust counterpart is proposed to determine the total compensation cost and the optimal consensus opinion of every expert.

Theorem 2. The robust counterpart of the robust optimization consensus model (16) can be written as

$$Minmax\sum_{i=1}^{m}c_i^* \times t_i + \sqrt{\sum_{i=1}^{m}\delta_i^2 t_i^2} \tag{17}$$

$$s.t.\ o'_i - o'_c \leqslant \varepsilon \tag{17-1}$$
$$o'_c - o'_i \leqslant \varepsilon \tag{17-2}$$
$$o'_c = \sum_{i=1}^{m} l_i o'_i \tag{17-3}$$
$$o_i - o'_i = x_i \tag{17-4}$$
$$x_i \leqslant t_i \tag{17-5}$$
$$-x_i \leqslant t_i \tag{17-6}$$
$$|c_i - c_i^*| \leqslant \delta_i \tag{17-7}$$
$$t_i \geqslant 0, i = 1, 2, \ldots, m \tag{17-8}$$

Proof. Due to the unit adjustment cost, c_i belongs to ellipsoidal set Γ, that is $c_i = c_i^* \pm \delta_i$. The objective function of the robust optimization consensus model consists of two parts: a certain part and an uncertain part. Thus, the corresponding objective function $Minmax\sum_{i=1}^{m}c_i \times t_i$ can be expressed as

$$Minmax\sum_{i=1}^{m}c_i^* \times t_i + \xi,\ \ \xi = \sum_{i=1}^{m}t_i|c_i - c_i^*| \tag{18}$$

where the variance of random part ξ is

$$Var(\xi) = \sum_{i=1}^{m}(t_i)^2(c_i - c_i^*)^2 \leqslant Var(t) = \sum_{i=1}^{m}(t_i)^2\delta_i^2 \tag{19}$$

Consequently, the robust optimization value of $Minmax\sum_{i=1}^{m}c_it_i$ will differ from the "nominal" value $Min\sum_{i=1}^{m}c_i^*t_i$. The idea of dealing with uncertainty is as follows. In Eq. (18), the greater the variance, the larger the deviation degree between the uncertain unit cost and the nominal value. It is the worst-case uncertainty. Hence, we take the worst situation where the value of ξ is equal to $Var^{1/2}(t)$ in Eq. (18). With this approach, the objective function of the robust optimization consensus problem is $\sum_{i=1}^{m}c_i^*t_i + \sqrt{\sum_{i=1}^{m}\delta_i^2t_i^2}$. Therefore, the robust counterpart of the robust optimization consensus is shown as model (17).

The robust counterpart of the robust optimization consensus can then be solved using MATLAB. Solving model (17), the optimal consensus opinion o_i' and total compensation B can be obtained. The moderator provides the cost B_i to expert e_i in the hope that the expert will change his opinion from o_i to o_i'. When expert e_i offers his new opinion, the adjustment suggestion belongs to the following set:

$$o_i' = \{o_i - t_i, o_i + t_i\}$$

In the following, we discuss how to estimate the unit adjustment cost of expert e_i.

3.4.3. Estimation of the unit adjustment cost of an expert

Let B_χ be the compensation cost provided by the moderator at iteration χ $(1 \leqslant \chi \leqslant \vartheta)$, and let $c_{i,\chi}$ be the unit adjustment cost of expert e_i at iteration χ. Let $o_{i,\chi}$ be the opinion of expert e_i at iteration χ, and let $o_{i,\chi}'$ be the consensus opinion of expert e_i at iteration χ. Clearly, $o_{i,\chi+1} = o_{i,\chi}'$. In the first iteration($\chi = 1$), we have three principles for estimating the unit adjustment cost of the expert: (1) The uncertain unit adjustment cost of experts belongs to an ellipsoidal set, it is constrained by $\Gamma = \{c_i||c_i - c_i^*| \leqslant \delta_i,\ i = 1, 2, \ldots, m\}$. (2) An expert who changes his opinion a lot should be given a higher unit adjustment cost. (3) The sum of the unit adjustment cost at iteration χ should not exceed the total compensation cost paid by the moderator, that is $\sum_{\chi=1}^{\vartheta}B_\chi \leqslant B$.

Based on the three principles, the unit adjustment cost of expert e_i at iteration χ is

$$c_{i,\chi} = B_\chi \times \frac{|o_{i,\chi}' - o_{i,\chi}|}{\sum_{i=1}^{m}|o_{i,\chi}' - o_{i,\chi}|} \tag{20}$$

Considering the first principle, the unit adjustment cost is limited to an ellipsoidal set, it has been bound. Therefore, we have the following result

$$c_{i,\chi} = \begin{cases} B_\chi \times \frac{|o_{i,\chi}' - o_{i,\chi}|}{\sum_{i=1}^{m}|o_{i,\chi}' - o_{i,\chi}|} & B_\chi \frac{|o_{i,\chi}' - o_{i,\chi}|}{\sum_{i=1}^{m}|o_{i,\chi}' - o_{i,\chi}|} < c_{i,\chi}^* + \delta_{i,\chi} \\ c_{i,\chi}^* \pm \delta_{i,\chi} & otherwise \end{cases} \tag{21}$$

In addition, the detailed CRP of LSGDM in social network is shown in Algorithm 2.

Algorithm 2 Detailed CRP of LSGDM in social network.

Input: The individual trust information $ts = (t_{ij}, s_{ij})$, the opinion o_i of expert e_i, and the parameters $\tau, \varepsilon, \delta_i, \varphi_i, \phi_i$.
Output: The iteration number χ, consensus index of subgroup CIC_χ^k, consensus index CI_χ, total compensation cost B_χ, and consensus opinion o_i'.
Step 1. Using Eq. (7) to calculate the missing value of *TS* matrix, then we obtain the complete *TS* matrix.
Step 2. Using Algorithm 1 to classify experts who have similar comprehensive evaluation into one subgroup. Then, large-scale experts can be classified into τ subgroups.
Step 3. Set $\chi = 0$, utilizing Eq. (14) to calculate the consensus index of subgroup CIC_χ^k.
Step 4. Utilize Eq. (15) to compute the consensus index CI_χ. If $CI_\chi \geqslant \sigma$, go to step 6. Otherwise, go to the next step.
Step 5. Select the subgroup with lowest CIC_χ^k to adjust. Using model (17) to determine the optimal consensus opinion $o_{i,\chi}'$ and total compensation cost B_χ, then estimate the unit adjustment cost $c_{i,\chi}$. In the next iteration, let $o_{i,\chi+1} = o_{i,\chi}', \chi = \chi + 1$ and $B = B_{\chi-1} + B_\chi$, then return to Step 2.
Step 6. When the acceptable consensus index is reached, output the consensus index of subgroup CIC_χ^k, consensus index CI_χ, optimal consensus opinion $o_{i,\chi}$ and total compensation cost B_χ.

4. Case study

In January 2020, COVID-19 broke out in Wuhan, China. The virus is highly infectious, and the number of infected people in China is soaring. According to data from dxy.cn (https://ncov.dxy.cn), in China, there were 74,281 infected people as of February 19, 2020, and 2,009 deaths. In Hubei province, the number of infections reached 61,682. The bar chart of the infected in China apart from Hubei province is depicted in Fig. 5.

The Chinese Center for Disease Control and Prevention confirmed that the Huanan seafood market was one of the important transmission sources. To shut off the source of infection, the Huanan seafood market was advised to close. The closing strategy plays an important role in decreasing the number of infected people. This type of strategy mainly involves compensation negotiations between the market manager and tenants. Generally, the market manager would take effective measures to encourage and persuade tenants to close. If the compensation obtained by tenants meets their expectations, and the expenses paid by market manager in keeping with their expectations.

In a compensation negotiation, the market manager is the moderator while tenants play the role of DMs. Naturally, the market manager needs to negotiate with the tenants about the compensation price. In the negotiation process, the market manager pays a unit negotiation cost to the tenants so as to change their initial compensation prices. However, due to the COVID-19 outbreak, the market manager lacks experience in determining the unit negotiation cost. The unit negotiation cost is difficult to obtain and is unknown. In this situation, each tenant seeks to receive the highest unit negotiation cost from the market manager. So, the total negotiation cost paid the market manager may be very high, which is the worst-case consensus problem. Therefore, we intend to minimize the total negotiation cost in the worst case, which is the consensus problem based on robust optimization.

As Liu et al. [22] mentioned, the number of experts in the LSGDM problem is generally considered to be greater than or equal to 20. Therefore, 20 tenants with social relationships were selected as experts. The 20 tenants' initial compensation prices are: $\{o_1, o_2, o_3 \ldots, o_{19}, o_{20}\} = \{0.4, 0.4, 0.9, \ldots, 0.8, 0.5\}$ (Unit: thousand RMB). The social information of the 20 tenants is depicted in Table 2. Then, some relevant parameters and assumptions are set as follows Table 3. (1) It is assumed that the propagation path greater than or equal to two mediators is invalid. This is because the relationship strength value ranges from 0 to 1. The more mediators involved, the lower the propagation efficiency. Therefore, in this case, there should be only one mediator involved in trust propagation. (2) The acceptable consensus threshold σ is 0.88, and the maximum iteration number χ^* is set to 7.

In the robust optimization consensus problem, 20 tenants in social network are classified into four subgroups. The detailed expert clustering result is shown in Table 4. From Table 4, we can see that tenant e_{11} is classified into a separate group. He does not have any neighbors who share similar opinions with him. Hence, we consider tenant e_{11} to be a non-cooperative expert. He was not invited to participate in this compensation negotiation.

By aggregating the harmony degree of cl_1, cl_3 and cl_4, we can get the harmony degree of the collective HC_0. It is

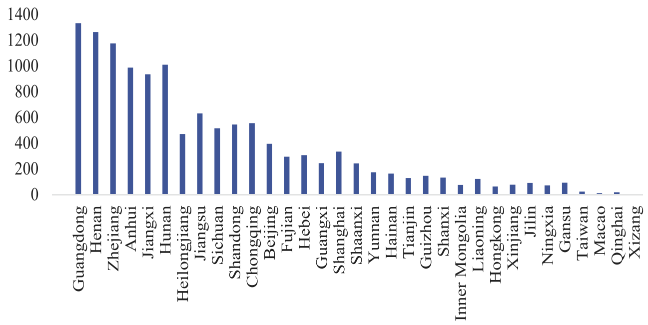


Fig. 5. The number of infections apart from Hubei province.

Table 2
The initial assessment information of social network about 20 experts.

Experts	e_1	e_2	e_3	...	e_{15}	...	e_{19}	e_{20}
e_1	(1,0)	(–,0)	(0.6,0.5)	...	(0.4,0.6)	..	(0.7,0.7)	(0.4,0.6)
e_2	(0.5,0.6)	(1,0)	(0.3,0.5)	...	(–,0)	..	(0.6,0.4)	(0.8,0.7)
e_3	(0.8,0.7)	(0.6,0.7)	(1,0)	...	(0.7,0.4)	..	(0.4,0.8)	(0.5,0.8)
e_4	(0.4,0.6)	(0.5,0.6)	(0.7,0.6)	...	(0.5,0.6)	..	(–,0)	(0.7,0.6)
e_5	(0.5,0.5)	(0.3,0.5)	(0.3,0.5	...	(0.3,0.5)	..	(0.7,0.6)	(–,0)
e_6	(0.2,0.6)	(–,0)	(0.6,0.7)	...	(–,0)	..	(0.2,0.5)	(0.4,0.4)
...								
e_{19}	(0.5,0.6)	(0.7,0.8)	(0.8,0.7)	...	(0.5,0.7)	..	(1,0)	(0.6,0.7)
e_{20}	(0.8,0.9)	(–,0)	(0.6,0.8)	...	(0.4,0.6)	..	(–,0)	(1,0)

Table 3
The complete assessment information of social network about 20 experts.

Experts	e_1	e_2	e_3	...	e_{15}	...	e_{19}	e_{20}
e_1	(1,0)	(0.15,0)	(0.6,0.5)	...	(0.4,0.6)	..	(0.7,0.7)	(0.4,0.6)
e_2	(0.5,0.6)	(1,0)	(0.3,0.5)	...	(0.12,0)	..	(0.6,0.4)	(0.8,0.7)
e_3	(0.8,0.7)	(0.6,0.7)	(1,0)	...	(0.7,0.4)	..	(0.4,0.8)	(0.5,0.8)
e_4	(0.4,0.6)	(0.5,0.6)	(0.7,0.6)	...	(0.5,0.6)	..	(0.10,0)	(0.7,0.6)
e_5	(0.5,0.5)	(0.3,0.5)	(0.3,0.5)	...	(0.3,0.5)	..	(0.7,0.6)	(0.13,0)
e_6	(0.2,0.6)	(0.14,0)	(0.6,0.7)	...	(0.12,0)	..	(0.2,0.5)	(0.4,0.4)
...								
e_{19}	(0.5,0.6)	(0.7,0.8)	(0.8,0.7)	...	(0.5,0.7)	..	(1,0)	(0.6,0.7)
e_{20}	(0.8,0.9)	(0.16,0)	(0.6,0.8)	...	(0.4,0.6)	..	(0.15,0)	(1,0)

Table 4
The experts clustering result in social network.

cl_k	e_i	ω_k	h_{ij}^k
cl_1	$\{e_1, e_3, e_{13}, e_{14}, e_{18}, e_{20}\}$	0.32	$\begin{bmatrix} 0 & 0.25 & 0.63 & 0.42 & 0.63 & 0.54 \\ 0.25 & 0 & 0.14 & 0.45 & 0.64 & 0.48 \\ 0.63 & 0.14 & 0 & 0.24 & 0.32 & 0 \\ 0.42 & 0.45 & 0.24 & 0 & 0.81 & 0.56 \\ 0.63 & 0.64 & 0.32 & 0.81 & 0 & 0.48 \\ 0.54 & 0.48 & 0 & 0.56 & 0.48 & 0 \end{bmatrix}$
cl_2	$\{e_{11}\}$	–	–
cl_3	$\{e_2, e_4, e_9, e_{16}, e_{17}, e_{19}\}$	0.32	$\begin{bmatrix} 0 & 0.42 & 0.72 & 0.35 & 0.6 & 0.24 \\ 0.48 & 0 & 0.42 & 0.6 & 0.42 & 0 \\ 0.72 & 0.42 & 0 & 0.42 & 0.63 & 0.3 \\ 0.35 & 0.6 & 0.42 & 0 & 0.28 & 0.36 \\ 0.6 & 0.42 & 0.63 & 0.28 & 0 & 0.3 \\ 0.24 & 0 & 0.6 & 0.36 & 0.3 & 0 \end{bmatrix}$
cl_4	$\{e_5, e_6, e_7, e_8, e_{10}, e_{12}, e_{15}\}$	0.36	$\begin{bmatrix} 0 & 0.15 & 0.36 & 0.54 & 0.12 & 0.48 & 0.8 \\ 0.15 & 0 & 0.35 & 0.28 & 0.45 & 0.4 & 0 \\ 0.36 & 0.35 & 0 & 0.35 & 0.3 & 0.48 & 0.48 \\ 0.54 & 0.28 & 0.35 & 0 & 0.18 & 0.54 & 0.45 \\ 0.12 & 0.45 & 0.3 & 0.18 & 0 & 0.2 & 0.08 \\ 0.48 & 0.4 & 0.48 & 0.54 & 0.2 & 0 & 0.56 \\ 0.8 & 0 & 0.48 & 0.45 & 0.08 & 0.56 & 0 \end{bmatrix}$

$$HC_0 = \begin{bmatrix} 0 & 0.268 & 0.562 & 0.441 & 0.437 & 0.422 & 0.288 \\ 0.268 & 0 & 0.305 & 0.437 & 0.501 & 0.298 & 0 \\ 0.562 & 0.305 & 0 & 0.337 & 0.412 & 0.269 & 0.173 \\ 0.441 & 0.437 & 0.337 & 0 & 0.414 & 0.489 & 0.162 \\ 0.437 & 0.501 & 0.412 & 0.414 & 0 & 0.322 & 0.029 \\ 0.422 & 0.298 & 0.269 & 0.489 & 0.322 & 0 & 0.202 \\ 0.288 & 0 & 0.173 & 0.162 & 0.029 & 0.202 & 0 \end{bmatrix}$$

The consensus index of subgroup CIC_0^k can then be calculated using Eq. (14). The values are CIC_0^1 = 0.867, CIC_0^3 = 0.865 and CIC_0^4 = 0.838.

Utilizing Eq. (15), we get CI_0 = 0.856. Due to $CI_0 < \sigma = 0.88$ and $\chi \neq \chi^*$, the feedback process is activated. That is, the opinions of tenants are adjusted by using the minimum cost model based on robust optimization until the robust optimization consensus is reached.

Since $CIC_0^4 = min\{CIC_0^k | k = 1, 3, 4\} = 0.838$, the consensus index of subgroup cl_4 is the lowest among them. Thus, the first consensus iteration should be conducted on subgroup cl_4.

In the first consensus iteration, the unit negotiation cost of each tenant is unknown, and it is deemed to be in an ellipsoidal set. In the robust counterpart model (17), the nominal values c_i^* of seven tenants in subgroup 4 are assumed to be $\{c_{5,1}^{4,*}, c_{6,1}^{4,*}, c_{7,1}^{4,*}, c_{8,1}^{4,*}, c_{10,1}^{4,*}, c_{12,1}^{4,*}, c_{15,1}^{4,*}\} = \{65, 50, 70, 80, 60, 50, 50\}$. In addition, the ellipsoidal parameters in subgroup 4 are $\{\delta_{5,1}^4, \delta_{6,1}^4, \delta_{7,1}^4, \delta_{8,1}^4, \delta_{10,1}^4, \delta_{12,1}^4, \delta_{15,1}^4\} = \{6, 8, 7, 5, 4, 6, 6\}$. Using $\{\varepsilon_{5,1}^4, \varepsilon_{6,1}^4, \varepsilon_{7,1}^4, \varepsilon_{8,1}^4, \varepsilon_{10,1}^4, \varepsilon_{12,1}^4, \varepsilon_{15,1}^4\} = \{0.1, 0.2, 0.4, 0.1, 0.1, 0.2, 0.2\}$ as the input of model (17), both the opinion deviation $t_{i,1}^4$ and total negotiation cost B_1 can be obtained. They are $\{t_{5,1}^4, t_{6,1}^4, t_{7,1}^4, t_{8,1}^4, t_{10,1}^4, t_{12,1}^4, t_{15,1}^4\} = \{0.2, 0.3, 0.1, 0.1, 0.4, 0, 0.2\}$ and $B_1 = 79.05$. After adjustment, the opinions of the tenants are $\{o_{5,1}^4, o_{6,1}^4, o_{7,1}^4, o_{8,1}^4, o_{10,1}^4, o_{12,1}^4, o_{15,1}^4\} = \{0.7, 0.5, 0.4, 0.9, 0.5, 0.7, 0.7\}$. By using the method proposed in Eq. (20), the unit negotiation costs of the tenants are $\{c_{5,1}^4, c_{6,1}^4, c_{7,1}^4, c_{8,1}^4, c_{10,1}^4, c_{12,1}^4, c_{15,1}^4\} = \{12.16, 18.24, 6.08, 6.08, 24.32, 0, 12.16\}$.

Table 5
The detailed iteration process.

χ	$c_{i,\chi}^k$	$o_{i,\chi}^k$	CI_χ	$min\{CIC_\chi^k\}$
0			0.856	$CIC_0^4 = 0.838$
1	$\{12.16, 18.24, 6.08, 6.08, 24.32, 0, 12.16\}$	{0.7,0.5,0.4,0.9,0.5,0.7,0.7}	0.863	$CIC_1^3 = 0.853$
2	$\{0, 5.60, 5.60, 5.60, 11.20, 11.20\}$	{0.4,0.6,0.2,0.8,0.6,1}	0.877	$CIC_2^4 = 0.850$
3	$\{0, 0, 6.31, 12.63, 6.31, 0, 12.63\}$	{0.7,0.5,0.3,1,0.6,0.7,0.9}	0.880	

Afterwards, we integrate the new compensation prices of subgroup 4 into the collective harmony degree. Then the $CIC_1^1 = 0.877, CIC_1^3 = 0.853, CIC_1^4 = 0.860$ can be calculated using Eq. (14). Using Eq. (15), we get $CI = 0.863 < \sigma = 0.88$. Hence, the next consensus feedback process continues.

Finally, consensus process is terminated after three iterations. The detailed iteration process is described in Table 5.

5. Comparative analysis and discussion

In this section, some comparisons with existing methods are provided to examine the superiority of the proposed method. A discussion of the comparison results is given. Finally, cost comparisons of three cases are analyzed to investigate the advantages of the robust optimization method.

5.1. Some comparisons with existing methods

In this subsection, to demonstrate the effectiveness of the proposed robust consensus problem, we compare the result with existing methods and algorithms. The comparative analysis is conducted in the context of the same case study. The opinions on alternatives and the relationship information are expressed by o_i and (t, s), respectively. Because the other information type is used in the following methods, we make small changes to each method to fit the case. The performances of six methods are compared. M1, M2, M3, M4, M5 and M6 are used to denote the six methods.

(1) M1: this method is proposed by Wu and Chiclana [32]. When a consensus is achieved in social network, the trust information between experts is considered. A trust consensus is proposed by Wu and Chiclana [32] in which a consensus is reached in an incomplete social network. However, this only considers the trust degree in social network; relationship strength is not taken into account. In the proposed robust consensus model based on trust propagation, the initial incomplete trust network can be extended to a complete trust network. Nevertheless, in the method proposed by Wu and Chiclana [32], both trust propagation and relationship strength are ignored. By applying this method to the case, a detailed consensus iteration result can be obtained (see Table 6).

(2) M2: this model, proposed by Wu et al. [33], explores social network GDM with trust propagation. It builds trust relationships between experts and assumes that trust can be propagated. However, it does not consider relationship strength in trust propagation and consensus processes. By applying Wu et al.'s [33] method to the case, we can derive a detailed consensus iteration result (see Table 7).

Table 6
The detailed iteration process of Wu and Chiclana [32] (M1).

χ	$c_{i,\chi}^k$	CI_χ	B_χ
0		0.822	
1	$\{8.91, 8.91, 20.06, 4.46, 2.23, 17.83, 15.60\}$	0.833	78.00
2	$\{0, 10.06, 0, 20.12, 0, 0, 10.06\}$	0.849	40.23
3	$\{0, 4.82, 4.82, 4.82, 4.82, 0, 0\}$	0.863	19.28
4	$\{22.19, 0, 14.79, 7.40, 7.40, 14.79\}$	0.874	66.56
5	$\{0, 11.58, 11.58, 0, 5.79, 5.79, 11.58\}$	0.880	46.31

Table 7
The detailed iteration process of Wu et al. [33] (M2).

χ	$c_{i,\chi}^k$	CI_χ	B_χ
0		0.855	
1	$\{13.66, 20.49, 0, 6.83, 0, 6.82\}$	0.862	47.80
2	$\{13.86, 9.24, 0, 0, 9.24, 4.62\}$	0.867	36.96
3	$\{10.32, 15.48, 0, 0, 10.32, 5.16\}$	0.871	41.28
4	$\{5.25, 15.75, 0, 0, 10.50, 10.50\}$	0.877	42.00
5	$\{12.30, 6.15, 0, 0, 6.15, 12.31\}$	0.885	36.91

Table 8
The detailed iteration process of Liu et al. [24] (M3).

χ	$c_{i,\chi}^{k}$	CI_{χ}	B_{χ}
0		0.863	
1	$\{11.55, 11.55, 11.55, 5.77\}$	0.867	40.41
2	$\{0, 6.27, 25.07, 0, 12.54, 18.80\}$	0.870	62.68
3	$\{4.27, 0, 8.54, 8.54, 4.27\}$	0.879	25.63
4	$\{11.80, 0, 17.80, 11.87, 5.93\}$	0.885	47.46

Table 9
The detailed iteration process of Liu et al. [25] (M4).

χ	$c_{i,\chi}^{k}$	CI_{χ}	B_{χ}
0		0.859	
1	$\{6.04, 0, 12.07, 18.11\}$	0.869	36.22
2	$\{14.08, 0, 14.08, 0, 7.05\}$	0.871	35.21
3	$\{21.72, 0, 7.24, 0, 21.72\}$	0.874	50.68
4	$\{13.63, 0, 13.63, 0, 13.64\}$	0.880	40.90

(3) M3: this grey clustering algorithm method developed by Liu et al. [24] classifies experts with similar similarity degrees into a subgroup. In this algorithm, similarity degree is based only on fuzzy preference values and self-confidence levels; it does not consider the harmony degree between experts. We can take the harmony degree into account in the grey clustering algorithm [24] and implement it in the case. The expert clustering result is: $cl_1 = \{e_1, e_{14}, e_{18}, e_{20}\}$, $cl_2 = \{e_2, e_4, e_5, e_{11}, e_{13}\}, cl_3 = \{e_6, e_7, e_{10}, e_{15}, e_{19}\}, cl_4 = \{e_3, e_8, e_9, e_{12}, e_{16}, e_{17}\}$. The detailed consensus iteration result are shown in Table 8.

(4) M4: this alternative ranking-based clustering (ARC) method proposed by Liu et al. [25], involves conducting expert clustering based on the alternative ranking of hesitant fuzzy preference relation. The similarity degree of experts can be calculated using the alternative position order. It ignores the relationship strength among experts. By incorporating relationship strength into the ARC method, we can implement it in the case. We obtain the expert clustering result: $cl_1 = \{e_1, e_2, e_3, e_4, e_9, e_{17}\}$, $cl_2 = \{e_{11}, e_{14}, e_{18}, e_{20}\}, cl_3 = \{e_5, e_6, e_7, e_{15}, e_{19}\}, cl_4 = \{e_8, e_{10}, e_{12}, e_{13}, e_{16}\}$. The detailed consensus iteration result are exhibited in Table 9.

(5) M5: this method, a product line design proposed by Bertsimas and Mišić [5], uses the nominal model and robust model to discuss the expected revenue. The worst-case revenue also is explored in this method. When the worst case occurs, the robust optimization method is used to minimize the total negotiation cost. The main difference between our method and M5 is the utilization of the worst case in the goal function. In this paper, we adjust the opinions of experts based on robust optimization without implementing a worst-case method into the process.

To carry out the worst-case method proposed by Bertsimas and Mišić [5] in this case, the goal function in model (16) is changed to

$$Minmax \quad (c_i^* + \delta_i)|o_i' - o_i|$$

where c_i^* is the nominal value of an uncertain unit cost, δ_i is the ellipsoidal parameter. Using the worst-case method to adjust the opinions of tenants, the detailed consensus iteration result are depicted in Table 10.

(6) M6: based on the minimum cost consensus model, Han et al. [13] proposed a new minimum cost consensus model with distributionally robust chance constraints. It is assumed that the unit adjustment cost is in a distribution uncertainty set. The main difference between our model and M6 is the use of distributionally robust chance-constrained optimization to deal with the minimum cost model.

By utilizing the distributionally robust chance-constrained optimization in the case, model (16) can be replaced by

Table 10
The detailed iteration process of Bertsimas and Mišić [5] (M5).

χ	$c_{i,\chi}^{k}$	CI_{χ}	B_{χ}
0		0.856	
1	$\{12.78, 19.17, 6.39, 6.39, 25.56, 0, 12.78\}$	0.869	83.06
2	$\{19.03, 0, 6.34, 6.34, 12.68, 12.68\}$	0.876	57.08
3	$\{0, 20.49, 13.66, 6.83, 0, 0\}$	0.881	40.98

$$Minmax \quad E[\sum_{i=1}^{m} c_i \times t_i] \tag{22}$$

$$\begin{aligned} s.t. \quad min \quad P\{\sum_{i=1}^{m} c_i \times t_i < B\} \geqslant 1-\alpha \\ |o'_i - o'_c| \leqslant \varepsilon \\ o'_c = \sum_{i=1}^{m} l_i o'_i \\ o_i - o'_i = x_i \\ |x_i| \leqslant t_i \end{aligned}$$

where E is the expectation and α represents the probability. B is the fixed budget limitation predefined by the moderator. By solving model (22) to adjust the opinions of the tenants, the detailed consensus iteration result are shown in Table 11.

5.2. Discussion

To verify the effectiveness and necessity of the proposed method further, we obtained the consensus iteration and cost comparison for different methods. Based on the detailed results shown in Tables 6–8, comparison results are shown in Figs. 6–7.

In Fig. 6, it is clear that the consensus-reaching speed for the proposed method is dramatically faster than methods (1)-(6). Moreover, the proposed method needs only three iterations to achieve consensus, while the number of iterations for methods (1)-(4) are always greater than three. This indicates that in real LSGDM, considering trust propagation and relationship information is more favorable for achieving consensus. That is to say, using the social network of experts to reach consensus is suitable for real LSGDM cases.

In addition, Fig. 6 also reflects how, compared with the grey clustering algorithm [24] and ARC method [25], the new K-means clustering algorithm is more efficient at classifying experts and improves the consensus negotiation process. Regarding methods (5) and (6), although the number of iterations is the same as in the proposed method, the moderator will pay

Table 11
The detailed iteration process of Han et al. [13] (M6).

χ	$c^k_{i,\chi}$	CI_χ	B_χ
0		0.856	
1	{12.82, 12.82, 12.82, 6.44, 12.82, 12.82, 12.82}	0.864	83.36
2	{6.14, 6.14, 6.14, 6.14, 6.14, 6.15, 6.15}	0.874	43.00
3	{11.25, 11.25, 11.25, 11.25, 11.25, 11.25}	0.884	67.50

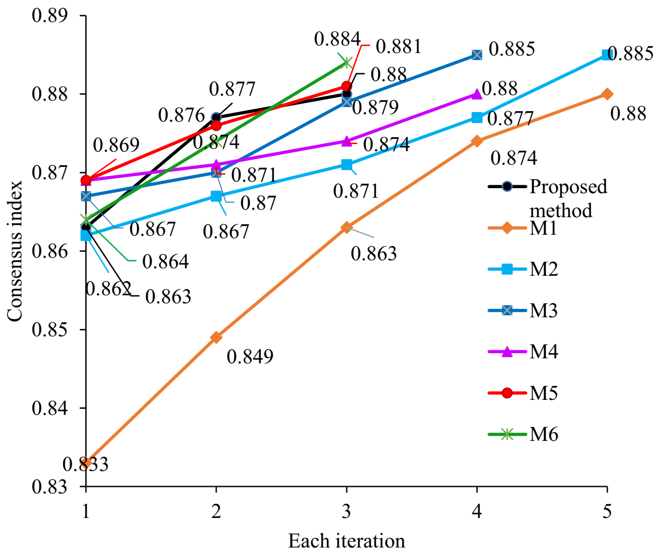


Fig. 6. The consensus iteration using different methods.

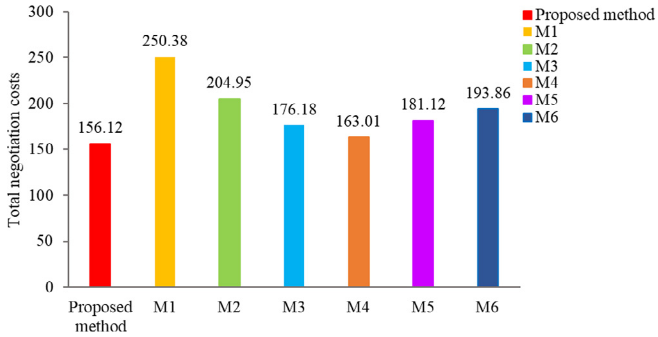


Fig. 7. The cost comparison using different methods.

more total negotiation costs to the experts. This means that, when facing the same consensus task, the proposed method can greatly decrease the cost expenditure for moderator.

As can be seen in Fig.7, the total negotiation cost of the proposed method is lower than in methods (1)-(6). To achieve consensus using methods (1)-(6), the resource will occur huge consumption. This signifies that considering the social information of experts can motivate their positivity in consensus and reduce the consensus cost. Moreover, reasonable utilization of relationship strength can effectively reduce the total negotiation costs of the moderator. Selecting the appropriate clustering algorithm can also reduce the consensus cost. Compared with the worst-case consensus that requires the maximum unit adjustment cost, the robust optimization method saves more costs in the real consensus of LSGDM in social network. In the case of method (6), although it overcomes the conservatism of robust optimization [13], the moderator will pay more in total negotiation costs than he would with robust optimization if it is used to deal with LSGDM consensus in social network.

Therefore, these advantages can mainly be contributed to the unique characteristics of the proposed method.

(1) In the expert clustering process, we propose a definition for comprehensive evaluation to express opinion information and a Euclidean distance to measure the similarity degree between experts. The new K-means clustering algorithm is conducted based on the social relationships of experts. It can be used to classify large-scale experts with society. In comparison, traditional expert clustering methods [24,47] based on the opinion similarity between experts are more restrictive. They do not motivate the social relationship of experts in the CRP. Although Dong et al.'s [8] method analyzes the relationship between experts through a network partition algorithm, it has the following drawbacks: (1) this algorithm does not consider that relationship strength among experts may affect the clustering process, and (2) the clustering result obtained through this algorithm has an intersection between two different subgroups and is not suitable for our consensus model.

(2) When measuring the consensus degree of a collective, our method not only takes into account the opinion differences among experts but also incorporates relationship strength information. Most traditional consensus measurement methods [30,25,26] only emphasize managing opinion differences among experts; they are not suitable for solving the consensus problem among large-scale experts with social relationships. As demonstrated by comparison, effectively utilizing the relationship between experts can motivate CRP among large-scale experts.

(3) Facing the uncertain unit adjustment cost in LSGDM, we deem it in an ellipsoidal set and propose a minimum cost model based on robust optimization to solve it. If the worst-case consensus occurs, the robust optimization method can be used to decrease the loss of the moderator. However, Li et al.'s [20] method considers the uncertain cost in an interval and utilizes a nonlinear optimization model to solve the opinions. Although their method can obtain the unit adjustment cost and opinions, the worst case is not considered in the consensus, and there is no rule about how to optimize the total negotiation cost in the worst case. The uncertain cost method is not suitable for LSGDM. In summary, the proposed model presents a reasonable opinion modification method, which minimizes the total cost compensation when the unit adjustment cost is uncertain. The superiority of the proposed method in realizing LSGDM consensus in social network has been demonstrated and validated.

From Fig. 6, we can see that the CI of the proposed method will continually increase if the next iteration is conducted. In comparison, the proposed method requires fewer iterations than method (3) to achieve consensus. However, method (3) can reach a higher CI than the proposed method. Although the proposed method has a faster consensus speed, it is not reasonable for decisions that only require a higher consensus threshold. For example, in large-group emergency decision making, an optimal decision usually require a relatively high consensus threshold [42]. Thus, how to use the proposed method to achieve consensus in large-group emergency decision making is a problem worth discussing further.

5.3. The cost comparison in three cases

In this paper, to realize LSGDM consensus in social network, we describe three consensus cases. The first is the nominal case. It is assumed that the unit adjustment cost is the nominal value c_i^*. The second is the worst case, which deems that when the unit adjustment cost is uncertain, and each expert intends to get the maximal unit adjustment cost, the total compensation cost paid by the moderator will reach the maximum. The third is the robust optimization case. Its goal is to minimize the total compensation cost in the worst case. Thus, the total compensation costs of the three cases are calculated as

$$\begin{cases} B^n = c_i^* |o_i' - o_i| & \textit{nominal case} \\ B^w = (c_i^* + \delta_i)|o_i' - o_i| & \textit{worst case} \\ B^r = c_i |o_i' - o_i| & \textit{robust optimization case} \end{cases} \tag{23}$$

In order to discuss the advantage of robust optimization in dealing with uncertain cost, we propose two indexes to quantify the benefit of robustness. The first is the worst-case consensus loss (WCCL). The WCCL is defined by the nominal case and the worst case as

$$WCCL = 100\% \times \frac{B^w - B^n}{B^n} \tag{24}$$

where B^n is the total compensation cost in the nominal case. Additionally, B^w denotes the total compensation cost paid by the moderator when the worst-case consensus happens. The WCCL measures how the moderator's total compensation cost changes when the consensus transfers from the nominal case to the worst case.

The second index we discuss is the relative improvement (RI). The RI is defined by the worst case and the robust optimization case as

$$RI = 100\% \times \frac{B^w - B^r}{B^w} \tag{25}$$

where B^r is the total compensation cost in the robust optimization case. Compared with the WCCL, RI measures the cost improvement of the robust optimization case.

When the ellipsoidal parameter δ is set as different values, we compute the cost of each iteration under three cases. In addition, for each δ, we compute the WCCL of the nominal case and the RI of the robust optimization case. The results are presented in Fig. 8 and Table 12.

As can be seen in Fig. 8, when the consensus cases are fixed in the same iteration, the consensus cost increases as the value of the ellipsoidal parameter δ increases. This finding implies that resource compensation will increase if the uncertain cost is in a bigger ellipsoidal set. Moreover, in each iteration, the nominal case has the lowest compensation cost. When the ellipsoidal parameter δ is the same, the total compensation cost in the worst case is higher than the robust optimization case. This shows that the robust optimization method can effectively reduce the loss of total compensation cost when the unit adjustment cost is uncertain.

Table 12 shows how WCCL and RI vary with the value of δ. We can see from the table that for the small uncertainty ($\delta < 10$), the loss generated in the worst case is relatively lower (e.g., for $\delta = 7$, the WCCL is 11.67%.), and the robust optimization method provides a slight advantage in reaching the consensus process. However, for the larger value of the ellipsoidal parameter δ ($\delta \geqslant 15$), the WCCL increases distinctly, and the robust optimization method has huge a advantage in reducing the total compensation cost. Therefore, when the unit adjustment cost is uncertain, the robust optimization method can minimize the total compensation cost of the worst case.

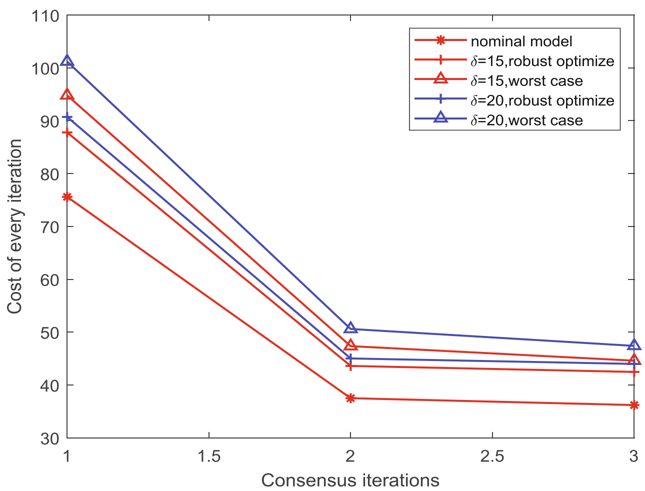


Fig. 8. The cost of each iteration under three cases.

Table 12
The WCCL of nominal case and the RI of robust case for different δ.

Uncertain parameter δ	B^n	B^r	B^w	WCCL(%)	RI(%)
$\delta = 0$	149.30	149.30	149.30	0.00	0.00
$\delta = 6$	149.30	163.21	164.24	10.01	0.63
$\delta = 7$	149.30	164.36	166.73	11.67	1.42
$\delta = 10$	149.30	167.94	174.20	16.68	3.59
$\delta = 11$	149.30	169.14	176.69	18.35	4.27
$\delta = 15$	149.30	173.86	186.75	25.08	6.90
$\delta = 20$	149.30	179.73	199.20	33.42	9.77

6. Conclusions

In this paper, we researched LSGDM in social network and proposed a novel consensus model based on robust optimization to deal with the uncertain unit adjustment cost. The main innovations and contributions of this paper are summarized as follows:

(1) We focus on LSGDM, considering the relationship information between experts, which can allow experts to express both trust value and relationship strength. A new expert clustering method is proposed to reduce the complexity of LSGDM. In this method, we classify experts with similar comprehensive evaluation into a subgroup by combining trust degree and relationship strength. This guarantees that all the social information of experts can be incorporated, which ensures that the clustering result is effective and reasonable. The proposed expert clustering approach can improve the efficiency of the CRP in social network.

(2) Considering the relationship strength between experts, a harmony degree measure method considering both opinion value and relationship strength is presented. A CI reflecting the harmony degree between experts is devised to measure the collective consensus level of LSGDM in social network. This ensures that LSGDM can better reflect the actual social network decision-making situation.

(3) To deal with the uncertainty of the unit adjustment cost, an ellipsoidal set in the feedback is proposed. To minimize the total compensation cost in the worst case, a consensus model based on robust optimization is constructed. A robust counterpart to solving the optimal consensus opinion is transformed. To estimate the unit adjustment cost, the rules are proposed to give suggestions to the moderator. The minimum cost model based on robust optimization makes the CRP more economical in the worst case for the LSGDM problem.

In this paper, the CRP of offline LSGDM is researched. However, in online LSGDM, the efficiency of the proposed method is impacted by the different decision-making scenarios. This paper does not consider the consensus of LSGDM in the online interaction situation. Thus, in an online situation, how a high level of LSGDM consensus may be obtained is still a topic worth discussing further.

In addition, in some real LSGDM problems, experts may exhibit the game behavior for pursuing their personal interest. This paper only discusses the influence of social relationship on the CRP of LSGDM; however, the game behavior of experts is ignored in the CRP. Therefore, in future work, another interesting issue to discuss would be the game behavior of experts and how that influences CRPs.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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