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Free contract environment for big electricity consumer in Brazil considering correlated scenarios of energy, power demand and spot prices

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ABSTRACT

In Brazil, big consumers can choose the free market to purchase their energy. The main advantage is the possibility of negotiating a lower price and the main risk is the difference between the energy contracted and consumed in each month, which should be settled by the spot price.

This paper proposes to optimize the parameters of a contract by a statistical and optimization model. In the statistical part, the parameters for energy and peak demand time series are estimated in a correlated way with the simulated scenarios of spot price, which is an input in the proposed methodology.

In the optimization part, a stochastic model is applied using a convex combination of the Expected Value and Conditional Value-at-Risk to find the optimum contract parameters, which are the energy contracted, the upper and lower bounds of the contract and peak demand contracted. To summarize, the main contribution of the proposed approach is to provide a methodology for big electricity consumers in the free energy market in Brazil, which considers a statistical model, that correlates and simulates scenarios of energy and peak demand with the spot price scenarios, and optimizes it based on a stochastic optimization model combining the Expected Value and Conditional Value at Risk as risk metrics.

The results indicate that the methodology can be a powerful tool for consumers in the free market and, due to the nature of the model, it can be generalized for different energy markets.

1. Introduction

In the Brazilian electricity market, big consumers are characterized by installed load capacity equal or greater than 2 MW [1]. In contrast with small consumers, they have two different options for contracting energy: the Regulated Contract Environment (RCE) and the Free Contract Environment (FCE) [2]. The tradeoff for being in one or other involves a higher cost in RCE and a higher risk in FCE.

The cost and risk are related with decision variables in the contract. While in the RCE market the power demand is the variable computed by consumers in contract with the utility, in the FCE market, beyond the power demand contract with utility, energy and the duration of the contract with commercialization company are mandatory variables to be defined. In addition to that, the limits of energy contract are also important to mitigate the risk due to the difference between the energy consumed and energy contracted settled by the spot price [3].

Recently, in [4], the authors presented an optimization model to address the problem in RCE by computing the optimal value of the power demand contracted. To address the problem, firstly the parameters of the power demand were estimated based on the historic time series data of power demand from a real consumer. Then, scenarios of power demand were simulated to, finally, be applied in a stochastic optimization model to compute the power demand contract. This model considers metrics of risk such as Conditional Value at Risk (CVaR) and EV (Expected Value) to find the best power demand contract.

Partially, the aforementioned methodology is being used as inspiration for this paper. Here, we propose to adapt the methodology applied in [4] to compute the value of the energy contract, limits of this contract and power demand contracted for the FCE market. This adaptation involves the simulation of combined scenarios of energy, power demand and spot prices in a statistical model and a stochastic optimization model to minimize the total cost of the electricity bill.

The spot price in Brazil is a systemic variable. As stated in [5], the Brazilian system is strongly based on hydrothermal generation, and the short-term operation is centrally coordinated by an Independent System Operator (ISO). The coordination is based on the minimal cost dispatch model and a by-product of this model is the operational marginal cost, which defines the spot price. Because most of the energy in Brazil comes from hydro power plants, when inflows are favourable, the demand is covered by hydro generation and the spot price tends to be lower. On

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Notation

B	Lag operator
B^S	Seasonal lag operator
B^+	Contract's upper bound
B^-	Contract's lower bound
$C_{s,t}^D$	Peak demand cost in the scenario s and month t
$C_{E_{s,t}}$	Use of distribution energy cost in the scenario s and month t
C_{ANNUAL}	Annual optimum cost
$C_{s,t}$	Function that calculates total cost in the scenario s and month t
$C_{s,t}^1$	Cost above the upper bound in the scenario s and month t
$C_{s,t}^2$	Cost below the lower bound in the scenario s and month t
$C_{s,t}^3$	Cost within the lower and upper bounds in the scenario s and month t
D_t	Peak demand time series
$D_{s,t}$	Peak demand simulated in the scenario s and month t
D_{cont}	Peak demand contracted
$d_{s,t}^u$	Exceeding demand in the scenario s and the month t
E_t	Energy consumption time series
$E_{s,t}$	Energy simulated in the scenario s and month t
$E_{s,t}^{op}$	Off-peak energy in the scenario s and the month t
$E_{s,t}^p$	Peak energy in the scenario s and the month t
M	Big M used as an auxiliary parameter
P_e	Contracted energy price
Q	Contracted energy quantity
Q^+	Upper bound in terms of energy
Q^-	Lower bound in terms of energy
S	Total scenarios
$\pi_{s,t}$	Spot price in the scenario s and the month t
T	Analysis period
$TUSD_D$	Distribution system use tariff for peak demand
$TUSD_D^{exc}$	Distribution system use tariff for exceeding peak demand
$TUSD_E^p$	Distribution system use tariff for peak energy consumed
$TUSD_E^{op}$	Distribution system use tariff for peak energy consumed
T_E^p	Peak energy tariff
T_E^{op}	Peak-off energy tariff
u	Tolerated exceeding demand percentage. In the case of this paper, 5%.
w_t	Auxiliary variable that reaches the value-at-risk (VaR) of the distribution costs in the month t for the period of analysis.
x_t	Explanatory variable of a time series
$X_{s,t}^1$	Main binary variable that indicates if energy is above upper bound in the scenario s and the month t
$X_{s,t}^2$	Secondary binary variable that indicates if energy is within the bounds in the scenario s and the month t
$X_{s,t}^3$	Secondary binary variable that indicates if energy is within the bounds in the scenario s and the month t
$X_{s,t}^{23}$	Main binary variable that indicates if energy is within the bounds in the scenario s and the month t
$X_{s,t}^4$	Main binary variable that indicates if energy is below lower bound in the scenario s and the month t
$Y_{s,t}$	Main binary variable that indicates exceeding demand in the scenario s and the month t
Z_t	Current value of the time series
$\Delta_{s,t}^{\mathrm{D}}$	Peak demand tolerance to avoid penalty in the scenario s and the month t
$\Delta_{s,t}^+$	Difference between energy and upper bound in the scenario s and the month t
$\Delta_{s,t}^-$	Difference between lower bound and energy in the scenario s and the month t
α_t	White noise of the time series
α	Aversion to risk parameter that defines the confidence level of the CVaR
γ	Parameter associated with the explanatory variable
γ_π	Parameter associated with the Spot Price variable
$\delta_{s,t}$	Auxiliary variable that represents the left side of the distribution costs in the scenario s and the month t
λ	Constant that makes the balance between Expected Value (EV) and the Conditional Value-at-Risk (CVaR).
ϕ_p	Autoregressive parameter of order p
Φ_P	Seasonal autoregressive parameter of order P
θ_q	Moving average parameter of order q
Θ_Q	Seasonal moving average parameter of order Q
∇^d	Difference operator of order d
∇_S^D	Seasonal difference operator of order D

the other hand, when the system's future simulated scenarios of hydro generation are not favourable, the spot price can skyrocket. For big consumers in FCE, the difference between the energy contracted and energy consumed is settled by the spot price.

In Brazil, the ISO simulates spot prices scenarios for 5 years ahead and, because of this, we can use it as input data in this paper. After the combined simulation of the spot prices and the interested variables with the statistic model, the values of power demand, energy and limits of this energy are computed to set the contract in the optimization model in order to mitigate the risk for the consumer. The metrics used in the optimization model are CVaR and EV.

Related to the statistical part, many papers have been proposed in the last few years to estimate load scenarios in long- and short-term analysis. In long term analysis, classical approaches are still quite often used. In [6], a mid-term electricity load demand database is considered to fit an Autoregressive Moving Average (ARMA) model using Corrected Akaike Information Criterion (AICC) as metric of analysis. Refs [4,7] considered SARIMA (Seasonal Autoregressive Model integrate with Moving Average) to estimate power demand and energy time series for a big consumer on a monthly basis.

On the other hand, for short-term analysis, Artificial Intelligence, optimization models and machine learning techniques have been replacing the classical approach. In [8,9], neural networks are used to predict daily and quarter-hourly electricity load power, respectively. The results are compared to actual load data in order to verify the accuracy of the prediction.

Optimization models for energy load management and power demand contracts have been proposed in many projects. In Ref. [10] an optimal operation of an industrial system is established to minimize the energy cost of an industrial process. In Ref. [11], the optimization model is applied to minimize the energy cost by adjusting an industrial process and takes into account the portfolio effect of renewable energy. In Ref. [12] a real case application in the context of the Brazilian rules for reducing costs of a large consumer is presented. In Ref. [13], a stochastic optimization model with CVaR and EV is used for reducing the energy cost of a consumer. The CVaR and EV is also applied in Refs. [4,7] for big consumers to compute power demand and the optimal number of PV panels applying Brazilian rules.

The Ref. [4] can be used as an inspiration for the proposed model in this paper. The main difference is the statistical model that considers the correlation between spot prices, energy consumption and power demand. To combine those variables, the monthly energy consumed is simulated taking into account the spot prices as explanatory variables. Additionally, peak demand is simulated using scenarios of energy as explanatory variables. At the conclusion all variables are combined in the simulation process.

The optimization model is also inspired by Lima et al. [4], but in this paper it consists in a convex combination between EV and CVaR to minimize the annual cost of the consumer in FCE instead of RCE. To summarize, the main contribution of the proposed approach is to provide a methodology for big electricity consumers in the free energy market in Brazil, which considers a statistical model, that correlates and simulates scenarios of energy and peak demand with the spot price scenarios, and optimizes it based on a stochastic optimization model combining the Expected Value and Conditional Value at Risk as risk metrics.

In order to detail the methodology, this article is organized as follows: Section 2 presents a short description of the energy contracting rules in Brazil; Section 3 presents the historical data of the variables involved in this problem such as energy consumption, spot price and peak demand; Section 4 presents the statistical model used to simulate future scenarios and the treatment proposed to correlate spot price, peak demand and energy consumed; Section 5 presents the optimization model that computes peak demand contracted value, energy contracted and the limits of this contract using EV and CVaR as risk metrics; Section 6 presents the results of the proposed problem and, finally; Section 7 presents the conclusion of the paper.

2. Brazilian contracting environments

In the RCE, consumers can freely choose between two different modalities of contract, the Green Rate (GR) and the Blue Rate (BR) [2]. The main difference between them is the cost structure associated with peak and off-peak demand. In GR only the maximum peak demand over the month is considered to compute the bill, whereas in the BR the peak and off-peak demands are separately charged. The price of energy consumption is separated in peak and off-peak hours for both modalities. In this paper, for the sake of simplicity, only GR will be used, but it is important to highlight that the proposed approach can be generalized for any modality.

The cost computed in GR for peak demand combines the greatest value between the peak demand ($D_t^{\max}$) and the peak demand contracted (D_t^c), for each month t. The rule to compute the total cost of this contract can be written as follows:

$$C_t^{GR} = T_E^p E_t^p + T_E^{op}\cdot E_t^{op} + TUSD_E^p\cdot E_t^p + TUSD_E^{op}\cdot E_t^{op} + C_t^D \tag{1}$$

The demand cost can change according to $D_t^{\max}$ and D_t^c. If $0 < D_t^{\max} \le 1.05 D_t^c$:

$$C_t^D = \max(D_t^c, D_t^{max})\cdot TUSD_D \tag{2}$$

Otherwise, if $D_t^{max} > 1.05D_t^c$:

$$C_t^D = D_t^{max}\cdot TUSD_D + (D_t^{max} - D_t^c)\cdot TUSD_D^{exc} \tag{3}$$

Observe that the tolerance for violating the value of the peak demand contracted is 5%, which means that if the D_t^{max} value is greater than $105\% D_t^c$, a cost penalty will take place. As stated in [4], the cost function changes in each stage and the main challenge to model it as an optimization model is the changing of the rule when D_t^{max} value violates the tolerance of 5%, creating a non-convexity for the optimization problem. Thus, a MILP (Mixed integer Linear Programming) formulation should be used.

In FCE, the energy contract is freely agreed between the consumer and commercialization company. A typical agreement involves the energy to be contracted and the limits used to mitigate the spot price risk. In order to illustrate this problem, a contract in FCE can consider the following rule:

$$C_t^{FCE} = Q_t\cdot P_t + (Q_t - E_t)\cdot \pi_t + C_t^D \tag{4}$$

According to (4), the higher is the difference between the energy contracted and the energy consumed is, the higher the risk associated with spot price will be.

In order to minimize this risk, the contract can be bounded in such a way that it minimizes the settlement by the spot price as follows:

If $Q_t^- \le E_t \le Q_t^+$:

$$C_t^{FCE} = E_t\cdot P_t + C_t^D \tag{5}$$

If $Q_t^+ < E_t$:

$$C_t^{FCE} = Q_t^+\cdot P_t + (Q_t^+ - E_t)\cdot \pi_t + C_t^D \tag{6}$$

If $E_t < Q_t^-$:

$$C_t^{FCE} = Q_t\cdot P_t + (E_t - Q_t^-)\cdot \pi_t + C_t^D \tag{7}$$

The proposed problem also creates non-convexity in an optimization approach that should be addressed by a MILP formulation. In conclusion, the larger the bounds of the contract are the less risk in the spot price will take place. It is important to highlight that due to the large variation of the spot price over the year, the cost can increase or decrease significantly when the consumer is exposed to this price.

2.1. Data analysis

In this paper, three time series are considered as inputs for the statistical model: energy consumption, peak demand and spot prices expressed in MWh, kW and R$/MWh, respectively. The historical data of the series are monthly based from January 2002 to December 2018, totaling 204 observations for each series. In Brazil, the spot price for long term simulation is on a monthly basis. However, the proposed approach in this paper could be applied for a week, day, an hour, or any time discretization.

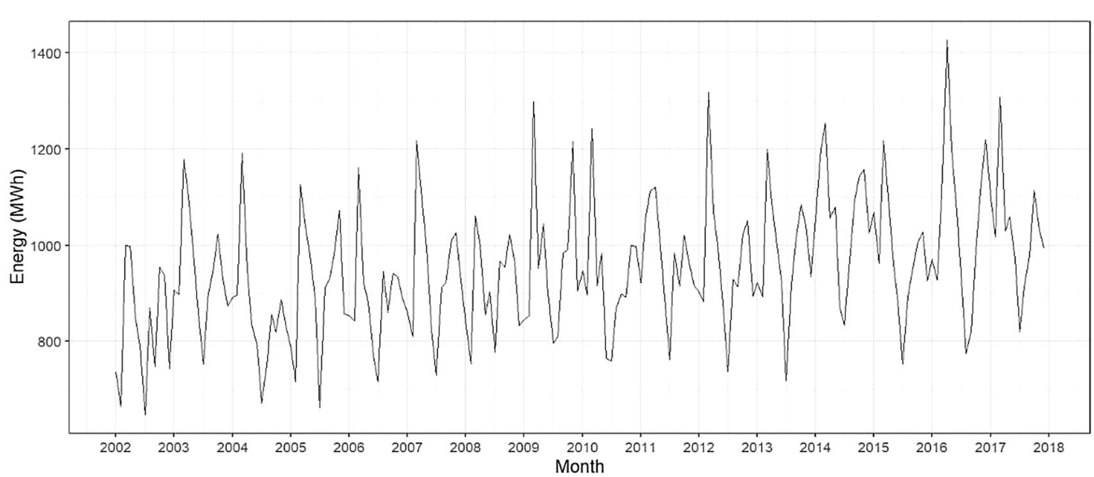

Fig. 1. Energy time series from 2002 to 2018.

The energy consumption and peak demand data are from a big consumer located in Rio de Janeiro, Brazil, and the spot prices data were gathered from [14]. Figs. 1–3 show the energy consumption, peak demand and spot price time series, respectively.

2.2. Time series correlation

The time series correlation, in this paper, can be justified by the Pearson Correlation Test [15]. In this test, the covariance and the standard deviation of two time series are combined in a way that produces a value between -1 and indicates the level of correlation between these series. If the results are close to 1 or -1, it indicates a strong positive and negative correlation, respectively, between the series. On the other hand, if the result is close to zero, there is a weak correlation. For the time series presented in this paper, the correlation of the time series is presented in Table 1. As it can be observed, the spot price and energy have a weak correlation compared with Energy and Peak demand. However, to be conservative, the statistical model used in this paper will take into account the correlation between all variables.

2.3. Statistical models

The approach chosen to estimate the appropriate model to fit the data was the SARIMA model. The general expression of a *SARIMA* (*p, d, q*) *x* (*P, D, Q*)$_S$ model considers the use of an explanatory variable as in the following expression [16]:

$$\phi_p(B)\Phi_P(B^S)\nabla^d\nabla_S^D Z_t = \gamma x_t + \theta_q(B)\Theta_Q(B^S)a_t \tag{8}$$

There are several evaluation metrics that can be used to decide the best SARIMA model. In Ref. [7] the AIC (Akaike Information Criteria), BIC (Bayesian Information Criteria), MAPE (Mean Absolute Percentage Error) and R^2 (Coefficient of Determination) were combined to create a metric of evaluation. The AIC and BIC are estimators of a mathematical model adherence by penalizing the difference of the model with a set of models [17,18]. The difference between AIC and BIC is the level of penalization. Similarly, R^2 indicates how much the total data variation is explained by the model [19]. Finally, MAPE is one of the most used forecast accuracy measures and is calculated by the difference between estimated values and real values [20]. As in [7], the coefficient proposed in this paper is calculated as follows:

$$K_i = \frac{AIC_i}{\sum_i AIC_i} + \frac{BIC_i}{\sum_i BIC_i} + \frac{MAPE_i}{\sum_i MAPE_i} + \frac{(1-R_i^2)}{\sum_i (1-R_i^2)} \tag{9}$$

According to the numerical meaning of the evaluation metrics, the smaller the value of K_i, the better the model i is. Based on this coefficient, the appropriate statistical model was identified for each time series. In this paper, the software used to perform this analysis was RStudio, version 1.2 [21].

The chosen model was SARIMA (5,1,4) × (0,1,2)$_{12}$, with $K_i = 0.4126$, and the mathematical representation of this model is as follows:

$$\begin{aligned} E_t = {} & \gamma_\pi \pi_t + (1-\phi_1)E_t - (\phi_1-\phi_2)E_{t-2} - (\phi_2-\phi_3)E_{t-3} - \\ & (\phi_3-\phi_4)E_{t-4} - (\phi_4-\phi_5)E_{t-5} - \phi_5 E_{t-6} + E_{t-12} - (1+\phi_1)E_{t-13} - \\ & (\phi_2-\phi_1)E_{t-14} - (\phi_3-\phi_2)E_{t-15} - (\phi_4-\phi_3)E_{t-16} - (\phi_5-\phi_4) \\ & E_{t-17} + \phi_5 E_{t-18} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_2 a_{t-3} - \theta_3 a_{t-4} - \\ & \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + \theta_2\Theta_1 a_{t-14} + \theta_3\Theta_1 a_{t-15} + \theta_4\Theta_1 a_{t-16} - \\ & \Theta_2 a_{t-24} + \theta_1\Theta_2 a_{t-25} + \theta_2\Theta_2 a_{t-26} + \theta_3\Theta_2 a_{t-27} + \theta_4\Theta_5 a_{t-28} \end{aligned} \tag{10}$$

For the case of peak demand, the model chosen was SARIMA (4,1,3) x(1,1,1)$_{12}$, with $K_i = 0.4367$ and the following mathematical model:

$$\begin{aligned} D_t = {} & \gamma_E E_t + (1+\phi_1)D_{t-1} - (\phi_1-\phi_2)D_{t-2} - (\phi_2-\phi_3)D_{t-3} - \\ & (\phi_3-\phi_4)D_{t-4} - \phi_4 D_{t-5} + (1+\Phi_1)D_{t-12} - (\phi_1+\phi_1\Phi_1+1+\Phi_1) \\ & D_{t-13} - (\phi_2+\phi_2\Phi_1-\phi_1-\phi_1\Phi_1)D_{t-14} \\ & - (\phi_3+\phi_3\Phi_1-\phi_2-\phi_2\Phi_1)D_{t-15} \\ & - (\phi_4+\phi_4\Phi_1-\phi_3-\phi_3\Phi_1)D_{t-16} + (\phi_4+\phi_4\Phi_1)D_{t-17} - \Phi_1 D_{t-24} + \\ & (\phi_1\Phi_1-\Phi_1)D_{t-25} - (\phi_1\Phi_1-\phi_2\Phi_1)D_{t-26} - (\phi_2\Phi_1-\phi_3\Phi_1)D_{t-27} - \\ & (\phi_3\Phi_1-\phi_4\Phi_1)D_{t-28} + \phi_4\Phi_1 D_{t-29} + a_t - \theta_1 a_{t-1} - \theta_1 a_{t-2} - \\ & \theta_3 a_{t-3} - \Theta_1 a_{t-12} + \theta_1\Theta_1 a_{t-13} + \theta_2\Theta_1 a_{t-14} + \theta_3\Theta_1 a_{t-15} \end{aligned} \tag{11}$$

The coefficients computed for energy and peak demand of the proposed model can be summarized in Tables 2 and 3. In order to evaluate the proposed model, some metrics such as lag values of ACF (autocorrelation function) and PACF (partial autocorrelation function) must be non-significant to guarantee that the residuals are not correlated. In the Figs. 4 and 5 it can be seen that there are no correlation patterns between the residuals, which indicate that the models are appropriate.

In addition to that, in Figs. 6 and 7 the normality of the residuals in each time series is checked. Both figures indicate that there is a normal distribution of the residuals, which is highly desirable for the proposed approach in this paper.

Finally, to validate the quality of both SARIMA models, a Pseudo Out-of-Sample analysis can be performed. The available time series were divided in 2 different groups: training data, starting from 2002 to 2014, and test data, from 2015 to the end of 2017.

Figs. 8 and 9 present the models applied for In-Sample and Pseudo Out-of-Sample analysis. The tests indicated that both models could be used to explain the behavior of the related time series.

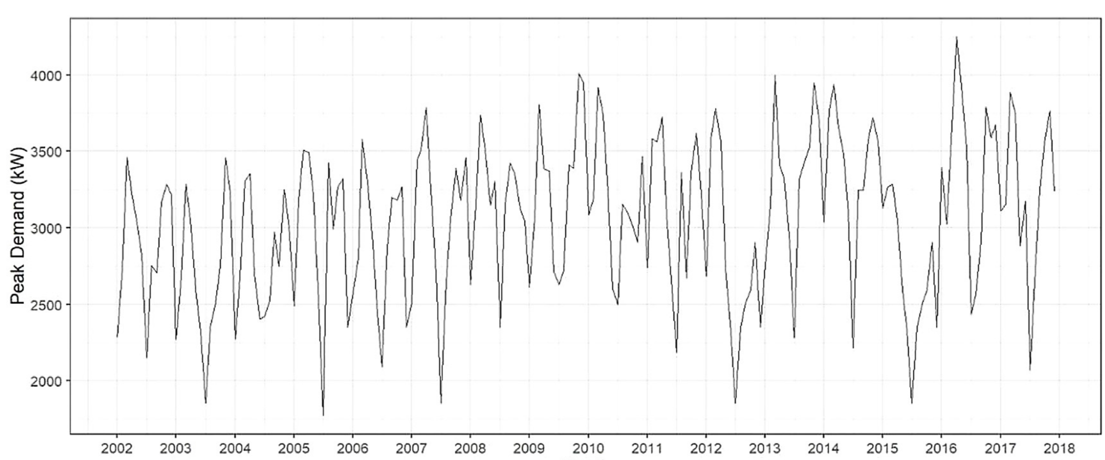

Fig. 2. Peak Demand time series from 2002 to 2018.

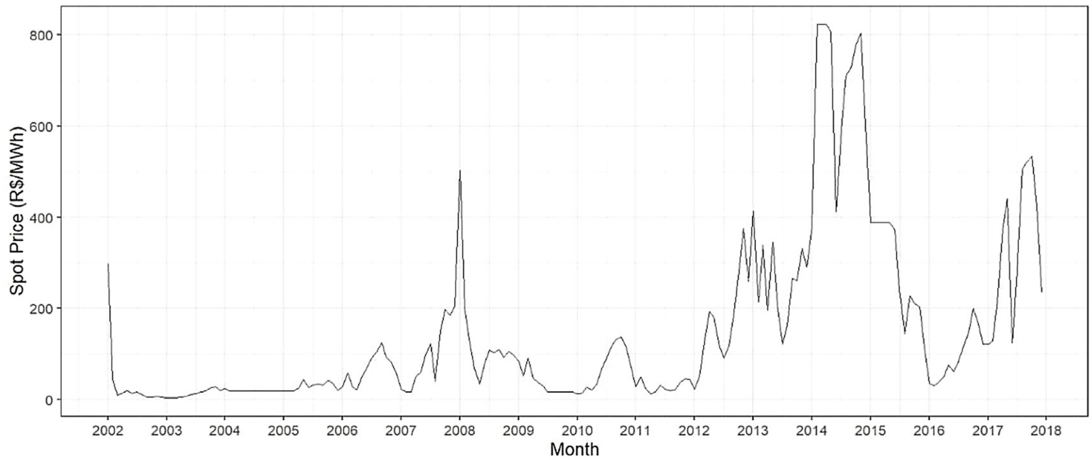

Fig. 3. Spot Price time series from 2002 to 2018.

Table 1
Pearson correlation test results.

Time series correlated	Pearson coefficient
Spot price and Energy	0.2871553
Energy and peak demand	0.7258662

Table 2
SARIMA coefficients for energy time series.

Coefficient	Value	Coefficient	Value
γ_{π_t}	0.0529	$\boldsymbol{\theta}_1$	−0.8603
ϕ_1	0.0836	$\boldsymbol{\theta}_2$	0.2382
ϕ_2	−0.2354	$\boldsymbol{\theta}_3$	0.3251
ϕ_3	−0.6739	$\boldsymbol{\theta}_4$	−0.7029
ϕ_4	0.1455	$\boldsymbol{\Theta}_1$	−1.0612
ϕ_5	−0.0395	$\boldsymbol{\Theta}_2$	0.2300

Table 3
SARIMA coefficients for peak demand time series.

Coefficient	Value	Coefficient	Value
γ_E	1.5516	$\boldsymbol{\theta}_1$	−1.2545
ϕ_1	0.5605	$\boldsymbol{\theta}_2$	1.2546
ϕ_2	−0.7972	$\boldsymbol{\theta}_3$	−0.9999
ϕ_3	0.2983	$\boldsymbol{\Phi}_1$	0.0430
ϕ_4	0.2562	$\boldsymbol{\Theta}_1$	−0.9999

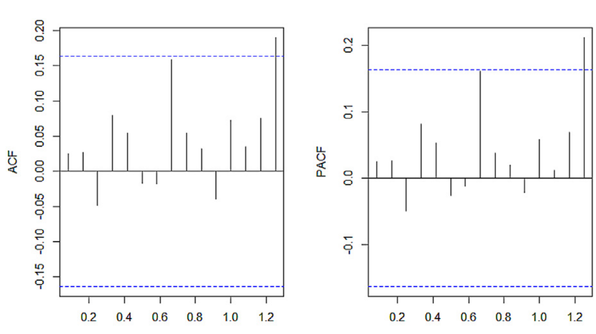

Fig. 4. ACF and PACF of energy time series.

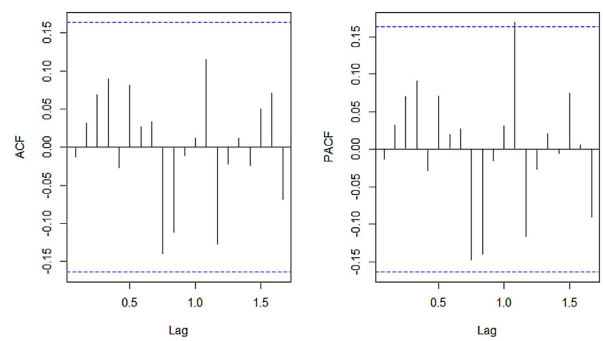

Fig. 5. ACF and PACF of peak demand time series.

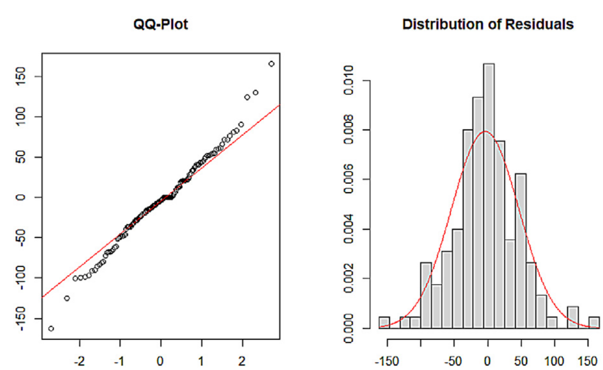

Fig. 6. Residual normality of energy time series.

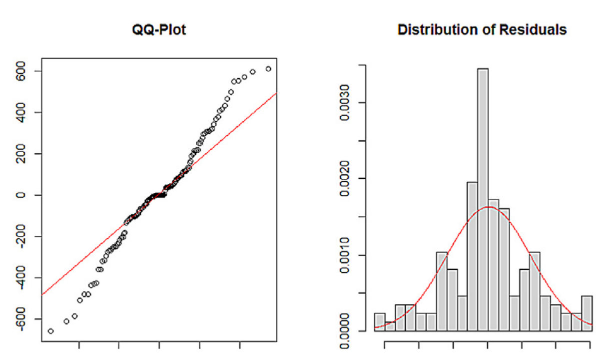

Fig. 7. Residual normality of peak demand time series.

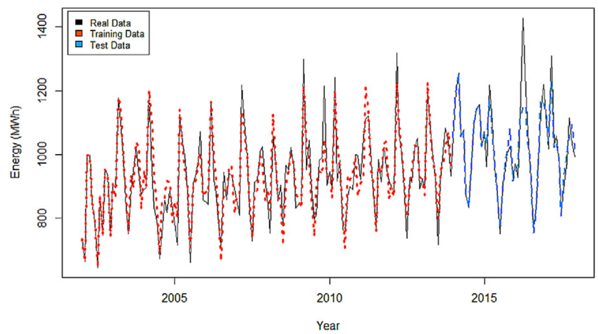

Fig. 8. In-sample and out-of-sample analysis for energy time series.

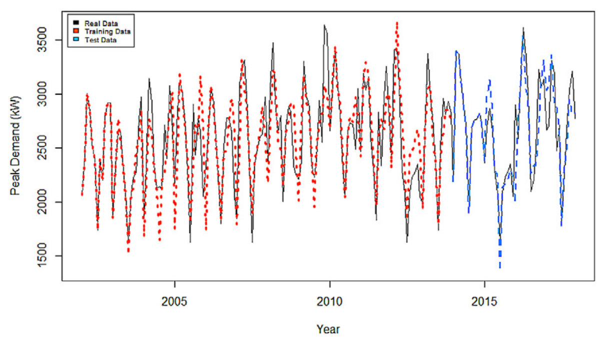

Fig. 9. In-sample and out-of-sample analysis for peak demand time series.

2.4. Scenarios generation

In Brazil, the ISO simulates many future scenarios of spot price as a result of the system operation simulation. Because the spot price is an explanatory variable for energy, we can simulate many scenarios of energy consumed correlated with the spot price and, finally, simulate many scenarios of peak demand with the energy scenarios. Fig. 10 presents 200 scenarios of spot price simulation randomly chosen from the optimization system in Brazil. The dataset used in this work is available in [22].

After the simulation of energy scenarios by the 200 scenarios of spot prices, a weak variability of energy scenarios was observed due to a weak correlation among the spot price and energy. In order to resolve this problem, the energy scenarios were resampled keeping the mean of the data and introducing a variability based on the historical data.

Figs. 11 and 12 presents the historical data and the final simulation of energy and peak demand after introducing the resample process.

3. Optimization model

The proposed model considers similar objective functions presented in [4]. However, in this paper, the objective function is based on the FCE cost function presented in Eqs. (4) to (7) in order to compute the optimal contract using the previously generated scenarios. The model uses EV and CVaR as measurements to consider different levels of risk, as follows:

$$C_{ANNUAL} = \min_{\substack{\Delta^{+}_{s,t},\Delta^{-}_{2,s,t},B^{+},B^{-},X^{1}_{s,t},X^{2}_{s,t},X^{3}_{s,t},X^{4}_{s,t},X^{23}_{s,t},\\ C^{1}_{s,t},C^{2}_{s,t},C^{3}_{s,t},D_{cont},\Delta^{D}_{s,t},du_{s,t},y_{t},C^{D}_{s,t},C_{s,t},\delta_{s,t},w_{t}}} (1-\lambda)\cdot\frac{1}{S}\cdot\sum_{t\in T}\sum_{s\in S}C_{s,t} + \lambda\cdot\left[\sum_{t\in T}w_t + \sum_{s\in S}\left(\frac{\delta_{s,t}}{1-\alpha}\right)\cdot\frac{1}{S}\right] \tag{12}$$

Subject to:

$$0 \le B^{+} \le 1 \tag{13}$$

$$0 \le B^{-} \le 1 \tag{14}$$

$$Q^{+} = (1 + B^{+})\cdot Q \tag{15}$$

$$Q^{-} = (1 - B^{-})\cdot Q \tag{16}$$

$$\Delta^{+}_{s,t} = (E_{s,t} - Q^{+})\cdot X^{1}_{s,t} \tag{17}$$

$$\Delta^{+}_{s,t} \ge 0 \tag{18}$$

$$(Q^{+} - E_{s,t})\cdot X^{2}_{s,t} \ge 0 \tag{19}$$

$$\sum_{s\in S}\sum_{t\in T} X^{1}_{s,t} + X^{2}_{s,t} = 1 \tag{20}$$

$$(E_{s,t} - Q^{-})\cdot X^{3}_{s,t} \ge 0 \tag{21}$$

$$\Delta^{-}_{s,t} = (Q^{-} - E_{s,t})\cdot X^{4}_{s,t} \tag{22}$$

$$\Delta^{-}_{s,t} \ge 0 \tag{23}$$

$$\sum_{s\in S}\sum_{t\in T} X^{3}_{s,t} + X^{4}_{s,t} = 1 \tag{24}$$

$$\sum_{s\in S}\sum_{t\in T} X^{2}_{s,t} + X^{3}_{s,t} - 1 = X^{23}_{s,t} \tag{25}$$

$$X^{1}_{t}, X^{2}_{t}, X^{3}_{t}, X^{4}_{s,t}, X^{23}_{s,t} \in \{0, 1\} \tag{26}$$

$$C^{1}_{s,t} = Q^{+}\cdot P_e + \Delta^{+}_{s,t}\cdot\pi_{s,t} \tag{27}$$

$$C^{2}_{s,t} = Q\cdot P_e - \Delta^{-}_{s,t}\cdot\pi_{s,t} \tag{28}$$

$$C^{3}_{s,t} = E_{s,t}\cdot P_e \tag{29}$$

$$D_{cont} + \Delta^{D}_{s,t} + du_{s,t} \ge D_{s,t} \tag{30}$$

$$\Delta^{D}_{s,t} \le D_{cont} \times u \tag{31}$$

$$du_{s,t} \ge (D_{t,s} - (1 + u) \times D_{cont})\cdot Y_{s,t} \tag{32}$$

$$Y_{s,t} \in \{0, 1\} \tag{33}$$

$$du_{s,t} \le M\cdot Y_{s,t} \tag{34}$$

$$C^{D}_{s,t} = (D_{cont} + \Delta^{D}_{s,t})\cdot TUSD_{D} + du_{s,t}\cdot TUSD^{exc}_{D} \tag{35}$$

$$C^{E}_{s,t} = E^{p}_{s,t}\cdot TUSD^{p}_{E} + E^{op}_{s,t}\cdot TUSD^{op}_{E} \tag{36}$$

$$C_{s,t} = C^{1}_{s,t}\cdot X^{1}_{s,t} + C^{2}_{s,t}\cdot X^{23}_{s,t} + C^{3}_{s,t}\cdot X^{4}_{s,t} + C^{D}_{s,t} + C^{E}_{s,t} \tag{37}$$

$$\delta_{s,t} \ge C_{s,t} - w_t \tag{38}$$

$$\delta_{s,t} \ge 0 \tag{39}$$

The expression (12) optimizes the energy cost in FCE weighting the

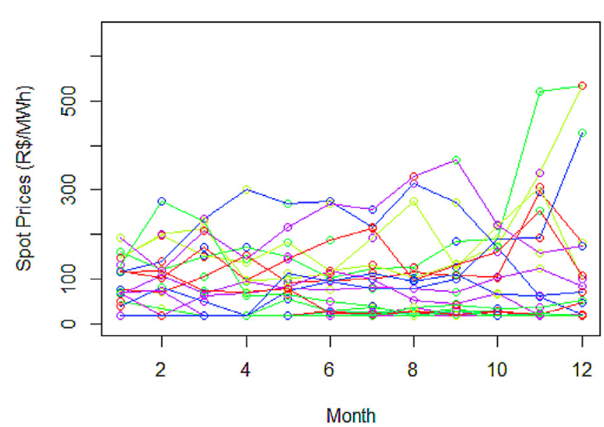

Fig. 10. 200 Spot Prices scenarios.

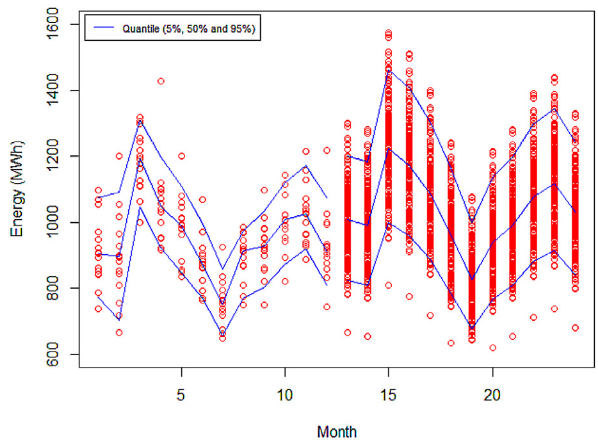

Fig. 11. Energy historical and generated data side by side.

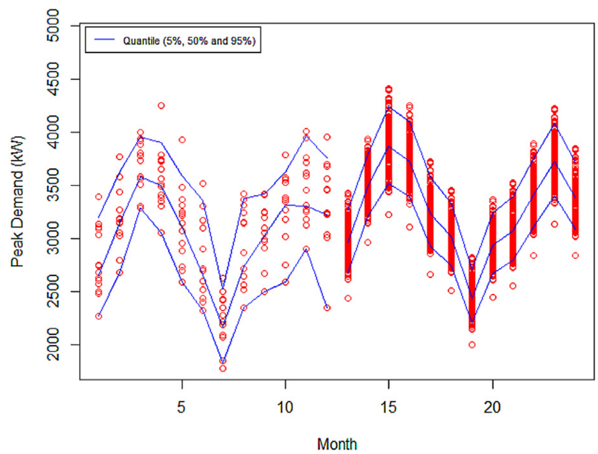

Fig. 12. Peak demand historical and generated data side by side.

EV and CVaR by the λ parameter. The expressions from (13) to (16) establishes the limits of the contract (in MWh), and Q^{+} and Q^{-} are the upper and lower bounds expressed in MWh. Constraints from (17) to (26) are modelled to choose the rules of the contract defined from (5) to (7), according to the energy consumption in a scenario *s* and month *t*.

Constraints from (27) to (29) present the energy cost computed for each rule that will be activated by the binary auxiliary variables computed from (17) to (26).

Constraints from (30) to (34) are related to the peak demand contract, where the tolerance is 5% to avoid penalties. Once more, binary variables are used to indicate whether or not the penalty is applied for each peak demand scenario *s* and month *t*.

The expressions (35) and (36) are the cost of peak demand and energy for each scenario *s* and month *t*. The expression (37) represents the total cost of the consumer for each scenario *s* and month *t* and, finally, constraints (38) and (39) defines the CVaR for the $(1-\alpha)\%$ worst case scenarios to be applied in the objective function (12).

4. Model simulations

The proposed optimization model described from (12) to (39) was implemented in the software Xpress [23] though the use of a built-in MILP [24] solver. The period of analysis considered was one year and the energy contract *Q* was computed *ex-ant* by optimizing the model without the limits (upper and lower bound) for an energy price of 200 R $/MWh.[1] After that, the upper and lower bound contract was optimized for energy price equal to 200 R$/MWh and 100 R$/MWh.

The risk aversion parameter α was chosen to be equal to 0.95, which means that 5% worst-case cost scenarios will be used by CVaR in the optimization process. Besides that, the λ parameter was used to simulate the risk aversion profiles. If λ = 0, the simulation considers only EV; if λ = 0.5, the simulation weighs 50% for EV and 50% for CVaR; finally, if λ = 1, the simulation considers only CVaR.

For the appropriate interpretation of the results, it is important to observe the energy price in the contract. In a simplified way, if the average of the spot price is cheaper than the energy price contracted in a scenario *s* and month *t*, a reduced energy and upper bound contracts are obtained, as well as a greater lower bound contract. This happens because the settlement (purchase) by the spot price is desirable for the consumer in this scenario. On the other hand, for higher spot prices, the best solution would be to increase the energy contract and upper bound and reduce the lower bound in order to sell the energy by the spot price. Obviously, in a stochastic context, the proposed analysis is more complex due to the many scenarios of spot price, energy and peak demand to be considered. Tables 4 and 5 present the results of the optimization model considering energy prices of 200 R$/MWh and 100 MWh, respectively. The results present the energy contract, average energy, which is the average value of energy scenarios, peak demand contract and average peak demand, which is the average value of peak demand scenarios. In addition, the average value of the spot price in risky scenarios, the lower and upper bound of the contract and, finally, the cost associated with the risk parameters are also presented.

In the first situation (energy price = 200 R$/MWh), the average value of the spot price (R$ 120.85) for λ = 0 is lower than the energy price, which means that to purchase energy by the spot price can be an advantage. Because of this, a lower energy contract (720 MWh) and upper bound (0%) was established. The lower bound (15.4%) aims to avoid selling by the spot price. For λ = 0.5, as expected, the energy contract (864 MWh) and upper bound (82.23%) increased indicating more aversion of risk for purchasing by the spot price, since the average spot price in risky scenarios (307.05 R$/MWh) is greater than the energy price. The lower bound also increased, but much less than the upper bound because to sell energy by the spot price can be an advantage in some cases. Finally, for and λ = 1 the energy contract (1296 MWh) increased even more. For this case, the upper and lower bound can be reduced because the higher energy contract will result in selling by the spot prices, which is desirable when this risk aversion parameter is applied.

In the second case (energy price = 100 R$/MWh), keeping the energy contract equal to the first case, for λ = 0, the upper bound (100%) is much higher than the lower bound (19.39%). It means that to purchase energy by the spot price (R$ 120.85) could be a drawback. For λ = 0.5 and λ = 1, the amplitude (difference between the upper and lower bound) is reduced mainly to sell the energy by the spot price.

In all cases, peak demand contract follows the combination with energy scenarios and, consequently, the spot prices scenarios, indicating the importance to correlate the variables in the simulation process.

5. Conclusion

In this paper, a statistical and optimization model is proposed to be applied in the FCE market in Brazil. The main contribution of the proposed paper is to provide a methodology for big electricity consumers in the free energy market in Brazil, which considers a statistical model, that correlates and simulates scenarios of energy and peak demand with the spot price scenarios, and optimizes it based on a stochastic optimization model combining the Expected Value and Conditional Value at Risk as risk metrics. By the generated scenarios produced by the statistical approach, the optimization model provides the best contract and its limits according to the level of risk taken by the consumer, indicating more or less exposure to the spot price. In

[1] According to Ref [25], US$ 1 corresponds to R$ 5.28 in April of 2020.

Table 4
Results of the optimization model for energy price equal to 200 R$/MWh.

	Energy price = 200 R$/MWh		
	λ = 0	λ = 0.5	λ = 1
Energy Contract (MWh)	720	864	1296
Average Energy (MWh)	1038.10	1162.06	1285.65
Peak demand contract (kW)	3474.06	3691.59	3752
Average Peak Demand (kW)	3278.23	3470.57	3662.34
Average Spot Price in risky scenarios (R $/MWh)	120.85	307.05	490.43
Upper bound	0.00%	82.23%	21.45%
Lower bound	15.14%	31.63%	0.00%
Annual cost	R$ 4118,510.62	R$ 5639,075.00	R$ 6022,629.35

Table 5
Results of the optimization model for energy price equal to 100 R$/MWh.

	Energy price = 100 R$/MWh		
	λ = 0	λ = 0.5	λ = 1
Energy Contract (MWh)	720	864	1296
Average Energy (MWh)	1038.10	1162.06	1286.02
Peak demand contract (kW)	3474.06	3691.59	3752.30
Average Peak Demand (kW)	3278.23	3470.57	3662.9
Average Spot Price in risky scenarios (R $/MWh)	120.85	307.05	490.43
Upper bound	100.00%	82.22%	21.49%
Lower bound	19.39%	31.57%	0.00%
Annual cost	R$ 3814,278.10	R$ 4244,521.28	R$ 4410,612.31

addition to the numerical results, due to the nature of the MILP problem, the proposed approach can be adapted for other modalities of contract in Brazil or other countries. It is important to highlight that, in Brazil, according to ([3]), the migration of big electricity consumers to FCE provides 29% of reduction, on average, in the electricity bill compared with RCE. However, in this paper, the comparison between FCE and RCE is not trivial, once the risk component associated with FCE takes into account energy, spot price and peak demand. In contrast, RCE only considers peak demand in the risk component. For future projects, other modalities of contract will be incorporated as well as the integration in a bilevel problem of the energy contract and its limits.

CRediT authorship contribution statement

Delberis A. Lima: Conceptualization, Methodology, Validation, Formal analysis, Writing - review & editing, Supervision. **Daniel Niemeyer Teixeira Paula:** Software, Investigation, Data curation, Visualization.

Declaration of Competing Interest

None.

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