



## Review

# A mini review of recent progress on vortex-induced vibrations of marine risers

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## ABSTRACT

Marine risers are subject to vortex-induced vibration (VIV) caused by a combination of various external and internal excitations. The external excitations include the current-induced hydrodynamic forces and the wave-induced motions of the floating platform, while the internal excitation refers to the effect of internal flow. With significant increase of the riser aspect ratio, the VIV becomes much more complicated, resulting in more frequent fatigue damages to the risers. Hence, VIV is an important research area in marine engineering. This review surveys the latest progress on the VIV research, including the multi-mode response, VIV response at high Reynolds numbers, flow-induced vibrations between multiple marine risers, VIV of inclined risers and intermittent VIV in oscillatory flow. Particularly, the impacts of the floating platform and internal flow that have not been widely considered in previous studies are discussed to comprehensively understand the VIV of marine risers. Moreover, typical experimental and numerical investigations on the VIV of marine risers are also introduced.

## 1. Introduction

A marine riser system is a key part in the offshore oil and gas production system to connect the floating platform and underwater equipment. However, due to harsh operating environments (such as varying waves and flows), the marine riser is vulnerable to fatigue failures. VIV, generated by the interactions between the platform, current and riser, is the main cause of riser failures. Fig. 1 portrays the VIV of a riser system under internal and external excitations. As a typical fluid-structure coupling phenomenon, the VIV is involved with the response of complex multi-frequency and multi-degree of freedom vibrations. For instance, the simplest form of the VIV is generated by the interaction between an elastically mounted rigid cylinder and the surrounding uniform flow. When the flow passes over a cylinder, an unsteady wake will be generated in the form of shedding vortices. The vortices alternately fall off from both sides of the cylinder, producing a periodically varying lift force along the cross-flow and in-line directions. The VIV is then raised by the lift force (Violette et al., 2007). Because the VIV is a quite complicated nonlinear and asymmetrical phenomenon (Bernitsas et al., 2008, 2009), its developing and behaving mechanisms

are still not clear. In the past decades abundant scientific studies have investigated the mechanisms of the VIV. Some comprehensive reviews can be found in Sumer and Fredsøe (1997) and Williamson and Govardhan (2004, 2008). Furthermore, the VIV problem in the marine riser system has attracted extensive attention in recent years.

Innovative designs have been applied to developing different types of marine risers in order to enable their survival in complex ocean environments. There are four basic riser types: the top tension risers, steel catenary risers, hybrid risers and flexible risers. Among these four types, the steel catenary risers and flexible risers have been adopted in the exploitation of marine oil and gas resources from offshore to deep sea because of their availability to deep-sea environment (Sun et al., 2012). Many studies have been performed to investigate the complex characteristics of the VIV in the flexible and steel catenary risers, where the riser models are usually simplified in the form of a long flexible circular cylinder immersed in the unsteady flow. Due to large scale of marine risers, in practice it is difficult to directly measure the VIV of the risers. To this avail, laboratory experimental test is an effective tool to inspect the VIV of marine risers. Biermann and Herrnstein (1934) conducted experimental research on two circular cylinders in the tandem and

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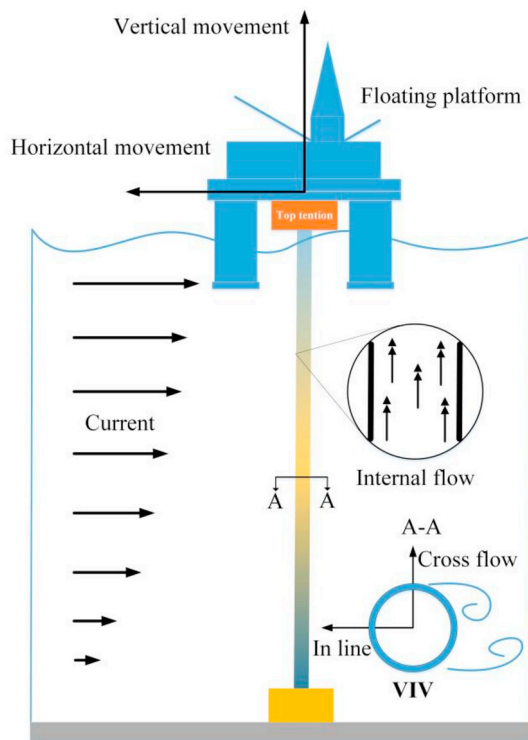


Fig. 1. The VIV of the riser system under internal and external excitations.

side-by-side configurations. The results showed that the proximity interference may have a significant effect on the VIV. In 1955, based on the relationship between the Strouhal number and Reynolds number, Roshko (1955) studied the flow field around the cylinder and defined the regime of the flow field. Feng (1968) conducted typical free vibration experiments and measured the stress and VIV response of a flexibly mounted cylinder. In 1975, in order to study the effect of axial tension on the VIV of marine risers, Hsu (1975) investigated the response of a suspended slender cylinder subjected to the parametric excitation. Followed research focused on the vortex characteristics in order to reveal the root mechanism of the VIV of marine risers. In 1993, Brika and Laneville (1993) studied the vortex wake pattern of the free vibration by testing vibration cables in a wind tunnel. Gu et al. (1994) investigated the vorticity dynamics using the particle image velocimetry (PIV) to control the motion at a cylinder. Then Govardhan and Williamson (2000) found that the upper and lower branches of the amplitude curve corresponded to the two-single and two-pair vortex wake patterns, respectively. To compare the effects of cross-flow and in-line vibrations on the fatigue damage of marine risers, Trim et al. (2005) conducted an experimental study to check the VIV response in a uniform and shear flow, and found that the degree of fatigue damage along the in-line direction was the same as the cross-flow direction.

In order to predict the VIV, semi-empirical models and computational fluid dynamics (CFD) methods have been used. In terms of semi-empirical model based predictions, Birkhoff and Zarantonello (1957) used the van der Pol oscillator equation to simulate the wake motion. Then, Bishop and Hassan (1964) and Harten and Curie (1970) proposed the wake oscillator model that used the van der Pol oscillator equation to simulate the hydrodynamic force of the wake acting on the cylinder. Recently, nonlinear fluid models have been developed by considering the effects of multi-mode interactions. Srinil (2010) proposed a low-order fluid-structure coupling model to evaluate the multi-mode VIV response of flexible cylinders, as well as the multimodal characteristics of flexible risers. In the CFD models, the computing approaches that handle the turbulent flows can be categorized into: direct numerical simulation (DNS), Reynolds-averaged Navier-Stokes (RANS), large eddy

simulation (LES) and discrete vortex methods. Because the axial scale of marine risers is very large, the calculation effort is quite heavy when performing three-dimensional CFD simulation (Liu et al., 2019a). An alternative solution is the strip method. Herfjord et al. (1999) used the strip method to simulate the VIV in deep-water risers. Based on the strip theory, Duan et al. (2018) and Fu et al. (2018) recently developed a new CFD solver, *viv-FOAM-SJTU*, to predict the VIV in marine risers.

Furthermore, in order to pursue a better understanding of the VIV in marine risers, new findings have been obtained in recent years. Compared to the previous results, recent studies show the following changes.

- (1) The VIV models have progressed from one-dimension to two-dimension and then to three-dimension; viz from one-dimensional cross-flow direction to two-dimensional cross-flow and in-line directions, and then towards three-dimensional cross-flow, in-line and axial directions. At arbitrary flow velocities, the amplitude in the cross-flow direction is greater than that in the in-line direction, but the vibration frequency of the in-line direction is nearly twice that of the cross-flow direction. Thus, the fatigue damage along the in-line and cross-flow directions is at the same level and it is necessary to propose a reliable in-line and cross-flow coupled VIV prediction method (Song et al., 2011; Duanmu et al., 2018; Wu et al., 2019). In addition, considering the influence of viscoelasticity on the dynamic response of marine risers, a fully-coupled three-dimensional VIV model is essentially required (Yang et al., 2018).
- (2) The flow field around the risers has been investigated from the uniform flow to the shear flow and the oscillatory flow. When the riser is active, the relative incoming flow speed is usually unstable due to the unstable current and the movement of the floating platform. Hence, the VIV of marine risers often presents complicated features.
- (3) Nonlinear simulation models in the time domain have been found suitable for the calculation of the riser VIV in several cases, including the cross-flow and in-line directions and various current conditions (Thorsen et al., 2017; Xue et al., 2015; Wang et al., 2015; Lu et al., 2019).
- (4) With the widespread use of the long flexible cylinder, the prediction of the riser force and response is an important topic in the VIV research. A unique feature to make the prediction of deep-water risers different from other applications is that the aspect ratio of marine riser is very large (at least  $10^3$ ).

This review summarizes the recent new discoveries on the VIV research. The reminders are organized as follows. In Section 2, recent progress on VIV of marine risers is introduced. Section 3 surveys the VIV analysis techniques. Section 4 draws the conclusions.

## 2. VIV of marine risers

### 2.1. VIV induced by current

VIV of marine risers has been extensively investigated in the past decades. The VIV dynamics of a marine riser are irregularly affected by the non-perpendicular current (Srinil, 2010), leading to multi-mode and high-order harmonic characteristics. This irregularity feature of the VIV is particularly important for investigating real offshore oil and gas production systems because the factors such as the configuration and inclination of the risers, the movement of the floating platform and the internal flow of the risers may intensify the uncertainty of the current. As a result, considerable efforts have been made to understand the VIV in different current conditions.

#### 2.1.1. Multi-mode response

Multi-mode vibration characteristics have been observed in long

flexible risers in previous studies, and the multi-mode characteristics are the main difference between long flexible risers and rigid cylinders; the VIV of the latter exhibits single-mode lock-in vibration. Due to large aspect ratio, the riser may vibrate at high-frequencies and multi-mode conditions. Even in the simplest case where the vertical riser is placed in a uniform flow field, the multi-mode VIV is still possibly generated (Wang and Xiao, 2016). Fig. 2(a) shows the orbital trajectories at different riser positions and Fig. 2(b) shows the vortex shedding in the uniform flow. Useful attempts have been made to address the multi-mode and multi-frequency VIV of a long flexible cylinder that is mounted in different forms (such as curved, inclined, and straight) and immersed in different flows (such as uniform flow, linear shear flow, exponential shear flow, and stepped flow). Trim et al. (2005) carried out a series of riser model tests in uniform and shear flows to investigate the VIV of marine risers. Multi-mode VIV response was observed in the experimental tests and the number of dominating modes in both uniform and shear flows increased with the increase of the flow velocity. Similarly, Lie and Kaasen (2006) carried out a large-scale riser model test in a shear flow to investigate the modes of the VIV response. The experimental results showed an irregular VIV response and the mode number increased with the flow velocity. Moreover, the single-mode VIV response was not observed in the experimental test, i.e., the lock-in VIV response did not occur in the shear flow. Chaplin et al. (2005a) investigated the VIV of a marine riser in stepped flow and found that the VIV response of the riser mainly occurred at the frequency corresponding to the dominating mode. More recently, Srinil (2010, 2011), Pavlovskaja et al. (2016), Song et al. (2016) and Duanmu et al. (2018) have drawn similar conclusions by numerical simulations. The VIV response of a long flexible riser shows multi-mode characteristics no matter how it is placed in the flow field (only except when the angle between the axial direction of the riser and the flow direction of the flow field is very small), and the higher the flow velocity the stronger the multi-mode response.

Moreover, Behara et al. (2011) studied the VIV of the elastically mounted sphere and discovered the spiral vortices. The VIV amplitude caused hysteresis at the beginning of the lock-in state in the spiral vortex shedding mode. Srinil (2010, 2011) investigated the multi-mode

response of a geometrically nonlinear flexible cylinder and observed the nonlinear characteristics of the VIV such as multi-mode lock-in, shifting, sharing and interaction characteristics. Besides, the multi-modal dynamic response of the tensioned beam was highly aperiodic and the effect of Reynolds number on the VIV prediction was reduced, as shown in Fig. 3. Similar results were found by Vandiver et al. (2009) and Duanmu et al. (2018). Accordingly, the oncoming flow profile may affect the cylinder mode characteristics, resulting in different mode response in different flows. For instance, the number of modes of a cylinder in a uniform flow is generally lower than that in a shear flow (Bourquet et al., 2012, 2013b; and Duanmu et al., 2017). Song et al. (2011) and Wang and Xiao (2016) also observed that the dominant mode and the vibration frequencies of excited modes increased with the flow velocity. The multi-mode VIV response was generally controlled by traveling waves, which was positively proportional to the flow velocity (Zhu et al., 2019; Chen and Rhee, 2019). Furthermore, as the aspect ratio increased, the vibration mode of the cylinder transitioned from a single mode to multiple modes (Duanmu et al., 2017). In addition, the Reynolds number and the axial tension of a marine riser affected the multi-mode vibration of the VIV.

Chaos is a generic feature of the multi-mode VIV response in flexible risers. The dominant frequency of the multi-mode VIV is consistent with the Strouhal frequency which refers to the vortex shedding frequency calculated based on the Strouhal number (Song et al., 2011). The multi-mode VIV of a riser is more difficult to predict due to its complex chaotic characteristics. In 2010, Violette et al. (2010) studied the mode conversion and simultaneity of the cylinder VIV using a linear stability method. However, the linear method cannot accurately predict the amplitude due to nonlinear characteristics of the VIV. It is well known that the vibration amplitude of a riser in the primary mode is mainly determined by the relationship between the vortex shedding frequency and the riser natural frequency (Duanmu et al., 2018). If these two frequencies are too close, the resonance will happen to significantly increase the vibration amplitude (Duanmu et al., 2018). Moreover, factors such as the drag amplification, asymmetry and higher harmonics also contribute to the nonlinear characteristics of the VIV. For instance, the drag amplification may increase the deflection of a riser and the

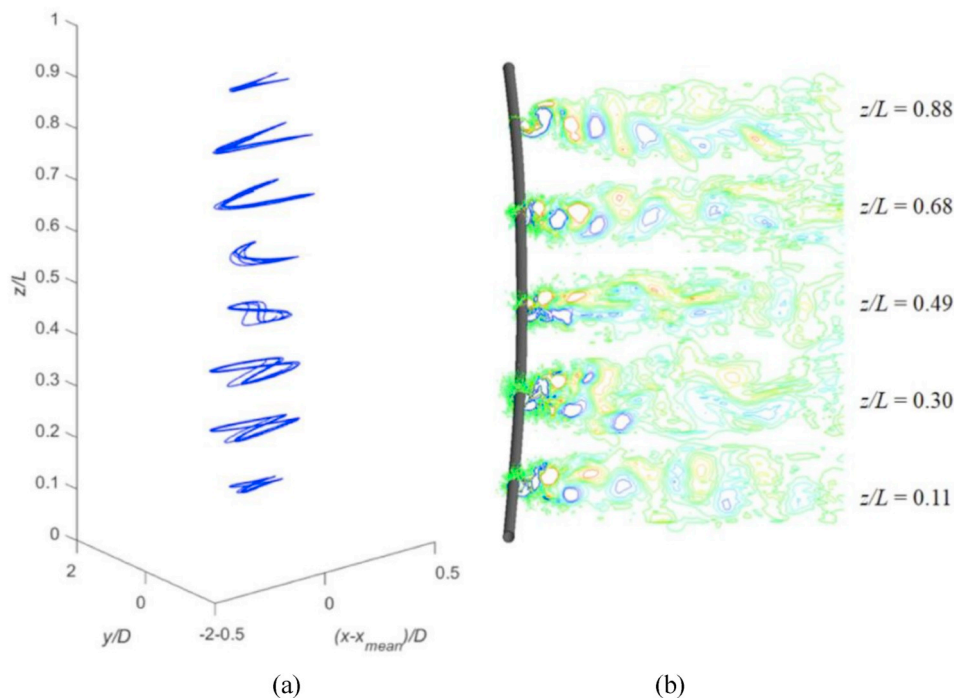


Fig. 2. (a) Comparison of the orbital trajectories at various positions of a riser; (b) Vortex shedding under current profiles; (a) (b): velocity equal to 0.2 m/s in uniform flow (Wang and Xiao, 2016).

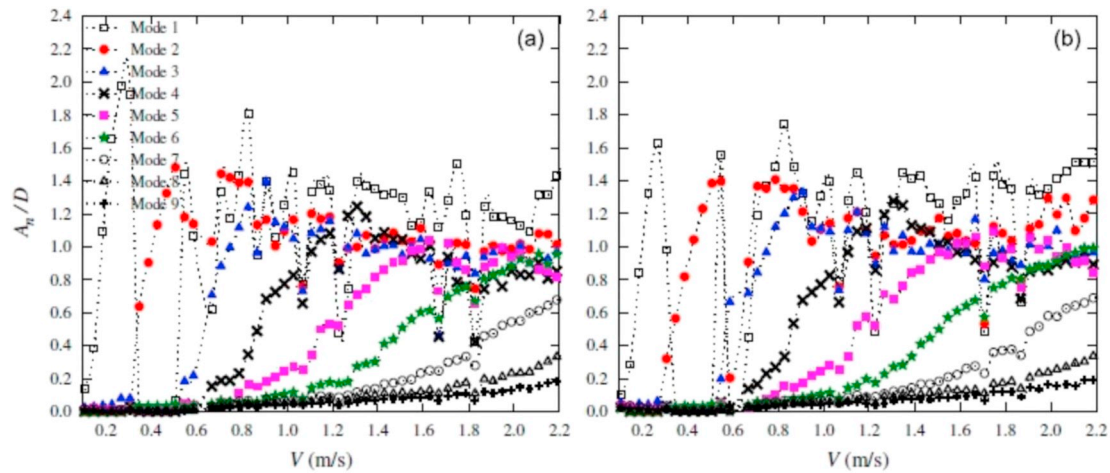


Fig. 3. Maximum modal amplitude diagrams of nonlinear beam with  $N = 9$  ( $n = 1-9$ ), varying  $V$  and models (a) accounting for or (b) neglecting the Reynolds number effect (Srinil, 2010).

loading at both ends of a riser (Huang et al., 2011b; Song et al., 2011). All these factors will enhance the multi-mode characteristics in the VIV response of long flexible risers.

### 2.1.2. Response at high Reynolds numbers

Experimental studies on the VIV at high Reynolds numbers are quite scarce due to huge difficulties in performing experimental investigations. Early in 1997, Kalro and Tezduyar (1997) investigated the VIV characteristics of a cylinder immersed in the uniform flow and the results presented weak three-dimensional features at the Reynolds number of 300. Once the Reynolds number reached up to  $1 \times 10^5$  and even higher, its influence on the synchronization and amplitude of the VIV is much clearer (Raghavan and Bernitsas, 2011). In order to obtain aquatic clean energy from the VIV, the model tests in a wide Reynolds number range were conducted by Raghavan and Bernitsas (2011) and Modir and Goudarzi, 2019. The results showed that high Reynolds numbers (in the range of  $8.00 \times 10^3$  to  $1.50 \times 10^5$ ) impacted the VIV amplitude greater than the mass ratio, and the VIV amplitude highly depended on the natural frequency and lock-in. As a result, a high Reynolds number flow will increase the vortex shedding frequency and the hydrodynamic force on marine risers, and generally increase the amplitude and frequency of the riser VIV.

Traditional method for predicting the VIV amplitude is the Griffin curve (Griffin, 1980). Recently, Williamson and Govardhan (2004, 2008) and Raghavan and Bernitsas (2011) found that the VIV amplitude at high Reynolds numbers was as twice as that predicted by the modified Griffin curve. In the upper branch of the amplitude curve, the synchronization range and amplitude ratio were positively correlated with the Reynolds number. Similarly, Behara et al. (2016) investigated the effects of Reynolds number and reduced velocity on the VIV and found that the riser maintained a synchronous vibration at a Reynolds number between 300 and 1000. Wanderley and Soares (2015) found that the Reynolds number greatly influenced the amplitude, frequency and phase angle of the low mass damping system. The numerical results showed that the wider the Reynolds number range, the higher the amplitude ratio and the closer the response frequency in the lock-in area to the riser natural frequency, as shown in Fig. 4. Similar results can be found in Nguyen and Nguyen (2016) with the Reynolds number ranging from  $3 \times 10^3$  to  $3 \times 10^4$  and in Gao et al. (2015) with the Reynolds number ranging from  $2.5 \times 10^4$  to  $1.8 \times 10^5$ . As a result, the Reynolds number is an important parameter that must be considered in VIV simulations. The influence of high Reynolds numbers on the VIV of flexible risers needs to be further studied.

### 2.1.3. Flow-induced vibration of multiple marine risers

The configuration of multiple marine risers is of great significance for marine oil and gas production systems. The VIV response of multiple marine risers is more complex than that in a single riser. For instance, the two-cylinder system can be configured as tandem, side-by-side and staggered configurations (see Fig. 5). The characteristics of flow fields around these three configurations are quite complicated in terms of flow patterns, fluid forces, vortices, wake and so on. Sumner (2010) reviewed in details the research on two cylinders in the stable cross-flow. Following, many new research has been conducted to address the VIV response in multiple marine risers.

The flow field around two tandem cylinders is influenced by the Reynolds number and spacing ratio ( $L/D$ ), and the vortex interaction mechanism of two tandem circular cylinders is very complicated. In order to study the effect of the Reynolds number and spacing ratio on the flow field around the cylinder and the cylinder vibration, Armina et al., 2018 conducted a series of experimental studies on the tandem configured multiple risers and found that the vibration frequencies of the upstream and downstream risers were different. This observation is contrary to the previous conclusions on the fixed risers. Carmo et al. (2010a, 2010b) divided the flow patterns of the tandem cylinders under low Reynolds numbers into three basic types: (1) symmetric at  $L/D = 1.5$ ; (2) alternating at  $L/D = 1.8$  and  $2.3$ ; and (3) wake at  $L/D = 5$ . Sewatkar et al. (2012) conducted a comprehensive study of flow fields around six square cylinders in the tandem configuration. By investigating the power spectrum and the influence of the secondary frequency on the lift coefficient, they proposed four primary flow regimes with different spacing ratios at the Reynolds number of 100, including the synchronous ( $0.5 \leq L/D \leq 1.1$ ), quasi-periodic-I ( $1.2 \leq L/D \leq 13$ ), quasi-periodic-II ( $1.4 \leq L/D \leq 5.0$ ), and chaotic ( $6.0 \leq L/D \leq 10.0$ ) regimes. They further discussed the underlying mechanisms of these four flow regimes. The flow is synchronized when a single wake is formed behind all cylinders, while the flow transitions to the quasi-periodic I, quasi-periodic II or chaotic regimes when the vortices are generated and shed in the gap of the cylinders. The observations from Huera-Huarte and Bearman (2011) and Huera-Huarte et al. (2016) showed that when the spacing ratio was from 2 to 6, the response of the cylinders displayed classical VIV resonance. Moreover, the upstream cylinder underwent larger vibrations than the downstream cylinder and the VIV amplitude of the upstream cylinder was negatively proportional to the spacing ratio. To investigate the vortex interaction mechanism of the tandem circular cylinders, Borazjani and Sotiropoulos (2009) carried out numerical simulations to study the VIV of two identical elastically mounted cylinders at the Reynolds number of 200 and found that when the reduced velocity was slow, the initial dynamic of the cylinder vibration



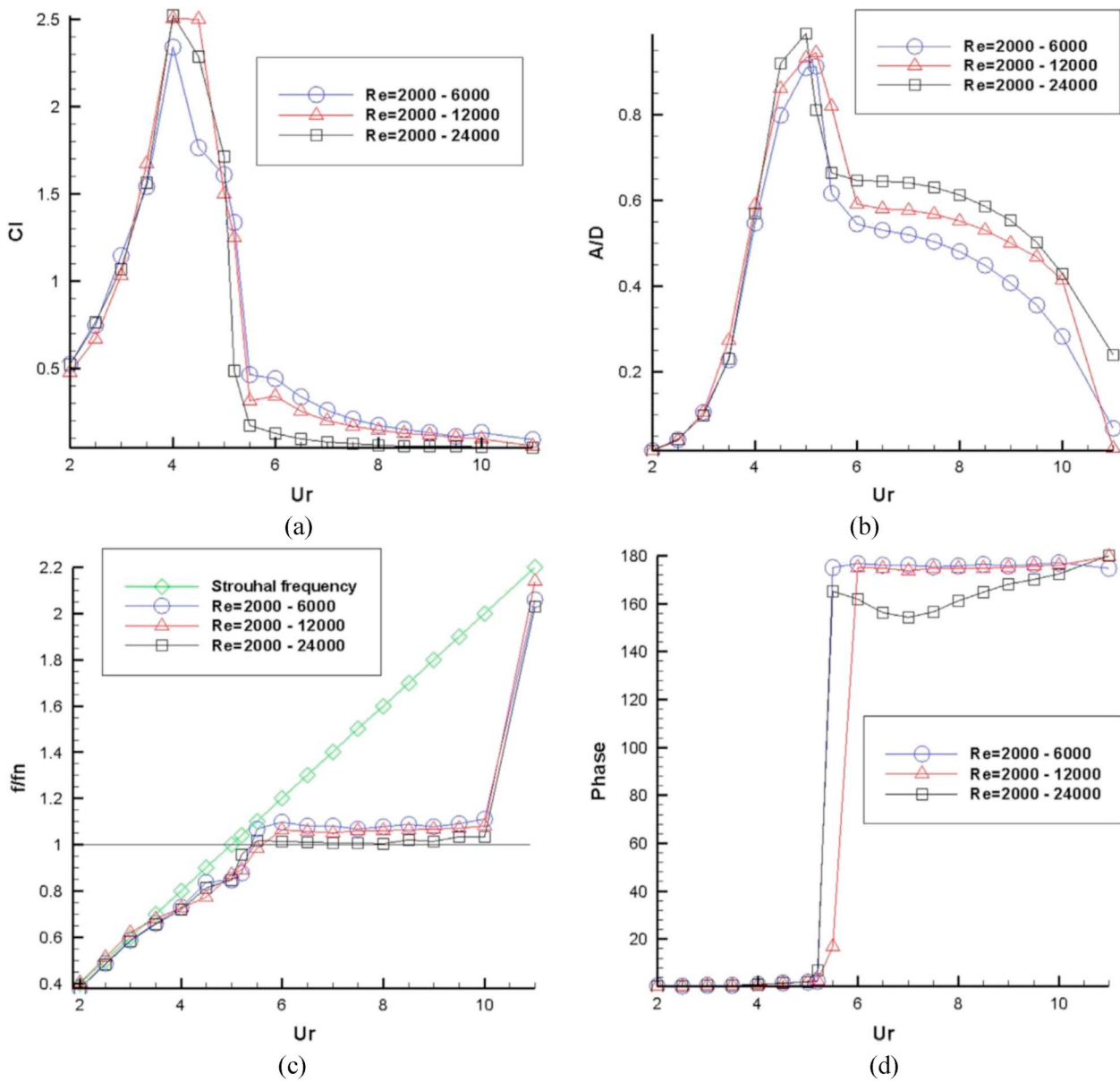


Fig. 4. Response parameters as a function of reduced velocity. (a) Lift coefficient amplitude; (b) Response amplitude; (c) Frequency of vibration; (d) Phase angle (Wanderley and Soares, 2015).

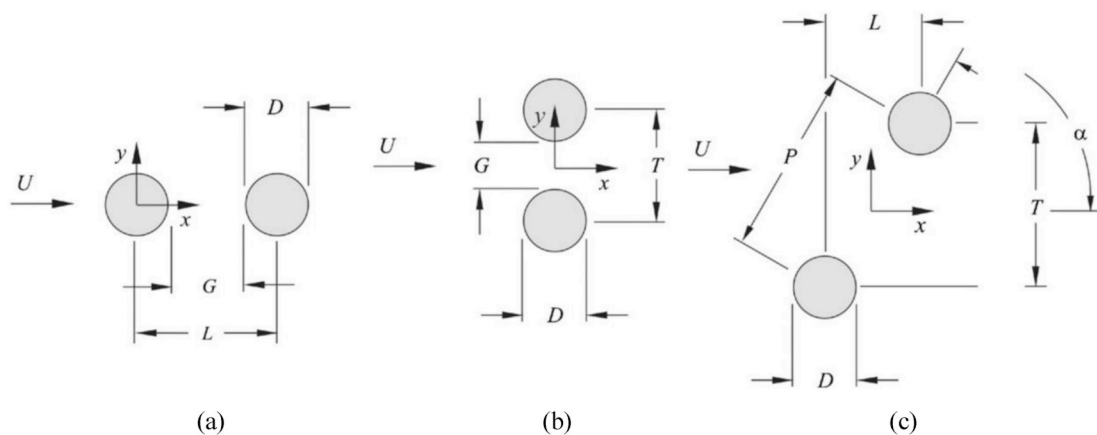


Fig. 5. Two circular cylinders of equal diameter in cross-flow: (a) tandem configuration; (b) side-by-side configuration; (c) staggered configuration (Sumner, 2010).

generated the shedding vortex; when the vertical distance between the two cylinders caused by the initial VIV exceeded the diameter of the cylinder, the gap flow was generated to dominate the vibration of the riser and greatly increase the VIV amplitude. Assi et al. (2010, 2013) stated that the wake-induced vibration of the multiple cylinders cannot be predicted by the classical galloping theory, and hence, introduced the wake stiffness to explain the vibration of the downstream cylinder. While most of existing research has focused on two cylinders, very limited work considered three or more cylinders. Wang et al. (2019a,b,c) carried out an experimental study on three and four long flexible cylinders with tandem configuration and found that the downstream riser produced a small effect on the amplitude of the upstream riser, and the wake induced vibration of the downstream cylinders occurred at a high reduced velocity. Besides, the last downstream cylinder in the three cylinders presented the same in-line and cross-flow main frequency, and the response of the other two cylinders was of dual resonance. Ding et al. (2018) carried out an experimental study on the flow-induced vibration of three tandem cylinders with rough strips. The results showed that the vortex of the downstream riser was disturbed by the upstream riser, leading to a lower amplitude in the downstream riser. In the galloping branch of the amplitude curve, the shear layer motion was consistent with the riser motion, and the amplitude of the first upstream riser is largest among the three risers.

In the side-by-side configuration, the flow pattern between the two risers dominates the riser vibration. There are four flow patterns in the side-by-side configuration (Sumner, 2010; Chen et al., 2015a), i.e., (1) single wake vortex street without vortex shedding at small spacing ratio ( $L/D < 1.1-1.2$ ); (2) biased flow pattern at intermediate spacing ratio ( $1.1-1.2 < L/D < 2.0-2.5$ ); (3) two parallel wake vortex streets at high spacing ratio ( $2.0-2.5 < L/D < 4$ ); and (4) large spacing ratio ( $L/D > 4$  and  $Re = 100$ ) at which the VIV between the two cylinders does not interact with each other and is similar to that of a single cylinder (Zhao, 2013a; Chen et al., 2015b). Moreover, the flow pattern of the VIV in the side-by-side configuration in a uniform flow with low Reynolds numbers can be divided into six near-wake patterns, namely the irregular pattern, the in-phase flip-flopping pattern, the out-of-phase flip-flopping pattern, the in-phase synchronized pattern, the anti-phase synchronized pattern and the biased anti-phase synchronized pattern (Chen et al., 2015a, 2015b; Carini et al., 2014; Bao et al., 2013; Huera-Huarte and Gharib, 2011). Fig. 6 portrays these six flow patterns. For the near-wake patterns, Chen et al. (2015a) further analyzed the asymmetric vibration and symmetry hysteresis phenomena of two side-by-side risers. They found that the asymmetric vibration was related to the stable biased flow and the near-wake mode around the cylinder, and the symmetry hysteresis was induced by the combination of the in-phase synchronized pattern and the biased anti-phase synchronized pattern. For the flow field between multiple risers, Sewatkar et al. (2012a,b) investigated the energy transfer in the flow around six side-by-side cylinders and found that the energy conversion was more intense in the resonance region of the cylinder. Xu et al. (2018) performed the flow-induced vibration experiments of three and four side-by-side cylinders and compared the flow-induced vibration with different number of cylinders. The comparison demonstrated that the dominant frequencies and mode numbers of the three- and four-cylinder systems were the same as those of the isolated- and two-cylinder systems, but the response amplitude and multi-mode response characteristics between the three- and four-cylinder systems were significantly different due to different wake actions.

In reality, the staggered configuration is commonly used in multiple marine risers. However, comprehensive studies on the staggered configuration are relatively few in literature. The relative position of two staggered cylinders can be characterized by non-dimensional center-to-center spacing ratio ( $L/D$ ) and incident angle ( $\alpha$ ) (Sumner et al., 2010). The interaction of the free shear layer and the two Kármán vortex streets in a staggered configuration may induce different flow patterns. Based on the Strouhal number and the flow form, Hu and Zhou (2008) divided

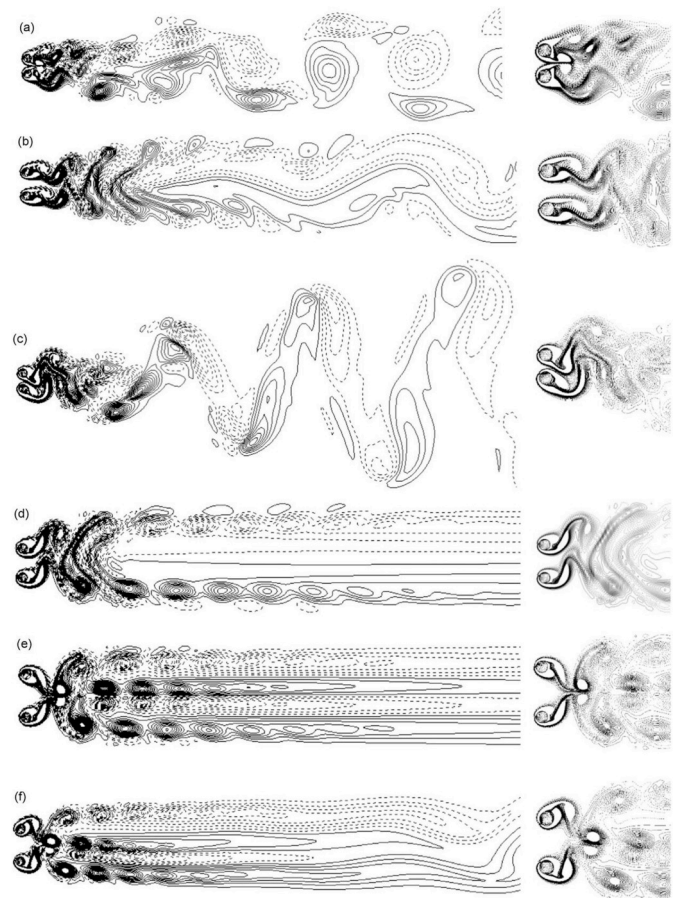


Fig. 6. Wake patterns in two side-by-side cylinders: (a) irregular pattern; (b) in-phase flip-flopping pattern; (c) out-of-phase flip-flopping pattern; (d) in-phase synchronized pattern; (e) anti-phase synchronized pattern; and (f) biased anti-phase synchronized pattern (Chen et al., 2015a).

the wake patterns into two single-street modes (S-I and S-II) and two double-street modes (T-I and T-II). Recently, many new achievements have been made on flow-induced vibration analysis of more than two risers in the staggered arrangement. Han et al. (2018) numerically investigated the flow-induced vibration characteristics of three cylinders in the equilateral-triangular arrangement at different flow incident angles and reduced velocities, as shown in Fig. 7. The results showed that the flow-induced vibration response and the vortex shedding mode were greatly affected by the flow incident angle and the reduced velocity, and the flow-induced vibration reached a minimum amplitude at an incident angle  $\alpha = 30^\circ$ . Gao et al. (2019a,b) simulated the fluid-induced vibration of four cylinders arranged in a square with different inflow angles and reduced velocities. They found that the flow incident angle had great influence on the orbital trajectory and shedding mode of the four cylinders as well as the amplitude and lift coefficient of the downstream risers. In addition, the drag coefficient of all cylinders increased first and then decreased with the reduced velocity. More details with respect to the vibration response of the staggered configuration can refer to Sumner et al. (2005, 2010) and Lee et al. (2009).

#### 2.1.4. VIV of inclined risers

Marine risers are usually inclined to the direction of the incoming current, which makes the VIV more complex than that in the vertical risers. In order to simplify the mathematical model of inclined risers, the independence principle has been proposed to calculate the VIV of an inclined cylinder analogously as that of a vertical cylinder. By projecting the inclined cylinder onto a plane perpendicular to the direction of the incoming flow, the VIV response can be calculated using the excitation

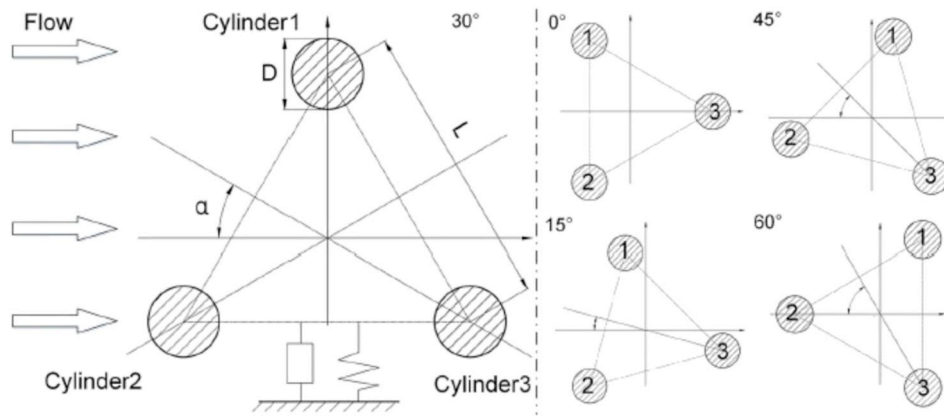


Fig. 7. Simplified physical model of three rigidly coupled cylinders (Han et al., 2018).

of the normal component of the incoming flow relative to the inclined riser. The independence principle has been proven effective for predicting the VIV response and hydrodynamic forces with an inclined angle  $\alpha \leq 65^\circ$ . However, there are also some shortcomings such as that the vibration amplitude of the cylinder is overestimated, and the conversion process cannot be measured from a single frequency to multi-frequency response (Franzini et al., 2013; Jain and Modarres-Sadeghi, 2013; Bourguet et al., 2015; Zhao, 2015). Recently, Bourguet and Triantafyllou, 2015 investigated the VIV response of a flexible cylinder with different inclined angles (see Fig. 8). The results showed that the vibration and fluid force of the inclined flexible cylinder deviated from the independence principle while the independence principle greatly underestimated the frequency of the VIV and failed to capture the asymmetry of the riser vibration. In addition, Bourguet et al. (2015) confirmed that the effectiveness of the independence principle can be affected by the bending of the cylinder along the in-line direction. As a result, the independence principle reduces the calculating cost but decreases the prediction accuracy of the VIV of inclined risers.

The VIV frequency ratio of in-line and cross-flow directions in the inclined riser is approximately equal to 2, which is similar to that of the vertical riser (Bourguet et al., 2012, 2015a). In order to study the intensity of the VIV response for the inclined risers, Franzini et al. (2009) and Seyed-Aghazadeh et al. (2015) carried out inclined cylinder experiments and observed that the oscillation amplitude, hydrodynamic forces and added mass of the inclined cylinder were smaller than that of the vertical cylinder. Han et al. (2017) conducted an experimental test to investigate the dynamic characteristics of an inclined flexible cylinder with the inclined angle  $\alpha = 45^\circ$ . They observed that the VIV response

presented obvious multi-mode characteristics, the dominating modes ranged from 1 to 3 in the cross-flow direction and 1 to 5 in the in-line

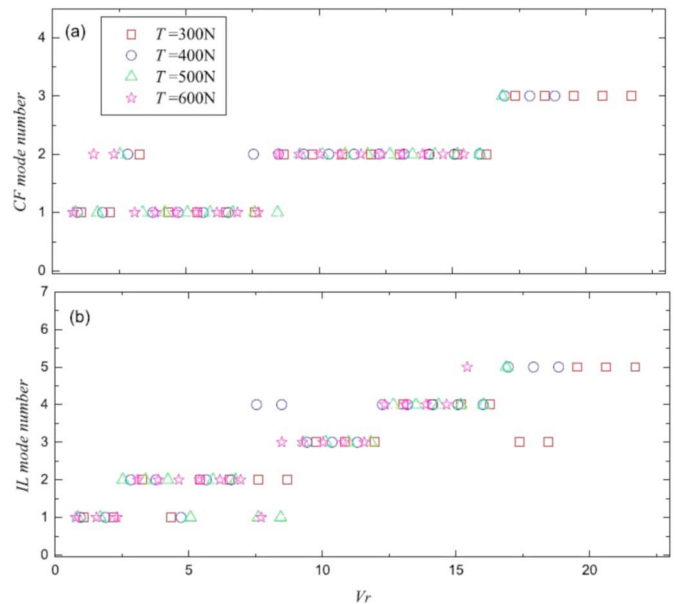


Fig. 9. Dominant mode Number with different flow velocity in the (a) cross-flow and (b) in-line directions (Han et al., 2017).

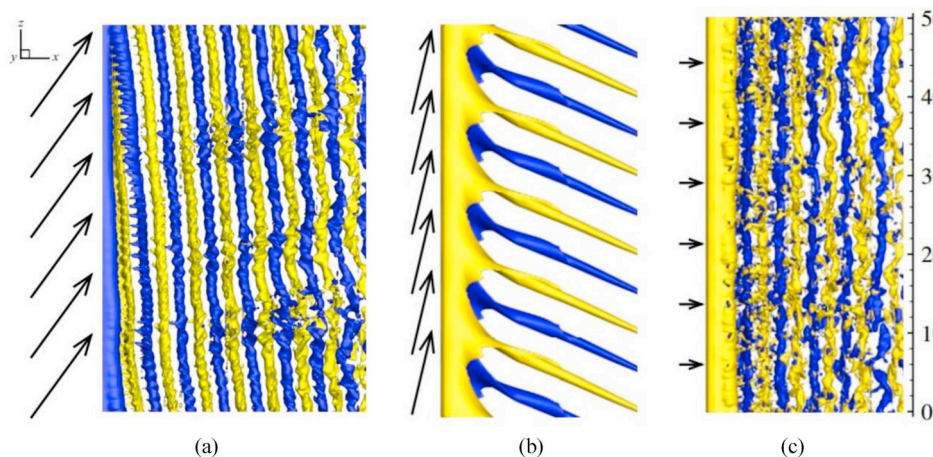


Fig. 8. Instantaneous iso-surfaces of the spanwise vorticity downstream of a stationary cylinder inclined at (a)  $60^\circ$ , (b)  $80^\circ$ , (c)  $0^\circ$ . (Bourguet et al., 2015; Bourguet and Triantafyllou, 2015).



direction (see Fig. 9), and the dominating frequency was linearly correlated with the flow velocity (see Fig. 10). Moreover, the results also showed that the drag coefficient was between 0.9 and 2.6, rather than being dispersed like in a vertical cylinder.

### 2.1.5. Intermittent VIV in oscillatory flow

Due to large amplitude motions of the floating platform caused by the ocean wave and current, an oscillatory flow around the marine risers may take place, leading to vortex shedding and intermittent VIV. A basic feature of the VIV response in an oscillating flow is that the response is strongly dependent on the Keulegan-Carpenter (KC) number and reduced velocity. The dominant vibration frequency of the intermittent VIV is positively correlated with KC number and negatively correlated with the reduced velocity in general. In the lock-in situation, when the KC number increases, the lock-in area will reach a stable point and expand with the increase of the maximum shedding frequency; when the maximum reduced velocity increases, the lock-in area widens. With the same reduced velocity, large amplitude of the VIV appears at a low KC number (Zhao, 2013b; Fu et al., 2013; Thorsen et al., 2016, 2017; and Yuan et al., 2018). Furthermore, Wang et al. (2015a) revealed that both the KC number and maximum reduced velocity produced a profound influence on the riser fatigue damage, which is a unique characteristic of intermittent VIV caused by the oscillatory flow. With the same maximum reduced velocity, the VIV amplitude and fatigue damage were negatively correlated to the KC number.

There are numerous different characteristics between the oscillating flow and the steady flow on the VIV of marine risers, including the amplitude modulation and high-order harmonics. As shown in Fig. 11, the phenomena of the amplitude modulation and high-order harmonics appear in the intermittent VIV under the oscillatory flow (Wang et al., 2015). The amplitude modulation would become inconspicuous at a large reduced velocity and the high-order harmonics was approximately three times of the maximum vortex shedding frequency. Hysteresis and mode conversion, as observed in a steady flow in Fig. 11, occurred in the oscillating flow. The mode conversion occurred if the reduced velocity was large enough (Fu et al., 2013, 2018). Wang et al. (2015a) carried out an experimental test to study the fatigue failure of flexible risers caused by the intermittent VIV, and found that the amplitude modulation and mode conversion had a significant impact on the riser fatigue damage. Furthermore, a sharp increase about two orders of magnitude in fatigue

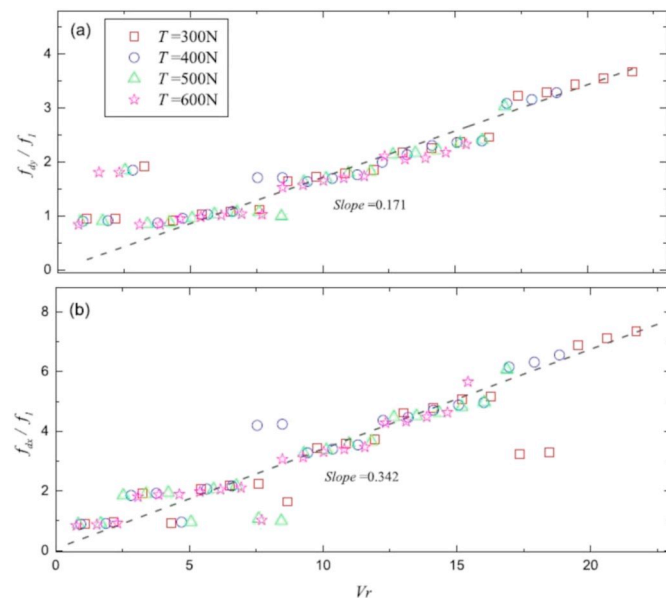


Fig. 10. Dominant frequency with different flow velocity in the (a) cross-flow and (b) in-line directions (Han et al., 2017).

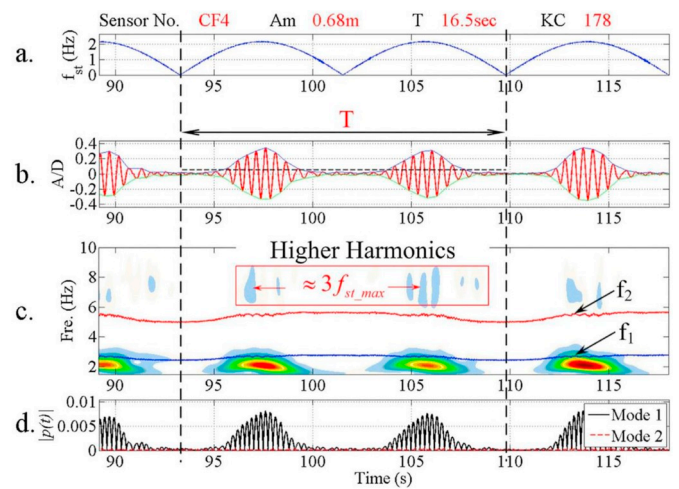


Fig. 11. VIV response test results at  $T = 16.5s$ ,  $KC = 178$ . (a) The shedding frequency; (b) Amplitude ratio; (c) Response frequency; (d) Modes (Wang et al., 2015).

damage was discovered in the region of high-order harmonics, and the damage increased by nearly 50% compared to that caused by a single harmonic stress (Zheng et al., 2014).

## 2.2. VIV affected by floating platform and internal flow

### 2.2.1. VIV affected by floating platform

As marine oil mining activities extend to deep sea areas (Liu et al., 2019b), the form of the riser system is quite different from the traditional fixed platform. The movement of the floating platform becomes complicated, and the dynamic coupling between the platform and underwater risers is strengthened, causing new problems such as additional lock-in regions, parameter excitation and nonlinear amplification (Garrett, 2005; Yang et al., 2013; Chen et al., 2015; Fan et al., 2017). The platform motions can cause the VIV of marine risers with high-order modes and large vibration amplitude. Recently, Yin et al. (2019) studied the influence of the platform motions on the vibration of the riser. Interestingly, they found that the in-line vibration caused by the platform motions was stronger than that of the cross-flow VIV caused by the oscillatory flow.

Furthermore, the dynamics and vibration response of the VIV affected by the floating platform are very different from that without considering the movement of the floating platform because of horizontal and vertical platform motions (Chen et al., 2015). The horizontal movement of the platform, such as sway and surge, would spread along the riser toward to the bottom and interact with the VIV of the riser, then further cause the nonlinear coupling on the motion boundary (Chen et al., 2014). In addition, the vertical/heave motion mainly affects the dynamic tension of the riser, which periodically changes the structure parameters of the riser, resulting in the problem of parameter excitation. Moreover, compared with the horizontal movement of the platform, the vertical motion can cause dangerously dynamic response on the riser.

For the horizontal motion, the floating platform may suffer from wave induced motions. The phenomena such as the oscillation flow and top-end sway would cause the intermittent VIV of marine risers. When the floating platform sways, the displacement of the riser will be enlarged. Chen et al. (2015) observed the nonlinear response amplification phenomenon, which indicated that the platform motion could be amplified during the motion spreading along the riser. Moreover, the displacement of the riser would increase with the swaying amplitude; both the displacement and response amplification would increase with the drop of the sway frequency. Wang et al. (2016) also found obvious drag amplification caused by the horizontal motion under small KC



number, and the experiment results showed that the platform motion-induced VIV presented strong time consistency in terms of both amplitude and frequency, as shown in Fig. 12.

As for the vertical movement (heave) of the platform, the foremost problem is the fluctuation in the axial tension of the riser, which can cause varies of the geometric stiffness. Although the fluctuation is greatly suppressed by the heave compensators, the linear balance of the riser may be broken down, resulting in intensified VIV. Moreover, when the vibration amplitude of the platform is sufficiently large, the tension may be transformed into pressure at a certain part of the riser to produce dangerous dynamic stresses (Kuiper et al., 2008). Varying structural parameters can also significantly affect the modal properties of the VIV. Focusing on the VIV of a flexible cylinder with axially varying structural parameters, Sanaati and Kato (2012) found that the fluctuating tension ratio and lock-in bandwidth of the VIV increased with the axial stiffness while decreased with the pre-tension. In addition, high pre-tension can greatly enlarge the lift force coefficient and reduce the vibration amplitude. In their experimental investigation, when the pre-tension increased by four times, the lift coefficient increased by about 57% while the vibration amplitude reduced by about 30%. Yang and Xiao (2014) further discovered that the pre-tension was effective in suppressing the instability of marine risers and preventing extreme vibrations.

In order to completely understand the effect of the floating platform motion on the VIV of marine risers, Wang et al. (2014, 2015b, 2016) carried out a lot of experimental investigations. The analysis results showed good agreement with earlier observations on flexible risers in oscillatory flow, and the VIV caused by the movement of the floating platform was strongly associated with the KC number. Wang et al. (2015b) stated that due to extensive changes in KC number and the tension, the time-varying characteristics of the system dynamics were more obvious with larger movement of the platform. In addition, the fatigue damage caused by the VIV in the cross-flow direction was

significantly influenced by both the amplitude and period of the platform movement. The most severe damages occurred in the touchdown point, the upper bend point and the top of the riser. Recently, in order to predict the VIV induced by the floating platform for a free-hanging riser under low KC number, Wang et al. (2019a,b,c) proposed a new numerical method. This new method was implemented in VIVANA and validated by experimental tests to show good performance in predicting the frequency, stress and amplitude of the VIV.

Generally, the horizontal and vertical movements of the platform never occur independently. If take both movements into consideration, not only a time-varying moving boundary and periodically parametric excitation but also a nonlinear dynamic response would be imposed to a marine riser. Consequently, a small platform movement may result in a large response of the riser accompanied by complex VIV.

### 2.2.2. VIV affected by internal flow

In addition to the external excitation loads such as the hydrodynamics forces and floating platform motions caused by the wind, wave and current, most of the VIV studies neglect the effect of the internal flow within the marine risers. Due to large aspect ratio and flexibility of marine risers, the effect of the internal flow cannot be neglected in the VIV analysis (Wu and Lou, 1991; Lee et al., 1995; Guo and Lou, 2008).

When the fluid flows within the riser, the centrifugal and Coriolis forces are generated by the relative movement between the fluid and the riser. In the past decade, the influence of the centrifugal and Coriolis forces has been explored in the VIV analysis (Meng et al., 2018; Chatjigeorgiou, 2010a; Montoya-Hernández et al., 2014). On the one hand, the effect of the centrifugal force is manifested as a pressure load, which can reduce the tension and natural frequency of the riser. The centrifugal force also can amplify the VIV in unstable regions, where more complex flow patterns would be induced (Meng et al., 2018). On the other hand, the Coriolis force introduces an extra damping component to narrow the resonance regions of the riser; but it also complicates the coupling

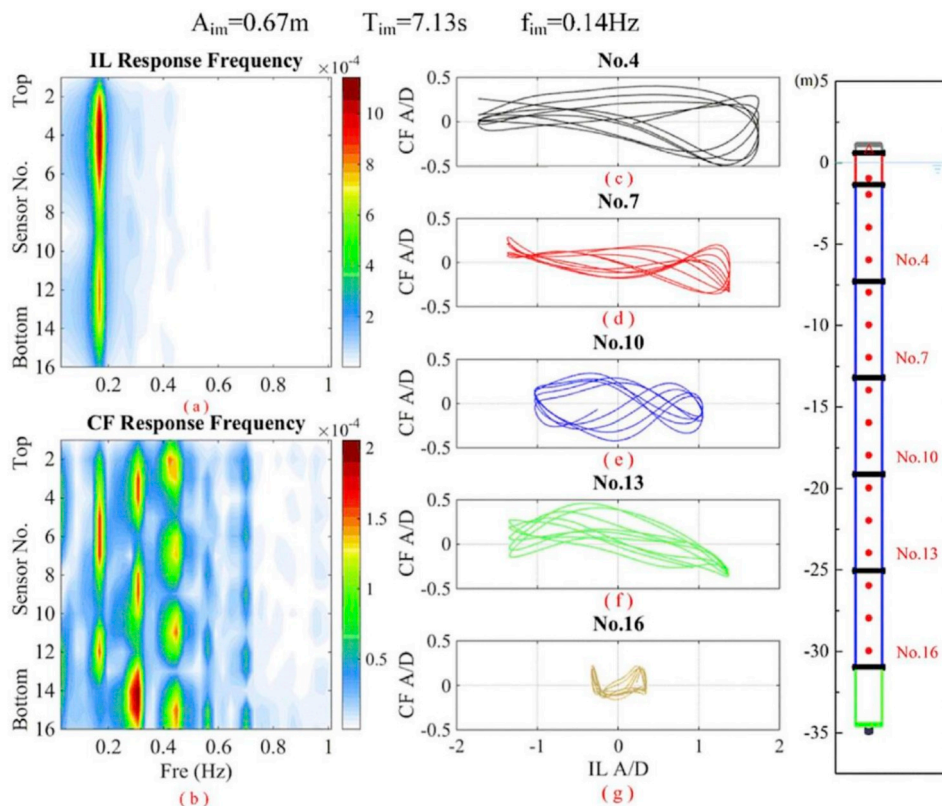


Fig. 12. Response frequency and section trajectory: (a) in-plane response frequency along the riser; (b) out-of-plane response frequency along the riser; (c–g) section trajectory at different stations. (Wang et al., 2016).

resonance. As a result, the VIV in the cross-flow direction may be affected while the in-line dynamics remain because of the damping effect of the Coriolis force (Meng et al., 2017a, 2017b).

The internal flow may greatly influence the dynamic response of marine risers in the forms of the centrifugal and Coriolis forces. Because the internal flow consists of water, oil and gas, the fluid density changes rapidly. Hence, the flow velocity and volume ratio between different fluid phases can cause fluctuations in mass, momentum, and pressure to increase the complexity to the VIV (Montoya-Hernández et al., 2014). In order to simplify the investigation of the impact of the internal flow on the VIV, the plug-flow concept was proposed to assume a single phase of the internal flow such that the flow velocity can be fixed (Chatjigeorgiou, 2010a, 2010b; Meng et al., 2017b). Under this assumption, the VIV analysis is significantly simplified. For two-phase (see Fig. 13) or multi-phase internal flow, the flow inside the riser can be divided into “plug-flow with bubbles”, “slug-flow”, “churn-flow”, “annular flow” and “wispy annular flow” (Ortega et al., 2013). The influence of the slug-flow was assessed by Chatjigeorgiou (2017). The slug-flow did not change the vibration patterns but it can greatly enlarge the vibration intensity. Moreover, Montoya-Hernández et al. (2014) treated the multi-phase internal flow as a pseudo-single phase flow and developed a homogenous model for multi-phase flow to investigate the natural frequency of marine risers. The results showed that the velocity and density of the internal flow greatly affected the riser response. Thorsen et al. (2019) numerically studied the influence of the internal flow density on the VIV of the offshore risers. The research results showed that when the wave length of the internal flow was close to the ratio of the riser length to the number of the VIV dominant modes, the fatigue damage of the riser was greatly worsened.

As internal flow velocity increases, the strain value of cross-flow and in-line response increases while the frequency decreases (Guo and Lou, 2008). Dai et al. (2013) and Meng et al. (2017a, 2017b) investigated the VIV of marine risers when the internal flow rate increased from subcritical to supercritical regions. It was found that (1) at low flow speed the internal flow absorbed the energy from the riser, resulting in significant reduction of the VIV; (2) in the subcritical region, the increase of the flow velocity may reduce the riser natural frequency, increase the vibration amplitude and switch the vibration modes; (3) in the supercritical region, the riser may lose stability and experience various motions including chaotic motion. Recent studies have found that when the inflow velocity is below the critical velocity, the riser response is dominated by the external currents; when the inflow velocity exceeds the critical value, three dimensional quasi-periodic and chaotic response are generated in both the cross-flow and in-line directions (Liu et al., 2019; Duan et al., 2018). In addition, it should emphasize that,

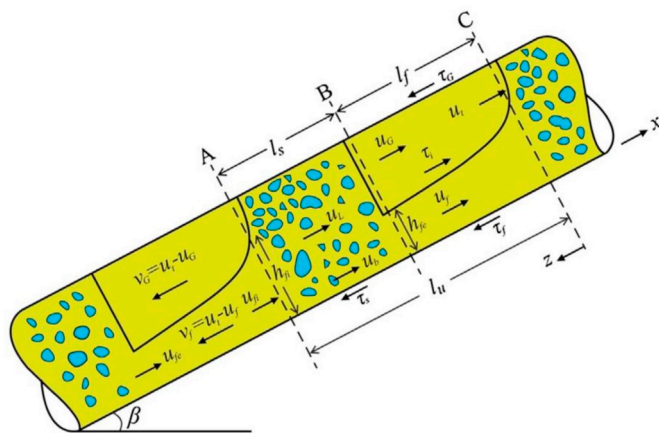


Fig. 13. The geometry of the slug-flow.  $l_u$  means the length of the slug unit;  $l_s$  means the length of the slug;  $l_f$  means the length of the liquid film (Chatjigeorgiou, 2017).

compared to previous work in the steady flow, the dynamic response are larger in the fluctuating flow (Dai et al., 2014a, 2014b).

In addition, the internal flow effect in the riser systems varies with different support modes, such as the cantilevered riser (Meng et al., 2017a, 2017b; Chatjigeorgiou, 2010a), the pinned-pinned riser (Guo and Lou, 2008), the hinged-hinged riser (Dai et al., 2013) and the clamped-clamped inclined risers (Alfossail et al., 2016). Hence, the influence of boundary conditions on the internal flow effect is worth investigating.

Besides the floating platform and internal flow, the effect of the surface roughness (Gao et al., 2015), the materials of structures (Xue et al., 2014) and the riser-soil interaction at touchdown zone (Wang et al., 2015) are also exciting research points.

### 3. VIV analysis approaches

The research approaches for the VIV analysis of marine risers can be divided into experimental investigation and numerical simulation groups. These two groups have been widely used to investigate the VIV in recent years.

#### 3.1. Experimental investigation

The experimental investigation is the most important tool to study marine risers. Due to the complexity of the environment and the chaotic response of marine risers, the experimental tests can directly provide the VIV response of real marine risers in the sea environments (Modarres-Sadeghi et al., 2011). However, limited to the experimental equipment, a very few experimental tests have been conducted in real seas while most of existing experiments have been performed in the laboratory. The experimental investigation on the marine risers generally includes the full-scale tests and scale tests.

##### 3.1.1. Full-scale test in the real ocean environment

Full-scale experiment tests were carried out using long riser models in the real seas. In 2005, Vandiver et al. (2005) conducted a riser VIV test with an aspect ratio up to 3673 at Seneca Lake in New York. The aspect ratio of the riser model was close to an actual marine riser. Since the boundary conditions of the test riser model were not completely controllable and the relative flow fields outside the riser was very complicated, the VIV was difficult to analyze using the experimental data. Srivilairit et al. (2009) monitored the riser acceleration and ocean current velocity at a depth of 1000 m in the deep sea drilling and analyzed the full-scale measurement data to calculate the VIV of the riser. To our best knowledge, the full-scale experiment tests have not reported much in literature.

##### 3.1.2. Scale test in the laboratory

The costs of the full-scale experiment tests are often very high and the experimental conditions are uncontrollable. To overcome these shortcomings, the scale tests in the laboratory are widely employed. In the scale tests an elastically mounted rigid riser is often restricted to the transverse flow in the towing tank or water tunnel (Chaplin et al., 2005a). Table 1 summarizes the scale tests in recent years. It can be seen that no one has taken all factors (such as the large aspect ratio, low mass ratio, and high Reynolds numbers) into account in a single scale test. As Chaplin et al. (2005a) suggested that in order to maintain a representative ratio of the high mode wavelength to its diameter, the aspect ratio of the riser model should not be less than 500. This aspect ratio is actually too large to be achieved using existing experimental equipment, and only a few tests can reach this standard such as 4198 in Vandiver et al. (2009), 1750 in Song et al. (2011), and 940 in Morooka et al. (2013).

In the scale tests, the material and structure of the riser model and the Reynolds number of the flow are the key factors that influence the VIV characteristics. Xue et al. (2014) found that the material of the riser

**Table 1**

Selected recent experimental studies of VIV. L/D = aspect ratio,  $m^*$  = mass ratio, dof = direction of VIV, Re = Reynolds number, A/D = amplitude ratio, SG = strain gauges, AC = accelerometers, FOSG = fiber optic strain gauges, FBG = fiber bragg grating strain sensors, U = uniform flow, SH = shear flow, ST = stepped flow, O = oscillatory flow.

Researchers	Type	Material	Instrumentation	Flow	L/D	$m^*$	dof	Re	A/D
Trim et al. (2005)	simple	fibre glass	SG, AC	U, SH	1400	/	CF, IL	/	0.5
Chaplin et al. (2005a)	simple	phosphor-bronze, fluoroplastic	SG, AC	ST	469	3.0	CF, IL	/	/
Lie et al. (2006)	simple	/	transducers	SH	3000	3.13	CF, IL	/	0.25
Franzini et al. (2009)	inclined	plexiglas	load cell, laser position sensor	U	18	2.5	CF	$2.0 \times 10^3$ – $8.0 \times 10^3$	0.7
Vandiver et al. (2009)	inclined	fiber glass	FOSG	U	4198	/	CF, IL	/	/
Raghavan et al. (2011)	simple	aluminum, polymer	/	U	6–14.4	/	CF	$8.00 \times 10^3$ – $1.50 \times 10^5$	1.9
Huera-Huarte and Gharib (2011)	tandem	aluminium, PVC	SG, DPIV	U	93.75	1.8	CF, IL	$<1.20 \times 10^4$	1.1
Huera-Huarte and Bearman (2011)	side-by-side	aluminium, PVC	SG, laser sensor	U	93.75	1.8	CF, IL	$<1.20 \times 10^4$	1.1
Huera-Huarte et al. (2014)	simple	brass, foam	SG	ST	158	1.1	CF, IL	$<3.74 \times 10^4$	3
Huera-Huarte et al. (2016)	tandem	brass, PVC	SG	ST	187	/	CF, IL	$<2.20 \times 10^4$	1.5
Song et al. (2011)	simple	steel	FOSG	U	1750	1.0	CF, IL	$3.0 \times 10^3$ – $1.0 \times 10^4$	2.0
Fu et al. (2013)	simple	/	FBG	O	167	/	CF, IL	$6.22 \times 10^3$ – $1.22 \times 10^4$	0.58
Gao et al. (2015)	roughness	steel, rubber	SG, force transducer	U	48.32	1.90	CF, IL	$2.5 \times 10^4$ – $1.8 \times 10^5$	1.2
Song et al. (2016)	simple	steel, plastic	FBG	U	263	2.5	CF, IL	/	1.1
Goncalves et al. (2012)	short	/	laser position sensors	U	1.0–2.0	1.0–2.62	CF, IL	$1.0 \times 10^4$ – $5.0 \times 10^4$	1.5
Morooka et al. (2013)	inclined	rubber, silicone	AC	U	940	1.25	CF, IL	$0.4 \times 10^3$ – $0.6 \times 10^3$	1
Chen et al. (2015)	inclined	steel, foam, PVC	AC, hot-wire anemometry	SH	144.7	/	CF, IL	$2.1 \times 10^4$	/
Han et al. (2017)	inclined	copper, silicon	load cell, SG	U	350	1.9	CF, IL	$0.80 \times 10^3$ – $1.6 \times 10^4$	3

affected the VIV response and fatigue damage in many ways. For example, despite vibrating in high-order modes, aluminum risers may have low natural frequencies and fatigue stress than these of the conventional steel risers. In order to obtain the large aspect ratio, the length of the cylinder is usually much larger than the diameter. However, this cylinder design requires high bending stiffness and tow speed, which is difficult to realize. To solve this problem, various riser models have been designed, e.g., the cylinder models made of plexiglass (Franzini et al., 2009) and fiber glass (Vandiver et al., 2009), aluminum metal (Raghavan et al., 2011) and steel (Song et al., 2011). A protective layer such as foam or PVC (Polyvinyl chloride) layer is wrapped to seal the cylinder model and to eliminate the protrusions of the riser model (Huera-Huarte and Bearman, 2011; Huera-Huarte and Gharib, 2011; and Huera-Huarte et al., 2014).

Similarly, because of the limitation of equipment, the Reynolds number in the laboratory experiments is not as close to the actual value as in practice. Recently, Raghavan et al. (2011) and Gao et al. (2015) achieved a Reynolds number in laboratory experiments as high as  $1.50 \times 10^5$  and  $1.80 \times 10^5$ , respectively.

In laboratory the uniform flow and oscillatory flow are easily prepared by changing the velocity of the current in the water tunnel and the motion of the model in the towing tank (e.g., Fu et al., 2013). In the uniform flow, the riser model is usually placed transversely and completely submerged in the water. Recently, Capell et al. (2019) studied the VIV characteristics of a cylinder near the water surface, and found a vibration region with two vibration modes (i.e., the lock-in vibration and non-zero vibration). To get the shear flow, Trim et al. (2005) used a crane to tow the rig in one direction to make a linear shear current, as shown in Fig. 14(a). Lie et al. (2006) connected the riser model to a floating vessel and pulled the device using a rope system. A

buoyancy device was used to maintain constant tension for the riser model, as shown in Fig. 14(b). The riser was exposed to a triangular current distribution by moving the vessel at a constant velocity. Chen et al. (2015) generated the shear flow in a wind tunnel using spires that were mounted in front of the cylinder model. As for the stepped flow, it is achieved by immersing the lower part of the cylinder in the uniform flow while keeping the upper part in still water (Chaplin et al., 2005a; Huera-Huarte et al., 2014) as shown in Fig. 14(c).

### 3.2. Numerical simulation

There is no doubt that the VIV has a great impact on the fatigue damage in the marine risers. Therefore, it is necessary to establish a model that can accurately predict the VIV. Nevertheless, the VIV prediction of deep-water risers is difficult because of the unsteady flow and extremely complicated fluid-structure interaction. There are two main ways to predict the VIV, i.e., the semi-empirical approach and CFD. Based on the structure of marine risers and empirical hydrodynamic parameters obtained from experiments, the semi-empirical approach is able to establish a mathematical model to predict the VIV. CFD can calculate the hydrodynamic force distributed along the structure to predict the VIV of the riser (Thorsen et al., 2017). The key parameters that are essential to calculate the VIV are given in Table 2.

In 2005, Chaplin et al. (2005b) compared the VIV predicted results of eleven different numerical models with the experimental measurements in the stepped flow. The results showed that the empirical models were more accurate than the CFD in predicting the cross-flow displacement and curvature; the in-line curvature induced by the vortices cannot be accurately predicted by neither empirical nor CFD models. Wanderley et al. (2008) obtained the similar conclusion that CFD was ineffective in



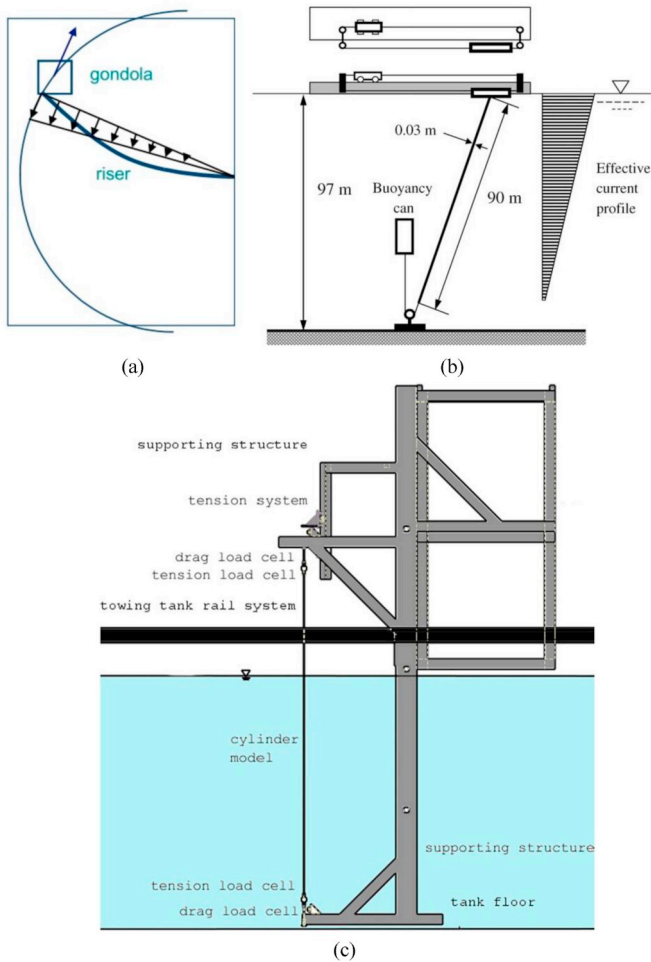


Fig. 14. Test schematics (a) shear flow by Trim et al. (2005); (b) shear flow by Lie et al. (2006); (c) stepped flow by Huera-Huarte et al. (2014).

Table 2

Key parameters.

Parameters	Symbol	Expression
Mass ratio	$m^*$	$\frac{m}{\pi \rho D^2 L / 4}$
Damping ratio	$\xi$	$\frac{\sqrt{k(m+m_A)}}{f_N D}$
Velocity ratio/reduced velocity	$U^*$	$\frac{U}{f_N D}$
Amplitude ratio	$A^*$	$\frac{A}{D}$
Frequency ratio	$f^*$	$\frac{f}{f_N}$
Streamwise force coefficient (the vortex drag coefficient)	$C_d$	$\frac{F_x}{\frac{1}{2} \rho U^2 DL}$
Transverse force coefficient (the vortex lift coefficient)	$C_L$	$\frac{F_y}{\frac{1}{2} \rho U^2 DL}$
Reynolds number	Re	$\frac{\rho U D}{\mu}$
aspect ratio	/	$\frac{L}{D}$

predicting the VIV amplitude at the low mass ratio condition due to the diffusive characteristics of the numerical approaches. With rapid development of numerical calculation techniques in recent years, the numerical simulation results nowadays can be used to qualitatively and quantitatively predict typical VIV characteristics of marine risers.

### 3.2.1. The semi-empirical approach

The semi-empirical approaches mainly include the wake oscillator

model and semi-empirical model. Since Bishop and Hassan (1964) presented the idea that using a wake oscillator to simulate the wake motion around a cylinder and Hartlen and Currie (1970) established the wake oscillator model, a large amount of wake oscillator models based on the van der Pol oscillator equation have been proposed. The oscillator models possess good properties such as self-excited, self-limiting and natural frequency that satisfy the Strouhal relationship (Gu et al., 2012). While for semi-empirical models, because they are based on the data acquired from forced oscillation experiments, these models can reliably predict a certain characteristics of the riser VIV. Furthermore, because semi-empirical models are able to address the three dimensional flow field conditions, they can be applied in practice (Srinil, 2010).

Based on the semi-empirical method, abundant nonlinear fluid models have been developed in recent years to solve the overestimation issue of the VIV amplitude that exists in the linear models and to consider the effect of the multi-mode interactions. Srinil (2010) proposed a low-order fluid-structure coupling model to evaluate the nonlinear multi-mode VIV response of flexible curved and linear cylinders. And then, Srinil (2011) developed a time-domain analysis model focusing on the VIV prediction of the cross-flow direction for the variable-tensioned risers in the linear shear flow. Aiming to study the VIV of marine risers under nonlinear conditions, Thorsen et al. (2015, 2017) combined a semi-empirical model with a nonlinear finite element model to simulate the dynamic response of a marine riser in the oscillatory flow. Recently, Gao et al. (2019a,b) proposed a VIV prediction method for a deep-sea long flexible riser based on a wake oscillator model, and found that the vibrations at the end and middle of the riser in the shear flow were dominated by the standing and traveling waves.

Relying on experimental data, the time-domain analysis method has been used to develop the VIV prediction models. By comparing with previous experimental studies, the time-domain analysis method can accurately predict the VIV in the steady flow and unstable flow (Wang et al., 2015; Lu et al., 2019). According to the theory of energy equilibrium, Xue et al. (2014, 2015) put forward a time domain prediction model for cross-flow and in-line vibrations of a marine riser under stepped and shear currents. The influence of the maximum shear current velocity, top tension, internal fluid density and structural material on the riser fatigue damage was discussed. Pang et al. (2019) and Lu et al. (2019) also proposed different time domain prediction methods to fit the experimental data, and predicted the vibration response and modal characteristics of the VIV at different flow rates. For an elastically fixed cylinder, as shown in Fig. 15(a), the structural oscillation model of the riser was modeled by the Euler-Bernoulli beam hypothesis (e.g., Pavlovskaja et al., 2016; Low and Srinil, 2016).

$$M \frac{\partial^2 x}{\partial t^2} + C \frac{\partial x}{\partial t} + EI \frac{\partial^4 x}{\partial z^4} - T \frac{\partial^2 x}{\partial z^2} = F_x \quad (1)$$

$$M \frac{\partial^2 y}{\partial t^2} + C \frac{\partial y}{\partial t} + EI \frac{\partial^4 y}{\partial z^4} - T \frac{\partial^2 y}{\partial z^2} = F_y \quad (2)$$

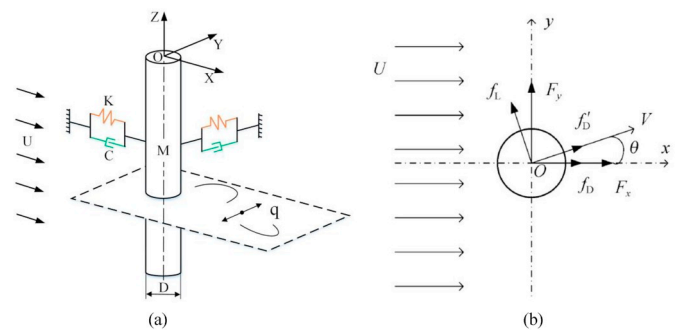


Fig. 15. (a) Model of coupled structure and wake oscillators. (b) The illustration of the cross-section of the circular cylinder in a cross flow and the fluid forces exerted on it when vibrating (Ge et al., 2009).

where  $EI$  denotes the flexural stiffness;  $M$ ,  $C$  and  $T$  are the mass, damping and effective axial tension, respectively;  $F_x$  and  $F_y$  are the VIV excitation forces in the in-line and cross-flow direction, respectively. Eqs. (1) and (2) can be discretized as (Xue et al., 2014; Gu et al., 2012, 2013; Yuan et al., 2018)

$$M\{\ddot{x}\} + C\{\dot{x}\} + K\{x\} = F \quad (3)$$

where  $K$  is the stiffness. Furthermore, based on empirical parameters obtained from the forced oscillation experiments, Thorsen et al. (2015, 2016) improved the model by making the right side of the formula with the excitation, damping, and added mass. Then the drag and lift forces (see Fig. 15(b)) can be expressed as (Ge et al., 2009)

$$F_x = f_D + f'_D \cos \theta - f_L \sin \theta \quad (4)$$

$$F_y = f_L \cos \theta + f'_D \sin \theta \quad (5)$$

where  $f_D$  is the average drag force,  $f'_D$  and  $f_L$  are respectively the drag and lift force, and  $\theta$  is the angle between the cylinder speed and the x-axis. Moreover, Teixeira and Morooka (2017) proposed a time domain analysis method for hydrodynamic force calculation based on the concept of energy balance. The mathematical equations of the nonlinear system of the riser VIV can be solved by the Newmark-beta method (Duanmu et al., 2017).

The VIV response of marine risers generally has two degrees of freedom. The vibration response in the in-line direction has always been ignored to simplify the calculation model. Recent studies have shown that the response in the in-line direction is very important for long flexible risers (Bai et al., 2014). Bai et al. (2014) established a reduced-order model based on the discrete point vortex to simulate the two-dimensional VIV of the cylinder, and the in-line and cross-flow fluid forces were quantified by the eddy current intensity. In actual applications, the marine risers are usually inclined. Thus, Meng and Chen (2012) presented a model to analyze the three-dimensional vibration of an inclined extensible steel catenary riser. Yang et al. (2018) investigated the VIV by a fully-coupled three-dimensional nonlinear dynamic model with Kelvin-Voigt viscoelasticity properties to pursue a suitable viscoelastic coefficient. Till now, there are numerous semi-empirical approaches to predict the riser VIV, but a generic model still remains (Srinil, 2010).

### 3.2.2. CFD simulation

The complex fluid-structure interaction problem of marine risers can be solved by the CFD method. As for turbulent flows, the CFD-based turbulent solvers can be categorized into: direct numerical simulation (DNS), Reynolds-averaged Navier-Stokes (RANS), large eddy simulation (LES) and discrete vortex method. The DNS method can predict the VIV response of long flexible structures by directly solving the three-dimensional incompressible Navier-Stokes equations (Violette et al., 2007). The DNS approach requires very fine grids to resolve small turbulent scales, which has high demand on computing resource. Currently, the DNS method can only calculate simple turbulent flows at low Reynolds numbers, and it is difficult to predict complex turbulent flows at high Reynolds numbers. The RANS method averages the Navier-Stokes equation in the time domain and models the Reynolds stress term due to the fluctuating velocity field using a suitable turbulence model. The most commonly used turbulence model is the eddy viscosity model, in which the Reynolds stress generated in the time-averaged process is modeled by the eddy viscosity coefficient. The RANS method can be applied to complex turbulent flows at high Reynolds numbers. However, the accuracy of RANS is lower than that of DNS. The LES method divides turbulent eddies into large-scale and small-scale eddies, and simulates them respectively. The large-scale eddies are directly calculated by solving the Navier-Stokes equation, and the small-scale eddies are modeled by establishing sub-grid scale models (Matin Nikoo et al.,

2018). In the LES calculation process, only the large-eddies are resolved while the sub-grid scale eddies are modeled, hence the computational efficiency of LES is much higher than that of DNS, but lower than that of RANS. Besides, the Reynolds number of the flow field that LES can calculate is much higher than that of the DNS method. The discrete vortex method uses a finite number of discrete vortex elements to represent the concentrated or continuous vortices in the flow field, and the Lagrange method is used to simulate the motion of the discrete vortex elements (Yamamoto et al., 2004). The discrete vortex method does not need to resolve the turbulence. Moreover, the discrete vortex method ignores the viscosity of the flow field.

Based on the DNS method, Bourguet et al. (2011, 2013a) studied the in-line and cross-flow VIV of a long tensioned beam immersed in the linear shear flow with a Reynolds number range from 110 to 1100 and an aspect ratio of 200. Khan et al. (2017) used the RANS equation to simulate the VIV of the cylinder with high Reynolds number  $Re = 10^4$  and low mass ratio, and verified the effectiveness of this method by comparing the numerical simulation results with experimental results. Wang and Xiao (2016) calculated the VIV of a marine riser under the action of the uniform flow and linear shear flow. The CFD simulation results agreed well with the experimental results and show that the dominant frequency, the dominant mode number and the root-mean-square amplitude are proportional to the flow speed. Nguyen and Nguyen (2016) improved the CFD model by combining RANS and LES to simulate the flow-induced vibration of the incompressible Navier-Stokes flow at high Reynolds numbers. Jung et al. (2014) and Guo et al. (2019) numerically simulated the VIV of twisted and wavy cylinders at subcritical Reynolds numbers and found that the VIV was significantly weakened if comparing with the smooth cylinders. Guo et al. (2015) and Wang et al. (2019a,b,c) used the LES method to study the effect of different design parameters on the VIV of the fiber reinforced composite riser. They found that the maximum equivalent stress of the riser occurs at both ends of the riser and the amplitude of the simply supported riser is larger than that of the fixed support riser.

Due to limitation of the computing techniques, direct three dimensional simulation of a marine riser with high aspect ratio and complex flow field is quite difficult. A feasible method is to use the strip theory for quasi-three-dimensional simulation. Based on the strip theory, Duanmu et al. (2018) and Fu et al. (2018) developed the *viv-FOAM-SJTU* solver to simulate the VIV of marine risers. Sun et al. (2012) presented a stripwise discrete vortex method to study the VIV of flexible risers, in which each strip was calculated by the discrete vortex method and the three-dimensional dynamics was calculated by the finite volume and the incremental method. Similarly, Lin and Wang, 2019 proposed a quasi-three-dimensional strip method based on the discrete vortex method and finite element method to predict the VIV of long flexible risers, and the numerical simulation results agree well with the experiment results.

Although the numerical simulations based on the strip theory produce high prediction accuracy for the VIV, the three-dimensional effect of the flow field is always ignored. To investigate the effect of the three-dimensional flow field, Labbe and Wilson (2007) developed a parallel three-dimensional incompressible Navier-Stokes solver. Huang et al., 2011a presented a finite difference scheme and an effective riser motion solver for three-dimensional VIV simulation with an aspect ratio of 3350. Wang et al. (2017) used the three-dimensional CFD method to study the effect of different in-line to cross-flow natural frequency ratios on the VIV of the cylinder at the Reynolds number of 500. The results showed that as the natural frequency ratio increased from 1 to 2, the VIV amplitude of the cylinder increased significantly while the maximum amplitude moved toward the high reduced velocity. In order to study the vibration suppression effect of a pipe-in-pipe model, Matin Nikoo et al. (2018) conducted the three-dimensional simulations using the bidirectional fluid-solid coupling CFD model. They found that the pipe-in-pipe model presented a good vibration suppression effect and the suppression effect reached maximum at the strongest VIV response. Following that,

Matin Nikoo et al. (2019) proposed a method for predicting the three-dimensional VIV of a marine riser using an improved RANS method at low Reynolds numbers. High prediction accuracy in the upper branch was obtained. Martins et al. (2019a, 2019b) conducted three-dimensional numerical simulations to investigate the impact of the Reynolds number and damping on the VIV response. The rigid body motion equation was used to deal with the moving mesh, which could not only reduce the computation cost, but also ensure the quality of the mesh. In addition, the commercial software package ANSYS MFX multi-field solver was developed to solve the three-dimensional fluid-structure interaction problem (Chen and Kim, 2010; Wang and Xiao, 2016).

Compared with the semi-empirical approach, the CFD method has been developed more rapidly in recent years. With fast development of computer and parallel-computing technologies, the CFD simulation can be gradually applied to the VIV prediction under various practical ocean conditions.

#### 4. Conclusion

The VIV of marine risers is a strong, nonlinear, self-exciting, self-limiting, and asymmetrical flow-structure coupling phenomenon. This review summarizes the recent basic research results of the VIV of marine risers, including the progress on the VIV of the multiple risers and the inclined risers, intermittent VIV caused by the oscillating flow and so forth. Different configurations of the multiple risers can result in different wake patterns, which significantly impact the VIV of the downstream risers. In general, the spacing between the risers and the Reynolds number of the incoming flow impact the VIV response of the risers. Furthermore, according to the independence principle, the VIV of the inclined riser can be analogous to the vertical riser, although the calculation accuracy needs improvement. The response amplitude of the inclined riser is smaller than that of the vertical riser, but the multi-mode characteristics of former are stronger. The intermittent VIV response characteristics are restricted by the Keulegan-Carpenter number and reduced velocity. The increase of the Keulegan-Carpenter number will increase the frequency of the VIV response while reduce the response amplitude. When the reduced velocity increases, high-order harmonics and mode conversion will appear. Moreover, the effects of the platform motion and internal multiphase flow on the VIV are also surveyed. The movement of the floating platform can cause the intermittent VIV under the oscillatory flow and increase the VIV amplitude. The internal flow can reduce the natural frequencies of the riser, which will make the riser more susceptible to the lock-in vibration; the internal flow is a heterogeneous multiphase flow that will cause the fluctuations in the parameters of the riser system, thereby increasing the complexity and uncertainty of the VIV.

Furthermore, various research approaches are reviewed in this paper. On the one hand, experimental research is performed in a real marine environment or a large-scale test pool. With continuous improvement of the experimental equipment, the experimental evaluation of large aspect ratio (over 4000) and high Reynolds numbers (over  $10^5$ ) is able to achieve. On the other hand, numerical simulation methods such as the semi-empirical method and CFD are developed to predict the VIV characteristics of marine risers. Till now, the semi-empirical method shows higher prediction accuracy than the CFD method. However, with fast development of the parallel-computing technologies, the CFD method will show better application prospects.

In the past decades, a lot of efforts have been made to improve the understanding of the riser VIV for the purpose of decreasing the operation cost and enhancing the reliability of the riser system. However, the complexity of external and internal excitations makes the VIV around a marine riser still an unsolved problem. Potential future research directions are discussed as follows.

- (1) The prediction of the VIV for risers with large aspect ratio is still an open research field. In previous research, the dominant parameters such as the Reynolds number, aspect ratio and mass ratio have been investigated; however, the combination effect of multiple factors is rarely reported. For example, the Reynolds number is individually studied without considering the aspect ratio (Raghavan and Bernitsas, 2011). Hence, it is crucial to establish a generic VIV analysis model that is capable for different applications.
- (2) The discrepancies between the model and actual marine riser system deserve further study. It is always difficult/costly to implement experimental tests in the conditions that are close to reality. Simplified testing models are often adopted. However, the effects of the model materials (often plastic) and structures (i. e., the simplified models often miss the mechanical joints) have not been deeply investigated (Meng and Chen, 2012; He et al., 2017). Furthermore, horizontally positioned pipes are used in many studies but the effect of the oil/gas fluid gravity is not considered.
- (3) The VIV suppression is an important research area. At the beginning of transporting the oil and gas production from deep sea, the VIV of marine risers tends to be more and more complex. The VIV suppression solution can fundamentally reduce/prevent the fatigue damages to marine risers; however, an accurate VIV prediction model is required before performing VIV suppression. As a result, the VIV suppression remains a challenging task (Assia et al., 2009; Huang, 2011; Lou et al., 2016).

With the improvement and development of experimental and simulation methods, the VIV mechanism for marine risers will become clear in the future.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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#### References

- Alfossail, F.K., Nayfeh, A.H., Younis, M.I., 2016. An analytic solution of the static problem of inclined risers conveying fluid. *Meccanica* 52, 1–13.
- Armata, Milad, Khorasanchi, Mahdi, Day, Sandy, 2018. Wake interference of two identical oscillating cylinders in tandem: an experimental study. *Ocean. Eng.* 166, 311–323.
- Assi, G.R.S., Bearman, P.W., Kitney, N., 2009. Low drag solutions for suppressing vortex-induced vibration of circular cylinders. *J. Fluids Struct.* 25, 666–675.
- Assi, G.R.S., Bearman, P.W., Meneghini, J.R., 2010. On the wake-induced vibration of tandem circular cylinders: the vortex interaction excitation mechanism. *J. Fluid Mech.* 661, 365–401.
- Assi, G.R.S., Bearman, P.W., Carmo, B.S., Meneghini, J.R., Sherwin, S.J., Willden, R.H.J., 2013. The role of wake stiffness on the wake-induced vibration of the downstream cylinder of a tandem pair. *J. Fluid Mech.* 718, 210–245.
- Bai, X., Qin, W., 2014. Using vortex strength wake oscillator in modelling of vortex induced vibrations in two degrees of freedom. *Eur. J. Mech. B.* 48, 165–173.
- Bao, Y., Zhou, D., Tu, J., 2013. Flow characteristics of two in-phase oscillating cylinders in side-by-side arrangement. *Comput. Fluids* 71, 124–145.
- Behara, S., Sotiropoulos, F., 2016. Vortex-induced vibrations of an elastically mounted sphere: the effects of Reynolds number and reduced velocity. *J. Fluids Struct.* 66, 54–68.



- Behara, S., Borazjani, I., Sotiropoulos, F., 2011. Vortex-induced vibrations of an elastically mounted sphere with three degrees of freedom at  $Re = 300$ : hysteresis and vortex shedding modes. *J. Fluid Mech.* 686, 426–450.
- Bernitsas, M.M., Raghavan, K., Ben-Simon, Y., 2008. VIVACE (Vortex induced vibrations aquatic clean energy): a new concept in generation of clean and renewable energy from fluid flow. *J. Offshore Mech. Arct. Eng.* 130 (4), 619–637.
- Bernitsas, M.M., Ben-Simon, Y., Raghavan, K., 2009. The VIVACE converter: model tests at high damping and Reynolds number around 105. *J. Offshore Mech. Arct. Eng.* 131 (1), 403–410.
- Biermann, D., Herrnstein, W.H., 1934. The Interference between Struts in Various Combinations. NACA Report No. p. 468.
- Birkhoff, G., Zarantonello, E.H., 1957. *Jets, Wakes and Cavities*. Academic Press, New York.
- Bishop, R.E.D., Hassan, A.Y., 1964. The lift and drag forces on a circular cylinder oscillating in a flowing fluid. *Math. Phys. Eng. Sci.* 277, 51–75.
- Borazjani, I., Sotiropoulos, F., 2009. Vortex-induced vibrations of two cylinders in tandem arrangement in the proximity - wake interference region. *J. Fluid Mech.* 621, 321–364.
- Bourguet, R., Triantafyllou, M.S., 2015. Vortex-induced vibrations of a flexible cylinder at large inclination angle. *Phil. Trans. R. Soc. A.* 373, 20140108.
- Bourguet, R., Karniadakis, G.E., Triantafyllou, M.S., 2011. Lock-in of the vortex-induced vibrations of a long tensioned beam in shear flow. *J. Fluids Struct.* 27, 838–847.
- Bourguet, R., Lucor, D., Triantafyllou, M.S., 2012. Mono- and multi-frequency vortex-induced vibrations of a long tensioned beam in shear flow. *J. Fluids Struct.* 32, 52–64.
- Bourguet, R., Karniadakis, G.E., Triantafyllou, M.S., 2013. Phasing mechanisms between the in-line and cross-flow vortex-induced vibrations of a long tensioned beam in shear flow. *Comput. Struct.* 122, 155–163.
- Bourguet, R., Karniadakis, G.E., Triantafyllou, M.S., 2013. Multi-frequency vortex-induced vibrations of a long tensioned beam in linear and exponential shear flows. *J. Fluids Struct.* 41, 33–42.
- Bourguet, R., Karniadakis, G.E., Triantafyllou, M.S., 2015. On the validity of the independence principle applied to the vortex-induced vibrations of a flexible cylinder inclined at  $60^\circ$ . *J. Fluids Struct.* 53, 58–69.
- Brika, D., Laneville, A., 1993. Vortex-induced vibrations of a long flexible circular cylinder. *J. Fluid Mech.* 250, 481–508.
- Capell, N.A., Carlson, D.W., Modares-Sadeghi, Y., 2019. Vortex-induced vibration of a single degree-of-freedom flexibly-mounted horizontal cylinder near the free surface. *J. Sound Vib.* 444, 161–175.
- Carini, M., Giannetti, F., Auteri, F., 2014. On the origin of the flip-flop instability of two side-by-side cylinder wakes. *J. Fluid Mech.* 742, 552–576.
- Carmo, B.S., Meneghini, J.R., Sherwin, S.J., 2010. Possible states in the flow around two circular cylinders in tandem with separations in the vicinity of the drag inversion spacing. *Phys. Fluids* 22, 054101.
- Carmo, B.S., Meneghini, J.R., Sherwin, S.J., 2010. Secondary instabilities in the flow around two circular cylinders in tandem. *J. Fluid Mech.* 644, 395–431.
- Chaplin, J.R., Bearman, P.W., Huera Huarte, F.J., Pattenden, R.J., 2005. Laboratory measurements of vortex-induced vibrations of a vertical tension riser in a stepped current. *J. Fluids Struct.* 21, 3–24.
- Chaplin, J.R., Bearman, P.W., Cheng, Y., Fontaine, E., Graham, J.M.R., Herford, K., Huera Huarte, F.J., Isherwood, M., Lambrakoc, K., Larsen, C.M., Meneghini, J.R., Moe, G., Pattenden, R.J., Triantafyllou, M.S., Willden, R.H.J., 2005. Blind predictions of laboratory measurements of vortex-induced vibrations of a tension riser. *J. Fluids Struct.* 21, 25–40.
- Chatjigeorgiou, I.K., 2010. On the effect of internal flow on vibrating catenary risers in three dimensions. *Eng. Struct.* 32, 3313–3329.
- Chatjigeorgiou, I.K., 2010. Three dimensional nonlinear dynamics of submerged extensible catenary pipes conveying fluid and subjected to end-imposed excitations. *Int. J. Non-Linear Mech.* 45, 667–680.
- Chatjigeorgiou, I.K., 2017. Hydroelastic response of marine risers subjected to internal slug-flow. *Appl. Ocean Res.* 62, 1–17.
- Chen, Z., Kim, W.J., 2010. Numerical investigation of vortex shedding and vortex-induced vibration for flexible riser models. *Int. J. Nav. Arch. Ocean.* 2, 112–118.
- Chen, Z., Rhee, S.H., 2019. Effect of traveling wave on the vortex-induced vibration of a long flexible pipe. *Appl. Ocean Res.* 84, 122–132.
- Chen, W.M., Li, M., Guo, S., Gan, K., 2014. Dynamic analysis of coupling between floating top-end heave and riser's vortex-induced vibration by using finite element simulations. *Appl. Ocean Res.* 48, 1–9.
- Chen, W.L., Zhang, Q., Li, H., Hu, H., 2015. An experimental investigation on vortex induced vibration of a flexible inclined cable under a shear flow. *J. Fluids Struct.* 54, 297–311.
- Chen, W.L., Ji, C., Xu, W., Liu, S., Campbell, J., 2015. Response and wake patterns of two side-by-side elastically supported circular cylinders in uniform laminar cross-flow. *J. Fluids Struct.* 55, 218–236.
- Chen, W.L., Ji, C., Wang, R., 2015. Flow-induced vibrations of two side-by-side circular cylinders: asymmetric vibration, symmetry hysteresis and near-wake patterns. *Ocean Eng.* 110, 244–257.
- Chen, W.M., Li, M., Zheng, Z., Guo, S., Gan, K., 2015. Impacts of top-end vessel sway on vortex-induced vibration of the submarine riser for a floating platform in deep water. *Ocean Eng.* 99, 1–8.
- Dai, H.L., Wang, L., Qian, Q., Ni, Q., 2013. Vortex-induced vibrations of pipes conveying fluid in the subcritical and supercritical regimes. *J. Fluids Struct.* 39, 322–334.
- Dai, H.L., Abdelkefi, A., Wang, L., 2014. Modeling and nonlinear dynamics of fluid-conveying risers under hybrid excitations. *Int. J. Eng. Sci.* 81, 1–14.
- Dai, H.L., Wang, L., Qian, Q., Ni, Q., 2014. Vortex-induced vibrations of pipes conveying pulsating fluid. *Ocean Eng.* 77, 12–22.
- Ding, L., Zou, Q., Zhang, L., Wang, H., 2018. Research on flow-induced vibration and energy harvesting of three circular cylinders with roughness strips in tandem. *Energies* 11 (11), 2977.
- Duan, J., Chen, K., You, Y., Wang, R., Li, J., 2018. Three-dimensional dynamics of vortex-induced vibration of a pipe with internal flow in the subcritical and supercritical regimes. *Int. J. Nav. Arch. Ocean. Eng.* 10 (6), 692–710.
- Duanmu, Y., Zou, L., Wan, D., 2017. Numerical simulations of vortex-induced vibrations of a flexible riser with different aspect ratios in uniform and shear currents. *J. Hydrodyn.* 29 (6), 1010–1022.
- Duanmu, Y., Zou, L., Wan, D., 2018. Numerical analysis of multi-modal vibrations of a vertical riser in step currents. *Ocean Eng.* 152, 428–442.
- Fan, H., Li, C., Wang, Z., Xu, L., Wang, Y., Feng, X., 2017. Dynamic analysis of a hang-off drilling riser considering internal solitary wave and vessel motion. *J. Nat. Gas Sci. Eng.* 37, 512–522.
- Feng, C.C., 1968. The Measurements of Vortex-Induced Effects in Flow Past a Stationary and Oscillating Circular and D-Section Cylinders. Master's thesis. University of British Columbia, Vancouver, Canada.
- Franzini, G.R., Gonçalves, R.T., Meneghini, J.R., Fajarra, A.L.C., 2013. One and two degrees-of-freedom vortex-induced vibration experiments with yawed cylinders. *J. Fluids Struct.* 42, 401–420.
- Franzina, G.R., Fajarra, A.L.C., Meneghini, J.R., Korkischko, I., Franciss, R., 2009. Experimental investigation of Vortex-Induced Vibration on rigid, smooth and inclined cylinders. *J. Fluids Struct.* 25, 742–750.
- Fu, S., Wang, J., Baarholm, R., Wu, J., Larsen, C.M., 2013. Features of vortex-induced vibration in oscillatory flow. *J. Offshore Mech. Arct. Eng.* 136, 1–10.
- Fu, B., Zou, L., Wan, D., 2018. Numerical study of vortex-induced vibrations of a flexible cylinder in an oscillatory flow. *J. Fluids Struct.* 77, 170–181.
- Gao, Y., Fu, S., Wang, J., Song, L., Chen, Y., 2015. Experimental study of the effects of surface roughness on the vortex-induced vibration response of a flexible cylinder. *Ocean Eng.* 103, 40–54.
- Gao, Y., Yang, K., Zhang, B., Cheng, K., Chen, X., 2019. Numerical investigation on vortex-induced vibrations of four circular cylinders in a square configuration. *Ocean Eng.* 175, 223–240.
- Gao, Y., Zou, L., Zong, Z., Takagi, S., Kang, Y., 2019. Numerical prediction of vortex-induced vibrations of a long flexible cylinder in uniform and linear shear flows using a wake oscillator model. *Ocean Eng.* 171, 157–171.
- Garrett, D.L., 2005. Coupled analysis of floating production systems. *Ocean Eng.* 32, 802–816.
- Ge, F., Long, X., Wang, L., Hong, Y., 2009. Flow-induced vibrations of long circular cylinders modeled by coupled nonlinear oscillators. *Sci. China Ser. G Phys. Mech. Astron.* 52 (7), 1086–1093.
- Goncalves, R.T., Rosetti, G.F., Fajarra, A.L.C., Franzini, G.R., Freire, C.M., Meneghini, J.R., 2012. Experimental comparison of Two degrees-of-freedom vortex-induced vibration on high and low aspect ratio cylinders with small Mass ratio. *J. Vib. Acoust.* 134, 061009-1-7.
- Govardhan, R., Williamson, C.H.K., 2000. Modes of vortex formation and frequency response for a freely vibrating cylinder. *J. Fluid Mech.* 420, 85–130.
- Griffin, O.M., 1980. Vortex-excited cross-flow vibrations of a single cylindrical tube. *ASME J. Press. Vessel Technol.* 102, 158–166.
- Gu, W., Chyu, C., Rockwell, D., 1994. Timing of vortex formation from an oscillating cylinder. *Phys. Fluids* 6, 3677–3682.
- Gu, J., An, C., Levi, C., 2012. Prediction of vortex-induced vibration of long flexible cylinders modeled by a coupled nonlinear oscillator: integral transform solution. *J. Hydrodyn.* 24 (6), 888–898.
- Gu, J., Vitola, M., Coelho, J., Pinto, W., Duan, M., Levi, C., 2013. An experimental investigation by towing tank on VIV of a long flexible cylinder for deepwater riser application. *J. Mar. Sci. Technol.* 18, 358–369.
- Guo, H., Lou, M., 2008. Effect of internal flow on vortex-induced vibration of risers. *J. Fluids Struct.* 24, 496–504.
- Guo, L., Zhang, X., He, G., 2015. Large-eddy simulation of circular cylinder flow at subcritical Reynolds number: turbulent wake and sound radiation. *Acta Mech. Sinica-proc* 32 (1), 1–11.
- Guo, C., Guo, H., Hu, J., Song, K., Zhang, W., Wang, W., 2019. Large eddy simulation of flow over wavy cylinders with different twisted angles at a subcritical Reynolds number. *J. Mar. Sci. Eng.* 7 (7), 227.
- Han, Q., Ma, Y., Xu, W., Lu, Y., Cheng, A., 2017. Dynamic characteristics of an inclined flexible cylinder undergoing vortex-induced vibrations. *J. Sound Vib.* 394, 306–320.
- Han, P., Pan, G., Tian, W., 2018. Numerical simulation of flow-induced motion of three rigidly coupled cylinders in equilateral-triangle arrangement. *Phys. Fluids* 30 (12), 125107.
- Hartlen, R.T., Curri, e I.G., 1970. Lift-oscillator model for vortex-induced vibrations. *J. Eng. Mech. Div. ASME.* 96 (5), 577–591.
- He, F., Dai, H., Huang, Z., Wang, L., 2017. Nonlinear dynamics of a fluid-conveying pipe under the combined action of cross-flow and top-end excitations. *Appl. Ocean Res.* 62, 199–209.
- Herfjord, K.K., Drange, S.O., Kvamsdal, T.T., 1999. Assessment of vortex-induced vibrations on deepwater risers by considering fluid-structure interaction. *J. Offshore Mech. Arct. Eng.* 121 (4), 207–212.
- Hsu, C.S., 1975. The response of parametrically excited hanging string in fluid. *J. Sound Vib.* 39, 305–316.
- Hu, J., Zhou, Y., 2008. Flow structure behind two staggered cylinders, Part I: downstream evolution and classification. *J. Fluid Mech.* 607, 51–80.
- Huang, A., 2011. VIV suppression of a two-degree-of-freedom circular cylinder and drag reduction of a fixed circular cylinder by the use of helical grooves. *J. Fluids Struct.* 27, 1124–1133.

- Huang, K., Chen, H., Chen, C., 2011. Numerical scheme for riser motion calculation during 3-D VIV simulation. *J. Fluids Struct.* 27, 947–961.
- Huang, S., Khorasanchi, M., Herford, K., 2011. Drag amplification of long flexible riser models undergoing multi-mode VIV in uniform currents. *J. Fluids Struct.* 27, 342–353.
- Huera-Huarte, F.J., Bearman, P.W., 2011. Vortex and wake-induced vibrations of a tandem arrangement of two flexible circular cylinders with near wake interference. *J. Fluids Struct.* 27, 193–211.
- Huera-Huarte, F.J., Gharib, M., 2011. Flow-induced vibrations of a side-by-side arrangement of two flexible circular cylinders. *J. Fluids Struct.* 27, 354–366.
- Huera-Huarte, F.J., Bangash, Z.A., González, L.M., 2014. Towing tank experiments on the vortex-induced vibrations of low mass ratio long flexible cylinders. *J. Fluids Struct.* 48, 81–92.
- Huera-Huarte, F.J., Bangash, Z.A., González, L.M., 2016. Multi-mode vortex and wake-induced vibrations of a flexible cylinder in tandem arrangement. *J. Fluids Struct.* 66, 571–588.
- Jain, A., Modarres-Sadeghi, Y., 2013. Vortex-induced vibrations of a flexibly mounted inclined cylinder. *J. Fluids Struct.* 43, 28–40.
- Jung, J.H., Yoon, H.S., 2014. Large eddy simulation of flow over a twisted cylinder at a subcritical Reynolds number. *J. Fluid Mech.* 759, 579–611.
- Kalro, V., Tezduyar, T., 1997. Parallel 3-D computation of unsteady flows around circular cylinders. *Parallel Comput.* 23, 1235–1248.
- Khan, N.B., Ibrahim, Z., Nguyen, L.T.T., Javed, M.F., Jameel, M., 2017. Numerical investigation of the vortex-induced vibration of an elastically mounted circular cylinder at high Reynolds number ( $Re = 104$ ) and low mass ratio using the RANS code. *PLoS One* 12 (10), e0185832.
- Kuiper, G.L., Brugmans, J., Metrikine, A.V., 2008. Destabilization of deep-water risers by a heaving platform. *J. Sound Vib.* 310, 541–557.
- Labbé, D.F.L., Wilson, P.A., 2007. A numerical investigation of the effects of the spanwise length on the 3-D wake of a circular cylinder. *J. Fluids Struct.* 23, 1168–1188.
- Lee, U., Park, C., Hong, S.C., 1995. The dynamics of a piping system with internal unsteady flow. *J. Sound Vib.* 180 (2), 297–311.
- Lee, K., Yang, K.S., Yoon, D.H., 2009. Flow-induced forces on two circular cylinders in proximity. *Comput. Fluids* 38, 111–120.
- Lie, H., Kaasen, K.E., 2006. Modal analysis of measurements from a large-scale VIV model test of a riser in linearly sheared flow. *J. Fluids Struct.* 22, 557–575.
- Lin, K., Wang, J., 2019. Numerical simulation of vortex-induced vibration of long flexible risers using a SDVM-FEM coupled method. *Ocean. Eng.* 172, 468–486.
- Liu, Z.Y., Wang, L., Dai, H.L., Wu, P., Jiang, T.L., 2019. Nonplanar vortex-induced vibrations of cantilevered pipes conveying fluid subjected to loose constraints. *Ocean. Eng.* 178, 1–19.
- Liu, P., Liu, Y., Huang, Z., Cai, B., Sun, Q., Wei, X., Xin, C., 2019. Design optimization for subsea gate valve based on combined analyses of fluid characteristics and sensitivity. *J. Pet. Sci. Eng.* 182, 106277.
- Liu, P., Liu, Y., Wei, X., Xin, C., Sun, Q., Wu, X., 2019. Performance analysis and optimal design based on dynamic characteristics for pressure compensated subsea all-electric valve actuator. *Ocean. Eng.* 191.
- Lou, M., Chen, Z., Chen, P., 2016. Experimental investigation of the suppression of vortex induced vibration of two interfering risers with splitter plates. *J. Nat. Gas Sci. Eng.* 35, 736–752.
- Low, Y.M., Srinil, N., 2016. VIV fatigue reliability analysis of marine risers with uncertainties in the wake oscillator model. *Eng. Struct.* 106, 96–108.
- Lu, Z., Fu, S., Zhang, M., Ren, H., 2019. An efficient time-domain prediction model for vortex-induced vibration of flexible risers under unsteady flows. *Mar. Struct.* 64, 492–519.
- Martins, F.A.C., Avila, J.P.J., 2019. Three-dimensional CFD analysis of damping effects on vortex-induced vibrations of 2DOF elastically-mounted circular cylinders. *Mar. Struct.* 65, 12–31.
- Martins, F.A.C., Avila, J.P.J., 2019. Effects of the Reynolds number and structural damping on vortex-induced vibrations of elastically-mounted rigid cylinder. *Int. J. Mech. Sci.* 156, 235–249.
- Matin Nikoo, H., Bi, K., Hao, H., 2018. Effectiveness of using pipe-in-pipe (PIP) concept to reduce vortex-induced vibrations (VIV): three-dimensional two-way FSI analysis. *Ocean. Eng.* 148, 263–276.
- Matin Nikoo, H., Bi, K., Hao, H., 2019. Three-dimensional vortex-induced vibration of a circular cylinder at subcritical Reynolds numbers with low-re correction. *Mar. Struct.* 66, 288–306.
- Meng, D., Chen, L., 2012. Nonlinear free vibrations and vortex-induced vibrations of fluid-conveying steel catenary riser. *Appl. Ocean. Res.* 34, 52–67.
- Meng, S., Kajiwara, H., Zhang, W., 2017. Internal flow effect on the cross-flow vortex-induced vibration of a cantilevered pipe discharging fluid. *Ocean. Eng.* 137, 120–128.
- Meng, S., Zhang, X., Che, C., Zhang, W., 2017. Cross-flow vortex-induced vibration of a flexible riser transporting an internal flow from subcritical to supercritical. *Ocean. Eng.* 139, 74–84.
- Meng, S., Song, S., Che, C., Zhang, W., 2018. Internal flow effect on the parametric instability of deepwater drilling risers. *Ocean. Eng.* 149, 305–312.
- Modarres-Sadeghi, Y., Chasparis, F., Triantafyllou, M.S., 2011. Chaotic response is a generic feature of vortex-induced vibrations of flexible risers. *J. Sound Vib.* 330, 2565–2579.
- Modir, Alireza, Goudarzi, Navid, 2019. Experimental investigation of Reynolds number and spring stiffness effects on vortex induced vibrations of a rigid circular cylinder. *Eur. J. Mech. B Fluid* 74, 34–40.
- Montoya-Hernández, D.J., Vázquez-Hernández, A.O., Cuamatzi, R., Hernandez, M.A., 2014. Natural frequency analysis of a marine riser considering multiphase internal flow behavior. *Ocean. Eng.* 92, 103–113.
- Morooka, C.K., Tsukada, R.I., 2013. Experiments with a steel catenary riser model in a towing tank. *Appl. Ocean. Res.* 43, 244–255.
- Nguyen, V.T., Nguyen, H.H., 2016. Detached eddy simulations of flow induced vibrations of circular cylinders at high Reynolds numbers. *J. Fluids Struct.* 63, 103–119.
- Ortega, A., Rivera, A., Larsen, C.M., 2013. Flexible riser response induced by combined slug flow and wave loads. In: Proceedings of the ASME 2013 32nd International Conference on Ocean, Offshore and Arctic Engineering, OMAE 2013. Nantes, France. June 2013.
- Pang, J., Zhu, B., Zong, Z., 2019. A numerical simulation model for the vortex induced vibration of flexible risers using dynamic stiffness matrices. *Ocean. Eng.* 178, 306–320.
- Pavlovskaja, E., Keber, M., Postnikov, A., Reddington, K., Wiercigroch, M., 2016. Multi-modes approach to modelling of vortex-induced vibration. *J. Non Linear Mech.* 8, 40–51.
- Raghavan, K., Bernitsas, M.M., 2011. Experimental investigation of Reynolds number effect on vortex induced vibration of rigid circular cylinder on elastic supports. *Ocean. Eng.* 38, 719–731.
- Roshko, A., 1955. On the wake and drag of bluff bodies. *J. Aeronaut. Sci.* 22, 124–132.
- Sanaati, B., Kato, N., 2012. A study on the effects of axial stiffness and pre-tension on VIV dynamics of a flexible cylinder in uniform cross-flow. *Appl. Ocean. Res.* 37, 198–210.
- Sewatkar, C.M., Patel, R., Sharma, A., Agrawal, A., 2012. Flow around six in-line square cylinders. *J. Fluid Mech.* 710, 195–233.
- Sewatkar, C.M., Sharma, A., Agrawal, A., 2012. On energy transfer in flow around a row of transversely oscillating square cylinders at low Reynolds number. *J. Fluids Struct.* 31, 1–17.
- Seyed-Aghazadeh, B., Budz, C., Modarres-Sadeghi, Y., 2015. The influence of higher harmonic flow forces on the response of a curved circular cylinder undergoing vortex-induced vibration. *J. Sound Vib.* 353, 395–406.
- Song, J., Lu, L., Teng, B., Park, H., Tang, G., Wu, H., 2011. Laboratory tests of vortex-induced vibrations of a long flexible riser pipe subjected to uniform flow. *Ocean. Eng.* 38, 1308–1322.
- Song, L., Fu, S., Cao, J., Ma, L., Wu, J., 2016. An investigation into the hydrodynamics of a flexible riser undergoing vortex-induced vibration. *J. Fluids Struct.* 63, 325–350.
- Srinil, N., 2010. Multi-mode interactions in vortex-induced vibrations of flexible curved/straight structures with geometric nonlinearities. *J. Fluids Struct.* 26, 1098–1122.
- Srinil, N., 2011. Analysis and prediction of vortex-induced vibrations of variable-tension risers in linearly sheared currents. *Appl. Ocean. Res.* 33, 41–53.
- Srivilairit, T., Manuel, L., 2009. Vortex-induced vibration and coincident current velocity profiles for a deepwater drilling riser. *J. Offshore Mech. Arct. Eng.* 131 (2), 1–11, 021101.
- Sumer, B.M., Fredsøe, J., 1997. Hydrodynamics Around Cylindrical Structures. World Scientific, Singapore.
- Sumner, D., 2010. Two circular cylinders in cross-flow: a review. *J. Fluids Struct.* 26, 849–899.
- Sumner, D., Richards, M.D., Akosile, O.O., 2005. Two staggered circular cylinders of equal diameter in cross-flow. *J. Fluids Struct.* 20, 255–276.
- Sun, L., Zong, Z., Dong, J., 2012. Stripwise discrete vortex method for VIV analysis of flexible risers. *J. Fluids Struct.* 35, 21–49.
- Teixeira, D.C., Morooka, C.K., 2017. A time domain procedure to predict vortex-induced vibration response of marine risers. *Ocean. Eng.* 142, 419–432.
- Thorsen, M.J., Sævik, S., Larsen, C.M., 2015. Fatigue damage from time domain simulation of combined in-line and cross-flow vortex-induced vibrations. *Mar. Struct.* 41, 200–222.
- Thorsen, M.J., Sævik, S., Larsen, C.M., 2016. Time domain simulation of vortex-induced vibrations in stationary and oscillating flows. *J. Fluids Struct.* 61, 1–19.
- Thorsen, M.J., Sævik, S., Larsen, C.M., 2017. Non-linear time domain analysis of cross-flow vortex-induced vibrations. *Mar. Struct.* 51, 134–151.
- Thorsen, M.J., Challabotla, N.R., Sævik, S., Nydal, O.J., 2019. A numerical study on vortex-induced vibrations and the effect of slurry density variations on fatigue of ocean mining risers. *Ocean. Eng.* 174, 1–13.
- Trim, A.D., Braaten, H., Lie, H., Tognarelli, M.A., 2005. Experimental investigation of vortex-induced vibration of long marine risers. *J. Fluids Struct.* 21, 335–361.
- Vandiver, J.K., Marcollo, H., Swithenbank, S., Jhingran, V., 2005. High mode number vortex induced vibration field experiments[C]. *Offshore Technol. Conf.* 2005 (5), 1083–1089.
- Vandiver, J.K., Jaiswal, V., Jhingran, V., 2009. Insights on vortex-induced, traveling waves on long risers. *J. Fluids Struct.* 25, 641–653.
- Violette, R., de Langre, E., Szydlowski, J., 2007. Computation of vortex-induced vibrations of long structures using a wake oscillator model: comparison with DNS and experiments. *Comput. Struct.* 85, 1134–1141.
- Violette, R., Langre, E.D., Szydlowski, J., 2010. A linear stability approach to vortex-induced vibrations and waves. *J. Fluids Struct.* 26 (3), 442–466.
- Wanderley, J.B.V., Soares, L.F.N., 2015. Vortex-induced vibration on a two-dimensional circular cylinder with low Reynolds number and low mass-damping parameter. *Ocean. Eng.* 97, 156–164.
- Wanderley, J.B.V., Souza, G.H.B., Sphaier, S.H., Levi, C., 2008. Vortex-induced vibration of an elastically mounted circular cylinder using an upwind TVD two-dimensional numerical scheme. *Ocean. Eng.* 35, 1533–1544.
- Wang, E., Xiao, Q., 2016. Numerical simulation of vortex-induced vibration of a vertical riser in uniform and linearly sheared currents. *Ocean. Eng.* 121, 492–515.
- Wang, J., Fu, S., Baarholm, R., Wu, J., Larsen, C.M., 2014. Fatigue damage of a steel catenary riser from vortex-induced vibration caused by vessel motions. *Mar. Struct.* 39, 131–156.

- Wang, K., Tang, W., Xue, H., 2015. Time domain approach for coupled cross-flow and in-line VIV induced fatigue damage of steel catenary riser at touchdown zone. *Mar. Struct.* 41, 267–287.
- Wang, J., Fu, S., Baarholm, R., Wu, J., Larsen, C.M., 2015. Fatigue damage induced by vortex-induced vibrations in oscillatory flow. *Mar. Struct.* 40, 73–91.
- Wang, J., Fu, S., Baarholm, R., Wu, J., Larsen, C.M., 2015. Out-of-plane vortex-induced vibration of a steel catenary riser caused by vessel motions. *Ocean. Eng.* 109, 389–400.
- Wang, J., Xiang, S., Fu, S., Cao, P., Yang, J., He, J., 2016. Experimental investigation on the dynamic responses of a free-hanging water intake riser under vessel motion. *Mar. Struct.* 50, 1–19.
- Wang, E., Xiao, Q., Incecik, A., 2017. Three-dimensional numerical simulation of two-degree-of-freedom VIV of a circular cylinder with varying natural frequency ratios at  $Re=500$ . *J. Fluids Struct.* 73, 162–182.
- Wang, C., Ge, S., Jaworski, J.W., Liu, L., Jia, Z., 2019. Effects of different design parameters on the vortex induced vibration of FRP composite risers using grey relational analysis. *J. Mar. Sci. Eng.* 7 (7).
- Wang, E., Xu, W., Yu, Y., Zhou, L., Incecik, A., 2019. Flow-induced vibrations of three and four long flexible cylinders in tandem arrangement: an experimental study. *Ocean. Eng.* 178, 170–184.
- Wang, J., Joseph, R., Ong, M., Jakobsen, J., 2019. Numerical investigation on vortex-induced vibration caused by vessel motion for a free hanging riser under small Keulegan-carpenter numbers. *J. Offshore Mech. Arct. Eng.* 141 (4), 041804-041804-8.
- Williamson, C.H.K., Govardhan, R., 2004. Vortex-induced vibrations. *Annu. Rev. Fluid Mech.* 36, 413–455.
- Williamson, C.H.K., Govardhan, R., 2008. A brief review of recent results in vortex-induced vibrations. *J. Wind Eng. Ind. Aerodyn.* 96, 713–735.
- Wu, M., Lou, J.Y., 1991. Effects of rigidity and internal flow on marine riser dynamics. *Appl. Ocean Res.* 13 (5), 235–244.
- Wu, J., Lekkala, M., Ong, M., Passano, E., Voie, P., 2019. Prediction of combined inline and crossflow vortex-induced vibrations response of deepwater risers. *J. Offshore Mech. Arct. Eng.* 141 (4), 041803-041803-8.
- Xu, W., Zhang, S., Liu, B., Wang, E., Bai, Y., 2018. An experimental study on flow-induced vibration of three and four side-by-side long flexible cylinders. *Ocean. Eng.* 169, 492–510.
- Xue, H., Tang, W., Qu, X., 2014. Prediction and analysis of fatigue damage due to cross-flow and in-line VIV for marine risers in non-uniform current. *Ocean. Eng.* 83, 52–62.
- Xue, H., Wang, K., Tang, W., 2015. A practical approach to predicting cross-flow and in-line VIV response for deepwater risers. *Appl. Ocean Res.* 52, 92–101.
- Yamamoto, C.T., Meneghini, J.R., Saltara, F., Fregonesi, R.A., Ferrari Jr., J.A., 2004. Numerical simulations of vortex-induced vibration on flexible cylinders. *J. Fluids Struct.* 19, 467–489.
- Yang, H., Xiao, F., 2014. Instability analyses of a top-tensioned riser under combined vortex and multi-frequency parametric excitations. *Ocean. Eng.* 81, 12–28.
- Yang, H., Xiao, F., Xu, P., 2013. Parametric instability prediction in a top-tensioned riser in irregular waves. *Ocean. Eng.* 70 (0), 39–50.
- Yang, W., Ai, Z., Zhang, X., Gou, R., Chang, X., 2018. Nonlinear three-dimensional dynamics of a marine viscoelastic riser subjected to uniform flow. *Ocean. Eng.* 149, 38–52.
- Yin, D., Passano, E., Lie, H., Grytøyr, G., Aronsen, K., Tognarelli, M., Kebabdz, E.B., 2019. Experimental and numerical study of a top tensioned riser subjected to vessel motion. *Ocean. Eng.* 171, 565–574.
- Yuan, Y., Xue, H., Tang, W., 2018. Numerical analysis of Vortex-Induced Vibration for flexible risers under steady and oscillatory flows. *Ocean. Eng.* 148, 548–562.
- Zhao, M., 2013. Flow induced vibration of two rigidly coupled circular cylinders in tandem and side-by-side arrangements at a low Reynolds number of 150. *Phys. Fluids* 25 (12), 355–381.
- Zhao, M., 2013. Numerical investigation of two-degree-of-freedom vortex-induced vibration of a circular cylinder in oscillatory flow. *J. Fluids Struct.* 39, 41–59.
- Zhao, M., 2015. The validity of the independence principle applied to the vortex-induced vibration of an inclined cylinder in steady flow. *Appl. Ocean Res.* 53, 155–160.
- Zheng, H., Price, R.E., Modarres-Sadeghi, Y., 2014. On fatigue damage of long flexible cylinders due to the higher harmonic force components and chaotic vortex-induced vibrations. *Ocean. Eng.* 88, 318–329.
- Zhu, H., Lin, P., Gao, Y., 2019. Vortex-induced vibration and mode transition of a curved flexible free-hanging cylinder in exponential shear flows. *J. Fluids Struct.* 84, 56–76.